YIELD TO MATURITY

ACCRUED INTEREST

QUOTED PRICE

INVOICE PRICE

September 1999

Quoted Rate Treasury Bills
[Called Banker's Discount Rate]

$$d = \left[\frac{P_1 - P_0}{P_1}\right] * \left[\frac{360}{N}\right]$$

d = Bankers discount yield

P_1 = face value

P_0 = Price

N = number of days until maturity

$$\frac{P_1}{P_0} * \frac{N}{360} * d = \frac{P_1}{P_0} - 1$$

$$\left[1 + \frac{P_1}{P_0}\frac{d * N}{360}\right] = \frac{P_1}{P_0} = \text{holding period return}$$

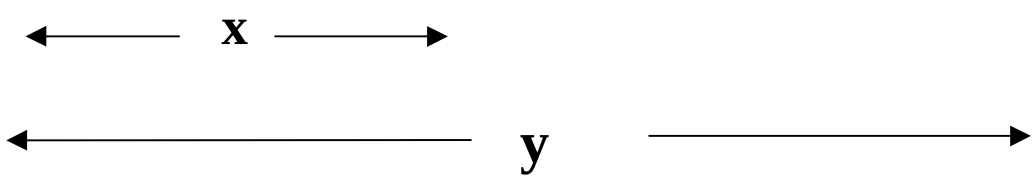
The invoice on a bond (what you pay) is quoted price plus accrued interest.

Government Bonds and Notes

Calculating Accrued Interest

Accrued interest is actual days/actual days

Last coupon date	settlement date	next coupon date



Accrued interest = x/y times interest payment

Example:

(1) **10% coupon**
(2) **$100 interest annually or $50 semi-annual per $1000 face**

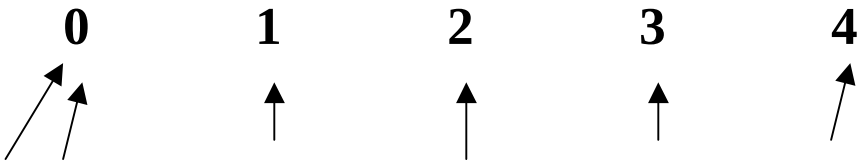
y = 183
x = 100

$$\frac{100}{183}[50] = \$27.32$$

Yield To Maturity

1. **One coupon left**
   [like T bill]

2. **At coupon paying date**



C **P_o** **C** **C** **C** **C + Prin**

$$P_0 = \frac{C}{[1+\frac{r}{2}]} + \frac{C}{[1+\frac{r}{2}]^2} + \frac{C}{[1+\frac{r}{2}]^3} + \frac{C+Prin}{[1+\frac{r}{2}]^4}$$

3. **Between Coupon Dates**

$$P_0 + A = \frac{C}{[1+\frac{r}{2}]^w} + \frac{C}{[1+\frac{r}{2}]^{1+w}} + \frac{C}{[1+\frac{r}{2}]^{2+w}} + \frac{C+prin}{[1+\frac{r}{2}]^{3+w}}$$

w = fraction of year to first coupon actual days over actual days

0 1 2 3 4

w ⟶

Note:

Ignore that intervals may be of uneven size because of Saturdays or end of month.

Eurobonds

- **will examine only bonds issued in dollars**

- **interest paid annually usually**

Calculating accrued interest

uses 30 day months 360 day years

Example 1:

1. **Issue date** **January 28**
2. **Settlement date** **March 5**

days	**January**	**[29, 30]**	**2**
	February	**30 day**	**30**
	March	**[1,2,3,4,5]**	**5**
			37

$$\frac{37}{360} \textit{ x interest = accrued interest}$$

Example 2:

1. **May 14** **issue date**

2. **Sept 17** **settlement date**

May 16
J, Ju Aug 90
Sept 17
123

The 31st is the same as the 30th.

$$\frac{123}{360} \text{ x interest = accrued interest}$$

Computing Yield To Maturity

1. **With one payment remaining like T-bill but usually with 30-day 360 calendar.**

2. **Multiple payments**

$$Price + accrued = \frac{C}{[1+r\ ]^{v}} + \frac{C}{[1+r\ ]^{v+1}} + \ldots \frac{C + Prin}{[1+r\ ]^{n-1+v}}$$

r is an annual rate

v is fraction of year until payment and is done on 360 days in year 30 day month calendar

Most have options. We will discuss this later.

