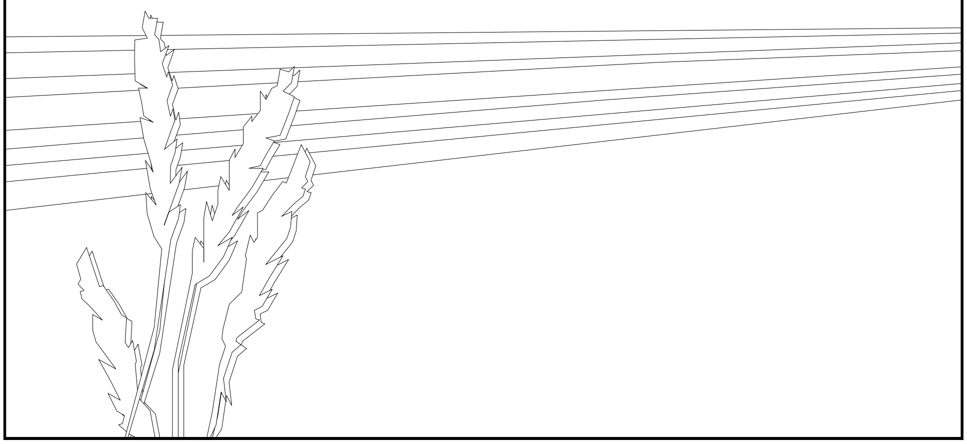
More on Callable Bonds



Concepts and Buzzwords

- Typical Provisions of Callable Bonds
- Interest Rate Sensitivity of Callable Bonds

– bullet, call protection, call schedule, negative convexity

Reading

- Tuckman, chapter 19.

Typical Provisions of Callable Bonds

- A callable bond may have a period of *call protection* when it is first issued during which the bond cannot be called.
- A typical structure is a "10-year noncall 5," meaning that the bond has a stated maturity of 10 years and is not callable for the first 5 years. Then the bond is callable from years 6 to 10.

Typical Provisions of Callable Bonds...

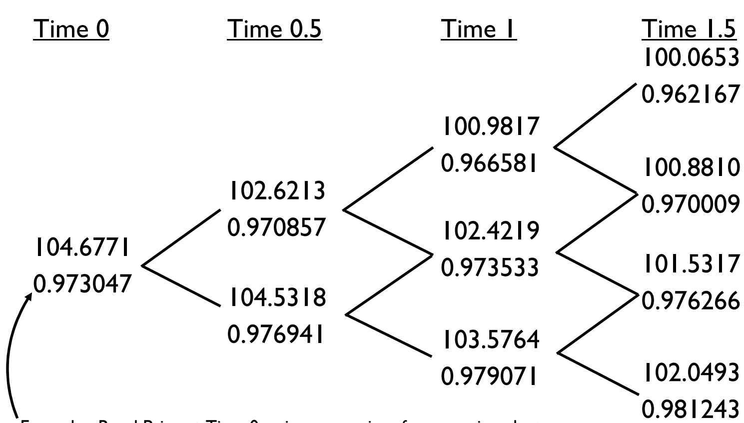
- Many bonds are callable at a strike price equal to par value. Often, however, the call price (strike price) starts out at a premium and declines over time according to a schedule.
- Typically, the bond is first callable at a price equal to par plus a coupon, and then the call price declines linearly over time to par value at maturity.
- For example, a 5% 10-year NC 5 might be callable at the end of year 5 at a price of 105, at the end of year 6 at a price of 104, at the end of year 7 at 103, and so on.

Example

- Let's value a 2-year 8%-coupon bond with call protection during the first year. The bond is callable at
  - 102 at time 1 and
  - 101 at time 1.5.
- We shall also value the noncallable "host" bond, the 2-year, 8%-coupon bond.
  - A noncallable bond is sometimes called a *bullet*.

2-Year, 8%-Coupon Noncallable Bond

Each node in the tree below lists the current ex-coupon price of the coupon bond, and the current price of a zero with 6 months to maturity.



