

# **CORPORATE BONDS**

**October 1999**

# Corporate Bonds

Spread depends on

1. Default Premium
2. State Taxes
3. Risk Premium
4. Liquidity

# Major Problems

1. Valuation
2. Size of Risk Premium
3. Classification

## Valuation

$$\text{Value} = \sum_{t=1}^T \frac{cf(t)}{(1 + r_{00t}^c)^t}$$

where

1.  $cf(t)$  is cash flow in  $t$  (Promised)
2.  $r_{00t}^c$  is corporate spot rate

## Determining Risk Premium

Basic Idea If no risk premium would discount expected cash flow at riskless rate and on average get invoice price. Risk premium is thus extra return so that on average invoice price is correct.

### Illustration

Let  $E[cf(t)]$  be expected cash flow in  $t$  then if no risk premium

$$\text{Model Price} = \sum_{t=1}^T \frac{E[cf(t)]}{(1 + r_{00t}^G)^t}$$

where

$r_{00t}^G$  is riskless rate

and Model Price = invoice price on average

Let  $P$  be Premium then find  $P$  such that

Model Price = Invoice Price

$$\text{Model Price} = \sum_{t=1}^T \frac{E[cf(t)]}{(1 + r_{00t}^G + P_t)^t}$$

Note actually estimate

$$\text{Model Price} = \sum_{t=1}^T \frac{E[cf(t)]}{(1 + r_{00t}^c)^t}$$

and

$$P_t = r_{00t}^c - r_{00t}^G$$

## Determining Expected Cash Flow

### A. Ignoring state taxes

Consider one Period Bond

<u>State</u>	<u>Cash Flow</u>
Doesn't Default	Principal + Interest
default	$a \bullet \text{Principal}$

where  $a$  = recovery rate

$$E[cf(1)] = (1 - P_1)(c + 100) + P_1 a \bullet 100$$

Consider two Period Bond

in one  $(1 - P_1)c + P_1(a \bullet 100)$

in two  $(1 - P_1)[(1 - P_2)(c + 100) + P_2(a \bullet 100)]$

Consider three Period Bond

in one  $(1 - P_1)c + P_1(a \bullet 100)$

in two  $(1 - P_1)((1 - P_2)c + P_2a \bullet 100)$

in three  $(1 - P_1)(1 - P_2)((c + 100)(1 - P_3) + P_3a \bullet 100)$



## B. Including state taxes

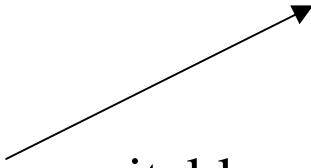
State taxes are deductible at federal level. Therefore, effective rate is  $t_s \bullet (1 - t_g)$

Also note cash flows are changed because of capital loss if bankrupt

Consider One Period Bond

$$E[cf(1)] = (1 - P_1)[ct_s(1 - t_g) + 100] + P_1(a \bullet 100) + \bullet P_1(1 - a)(100)t_s(1 - t_g)$$

tax saving on capital loss



Consider Two Period Bond

in one

$$(1 - P_1) c t_s (1 - t_g) + P_1 (a \bullet 100) + P_1 (1 - a) 100 t_s (1 - t_g)$$

in two

$$(1 - P_1) [(1 - P_2) (c t_s (1 - t_g) + 100) + P_2 (a \bullet 100) \\ + P_2 (1 - a) 100 t_s (1 - t_g)]$$

If options use

$$V_{opt} = V_{no\ opt} + \textit{option value}$$

**Table I**  
**Measured Spread From Treasury**

This table reports the average spread from treasuries for AA, A, and BBB bonds in the financial and industrial sectors. For each column, spot rates were derived using standard Gauss-Newton non-linear least squared methods as described in the text. Treasuries are reported as annualized spot rates. Corporates are reported as the difference between the derived corporate spot rates and the derived treasury spot rates. The financial sector and the industrial sector are defined by the bonds contained in the Lehman Brother's financial index and industrial index respectively. Panel A contains the average spot rates and spreads over the entire ten year period. Panel B contains the averages for the first five years and panel C contains the averages for the final five years.

