MULTI-INDEX MODELS IN THE BOND AREA

Fall 2000

Basic idea is to decompose total return into sources that affect total return.

1. **Common**

2. **Unique**

A. Default free non-callable government

Return = Expected + Unexpected due to + Random
Return to term Error
Structure Shift

$$R_{it} = \overline{R}_i - D_i\left[\frac{\Delta r}{1+r}\right] + e_{it}$$

Comments

(1). D_i **can be estimated numerically or from theory.**

(2). One must assume when term structure shifts.

When you assume it shifts is always a matter of taste. I've always used beginning.

(3). Can reformulate in terms of returns on indexes

$$R_I = \Sigma X_i R_i = \Sigma X_i \overline{R}_i - \Sigma X_i D_i \frac{\Delta R}{1+R} + \Sigma X_i e_{it}$$

For index it is reasonable to assume $\Sigma X_i e_i = 0$. After all e_i is:

a. **Pricing Error**

b. **Bid Ask Spread**

c. **Non-synchronous Trading**

$$R_I = \overline{R}_I - D_I \frac{\Delta R}{1+R}$$

$$\therefore \frac{-\Delta R}{1+R} = \frac{R_I - \overline{R}_I}{D_I}$$

And Equation (1) becomes:

$$R_{it} = \bar{R}_i + \frac{D_i}{D_I}\left(R_{It} - \bar{R}_I\right) + e_{it} \qquad \textbf{(2)}$$

(4). <u>Expected Return.</u> Most assume expectation theory so that $\bar{R}_i = R_F$. Nothing precludes adding liquidity premium.

With no liquidity premium the expression becomes:

$$R_{it} - R_{Ft} = \frac{D_i}{D_I}\left(R_I - \bar{R}_I\right) + e_{it}$$

With a liquidity premium the expression becomes:

$$R_{it} - R_{Ft} = l_i + \frac{D_i}{D_I}\left(R_i - \bar{R}_I\right) + e_{it}$$

(5). Can add additional factors

$$R_{it} = \overline{R}_i - D_1 \Delta r_1 - D_2 \Delta r_2 + e_{it}$$

We use two rates:

a. **Long**

b. **Spread**

Ho uses several "Key Rates"

Ways of estimating sensitivity coefficients:

a. **Use spots and aggregate**

b. **Numerically**

c. **Theory**

Can use two indexes to eliminate the $\Delta r's$

Trade-off

1. **More factors -→ fewer portfolios meet it. They are more costly.**

2. **Fewer factors -→ may not capture reality.**

B. Adding Options

1. Change in expectations about market volatility.

2. Added return due to bondholder issuing option.

$$R_{it} = \left[\overline{R}_i + OAS\right] - D_i \frac{\Delta r}{1+r} - V_i dV_i - P_i dt + e_{it}$$

$V_i dV_i$ **is change in value of option due to change in market volatility.**

$P_i dt$ **is loss in option premium.**

C. Corporate Bonds

1. Must deal with tax and spread considerations.

2. Generally, just add a term that captures spread changes like:

$$C_i \Delta \text{ spread}$$

However, since we know spread should be related to:

1. Default Risk

2. State Taxes

3. Risk Premium

Can be a little more creative; although not tested out research suggests:

1. **Default Risk**

   Economic Variables

2. **State Taxes**

   Difference in rates between government and corporate.

3. **Risk Premium**

   Stock market factors

D. Mortgage backed

Use government with option adjustment.

Uses:

1. **Covariance structure.**
2. **Selection of exposure.**
3. **Return attribution.**
4. **Portfolio evaluation.**