FORWARDS AND FUTURES

September 1999

FORWARD

DEFINE: AGREEMENT TO TRANSFER SOME ITEM AT A FUTURE DATE AT A FIXED PRICE.

EXAMPLE: ONE YEAR TREASURY BILL AT 88 FOR DELIVERY ON DECEMBER 18

PROFIT or LOSS ASSUME ONE YEAR TREASURY BILLS ARE $90 AS OF DECEMBER 18. LOSS/GAIN: BUYER +2, SELLER -2

NOTES:

1. ZERO SUM GAME
2. BOTH CAN BE RISK AVERSE BUYER AND SELLER AND HAVE SAME EXPECTATIONS

MARKETS EXIST: WHERE DEFAULT RISK IS SMALL

FUTURE

DIFFERENCE: SAME AS FORWARD WITH DAILY SETTLEMENT. DAILY SETTLEMENT ELIMINATES DEFAULT RISK.

MARK TO MARKET

Future	-3	-2	-1	0
Price	88	87	89	90

Day	Buyer	Seller
-2	-1	+1
-1	+2	-2
0	+1	-1
	+2	-2

NOTE: SAME DIFFERENCE AS FORWARD

FUTURES CONTRACTS INSTITUTIONAL DETAILS

1. **MARKET TO MARKET**

2. **MARGIN**

3. **MOVE LIMITS**

4. **CHEAPEST TO DELIVER**

   (SMALL OPTION VALUE)

5. **RELATIONSHIP OF PRICE TO ARBITRAGE MODEL**

TABLE 2

EFFECT OF MARKING TO THE MARKET
(for $1MM 3 Month Contracts)

DOLLAR SIZE	PERCENT OF OCCURRENCES
-$44 to -$39	1.00
-39 to -29	.67
-29 to -19	5.00
-19 to -9	8.90
-9 to +1	7.89
+1 to 11	29.37
11 to 21	31.81
21 to 31	7.45
31 to 41	4.11
41 to 51	.11
51 to 61	.00
61 to 71	.56
71 to 81	1.11
81 to 91	.44
91 to 102	1.56

AVERAGE EFFECT = $10.00

Which preferred - forward or future (depends on receipt of cash and the reinvestment rate).

1. **If interest rates rise, bond price falls and Buyer loses**

   Future better to Seller and high reinvestment rate

2. **If interest rates decrease, bond prices rises and Buyer gains**

   Future worse to seller and low reinvestment rate

Therefore, if increase and decrease equally likely, Buyer prefers forward

$$P_{fut} < P_{for}$$

$\therefore$ Model somewhat overstate price

PRICING OF FORWARD AND FUTURES CONTRACTS

1. PRICE IS PRIMARILY DETERMINED BY ARBITRAGE

EXAMPLE 1

STRATEGY A

BUY 45-DAY T-BILL

STRATEGY B

1. **BUY 136-DAY T-BILL**

2. **WRITE A FUTURES CONTRACT FOR DELIVERY OF A 91-DAY T-BILL IN 45 DAYS**

3. **DELIVER THE T-BILL IN 45 DAYS**

PRICING

1. **KNOW T-BILL PRICES. SOLVE FOR FUTURES PRICE.**

PRICING

R_{45} = RATE OF RETURN ON 45-DAY T-BILL

P_{136} = PRICE OF 136-DAY T-BILL

f = FUTURE PRICE OF 91-DAY T-BILL TO BE DELIVERED IN 45 DAYS

$$\frac{f - P_{136}}{P_{136}} = R_{45}$$

$$\frac{f}{P_{136}} - 1 = \frac{100}{P_{45}} - 1$$

$$f = \frac{P_{136}}{P_{45}} 100$$

SOME INSTITUTIONAL DETAILS

1. **Contract size 1 million**

2. **Delivery any of 3 days as 89, 90, 91 day bill**

3. **Price is quoted as 100 - BDY in dollars**

EXAMPLE:

a. **Price of 136-day T bill is 98**

b. **Price of 45-day T bill is 99.4**

Futures Price is

$$f = 100\left(\frac{98}{99.4}\right) = 98.5916$$

QUOTED FUTURES PRICE

$$\text{BDY} = \frac{360}{91}\left(\frac{100 - 98.5916}{100}\right) = .055717$$

or in dollars 5.5717

Thus theoretical quoted futures price is

100 - 5.5717 = 94.42831

EXAMPLE 2

(FUTURES ON CURRENCY)