Maturity	Treasuries	Financial Sector			Industrial Sector		
		AA	A	BBB	AA	A	BBB
Panel A: 1987-1996							
2	6.414	0.586	0.745	1.199	0.414	0.621	1.167
3	6.689	0.606	0.791	1.221	0.419	0.680	1.205
4	6.925	0.624	0.837	1.249	0.455	0.715	1.210
5	7.108	0.637	0.874	1.274	0.493	0.738	1.205
6	7.246	0.647	0.902	1.293	0.526	0.753	1.199
7	7.351	0.655	0.924	1.308	0.552	0.764	1.193
8	7.432	0.661	0.941	1.320	0.573	0.773	1.188
9	7.496	0.666	0.955	1.330	0.589	0.779	1.184
10	7.548	0.669	0.965	1.337	0.603	0.785	1.180
Panel B: 1987-1991							
2	7.562	0.705	0.907	1.541	0.436	0.707	1.312
3	7.763	0.711	0.943	1.543	0.441	0.780	1.339
4	7.934	0.736	0.997	1.570	0.504	0.824	1.347
5	8.066	0.762	1.047	1.599	0.572	0.853	1.349
6	8.165	0.783	1.086	1.624	0.629	0.872	1.348
7	8.241	0.800	1.118	1.644	0.675	0.886	1.347
8	8.299	0.813	1.142	1.659	0.711	0.897	1.346
9	8.345	0.824	1.161	1.672	0.740	0.905	1.345
10	8.382	0.833	1.177	1.682	0.764	0.912	1.344
Panel C: 1992-1996							
2	5.265	0.467	0.582	0.857	0.392	0.536	1.022
3	5.616	0.501	0.640	0.899	0.396	0.580	1.070
4	5.916	0.511	0.676	0.928	0.406	0.606	1.072
5	6.150	0.512	0.701	0.948	0.415	0.623	1.062
6	6.326	0.511	0.718	0.962	0.423	0.634	1.049
7	6.461	0.510	0.731	0.973	0.429	0.642	1.039
8	6.565	0.508	0.740	0.981	0.434	0.649	1.030
9	6.647	0.507	0.748	0.987	0.438	0.653	1.022
10	6.713	0.506	0.754	0.993	0.441	0.657	1.016

**Table II**  
**Average Root Mean Squared Errors**

This table contains the average root mean square error of the difference between theoretical price computed from the spot rates derived from the Gauss-Newton procedure and the actual bond invoice prices. For a given class of securities, the root mean squared error is calculated once per period. The number reported is the average of all the root mean squared errors within a class over the period indicated.

Period	Treasuries	Financial Sector			Industrial Sector		
		AA	A	BBB	AA	A	BBB
1987-1996	0.210	0.512	0.861	1.175	0.728	0.874	1.516
1987-1991	0.185	0.514	0.996	1.243	0.728	0.948	1.480
1992-1996	0.234	0.510	0.726	1.108	0.727	0.800	1.552



**Table IV**  
**Evolution of Default Probability**

Probability of default in year n conditional on (a) a particular starting rating and (b) not having defaulted prior to year n. These are determined using the transition matrix shown in Table IV. Panel (A) is based on Moody's transition matrix of table IV(A) and Panel (B) is based on Standard and Poor's transition matrix of table IV(B).

