Mortgage Pools,
Pass-Throughs, IOs, and POs

Concepts and Buzzwords

- Prepayment Risks
- Valuation of Mortgage Pools (Pass-Throughs)
- IOs and POs
- Interest Rate Sensitivity
- path-dependence, burnout, OAS, negative convexity, negative duration

Readings

- Tuckman, Chapter 21

Amortization Schedule for 2-Year, 5.5% Semi-Annual Mortgage

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.50	100.00	26.74	2.75	23.99	76.01
1.00	76.01	26.74	2.09	24.65	51.36
1.50	51.36	26.74	1.41	25.33	26.03
2.00	26.03	26.74	0.72	26.03	0.00

- We can think of this as
  - a single mortgage,
  - a pool of identical mortgages, or
  - a *pass-through* security that receives a fixed fraction of all cash flows that flow through the pool.

Mortgage Value Assuming No Prepayment

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.50	100.00	26.74	2.75	23.99	76.01
1.00	76.01	26.74	2.09	24.65	51.36
1.50	51.36	26.74	1.41	25.33	26.03
2.00	26.03	26.74	0.72	26.03	0.00

- With no prepayment, the mortgage would just be a stream of four fixed cash flows, each equal to 26.74.
- It could be valued as a package of zeroes:
  26.74*(0.9730+0.9476+0.9222+0.8972) = 100.02

Mortgagor's Prepayment Option

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.50	100.00	26.74	2.75	23.99	76.01
1.00	76.01	26.74	2.09	24.65	51.36
1.50	51.36	26.74	1.41	25.33	26.03
2.00	26.03	26.74	0.72	26.03	0.00

- The mortgagor has the option to pay off the mortgage at any time without penalty by paying the remaining principal balance.
  - For example, with the mortgage above, the mortgagor can pre-pay an additional 76.01 at time 0.5 (on top of his scheduled payment of 26.74) and remove his obligation to pay the remaining 3 payments.
  - Or he could pay 51.36 at time 1 and get out of the remaining 2 payments, etc.

Mortgagor's Prepayment Option

- Think of paying off the mortgage as buying back the remaining stream of payments.
- Then the prepayment option is an American call option where
  - the underlying asset is the remaining stream of payments
  - the strike price is the remaining principal balance.
- Thus, the underlying asset is "wasting away" and the strike price declines over time according to the pre-determined amortization schedule. Note that the option is
  - at the money when the market yield on the remaining monthly payments is equal to the original mortgage rate,
  - in the money when the market rate is below the mortgage rate
  - out of the money when the market rate is above the mortgage rate.

Valuation

- Mortgage = (Nonprepayable) Stream of Monthly Payments - Prepayment Option.
- Modeling the mortgage involves modeling the prepayment option.
- **If all borrowers prepaid according to a strategy that minimized the mortgage value** (maximized the option value), mortgage cash flows would be a function of interest rates and could be valued by replication and no arbitrage, just as we valued callable bonds.
- Alternatively, **if prepayments (option exercises) were uncorrelated with the market and independent across different loans in a given pool**, a well-diversified pool would just experience the ***average prepayment***, with little variance, by the law of large numbers, and MBSs would have nearly fixed cash flows which could be valued as a package of zeroes.

Valuation...

What makes valuation hard is that prepayments are random and subject to both market and non-market risks:

- **market**: a mortgage could prepay because rates fall (the prepayment option gets deep in the money)
- **non-market**: a mortgage could prepay even when rates are high because the mortgagor sells the property or the property is destroyed (the mortgagor may be forced to exercise the option when it is out of the money)
- **non-market**: a mortgage might not prepay even when rates fall because the mortgagor faces transaction costs, or cannot refinance because the property has lost value (the mortgagor fails to exercise a deep-in-the-money option)

Valuation...

- We can value non-market risks at their *true* expected value if they can be diversified away through pooling.
- Market risks can be hedged, and thus valued by no arbitrage (risk-neutral expected value).
- Conceptually, therefore, valuation is straightforward.
- Practically, however, we need to know the average prepayment along every interest rate path throughout the life of the mortgage to be able to value the mortgage exactly. Realistic valuation problems are very difficult.
- The examples in this lecture consider simple prepayment assumptions to illustrate some basic effects.

Mortgage with Value-Minimizing Prepayment Policy

At each state, the borrower can leave the loan outstanding, or else pay off the loan by paying the remaining principal balance in addition to the currently scheduled payment. Assume he chooses the action that minimizes the value of the mortgage.

Time 0

At each node, the mortgage value is the minimum of remaining principal and wait value.