STRATEGY A

BUY 91-DAY U.S. GOVERNMENT T-BILL

STRATEGY B

1) BUY 91-DAY BRITISH GOVERNMENT BILL

2) ENTER INTO FUTURES CONTRACT TO EXCHANGE POUNDS FOR DOLLARS IN 91 DAYS

DEFINE

1) r_{US} = **RETURN ON U.S. T-BILL**

2) $r_{£}$ = **RETURN ON BRITISH T-BILLS**

3) **F = FUTURES PRICE OF 1 POUND (e.g. $1.49)**

4) **S = CURRENT PRICE OF 1 POUND (e.g. $1.42)**

LAW OF ONE PRICE => SAME RETURN

ASSUME INVEST $1

$$(1 + r_{us}) = \frac{1}{S}(1 + r_{£}) \bullet F$$

SOLVE FOR F

EXAMPLE 3

I. STRATEGY A

BUY ONE-YEAR GOVERNMENT BILL

II. STRATEGY B

1. BUY LONG-TERM GOVERNMENTS

2. WRITE FUTURES CONTRACT FOR DELIVERY OF LONG GOVERNMENTS IN ONE YEAR

LAW OF ONE PRICE $\Rightarrow$ SAME RETURN

r_1 = RETURN ON ONE YEAR GOVERNMENT

F = FUTURES PRICE

PL = PRICE OF LONG GOVERNEMNT LESS PRESENT VALUE OF COUPONS

$$r_1 = \frac{F - P_L}{P_L}$$

$$r_1 = \frac{F}{P_L} - 1$$

$$F = P_L(1 + r_1)$$

SOME INSTITUTIONAL CONSIDERATIONS

· cheapest to deliver

· some options

· effect of accrued

· conversion factor

Prior formula used invoice prices futures are quoted without accrued interest and on a standard bond.

STEPS

1. **Calculate invoice price on cheapest to deliver.**

2. **Use formula to get invoice price on future.**

3. **Subtract off accrued interest from future price.**

4. **Divide by conversion factor to get quoted futures price.**

Example:

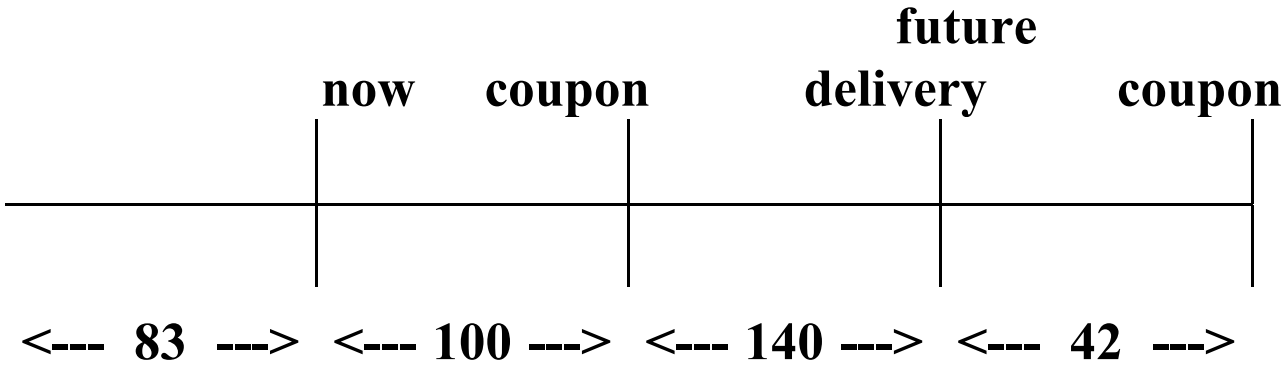
1. **Cheapest to deliver is 10% coupon bond with conversion factor of 1.2 and a price of 110.**