G N M A

Definition: bonds issued with mortgages as their backing.

Guaranties

1. **Issuer. If borrower fails to make a scheduled amortization payment in any month, issuer makes good. If borrower defaults, issuer must promptly remit remaining mortgage.**

2. **Government. Makes good payments if issuer fails.**

Rate is mortgage rate - 50 basis points

e.g. 13% mortgages 12.5% pool rate

44 basis points to issuer
6 basis points to gov.

Timing of Payments

January **February** **March**

[Home owners] **[Home owners]**

Pass through **Pass through**

If. No Prepayments constant amount paid each month

determining constant amount

$$100 = \frac{M}{[1+\frac{r}{12}]} + \frac{M}{[1+\frac{r}{12}]^2} + \ldots + \frac{M}{[1+\frac{r}{12}]^{360}}$$

M is scheduled amortized payment

r	m
8	.7338
12	1.0286

Quoted Prices on GNMA

Quoted as % of remaining principal balance

Assume quoted price is 95

1 million original Principal value
x .8 % still outstanding
x .95 Quoted Price
$760,000 Price

Accrued interest on GNMA

Settlement day. Two business days after trade date but first settlement is usually third Wednesday for GNMA less than 9.5% and following Monday for GNMA 9.5%.

Reason: Pool factors not available until 10th of month.

Accrued interest

$$\frac{coupon}{12} \; x \; \frac{number\ of\ days\ from\ 1st\ until\ settlement}{30}$$

Example:

13% GNMA

5 million face

Feb. l5 settlement date

.8 Pool factor

Accrued Interest $= \frac{14}{30} \; x \; [.8x5x \; \frac{13}{12}] = \$20{,}222$

Note:

Always use 30 days irrespective of days in the month.

Yield to Maturity

$$Price + Accrued = \frac{M}{(1+\frac{r}{12})} + \frac{M}{(1+\frac{r}{12})^2} + \frac{M}{(1+\frac{r}{12})^3} + ..$$

LIBOR

- **Interest rates are annual using "simple interest"**

Interest payment = principal x LIBOR x
actual days to payment
360

Example: One million dollar deposit made June 22 at 5.93750. Until December 22 we get

actual days to payment = 183

interest payment = 1,000,000 x .0593750 x 183
360

= 30,182

Later need semiannual rate. Adjustment is

$$\left(1+\frac{r}{2}\right)^2 = \left(1+Libor\ x\ \frac{actual\ days\ of\ life}{360}\right)^{\frac{365}{actual\ days}}$$

Pound LIBOR is quoted on 365-day year

WHY PROBLEM WITH

YIELD TO MATURITY

September 1999

Reinvestment Assumption
(Implicit in Yield to Maturity)

Example:

0	1	2	3	4
	10			
		10		
			10	
				110

$$Price = \frac{10}{1.05} + \frac{10}{[1.05]^2} + \frac{10}{[1.05]^3} + \frac{110}{[1.05]^4} = \$117.74$$

Assume 12%

$10\,[1.06]^3$	11.91
$10\,[1.06]^2$	11.24
$10\,[1.06]$	10.60
110	110.00
	143.75

$$143.75/(1.05)^4 = 118.26$$

Assume 10% Reinvestment
Ending Value

$10(1.05)^3$	11.58
$10(1.05)^2$	11.03
10(1.05)	10.50
11	110
	$143.11/(1.05)^4$
	= $117.74

Analytics:

$$Price = \frac{CF(1)}{(1+\frac{r}{2})} + \frac{CF(2)}{(1+\frac{r}{2})^2} + \frac{CF(3)}{(1+\frac{r}{2})^3} + \frac{CF(4)}{(1+\frac{r}{2})^4}$$