Example: Bond Price at Time 0: using zero prices from previous lectures,
4x(0.973047+0.947649+0.922242+0.897166)+100x0.891766=104.6771

Callable Bond at Time 1.5

Noncallable	Callable
100.0653	100.0653
100.8810	100.8810
101.5317	101 (called)
102.0493	101 (called)

Callable Bond at Time 1

Noncallable Bond and Discount Factor

Time 1

100.9817
0.966581

102.4219
0.973533

103.5764
0.979071

Callable

Time 1

called value=102
wait value=0.966581x
(4+0.5(100.0653+100.8810))
=100.9817
➔ callable bond value=100.9817

called value=102
wait value=0.973533x
(4+0.5(100.881+101))
=102.1631
➔ callable bond value=102

called value=102
wait value=0.979071x
(4+0.5(101+101)) = 102.8025
➔ callable bond value=102

Time 1.5

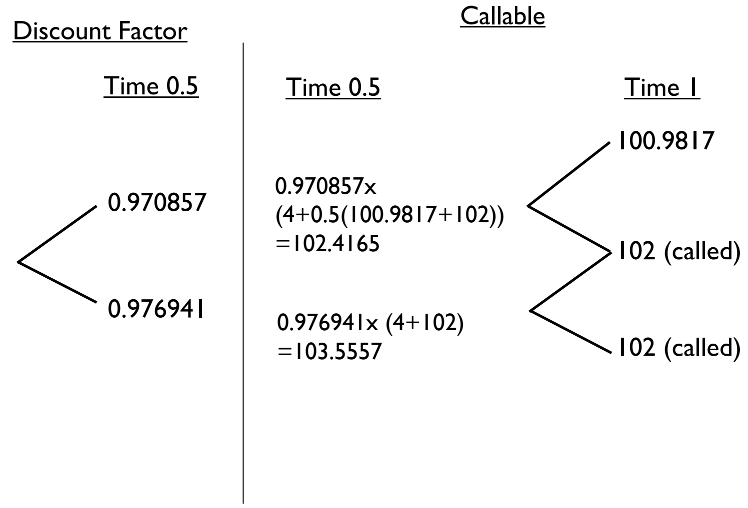
100.0653

100.8810

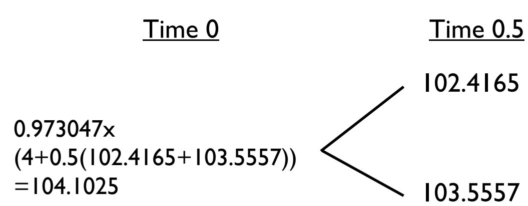
101 (called)

101 (called)

Callable Bond at Time 0.5



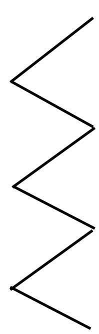
Callable Bond at Time 0



The Embedded Call Option

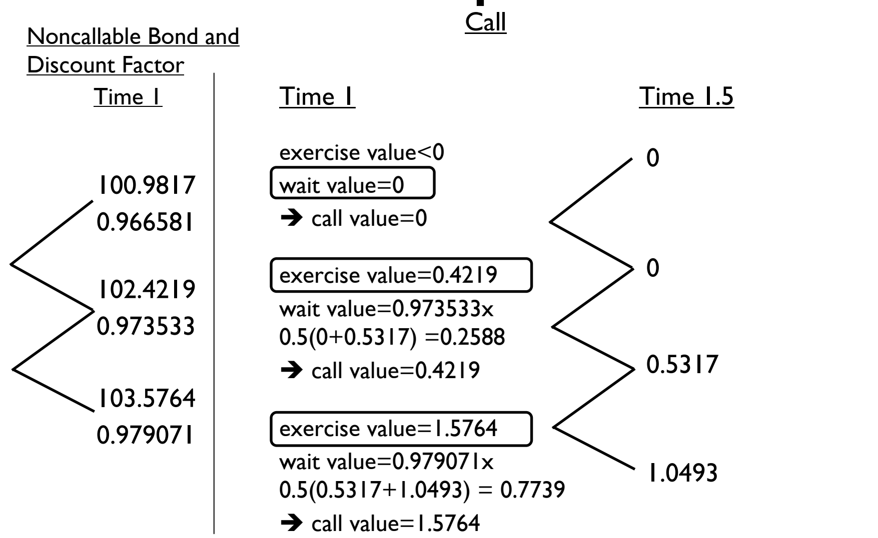
- Just as before, we can decompose the callable bond into a long position in the noncallable host bond, plus a short position in an embedded call option on that noncallable host.
- The call option, that investors have sold to the issuer, has the following features:
  - it is exercisable with a strike of 102 at time 1,
  - and exercisable with a strike of 101 at time 1.5.