2. **Future is for delivery in 240 days.**

3. **Interest rate to delivery day is 4% and 1.65% to next coupon date.**

4. **Time scale on cheapest to deliver bond is shown below.**

5. **Coupon payable in 100 days and 282 days.**



Invoice price on cheapest to deliver is

$$110 + \frac{83}{183} \times 5 = 112.268$$

Present value of interest is

$$\frac{5}{1.0165} = 4.919$$

Calculate futures invoice price

$$F = (112.268 - 4.919)\ (1.04) = 111.6429$$

Remove accrued interest

$$111.6429 \quad - \left(\frac{83}{183} \right) x\ 5 = 110.626$$

Use conversion factor to get Quoted Futures Price

Theoretical Quoted Price =

$$\frac{110.626}{1.2} = 92.189$$

USES

1. **MATURITY CHANGE (superior way of timing)**

2. **RISK REDUCTION**

3. **RETURN PICK UP**

4. **DURATION LENGTHENING**

5. **ALPHA FUND**

6. **RISK HEDGE**

EXAMPLE MATURITY CHANGE

CURRENT PORTFOLIO

LONG-TERM GOVERNMENTS

DESIRE

SHORTEN MATURITY

ACTION

SELL FUTURES FOR DELIVERY OF LONG-TERM GOVERNMENTS IN ONE YEAR

RESULTS

MATURITY EFFECTIVELY ONE YEAR

EXAMPLE RISK REDUCTION

POSITION

YOU ARE DEALER
OWN 1 BILLION OF GOVERNMENT BONDS

RISK

ADVERSE INTEREST RATE MOVEMENTS BEFORE SELL

RISK REDUCTION

SELL FUTURES

RETURN PICKUP

FOR MOST PERIODS RETURN ON LONG T-BILL PLUS SHORT FUTURE > RETURN ON SHORT T-BILL

WANT SHORT T-BILL HOLD "PSEUDO" T-BILL

DURATION LENGTHENING

- **OWNING BOND FUTURES IN CONJUNCTION WITH BONDS ↑ THE DURATION**

- **IN SOME PERIODS ONLY WAY TO IMMUNIZE**

DURATION OF FUTURE

RECALL

(1) DURATION OF PORTFOLIO IS WEIGHTED AVERAGE OF DURATION OF BONDS THAT COMPRISE IT.

(2) FUTURE CAN BE REPLICATED AS PORTFOLIO OF TWO BONDS (ONE SOLD SHORT).

(3) THUS DURATION ON FUTURE MUST BE SAME AS ITS REPLICATING PORTFOLIO.

(4) SAME WITH CONVEXITY.

I. Terms

a. Forward

b. Future

I. Concepts

a. What affects difference in price of a forward and future?

b. Pricing futures by arbitrage.

c. Uses of futures.

d. How to calculate duration of a future.

II. Calculations

a. Pricing T-bill future.

b. Pricing T-bond future.

c. Calculating the duration of a future.

EXAMPLE 2

1. **BUY LONG BOND**

2. **BUY FUTURE AND SUFFICIENT T BILLS TO TAKE DELIVERY**

3. **ASSUME PERIOD SO SHORT NO COUPON**

A	**B**
LONG BOND	**LONG T BILLS FUTURE**

OR

A	**B**
LONG BOND	**FUTURE**
SHORT T BILLS	

$$D_f = D_{CTD} - D_{TB}$$