99.76
=min(100,
99.76)

Time 0.5

min(76.01,
75.55)= 75.55

76.01
=min(76.01,
76.28)
(paid off)

Time 1

pay off:51.36
$\boxed{\text{wait: 0.9666*(26.74+ 0.5*(25.73+25.94))=50.82}}$
50.82

pay off: 51.36
$\boxed{\text{wait: 0.9735*(26.74+ 0.5*(25.94+26.03)=51.33}}$
51.33

$\boxed{\text{pay off: 51.36}}$
wait:0.9791*(26.74+
26.03)=51.66
51.36 (paid off)

Time 1.5

pay off:26.03
$\boxed{\text{wait:0.9622*26.74 =25.73}}$
25.73

25.94=min(26.03,
0.9700*26.74)

26.03=min(26.03,
0.9763*26.74)
(paid off)

26.03=min(26.03,
0.9812*26.74)
(paid off)

The number at each node represents the value of the remaining cash flows from the mortgage, excluding the currently *scheduled* payment.

Mortgage Value Assuming Deterministic Prepayments

- Under the value-minimizing prepayment policy,
  - there is a 50% (risk-neutral) chance of the mortgage prepaying at time 0.5, and given that it doesn't prepay at time 0.5,
  - there is a 25% chance of the mortgage prepaying at time 1.5.
- If we considered this as a pool of homogeneous value-minimized mortgages, all would follow exactly the same policy.
- What if we try to approximate the impact on value that this prepayment risk has by assuming that, with certainty,
  - 50% of the mortgages prepay in full at time 0.5,
  - of the remaining mortgages, 25% prepay at time 1.5.
- Using this approximation, we could value the mortgage as a stream of fixed cash flows.

Mortgage Value Assuming Deterministic Prepayments

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Prepayment	Ending Balance
0.50	100.00	26.74	2.75	23.99	38.00	38.00
1.00	38.00	13.37	1.05	12.33	0.00	25.68
1.50	25.68	13.37	0.71	12.66	3.25	9.76
2.00	9.76	10.03	0.27	9.76	0.00	0.00

- The mortgage would then be worth:
- (26.74+38.00)*0.9730 + 13.37*0.9476 + (13.37+3.25)*0.9222 + 10.03*0.8972 = 100.00
- By contrast, the correct value of the mortgage, under the value-minimizing prepayment policy, is 99.76.
- Replacing the interest rate-dependent optimal prepayments with their average value overstates the value of the mortgage.

Mortgage Price as a Percent of the Remaining Principal Balance

At each node below, the first number is the total pool value, and the second is the value as a percent of remaining principal.

Value-Minimized Mortgage:

Time 0	Time 0.5	Time 1
		50.82 98.96
	75.55 99.40	
99.76		51.33 99.95
	76.01 100	
		51.36 100

99.40 = 100*75.55/76.01
(note: price can't go above par)

Mortgage with No Prepayments:

Time 0	Time 0.5	Time 1
		50.82 98.96
	75.57 99.42	
100.02		51.37 100.03
	76.52 100.68	
		51.81 100.88

51.81=26.74*(0.9791+0.9583)
(value this as a package of zeroes)

Mortgage Pool with Both Types

Now consider a pool consisting half of mortgages that never prepay and half of mortgages that optimally prepay.

Time 0	Time 0.5	Time 1
	75.56 99.40	
99.89		51.35 99.99
		25.68 100.03
	76.27 100.34	

99.89=0.5*(99.76+100.02)

- In the beginning, the value of the pool is just the average of the value of the two types.
- However, over time, if the optimal prepayers do prepay, then the composition of the pool changes. The remaining pool consists of non-prepayers.
- Often, older mortgages prepay slower because the faster prepayers have dropped out. This slow down in prepayment speeds is called *burnout*.
- If prepayments depend on the level of interest rates, then the mortgage pool value becomes *path-dependent*. For example, at time 1 in the middle state, the mortgage price is higher if the path was down-up than up-down, because the optimal prepayers are gone.

Valuation Approach in Practice

- Rather than develop "rational" models of prepayment behavior that detail the decision-making process of mortgagors, practitioners usually take a different approach to valuation.
  - Estimate an empirical model of prepayments as a function of current and past interest rates, pool age and size, seasonality, and other variables.
    - prepayment rate this period = α*exp(β1*(coupon minus rate) + β2*(lagged rates) + β3*(percent of pool outstanding) + β4*(1 if summer) + ...