Embedded Call Option at Time 1.5

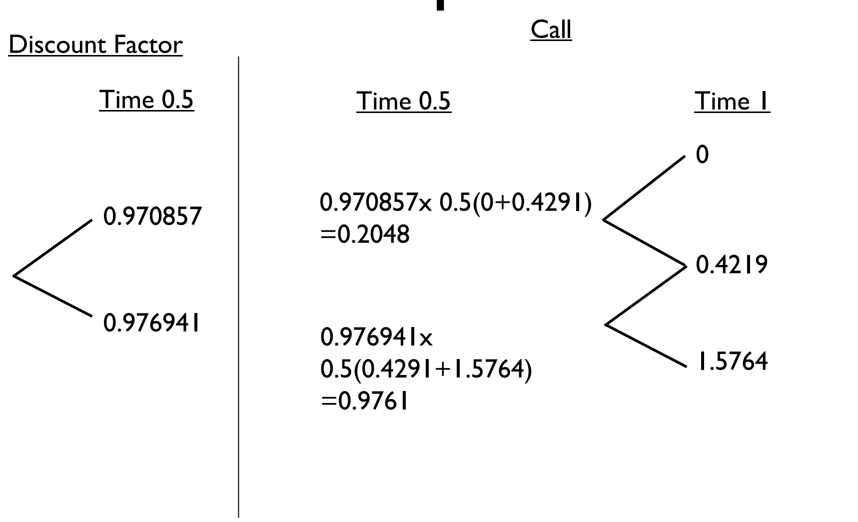


Noncallable	Call
100.0653	0
100.8810	0
101.5317	0.5317
102.0493	1.0493

Embedded Call Option at Time 1



Embedded Call Option at Time 0.5



Embedded Call Option at Time 0

Time 0

0.973047x 0.5(0.2048+0.9761)
=0.5745

Time 0.5

0.2048

0.9761

Price Tree

Each node in the tree below lists the ex-coupon price of the noncallable bond, the price of the embedded option, and the price the callable bond.

Time 0	Time 0.5	Time 1	Time 1.5
			100.0653 0 100.0653
		100.9817 0 100.9817	
	102.6213 0.2048 102.4165		100.8810 0 100.8810
104.6771 0.5745 104.1025		102.4219 0.4219 102	
	104.5318 0.9761 103.5557		101.5317 0.5317 101
		103.5764 1.5764 102	
			102.0493 1.0493 101

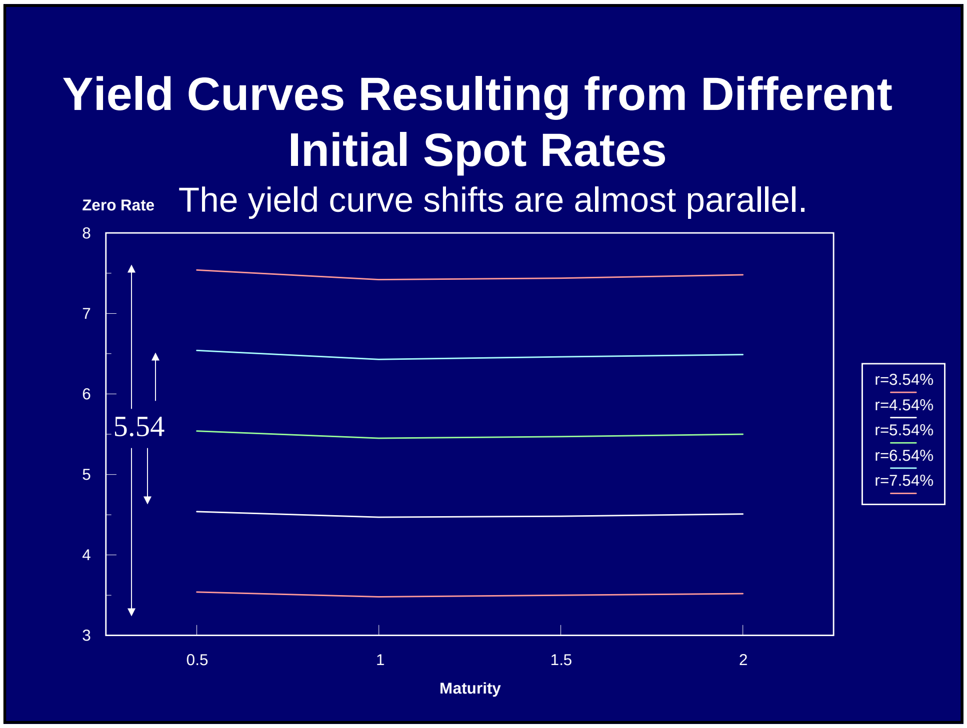
At every time and state, Callable = Noncallable - Option
Example at time 0: 104.1025 = 104.6771-0.5745