Panel (A) : Moody's

year	Aaa	Aa	A	Baa	Ba	B	Caa
1	0.000%	0.000%	0.000%	0.103%	1.594%	8.903%	22.052%
2	0.000%	0.004%	0.034%	0.274%	2.143%	8.664%	19.906%
3	0.001%	0.011%	0.074%	0.441%	2.548%	8.355%	17.683%
4	0.002%	0.022%	0.121%	0.598%	2.842%	8.003%	15.489%
5	0.004%	0.036%	0.172%	0.743%	3.051%	7.628%	13.421%
6	0.008%	0.053%	0.225%	0.874%	3.193%	7.246%	11.554%
7	0.013%	0.073%	0.280%	0.991%	3.283%	6.867%	9.927%
8	0.019%	0.095%	0.336%	1.095%	3.331%	6.498%	8.553%
9	0.027%	0.120%	0.391%	1.185%	3.348%	6.145%	7.416%
10	0.036%	0.146%	0.445%	1.264%	3.340%	5.810%	6.491%
11	0.047%	0.174%	0.499%	1.331%	3.312%	5.496%	5.743%
12	0.060%	0.204%	0.550%	1.387%	3.271%	5.203%	5.141%
13	0.074%	0.234%	0.599%	1.435%	3.218%	4.930%	4.654%
14	0.089%	0.265%	0.646%	1.474%	3.157%	4.678%	4.258%
15	0.106%	0.297%	0.691%	1.506%	3.092%	4.444%	3.932%
16	0.124%	0.329%	0.733%	1.532%	3.022%	4.229%	3.662%
17	0.143%	0.362%	0.773%	1.552%	2.951%	4.030%	3.435%
18	0.163%	0.394%	0.810%	1.567%	2.878%	3.846%	3.241%
19	0.184%	0.426%	0.845%	1.578%	2.806%	3.676%	3.074%
20	0.206%	0.457%	0.877%	1.585%	2.735%	3.519%	2.928%

Panel (B) : Standard and Poor's

year	AAA	AA	A	BBB	BB	B	CCC
1	0.000%	0.000%	0.103%	0.212%	1.209%	5.902%	22.526%
2	0.002%	0.017%	0.154%	0.350%	1.754%	6.253%	18.649%
3	0.007%	0.037%	0.204%	0.493%	2.147%	6.318%	15.171%
4	0.013%	0.061%	0.254%	0.632%	2.424%	6.220%	12.285%
5	0.022%	0.087%	0.305%	0.761%	2.612%	6.031%	10.031%
6	0.032%	0.115%	0.355%	0.879%	2.733%	5.795%	8.339%
7	0.045%	0.145%	0.406%	0.983%	2.804%	5.540%	7.095%
8	0.059%	0.177%	0.457%	1.075%	2.836%	5.280%	6.182%
9	0.075%	0.210%	0.506%	1.153%	2.840%	5.025%	5.506%
10	0.093%	0.243%	0.554%	1.221%	2.822%	4.780%	4.993%
11	0.112%	0.278%	0.600%	1.277%	2.790%	4.548%	4.594%
12	0.132%	0.313%	0.644%	1.325%	2.746%	4.330%	4.272%
13	0.154%	0.348%	0.686%	1.363%	2.695%	4.125%	4.006%
14	0.176%	0.383%	0.726%	1.395%	2.639%	3.934%	3.780%
15	0.200%	0.419%	0.763%	1.419%	2.581%	3.756%	3.583%
16	0.225%	0.453%	0.797%	1.439%	2.520%	3.591%	3.408%
17	0.250%	0.488%	0.830%	1.453%	2.460%	3.436%	3.252%
18	0.276%	0.521%	0.860%	1.464%	2.400%	3.292%	3.109%
19	0.302%	0.554%	0.888%	1.471%	2.341%	3.158%	2.979%
20	0.329%	0.586%	0.913%	1.475%	2.284%	3.033%	2.860%

**Table V**  
**Recovery Rates\***

The table shows the percentage of par that a bond is worth one month after bankruptcy, given the rating shown in the first column.

<u>Original Rating</u>	<u>Recovery Rate</u>
AAA	68.34%
AA	59.59%
A	60.63%
BBB	49.42%
BB	39.05%
B	37.54%
CCC	38.02%
Default	0%

\*From Altman and Vellore (1998)

**Table VI****Mean, Minimum and Maximum Spreads assuming Risk Neutrality**

This table shows the spread of corporate spot rates over government spot rates when taxes are assumed to be zero, but fractional recovery rates are taken into account. The corporate forward rates are computed using equation (6). These forward rates are converted to spot rates, which are then used to compute the spreads below.

