# Forward Rate Agreements

## Concepts

- Forward Rate Agreement (FRA)
- Forward Contract
- Valuation
- FRAs and Swaps

## Reading

- Veronesi, pp.162-167

## Forward Rate Agreement

- A forward rate agreement (FRA) is a contract between two counterparties to exchange a fixed interest payment for a floating interest payment on a single date.
- Large, liquid, over-the counter market.
- $47 trillion notional amount outstanding in 2009.
- Most contracts are linked to LIBOR or Eurobor.
- Contracts can be customized.

## FRA Payoff

- To take a simple example, consider a contract on a 0.5-year rate. The fixed receiver pays interest at some maturity date $t$ at the floating rate ${}_{t-0.5}r_t$ in exchange for interest at fixed rate $f$, on an agreed notional amount $N$.
- There would be a single cash flow at time $t$. The fixed payer would pay the fixed receiver

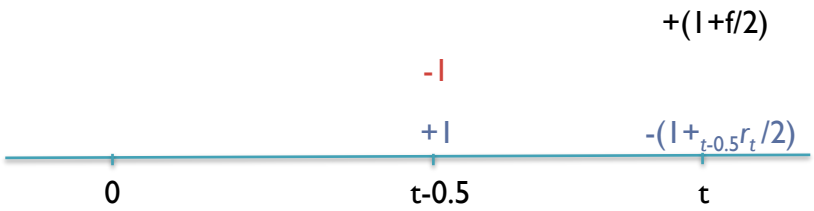
  $N \times (f - {}_{t-0.5}r_t)/2$
- Actually, in practice, the payoff is settled when it becomes known at time $t$-0.5 at

  $[N \times (f - {}_{t-0.5}r_t)/2]/(1+{}_{t-0.5}r_t/2)$

  but this does not change the value.

## FRA as Forward Contract on a Zero

- The fixed receiver's payoff $(f - {}_{t-0.5}r_t)/2$ at time $t$ is the same as $(1+f/2) - (1+{}_{t-0.5}r_t/2)$ at time $t$.
- This can be generated by
  - buying $(1+f/2)$ par of a zero maturing at time $t$,
  - selling 1 par of a zero maturing at time $t$-0.5, and
  - selling a 0.5-year par bond at time $t$-0.5:




## FRAs and Forward Rates

- The trade at time $t$-0.5 is self-financing, so the fixed receiver's side is equivalent to
  - long $(1+f/2)$ par of a zero maturing at time $t$,
  - short 1 par of a zero maturing at time $t$-0.5.
- Thus, the value of the FRA is $(1+f/2)\ d_t - d_{t-0.5}$
- At inception, the fixed rate $f$ is set to make the FRA worth zero, so

  $(1+f/2)\ d_t - d_{t-0.5} = 0 \Rightarrow 1/(1+f/2) = d_t/d_{t-0.5}$

  $\Rightarrow f = f_{t-0.5}{}^t$
- I.e., the forward rate is the fixed rate that makes the FRA worth zero.

## FRAs and Swaps

- Clearly, a swap can be viewed as a portfolio of FRAs.
- Each swap cash flow is the cash flow of an FRA with fixed rate equal to the swap rate $k$.
- Thus, there is a no-arbitrage relation between the LIBOR swap term structure and the term structure of forward rates in LIBOR-based FRAs.

## Swap Rates in Terms of Forward Rates

- The FRA valuation shows that replacing each floating rate in the swap with the corresponding fixed forward rate leaves the swap value unchanged.
- Thus the swap rate $k_T$ that makes the $T$-year swap worth zero must satisfy

  $(k_T - f_0^{0.5})d_{0.5} + (k_T - f_{0.5}^{1})d_1 + \ldots + (k_T - f_{T-0.5}^{T})d_T = 0$

  or $k_T = (f_0^{0.5}d_{0.5} + f_{0.5}^{1}d_1 + \ldots f_{T-0.5}^{T}d_T)/(d_{0.5} + \ldots + d_T)$

  i.e., the swap rate is the average forward rate weighted by the zero prices.
- This is an alternative par rate formula, equivalent to $k_T = 2(1-d_T)/(d_{0.5} + \ldots + d_T)$.