Introduction to Risk Management in Financial Institutions: the VaR approach

Prof. Eli Ofek

Types of risk

✦ Market risk
  » interest rate, currency, equity, commodity, spread, volatility,…
  » example: \( P(\text{bond}) \) decline as interest rates rise

✦ Credit risk
  » default, downgrade
  » example: \( P(\text{bond}) \approx \text{recovery upon default} \)

✦ Other
  » liquidity, regulatory, political, model, execution,…

✦ We focus primarily on market risk, and to a lesser extent on credit risk
  (especially the link btw market and credit risk for derivatives)
Risk Measurement

✦ Address the question:

“HOW MUCH CAN WE LOSE ON OUR TRADING PORTFOLIO BY TOMORROW’S CLOSE?”

✦ Risk MEASUREMENT <=> Risk MANAGEMENT

VaR: Example

Consider a spot equity position worth $1,000,000

✦ Suppose the daily standard deviation of the S&P500 is 100 basis points per day
✦ How do we make an informative statement about risk?

We can only make a probabilistic statement:

Assume $\Delta S_{t,t+1}$ is distributed normally (0, 100bp²)
“Value at Risk” (VaR):
(First Look)

✦ From the normal dist’n tables:
  » -1STD to +1STD 68.3%
  » -2STD to +2STD 95.4%

What is the “value” of one standard deviation?
What are the amounts on the X-axis?

V a R

Value at Risk

Prob
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

-2.7
-2.3
-1.9
-1.5
-1.1
-0.7
-0.3
0.1
0.5
0.9
1.3
1.7
2.1
2.5
2.9

SD x

Prob

0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

-2.33
-1.65

With probability 95% we will not see a loss greater than ___?____?____?____ on our position”

V a R

Value at Risk

Prob
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

-2.7
-2.3
-1.9
-1.5
-1.1
-0.7
-0.3
0.1
0.5
0.9
1.3
1.7
2.1
2.5
2.9

SD x
A two asset example

Consider the following FX position
(where vol($/Euro)=75bp ==> VaR=75*1.65=123,
vol($/GBP)=71bp ==> VaR=71*1.65=117)

<table>
<thead>
<tr>
<th>Position (FX in $MM)</th>
<th>95% move (in percent)</th>
<th>VaR (in $MM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro 100</td>
<td>1.23%</td>
<td>$1.23</td>
</tr>
<tr>
<td>GBP -100</td>
<td>1.17%</td>
<td>-$1.17</td>
</tr>
</tbody>
</table>

Undiversified risk $2.40
(absolute sum of exposures, ignoring the effect of diversification)

From vol to VaR

How do we move from vol to VaR?

Consider the $ volatility

\[
\text{STD} = \sqrt{\sigma_{Ra}^2 + \sigma_{Rb}^2 + 2\rho_{Ra,Rb}\sigma_{Ra}\sigma_{Rb}} \times 1.65
\]

and we get

\[
\text{VaR} = \sqrt{\text{VaR}_{Ra}^2 + \text{VaR}_{Rb}^2 + 2\rho_{Ra,Rb}\text{VaR}_{Ra}\text{VaR}_{Rb}}
\]

...or we could calculate the %vol and

\[
\text{VaR} = \%\text{STD} \times \text{value} \times 1.65
\]

The two approaches are EQUIVALENT
Portfolio VaR

- Suppose $\rho_{$/Euro$/$/GBP} = 0.80$

- For simplicity forget cont’ comp’ returns for the moment

$\text{VaR} = \sqrt{\text{VaR}_{R_a}^2 + \text{VaR}_{R_b}^2 + 2 \rho_{R_a,R_b} \text{VaR}_{R_a} \text{VaR}_{R_b}}$

$= \sqrt{1.23^2 + (-1.17)^2 + 2 \cdot 0.80 \cdot (-1.17)(1.23)}$

$= \sqrt{1.5129 + 1.3689 - 2.3025}$

$= \$0.76\text{Mil}$

The portfolio effect

- Compare: 
  - Undiversified VaR $\$2.40\text{MM}$
  - Diversified VaR $\$0.76\text{MM}$
  - $\Rightarrow$ Portfolio effect $\$1.64\text{MM}$

- Risk reduction due to diversification depends on the correlation of assets in the portfolio

- As the number of assets increases, portfolio variance becomes more dependent on the covariances and less dependent on variances

- The “marginal” risk of an asset when held as a small portion of a large portfolio, depends on its return covariance with other securities in the portfolio
The (approx) VaR of a 10yr AA bond

- VaR(treasury)=11.5bp
- VaR(spread)=2.5bp
- CORR(∆spread, ∆treasury)=0

\[ \text{VaR(AA Bond)} = \sqrt{11.5^2 + 2.5^2} = 11.77\text{bp/day} \]