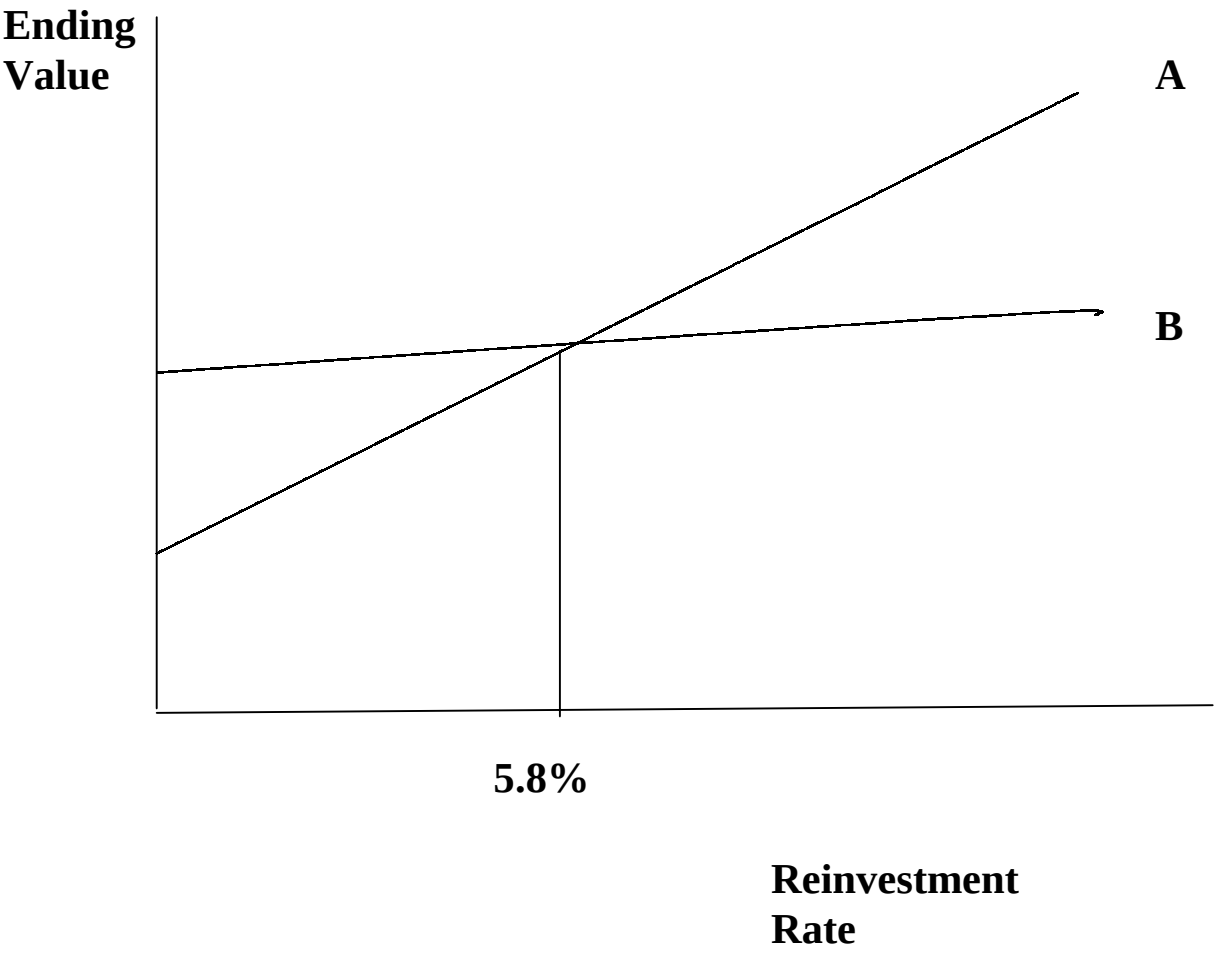
$$Price = \frac{CF(1)(1+\frac{y}{2})^3 + CF(2)(1+\frac{y}{2})^2 + CF(3)(1+\frac{y}{2}) + CF(4)}{(1+\frac{r}{2})^4}$$

Unless y = r, these don't match.

