Short Rate Dollar Duration, Duration, and Hedging

Measuring and Hedging Interest Rate Risk

- Duration and dollar duration indicate how sensitive a bond is to changes in interest rates and can be used to calibrate hedges between different bonds.
- We can use the interest rate model to calculate duration and dollar duration measures that apply to all fixed income instruments, not just securities with fixed cash flows.

Reading

Veronesi, pp. 372-375

Short Rate Dollar Duration

- Recall: dollar duration ≈ -Δprice/Δrates
- For securities like options that don't have fixed cash flows, there may not be an explicit price-rate function to differentiate.
- But the binomial tree gives prices at the end of the first period (time h) in two different interest rate environments, which we can use to estimate dollar duration with respect to the short rate:

	Time 0.5
Asset price	K_u
Short rate	${}_{0.5}r_1^{\,u}$
	K_d
	${}_{0.5}r_1^{\,d}$

$$\text{short rate \$dur} = -\frac{K_u - K_d}{{}_{0.5}r_1^{\,u} - {}_{0.5}r_1^{\,d}}$$

Class Problems

Recall the zero prices in our binomial tree below:

Time 0	Time 0.5 (up)	Time 0.5 (down)
	${}_{0.5}r_1^{\,u} = 6.004\%$	${}_{0.5}r_1^{\,d} = 4.721\%$
0.973047		
0.947649	0.970857	0.976941
0.922242	0.941787	0.953790

Calculate the SR dollar duration for \$1 par of each of these zeroes:

- \$1 par of 0.5- year zero:
- \$1 par of 1- year zero:
- \$1 par of 1.5- year zero:

Short Rate Dollar Duration…

- For ordinary bonds, the short rate dollar duration is like ordinary dollar duration after one period of time., i.e., 0.5-years later in our example.
- In practice, the time steps are much shorter, so the effect of the passage of time is small and the short rate dollar duration is very close to traditional dollar duration.
- The point of this risk measure is that it applies not only to bonds, but also to bond derivatives.

SR Dollar Duration for an Option

- Consider a call on $100 par of the zero maturing at time 1.5 with strike $95 and expiration date time 0.5.
- **Class Problems**

1) Fill in the tree of prices for this call and calculate its SR dollar duration.
2) Calculate the SR $duration for $100 par of the underlying zero.

•SR $dur of the call:

Time 0	Time 0.5
	0.941787 C_u = ?
0.973047 0.922242 C = ?	
	0.953790 C_d = ?

•SR $dur of $100 par of 1.5- year zero:

Hedging with SR Dollar Duration

- Like ordinary dollar duration, SR dollar duration measures the sign and magnitude of the dollar risk of a position. Here the risk is that of the dollar payoff of the position next period.
- To hedge that risk, we can combine short and long positions, using SR dollar duration to calibrate the hedge.
- Notice that a position with zero SR dollar duration has a riskless payoff, like the riskless asset.
- So hedging means setting the SR dollar duration of the net position equal to zero.

Class Problem

- Suppose a dealer writes the 95 call on $100 par of the zero maturing at time 1.5 from the last example.
- What par amount $N_{1.5}$ of the underlying zero must the dealer buy to hedge the position? That is, what par amount must the dealer buy to set the SR dollar duration of the combined position equal to zero?

SR Dollar Duration Hedging...

- Just as before, the hedge ratio between two fixed income securities is the ratio of their dollar durations.
- Before, the hedging was with respect to an instantaneous parallel shift in all zero rates, and the hedge was approximate.
- Here, the hedge is with respect to the risk of the bond market in one period.
- The hedge is exact in the model because there are only two possible future term structure states.

The Hedge Ratio

- Notice that the SR dollar duration hedge ratio between an asset to be hedged and the hedging instrument is just the ratio of the numerators. For example,

$$\text{hedge ratio } N_{1.5} = \left(-\frac{C_u - C_d}{{}_{0.5}r_1^u - {}_{0.5}r_1^d}\right) / \left(-\frac{{}_{0.5}d_{1.5}^u - {}_{0.5}d_{1.5}^d}{{}_{0.5}r_1^u - {}_{0.5}r_1^d}\right) = \frac{C_u - C_d}{{}_{0.5}d_{1.5}^u - {}_{0.5}d_{1.5}^d}$$

- This is the hedge ratio we get if we solve directly for $N_{1.5}$ that makes the net position riskless:
- Net position = short the call, long $N_{1.5}$ bonds
- Riskless net => up state net = down state net

$$\Rightarrow N_{1.5} \text{ x } {}_{0.5}d_{1.5}^u - C_u = N_{1.5} \text{ x } {}_{0.5}d_{1.5}^d - C_d \Rightarrow N_{1.5} = \frac{C_u - C_d}{{}_{0.5}d_{1.5}^u - {}_{0.5}d_{1.5}^d}$$

- This is why SR dollar duration uses the same denominator for all assets. The real dollar risk measure of interest is the numerator, but we divide by an interest rate change to give it the flavor of dollar duration.

Short Rate Duration

- For securities with positive prices, we can define
- short rate duration = short rate $duration/price
- This is essentially the effective duration of the security.
- Analogous to the traditional duration, this measures interest rate risk per dollar invested.
- It is essentially the same as the traditional duration of the security at time h. If the time step h is very small, it is virtually the same as the traditional duration.
- But this measure applies to derivatives as well.

Class Problems

- Calculate the SR duration of the following securities:

1) The zero maturing at time 0.5
2) The zero maturing at time 1
3) The zero maturing at time 1.5
4) The call