Note:
\[ \text{VaR(AA Bond) / VaR(treasury)} = \frac{11.77}{11.5} = 1.023, \]
only 2.3% higher VaR!
**Negative Correlation**

Ra → Rp → Rb

**Positive Correlation**

Ra → Rp → Rb

**Visual interpretation - corp’ bond example**

AA Bond → spread

treasury

AA Bond → treasury

Think, similarly, on the total risk of an FX equity investment
## Uses and applications

- **Corporates**
- **Financial Institutions**
  - Internal uses
    - Trading limits
    - Capital Allocation
  - External uses
    - Reporting
    - Capital requirements
    - Self regulation
- **Self regulation and market disclosure**

## Implementation difficulties

- Financial institution may hold at any point in time tens of thousands of securities
  - Equity – thousands of stocks around the world
  - Fixed income
    - Treasuries of different maturities
    - Corporate of different ratings
    - Swaps
    - Mortgage backs
  - Currencies
  - Commodities
  - Derivatives on all of the above
- Estimating correlations and volatilities between all these assets is very difficult.
  - Some helpful models
    - Barra data for Equity
    - Risk Metrics by JP Morgan for currencies
- What is correlation across assets?
  - LTCM
Trading Limits

✦ Full system may include VaR limits, notional limits, types of securities, types of exposures,…
✦ Management information and resource allocation
  » A unified measure of exposure at the trader, desk, group,... level
  » An input for capital allocation and reserve decisions
✦ Implementation isn’t simple
✦ Consider the case of using VaR for compensation/performance evaluation of trading divisions/desks

VaR and Performance Evaluation

✦ Example
  » Desk1: corr(P&L, IntRates) close to one
  » Desk2: corr(P&L, IntRates) close to zero
  » VaR1=VaR2
  » Performance/Compensation is a function of
    P&L - C * VaR
    *c* is the price of risk parameter
  BUT marginal VaR contribution of Desk1 >> than Desk2
  ==> P&L - C * MarginalVaR

... and what if P&L=0 and MarginalVaR<0???

✦ In reality P&L(realized/ex-ante VaR and MVaR)
Capital Requirements

\[ \text{CapRequ} = \text{MAX}[\text{VaR}_{t-1}, (\text{Mult} + \text{AddOn}) \times \text{AVG}(\text{VaR}_{t-60})] \]

✦ Mult = 3.
✦ AddOn related to past performance
  » Green zone - 4/250 exceptions on 1% VaR
  » Yellow zone - up to 9/250 AddOn = 0.3. To 1
  » Red zone - 10plus/250 -- Investigation starts
✦ Model must capture risk associated with options
✦ Must specify risk factors and a price-factor mapping process \textit{a priori}
✦ … for more, see \texttt{www.bis.org}

The Effect of Cyclical Vol.

✦ We measure vol as 7.3bp/day
✦ Suppose now that in fact

\[
\begin{align*}
\sigma & \quad \text{5} \\
7.3 & \\
15
\end{align*}
\]
✦ If in a given day $\Delta t = 22$bp, then do we interpret is as $22/7.3 = 3\text{sd}$, or $22/15 = 1.5\text{sd}$?
✦ This is the key goal of dynamic VaR engines
The Difficulty in Estimating Cyclical Vol.

- Need few days of data to realize change in vol. measure vol
- Question: how adaptable do you want to be

- Tradeoff exists, and we shall elaborate on it now
- “This is the key goal of dynamic VaR engines”
- ... And also the key difficulty...

Modeling time-variations in VaR

- **PARAMETRIC APPROACH:**
  estimate the parameters of a given distribution
  » STD - simple historical vol + Conditional normality
  » Declining weights + Conditional normality (RiskMetrics)
  » Mixture of normals, t-distribution, GARCH...

- **NONPARAMETRIC APPROACH:**
  let the data talk — estimate the entire distribution
  » Historical simulation

- **The Hybrid Approach**

- **Finance-based forecasts** (e.g., implied vol)
Why are tails so fat?

In trying to explain the fat tails, it could be the case that returns are simply fat tailed relative to the normal dist’n benchmark, or that returns are CONDITIONALLY NORMAL, but

**volatility varies through time**

» How does it affect risk?

» Is it enough to explain the tails?

Generating stress scenarios in practice

♦ Common practice: examine historical events
♦ Links to the historical simulation methodology:
  » use HS to generate the empirical distribution for the current portfolio
  » use HS to examine the “five worst weeks” given the current portfolio, when did they occur, and what were the circumstances
♦ Remember: there is no way to apply common statistical techniques, with so few (and economically different) data points
♦ However: extreme value theory is now commonly applied to the problem
♦ Its usefulness is very questionable