Different Bonds Assume Different Reinvestment Rates

Relative desirability depends on assumption about reinvestment

	Bond A	Bond B
Coupon	10%	3%
Principal	100	100
Price	$138.90	$70.22
Maturity	15 years	15 years
Frequency of payment	Annual	Annual
Yield to maturity	6%	6.1%



Usually one crossing point.

Yield to maturity on portfolio not weighted average on bonds that are in portfolio.

Illustrating the Nonadditivity of Yields

Bond	Outlay (Price)	Periods			Yield to Maturity	Weighted Average Yield
		1	2	3		
A	-100	15	15	115	15.00%	
B	-100	6	106		6.00%	
C	-92	9	9	109	12.35%	
A + B	-200	21	121	115	11.29%	10.50%
B + C	-192	15	115	109	9.65%	9.04%
A + C	-192	24	24	224	13.71%	13.73%

Review

I. Terms

- **a. Accrued interest**
- **b. Quoted price**
- **c. Invoice price**
- **d. Yield to maturity**

II. Concepts

- **a. That accrued interest calculations differ across types of bonds**
- **b. That yield to maturity is a poor measure of relative value**

III. Calculations

- **a. Accrued interest**
- **b. Yield to maturity**

Problems

Assume that the settlement date is December 1, 1997 and that the bond pays interest on September 15 and March 15. Further assume that the bond matures the following September. The coupon on the bond is ten percent and the face value is $1,000.

1. If the bond is a U.S. Treasury bond, what is the accrued interest?

Answer: For Treasuries, accrued interest is actual over actual

Since last interest payment

September	**15**
October	**31**
November	**30**
December	**1**
	77

In Period

September	**15**
October	**31**
November	**30**
December	**31**
January	**31**
February	**28**
March	**15**
	181

Accrued interest

$$\frac{77}{181} \text{ x } \$50 = \$21.27$$

2. Assume the bond is a corporate. What is the accrued interest?

Answer: For corporate bonds, months are 30 days and 360-day years.

Since last payment	
September	**15**
October	**30**
November	**30**
December	**1**
	76

In period	
September	**15**
October	**30**
November	**30**
December	**30**
January	**30**
February	**30**
March	**15**
	180

$$\frac{76}{180} \text{ x } \$50 = \$21.27$$

3. Assume bond is Italian government bond. What is the accrued interest?

Answer: Same as 2.

4. What is the yield to maturity for the bond in one if its price is 99?

Answer: Invoice Price = 990 + 21.27 = 1011.27
r = 11.317%

$$1011.27 = \frac{50}{(1+\frac{r}{2})^{\frac{104}{181}}} + \frac{1050}{(1+\frac{r}{2})^{1+\frac{104}{181}}}$$

Accrued Interest Calculations

Conventions

Different markets have different conventions for calculating accrued interest. The method used in each market is denoted by specifying the method of calculating two values:

- ***d*** is the number of days from the previous coupon payment date to settlement date (or from issue date to settlement date, if the next coupon payment if the first); that is, the number of days over which interest has accrued.
- A_y is the assumed number of days in one year.

The particular convention used can be Actual/Actual, Actual/365, Actual/360, or 30E/360.

In the name of the convention, the first part of the name denotes the method of computing *d*; and the second part of the name denotes the method of calculating A_y.

Computing the Accrued Interest

Once *d* and A_y have been calculated, accrued interest is computed by the formula:

$$I_A = C\frac{d}{A_y}$$

I_A is the accrued interest
C is the annual coupon payment

Calculating *d*

The three methods of calculating ***d*** are:

1. "Actual" - Calculate the actual number of days from the previous coupon payment date to the settlement date.