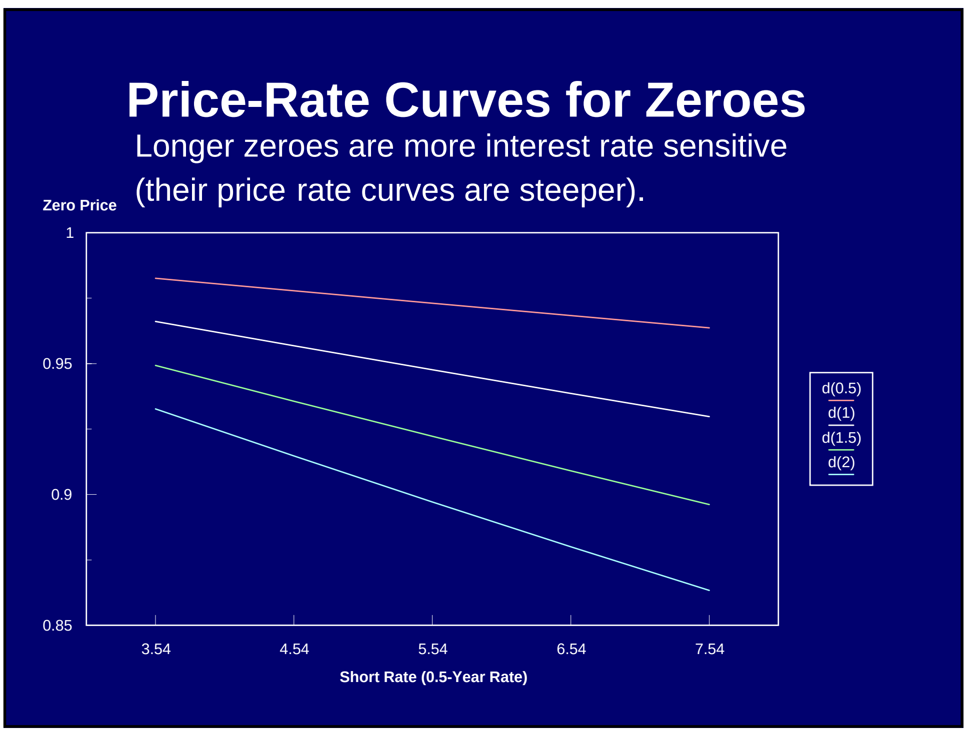
Interest Rate Sensitivity

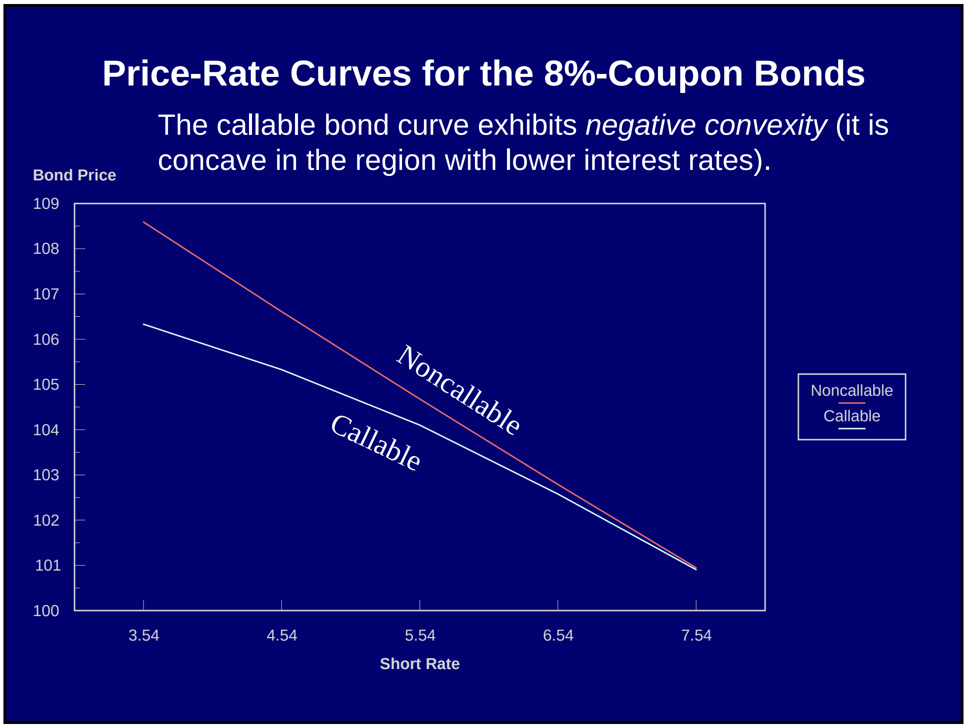
- An instrument's interest rate delta measures its **sensitivity to changes in the short rate in one period.**
- Another way to examine interest rate sensitivity is to see how price changes with **changes in the initial short rate**.
- As the step size in the tree grows small, these measures would converge.