## Panel (A) : Mean Spreads

years	1	2	3	4	5	6	7	8	9	10
AA	0.000%	0.004%	0.008%	0.012%	0.017%	0.023%	0.028%	0.034%	0.041%	0.048%
A	0.043%	0.053%	0.063%	0.074%	0.084%	0.095%	0.106%	0.117%	0.128%	0.140%
BBB	0.110%	0.145%	0.181%	0.217%	0.252%	0.286%	0.319%	0.351%	0.380%	0.409%

## Panel (B) : Minimum Spreads

years	1	2	3	4	5	6	7	8	9	10
AA	0.000%	0.003%	0.007%	0.011%	0.015%	0.020%	0.025%	0.031%	0.038%	0.044%
A	0.038%	0.046%	0.055%	0.063%	0.073%	0.083%	0.093%	0.104%	0.116%	0.128%
BBB	0.101%	0.132%	0.164%	0.197%	0.229%	0.262%	0.294%	0.326%	0.356%	0.385%

## Panel (C) : Maximum Spreads

years	1	2	3	4	5	6	7	8	9	10
AA	0.000%	0.004%	0.009%	0.014%	0.019%	0.025%	0.031%	0.038%	0.044%	0.051%
A	0.047%	0.059%	0.071%	0.083%	0.094%	0.106%	0.117%	0.129%	0.140%	0.151%
BBB	0.118%	0.156%	0.196%	0.235%	0.273%	0.309%	0.342%	0.374%	0.403%	0.431%



**Table VII**

**Mean, Minimum and Maximum Spreads with Taxes assuming Risk Neutrality**

This table shows the spread of corporate spot rates over government spot rates when taxes as well as fractional recovery rates are taken into account. The corporate forward rates are computed using equation (6). These forward rates are converted to spot rates, which are then used to compute the spreads below.

Panel (A) : Mean Spreads with 4.875%

years	1	2	3	4	5	6	7	8	9	10
AA	0.358%	0.362%	0.366%	0.370%	0.375%	0.379%	0.383%	0.388%	0.393%	0.398%
A	0.399%	0.410%	0.419%	0.429%	0.438%	0.448%	0.457%	0.466%	0.476%	0.486%
BBB	0.467%	0.501%	0.535%	0.568%	0.601%	0.632%	0.662%	0.691%	0.718%	0.744%

Panel (B) : Mean Spreads with 4% effective

years	1	2	3	4	5	6	7	8	9	10
AA	0.367%	0.371%	0.376%	0.380%	0.384%	0.388%	0.393%	0.397%	0.402%	0.407%
A	0.408%	0.419%	0.429%	0.438%	0.448%	0.457%	0.466%	0.476%	0.485%	0.495%
BBB	0.477%	0.510%	0.544%	0.578%	0.610%	0.642%	0.671%	0.700%	0.727%	0.752%

Panel (C) : Mean Spreads with effective tax rate of 6.7%

years	1	2	3	4	5	6	7	8	9	10
AA	0.496%	0.501%	0.505%	0.508%	0.512%	0.516%	0.520%	0.524%	0.528%	0.532%
A	0.537%	0.547%	0.557%	0.566%	0.575%	0.583%	0.592%	0.600%	0.609%	0.618%
BBB	0.606%	0.639%	0.672%	0.704%	0.735%	0.765%	0.794%	0.821%	0.847%	0.871%

Table VIII

**Relationship between Spreads and Treasury Term Structure**

This table shows the results of regression of Spreads on two variables summarizing the information contained in the treasury term structure. These two variables are (a) the term spread defined as the difference of 10 year treasury spot rate and 2 year treasury spot rate and (b) the two year treasury spot rate. The results reported are for Industrial Corporate bonds. Similar results were obtained for Corporate bonds issued by Financial firms. Values in parentheses are *t*-values.