Examples:

There are 10 days from 1/3/93 to 1/13/93
There are 41 days from 1/3/93 to 2/13/93
There are 31 days from 1/1/93 to 2/1/93
There are 28 days from 2/1/93 to 3/1/93
There are 29 days from 2/1/92 to 3/1/92

2. "30" - Calculate the number of days from the previous coupon payment date to the settlement date by assuming 30-day months, as follows:

Let the two dates by $M_1/D_1/Y_1$ and $M_2/D_2/Y_2$.
If D_1 is 31, change it to 30.
If D_2 is 31, change it to first of next month.
Then $d = 360(Y_2-Y_1) + 30(M_2-M_1) + (D_2-D_1)$.

(Note: According to this formula, February will always appear to be a 30-day month).

Examples:

There are 10 days from 1/3/93 to 1/13/93
There are 30 days from 1/1/93 to 2/1/93
There are 30 days from 2/1/93 to 3/1/93
There are 30 days from 2/1/92 to 3/1/92
There are 4 days from 2/27/93 to 3/1/93
There are 1 days from 5/30/93 to 5/31/93
There are 2 days from 5/29/93 to 5/31/93

This "30" method is used in the U.S. agencies, corporates, and municipals markets.

3. "30E" - Calculate the number of days from the previous coupon payment date to the settlement date by assuming 30-day months, as follows:

Let the two dates by $M_1/D_1/Y_1$ and $M_2/D_2/Y_2$.
If D_1 is 31, change it to 30.
If D_2 is 31, change it to 30.
Then $d = 360(Y_2-Y_1) + 30(M_2-M_1) + (D_2-D_1)$.

(Note: According to this formula, February will always appear to be a 30-day month.)

Examples:

There are 30 days from 2/1/93 to 3/1/93
There are 30 days from 2/1/92 to 3/1/92
There are 4 days from 2/27/93 to 3/1/93
There are 0 days from 5/30/93 to 5/31/93
There is 1 day from 5/29/93 to 5/31/93

This method, slightly different from the "30" method above, is used in the Eurobond market and many European government and corporate markets.

Calculating A_y

The three method of calculating A_y are:

(1) "365" - A_y is equal to 365.

(2) "360" - A_y is equal to 360.

(3) "Actual - A_y is equal to the number of days in the current coupon period times the number of coupon payments per year. For a semi-annual coupon, the number of days in the coupon period can range from 181 to 184, so A_y can range from 362 to 368.

Examples:

(1) A 6.75% bond paying semi-annually is traded to settle on July 2, 1993. The previous coupon paid on March 15, 1993, and the next coupon pays on September 15, 1993. If the bond accrues according to the Actual/Actual convention, what is the accrued interest at settlement?

There are 184 actual days in the current coupon period, and there are 109 actual days from the last coupon payment to settlement. Therefore, the accrued interest is:

$$\frac{109}{2x184} x\, 6.75 = 1.999321 \text{ per 100 of face value}$$

(2) Assume that the bond in example (1) accrues according to the 30/360 convention, instead of Actual/Actual. What would be the accrued interest?

There are 107 "30" days from the last coupon payment to settlement. Therefore, the accrued interest is:

$$\frac{107}{360} x\, 675 = 2.006250 \ per\ 100\ of\ \ face\ value$$

Accrued Interest - Market Conventions

Instrument Type	Accrual Convention	Coupons per Year	Notes
Domestic:			
US Govt. Treasury Bonds	Actual/Actual	2	
US Govt. Agency Bonds	30/360	2	
Corporate Bonds	30/360	2	
Municipal Bonds	30/360	2	
GNMA/FNMA/FLHMC	Actual/360	12	
International:			
Eurobonds	30E/360	1	
German Govt. and Corp. Bonds	30E/360	1	
Italian Govt. and Corp. Bonds	30E/360	2	
Japanese Govt. and Corp. Bonds	Actual/365	2	1
Swiss Govt. and Corp. Bonds	30/360	1	
UK Govt. and Corp. Bonds	Actual/365	2	

Notes:

1. In the Japanese markets, February 29 is never counted.