- The change in price per unit change in the short rate is like an effective dollar duration.
- One difference is that if we hold the model parameters – drift and volatility – constant, then as we change the initial short rate, the resulting yield curve shifts will not be exactly parallel.

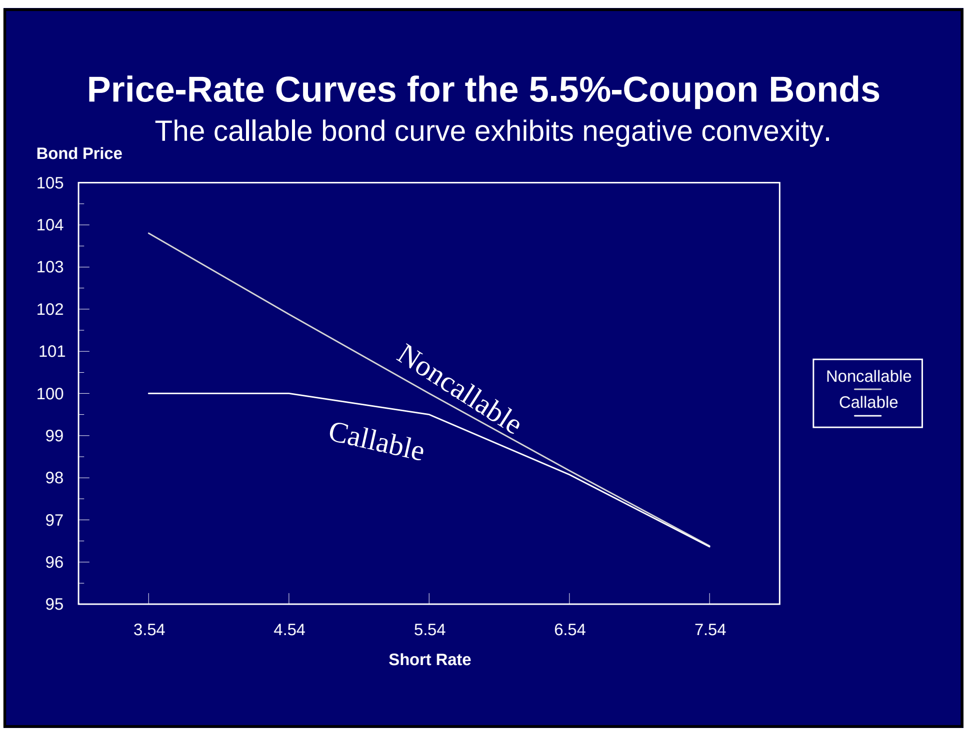
Sensitivity to Changes in the Short Rate

