Incomplete Markets and Portfolio Constraints: Optimal Consumption and Investment

Detailed outline

1. Constrained utility maximization problem
2. Unconstrained problem in auxiliary markets with adjusted appreciation rates
3. Characterization of optimal constrained solution as optimal unconstrained solution in auxiliary market with minimal value function
4. Convex dual functions

Readings

Karatzas and Shreve, 1998, chapter 6.

He, H. and N.D. Pearson, 1991, Consumption and portfolio choices with incomplete markets and short-sale constraints: The infinite-dimensional case, *Journal of Economic Theory* 54, 259-304.

Karatzas, I, J. Lehoczky, S. Shreve, and G.-L. Xu, 1991, Martingale and duality methods for utility maximization in an incomplete market, *SIAM Journal of Control and Optimization* 29, 702-730.

Cvitanic, J. and I. Karatzas, 1992, Convex duality in constrained portfolio optimization, *Annals of Applied Probability* 2, 767-818.

Cuoco, D., 1997, Optimal consumption and equilibrium prices with portfolio constraints and stochastic income *Journal of Economic Theory* 72, 33-73.

Problem

Consider a complete, standard continuous-time financial market with a single risky asset and assume that $\sigma(t) > 0$ is bounded and bounded away from zero, $r(t)$ is bounded below, and $\theta(t)$ is bounded. Suppose an investor with initial wealth x derives utility from consumption plan $\{c_t\}$ and terminal wealth W equal to

$$\mathrm{E} \int_0^T \log c_t \, dt + \log W \; .$$

Suppose the investor is required to keep the proportion of his wealth $p(t)$ that is invested in the risky asset in the range $[\alpha, \beta]$ where $-\infty < \alpha \leq 0 \leq \beta < \infty$. Solve for the optimal proportion process $p(t)$. Also, give the optimal consumption process in feedback form.