**Panel A : Industrial AA rated bonds**

Maturity in years	Constant	(10year-2year) Treasury rate	2 year Treasury rate	Adj-R2
2	0.1689 (1.785)	0.0681 (3.145)	0.0262 (2.277)	0.0625
3	0.2894 (2.977)	0.02 (0.901)	0.0166 (1.403)	0.0004
4	0.2068 (2.321)	0.0188 (0.923)	0.0354 (3.265)	0.1105
5	0.086 (1.049)	0.0301 (1.604)	0.0582 (5.837)	0.3054
6	-0.0254 (-0.315)	0.0429 (2.324)	0.0784 (7.985)	0.4496
7	-0.1176 (-1.404)	0.0542 (2.831)	0.0948 (9.309)	0.5225
8	-0.1917 (-2.163)	0.0636 (3.138)	0.108 (10.013)	0.5558
9	-0.2511 (-2.674)	0.0712 (3.316)	0.1184 (10.368)	0.5708
10	-0.2995 (-3.027)	0.0775 (3.423)	0.127 (10.548)	0.5776

**Panel B : Industrial A rated bonds**

Maturity in years	Constant	(10year-2year) Treasury rate	2 year Treasury rate	Adj-R2
2	-0.2101 (-1.461)	0.2063 (6.270)	0.0933 (5.328)	0.2418
3	-0.1549 (-1.252)	0.169 (5.974)	0.1004 (6.671)	0.2711
4	-0.1173 (-1.035)	0.1531 (5.906)	0.1028 (7.455)	0.3107
5	-0.0911 (-0.858)	0.1458 (6.002)	0.1035 (8.015)	0.3436
6	-0.0724 (-0.703)	0.1419 (6.027)	0.1037 (8.280)	0.3596
7	-0.0585 (-0.571)	0.1396 (5.950)	0.1037 (8.305)	0.3617
8	-0.048 (-0.462)	0.1381 (5.815)	0.1036 (8.199)	0.3561

9	-0.0397 -(0.376)	0.137 (5.663)	0.1035 (8.044)	0.3474
10	-0.0331	0.1362	0.1035	0.338

**Panel C : Industrial BBB rated bonds**

Maturity in years	Constant	(10year-2year) Treasury rate	2 year Treasury rate	Adj-R2
2	-0.6634 -(2.742)	0.5513 (9.961)	0.1881 (6.391)	0.4594
3	-0.5086 -(2.382)	0.5158 (10.562)	0.1761 (6.780)	0.4890
4	-0.3751 -(1.906)	0.4528 (10.059)	0.1672 (6.983)	0.4579
5	-0.2715 -(1.461)	0.3939 (9.270)	0.1607 (7.110)	0.4136
6	-0.1930 -(1.064)	0.3459 (8.341)	0.1559 (7.069)	0.3658
7	-0.1332 -(0.735)	0.3081 (7.430)	0.1524 (6.908)	0.3228
8	-0.0871 -(0.474)	0.2784 (6.629)	0.1496 (6.700)	0.2885
9	-0.0505 -(0.271)	0.2547 (5.961)	0.1475 (6.489)	0.2628
10	-0.0213 -(0.112)	0.2356 (5.414)	0.1458 (6.297)	0.2442

**Table IX**

**Relationship between Spread Changes and Fama-French Risk Factors**

This table shows the results of the regression of Changes in Spreads on the Fama-French Risk Factors, viz. (a) the market excess return (over T-bills) factor (b) the Small Minus Big factor and (c) the High Minus Low factor. The results reported below are for Industrial Corporate bonds of Industrial Category. Similar results were obtained for bonds of Financial Firms. The values in parentheses are *t*-values.

