MULTI-INDEX MODELS IN THE BOND AREA

October 1999

BASIC IDEA IS TO DECOMPOSE TOTAL RETURN INTO SOURCES THAT EFFECT TOTAL RETURN.

1. **COMMON**

2. **UNIQUE**

DEFAULT FREE NON CALLABLE GOVERNMENT

RETURN = EXPECTED + UNEXPECTED DUE RETURN TO TERM STRUCTURE SHIFT

+ RANDOM ERROR

$$R_{it} = \overline{R}_i - D_i \left[\frac{\Delta r}{1+r} \right] + e_i$$

COMMENTS

(1) D_i CAN BE ESTIMATED NUMERICALLY OR FROM THEORY

(2)   ONE MUST ASSUME WHEN TERM STRUCTURE SHIFTS.

   WHEN YOU ASSUME IT SHIFTS IS A MATTER OF TASTE. I'VE ALWAYS USED BEGINNING

(3)   CAN REFORMULATE IN TERMS OF RETURNS ON INDEXES.

$$R_I = \sum X_i R_i = \sum X_i \overline{R}_i - \sum X_i D_i \frac{\Delta R}{1+R} + \sum X_i e_{it}$$

FOR INDEX IT IS REASONABLE TO ASSUME $\Sigma X_i\, e_i = 0$. AFTER ALL e_i is

    a.   **PRICING ERROR**

    b.   **BID ASK SPREAD**

    c.   **NON SYNCHRONOUS TRADING**

$$R_I = \overline{R_I} - D_I \frac{\Delta R}{1 + R}$$

$$\therefore \; \frac{-\Delta R}{1 + R} = \frac{R_I - \overline{R_I}}{D_I}$$

AND PRIOR EQUATION IS

$$R_{it} = \overline{R_i} + \frac{D_i}{D_I}(R_I - \overline{R_I}) + e_i$$

(4) CAN ADD ADDITIONAL FACTORS

$$R_{it} = \overline{R_i} - D_1\Delta r_1 - D_2\Delta r_2 + e_{it}$$

WE USE TWO RATES

      a.   **LONG**

      b.   **SPREAD**

      HO USES USE SEVERAL "KEY RATES"

WAYS OF ESTIMATING SENSITIVITY COEFFICIENTS

      a.   **USE SPOTS AND AGGREGATE**

      b.   **NUMERICALLY**

      c.   **THEORY**

COUPON PAYING BOND AS PORTFOLIO OF PURE DISCOUNT

$$\text{Price} = \frac{Cf(1)}{(1+r001)} + \frac{Cf(2)}{(1+r002)^2} + \frac{Cf(3)}{(1+r003)^3}$$

$$\text{Price} = PV(1) + PV(2) + PV(3)$$

PROPORTIONS

AMOUNT IN	PROPORTION	SYMBOL
1	$\dfrac{PV(1)}{Price}$	X_1
2	$\dfrac{PV(2)}{Price}$	X_2
3	$\dfrac{PV(3)}{Price}$	X_3

(5) **EXPECTED RETURN.   M0ST ASSUME EXPECTATION THEORY SO THAT $R_i = R_F$. NOTHING PRECLUDES ADDING LIQUIDITY PREMIUM.**

TRADE-OFF

1.   **MORE FACTORS $\rightarrow$ FEWER PORTFOLIOS MEET IT.   THEY ARE MORE COSTLY**

2.   **FEWER FACTORS $\rightarrow$ MAY NOT CAPTURE REALITY**

ADDING OPTIONS. PRIOR MODEL DOES NOT CAPTURE:

1. **CHANGE IN EXPECTATIONS ABOUT MARKET VOLATILITY**

2. **CHANGE IN SPREAD DEMANDED BY MARKET**

3. **ADDED RETURN DUE TO BONDHOLDER ISSUING OPTION**

$$R_{it} = \overline{R_i} + OAS - D_i \frac{\Delta r}{1+r} - \sigma_i \, d \, OAS$$

$$- V_i \, dV_i - P_i \, dt + e_i$$

$$\uparrow$$

loss in option premium

ABOVE ALSO INCLUDES DEFAULT RISK ON OAS

USES

1.	**SELECTION OF EXPOSURE**

2.	**RETURN ATTRIBUTION**

3.	**PORTFOLIO EVALUATION**