SIMPLE RULES

Fall 2002

<u>Basic Ideas</u>:

1. **Covariances will be estimated via model.**

2. **With any standard model can develop easy selection rule.**

<u>Uses</u>:

1. **Makes clear why security enters.**

2. **Useful in sensitivity analysis.**

3. **Easy calculation.**

Example:

Assume $\rho_{jk} = \bar{\rho}$ **constant correlation.**

First order condition

$$\bar{R}_i - R_F = Z_i \sigma_i^2 + \sum_{\substack{j=1 \\ j \neq 1}}^{N} Z_j \sigma_{ij}$$

if $\sigma_{ij} = \bar{\rho} \sigma_i \sigma_j$

in what follows $\rho = \bar{\rho}$

$$\bar{R}_i - R_F = Z_i \sigma_i^2 + \sum_{\substack{j=1 \\ j \neq 1}}^{N} Z_j \rho \sigma_i \sigma_j$$

add and subtract $Z_i \rho \sigma_i \sigma_i$

$$\bar{R}_i - R_F = Z_i \sigma_i^2 (1 - \rho) + \rho \sigma_i \sum_{j=1}^{N} Z_j \sigma_j$$

Note $\sum_{j=1}^{N} Z_j \sigma_j$ is a constant.

solving for Z_i

$$Z_i = \frac{\bar{R}_i - R_F}{(1-\rho)\sigma_i^2} - \frac{\rho\sigma_i}{(1-\rho)\sigma_i^2} \sum_{j=1}^{N} Z_j \sigma_j$$

$$Z_i = \frac{1}{(1-\rho)\sigma_i} \left[\frac{\bar{R}_i - R_F}{\sigma_i} - \frac{\rho}{(1-\rho)} \sum_{j=1}^{N} Z_j \sigma_j \right]$$

let $C^* = \frac{\rho}{1-\rho} \sum_{j=1}^{N} Z_j \sigma_j$

Then

$$Z_i = \frac{1}{(1-\rho)\sigma_i}\left[\frac{\bar{R}_i - R_F}{\sigma_i} - C^*\right]$$

It can be shown that

$$C^* = \frac{\rho}{1-\rho+N\rho}\sum_{j=1}^{N}\frac{\bar{R}_j - R_F}{\sigma_j}$$

Note C^* is a constant. If short sales are not allowed, summation goes to number in set not N.

$$\frac{\rho}{1-\rho+N\rho} = \frac{1}{\frac{1}{\rho}+(N-1)} \cong \frac{1}{N}$$

$$\therefore C^* \cong \frac{1}{N}\sum_{j=1}^{N}\frac{\overline{R}_j - R_F}{\sigma_j}$$

If single-index model:

$$Z_i = \frac{\beta_i}{\sigma^2_{\varepsilon_i}} \left[\frac{\bar{R}_i - R_F}{\beta_i} - C^* \right]$$

where

$$C^* = \sigma^2_m \sum_{j=1}^{N} Z_j \beta_j$$

$$C^* = \frac{\sigma_m^2 \sum_{j=1}^{N} \frac{\left(\bar{R}_j - R_F\right)\beta_j}{\sigma_{\varepsilon_j}^2}}{1 + \sigma_m^2 \sum_{j=1}^{N} \frac{\beta_j^2}{\sigma_{\varepsilon_j}^2}}$$

If short sales are not allowed, summation goes to numbers that are held in positive proportion, not N.