Arbitrage and Martingales

Detailed outline

1. Consumption space – random variables in $L^p(\mathcal{P})$
2. Preferences – strictly monotone, convex, lower semicontinuous
3. Marketed payoffs
4. Prices – positive linear functionals
5. Existence of optimal demand, or viability of the price system
6. Free lunches, or arbitrage opportunities
7. Extension of the price functional
8. State-price density
9. Security prices – right-continuous stochastic processes
10. Trading strategies – simple, self-financing, tight
11. Dynamic completeness
12. Equivalent martingale measures

Readings

Domenico Cuoco's lecture notes, part III.

Duffie, chapter 6.

Harrison, J., and D. Kreps, 1979, Martingales and arbitrage in multiperiod securities markets, *Journal of Economic Theory*, 20, 381-408.

Harrison, J., and S. Pliska, 1981, Martingales and stochastic integrals in the theory of continuous trading, *Stochastic Processes and Their Applications*, 11, 215-260.

Dybvig, P, and C. Huang, 1989, Nonnegative wealth, absence of arbitrage, and feasible consumption plans, *Review of Financial Studies* 1, 377-401.

Back, K. and S. Pliska, 1991, On the fundamental theorem of asset pricing with an infinite state space, *Journal of Mathematical Economics* 20, 1-18.

Clark, S., 1993, The valuation problem in arbitrage price theory, *Journal of Mathematical Economics* 22, 463-478.

Cox, J., and S. Ross, 1976, The valuation of options for alternative stochastic processes, *Journal of Financial Economics*, 3, 145-166.

Vasicek, O., 1977, An equilibrium characterization of the term structure, *Journal of Financial Economics*, 5, 177-188.

Problems

Prove the following propositions adapted from part III of Domenico Cuoco's notes:

1. Let N be a simple trading strategy with trading dates $0 = t_1 < \ldots < t_J = T$ and $S = (S_0, S_1, \ldots, S_n)$ be (cum-dividend) security prices. Then N is tight iff

$$N(t)S(t) = N(0)S(0) + \sum_{i=1}^{j} N(t_i)(S(t_i \wedge t) - S(t_{i-1})), \text{ a.s.} \tag{8}$$

   for all $j = 1, \ldots, J$ and for all $t \in (t_{j-1}, t_j]$.

2. Suppose there are no free lunches. For every payoff x in the space $\mathcal{M}$ of marketed payoffs, let the price of x, $p(x)$, be the initial portfolio value $N(0)S(0)$ under a trading strategy N that finances x. Prove that p is a well-defined, strictly positive, linear functional on $\mathcal{M}$.

3. If the price system $(\mathcal{M}, p)$ (generated by the security prices S) is viable, then there are no free lunches.