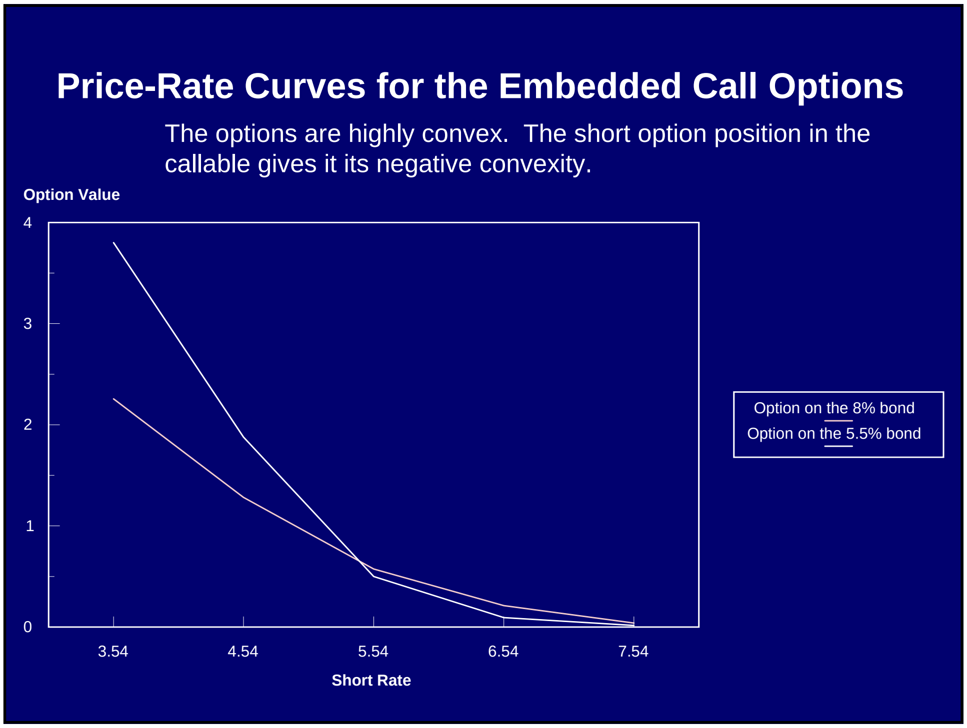
- To examine the interest rate sensitivity of different instruments, let's see how prices change as we vary the initial short rate *r*, while holding fixed the drift and volatility parameters of the model.
- Changing the initial short rate will
  - –change the future short rates,
  - –change the zero prices and their rates,
  - ➔ change the prices of coupon bonds and embedded options.


Yield Curves Resulting from Different Initial Spot Rates
The yield curve shifts are almost parallel.
Zero Rate
8
7
6
5.54
5
4
3
0.5
1
1.5
2
Maturity
r=3.54%
r=4.54%
r=5.54%
r=6.54%
r=7.54%


Price-Rate Curves for Zeroes
Longer zeroes are more interest rate sensitive
(their price rate curves are steeper).
Zero Price
1
0.95
0.9
0.85
3.54
4.54
5.54
6.54
7.54
Short Rate (0.5-Year Rate)
d(0.5)
d(1)
d(1.5)
d(2)


Price-Rate Curves for the 8%-Coupon Bonds
The callable bond curve exhibits negative convexity (it is concave in the region with lower interest rates).
Bond Price
109
108
107
106
105
104
103
102
101
100
3.54
4.54
5.54
6.54
7.54
Short Rate
Noncallable
Callable
Noncallable
Callable


Price-Rate Curves for the 5.5%-Coupon Bonds
The callable bond curve exhibits negative convexity.
Bond Price
105
104
103
102
101
100
99
98
97
96
95
3.54
4.54
5.54
6.54
7.54
Short Rate
Noncallable
Callable
Noncallable
Callable


Price-Rate Curves for the Embedded Call Options
The options are highly convex. The short option position in the callable gives it its negative convexity.
Option Value
4
3
2
1
0
3.54
4.54
5.54
6.54
7.54
Short Rate
Option on the 8% bond
Option on the 5.5% bond