**Panel A: Industrial AA rated bonds**

Maturity	Constant	Market	SMB	HML	Adj-R2
2	0.0023 (0.297)	-0.0386 (-2.197)	-0.0596 (-2.318)	0.0125 (0.404)	0.0986
3	0.0022 (0.286)	-0.0368 (-2.114)	-0.0682 (-2.680)	-0.0173 (-0.563)	0.0858
4	0.0014 (0.210)	-0.031 (-1.983)	-0.0656 (-2.877)	-0.0248 (-0.903)	0.0846
5	0.0007 (0.109)	-0.0252 (-1.791)	-0.0606 (-2.949)	-0.0252 (-1.018)	0.0801
6	0 (0.003)	-0.0204 (-1.463)	-0.0558 (-2.742)	-0.0236 (-0.961)	0.0608
7	-0.0005 (-0.077)	-0.0165 (-1.116)	-0.0517 (-2.391)	-0.0216 (-0.829)	0.0374
8	-0.0009 (-0.129)	-0.0135 (-0.839)	-0.0484 (-2.059)	-0.0198 (-0.700)	0.0195
9	-0.0012 (-0.163)	-0.0111 (-0.635)	-0.0458 (-1.798)	-0.0183 (-0.598)	0.0076
10	-0.0015 (-0.184)	-0.0091 (-0.489)	-0.0436 (-1.598)	-0.017 (-0.519)	-0.0002

**Panel B: Industrial A rated bonds**

Maturity	Constant	Market	SMB	HML	Adj-R2
2	0.0041 (0.437)	-0.0677 (-3.202)	-0.0916 (-2.965)	-0.0494 (-1.329)	0.1372
3	0.004 (0.534)	-0.0616 (-3.631)	-0.1024 (-4.134)	-0.0601 (-2.013)	0.2068
4	0.0031 (0.501)	-0.0544 (-3.904)	-0.0978 (-4.796)	-0.0655 (-2.666)	0.2493
5	0.0021 (0.403)	-0.0484 (-4.068)	-0.09 (-5.176)	-0.0685 (-3.270)	0.2754
6	0.0013 (0.262)	-0.0436 (-3.899)	-0.0825 (-5.050)	-0.0704 (-3.573)	0.2647
7	0.0006 (0.125)	-0.0399 (-3.480)	-0.0764 (-4.560)	-0.0716 (-3.549)	0.226
8	0.0001 (0.020)	-0.037 (-3.032)	-0.0714 (-4.003)	-0.0726 (-3.378)	0.1828
9	-0.0003 (-0.053)	-0.0347 (-2.654)	-0.0673 (-3.525)	-0.0733 (-3.185)	0.1469
10	-0.0006 (-0.105)	-0.0328 (-2.357)	-0.0641 (-3.149)	-0.0739 (-3.012)	0.1198

**Panel C: Industrial BBB rated bonds**

Maturity	Constant	Market	SMB	HML	Adj-R2
2	-0.0042 -(0.276)	-0.0556 -(1.626)	-0.1701 -(3.403)	-0.0629 -(1.045)	0.0969
3	-0.0031 -(0.255)	-0.0564 -(2.010)	-0.1552 -(3.787)	-0.0974 -(1.972)	0.1263
4	-0.0021 -(0.209)	-0.0595 -(2.601)	-0.1459 -(4.365)	-0.1151 -(2.858)	0.1798
5	-0.0012 -(0.153)	-0.0626 -(3.406)	-0.1397 -(5.199)	-0.1253 -(3.867)	0.2585
6	-0.0006 -(0.080)	-0.0653 -(4.025)	-0.1355 -(5.711)	-0.1317 -(4.607)	0.3126
7	-0.0001 -(0.008)	-0.0674 -(4.147)	-0.1323 -(5.567)	-0.136 -(4.750)	0.3122
8	0.0003 (0.045)	-0.0691 -(3.951)	-0.1299 -(5.084)	-0.1392 -(4.520)	0.2807
9	0.0007 (0.079)	-0.0705 -(3.685)	-0.1281 -(4.585)	-0.1417 -(4.209)	0.2445
10	0.0009 (0.101)	-0.0715 -(3.446)	-0.1266 -(4.173)	-0.1437 -(3.930)	0.2136

# Six Years Industrial Bonds Spreads