Valuation Approach in Practice

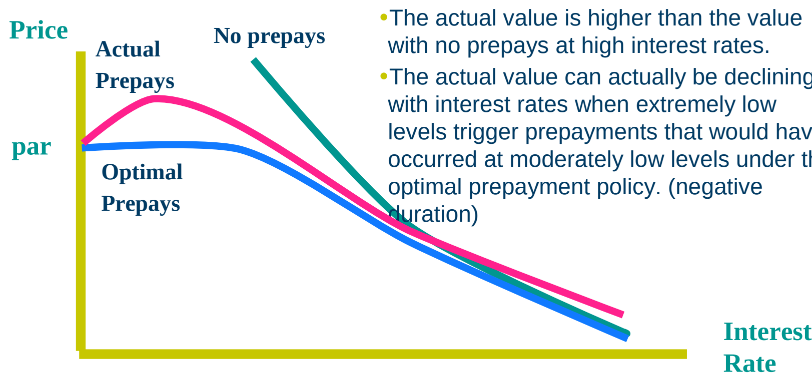
- Simulate mortgage cash flows along thousands of different paths in the interest rate tree using the estimated prepayment function to determine prepayments along each path.
- Compute, for each path, the present value of the cash flows discounted back at the short rates along the path.
- Value the pool as the average discounted payoff value across different paths, weighted by the risk-neutral probability of each path.

Option-Adjusted Spread (OAS)

- The option-adjusted spread on a given mortgage-backed security implied by its market price is the spread one would need to add to each of the short rates on the interest rate tree to make the model price equal the market price.
- In general, OAS is a measure of the market price of any security relative to its model value
  - OAS is positive: market price looks ‘cheap’ relative to model price
  - OAS is negative: market price looks ‘rich’

Price-Rate Curve for Pool

- The optimal value is always the lowest.
- The actual value is higher than the value with no prepays at high interest rates.
- The actual value can actually be declining with interest rates when extremely low levels trigger prepayments that would have occurred at moderately low levels under the optimal prepayment policy. (negative duration)



IOs and POs

- The most basic MBS is a pass-through, a security that receives a fixed fraction of all cash flows that flow through the pool.
- As an alternative, MBS issuers sometimes strip pass-throughs into two securities:
  - IOs -- Securities that receive interest only.
  - POs -- Securities that receive principal only.
- For the pass-through, optimal prepayments reduce value, but suboptimal prepayments can actually increase value.
- For the PO, prepayments are unambiguously good:
  - get the same cash flows earlier
- For the IO, prepayments are unambiguously bad:
  - get altogether less cash flow

IO and PO from the 2-Year 5.5% Mortgage With No Prepayments

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance	IO	PO
0.50	100.00	26.74	2.75	23.99	76.01	2.75	23.99
1.00	76.01	26.74	2.09	24.65	51.36	2.09	24.65
1.50	51.36	26.74	1.41	25.33	26.03	1.41	25.33
2.00	26.03	26.74	0.72	26.03	0.00	0.72	26.03

- With no prepayment, each security has fixed cash flows and can be valued as a package of zeroes:

  IO: 2.75*0.9730 + 2.09*0.9476 + 1.41*0.9222 + 0.72*0.8972 = 6.60.

  PO:
  23.99*0.9730+24.65*0.9476+25.33*0.9222+26.03*0.8972=93.42.
- The IO plus the PO reconstitute the whole mortgage, so their values must satisfy: IO value + PO value = Mortgage value.

  Indeed, 6.60 + 93.42 = 100.02.

IO and PO from the 2-Year 5.5% Mortgage With Deterministic Prepayments

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Prepaym't	Ending Balance	IO	PO
0.50	100.00	26.74	2.75	23.99	38.00	38.00	2.75	62.00
1.00	38.00	13.37	1.05	12.33	0.00	25.68	1.05	12.33
1.50	25.68	13.37	0.71	12.66	3.25	9.76	0.71	15.92
2.00	9.76	10.03	0.27	9.76	0.00	0.00	0.27	9.76

- With deterministic prepayment, each security has fixed cash flows and can be valued as a package of zeroes:

  IO: 2.75*0.9730 + 1.05*0.9476 + 0.71*0.9222 + 0.27*0.8972 = 4.56.

  PO: 62.00*0.9730+12.33*0.9476+15.92*0.9222+9.76*0.8972=95.44.
- IO value + PO value = Mortgage value

  4.56 + 95.44 = 100.00
- The prepayments reduce the IO value and increase the PO value.

IO from Mortgage with Value-Minimizing Prepayment Policy

Time 0 | Time 0.5 | Time 1 | Time 1.5

At each node, the IO value is the value of the remaining cash flows, excluding the current interest payment.

Time 0:
4.55 = (2.75+ 0.5(3.85+0)) *0.9730

Time 0.5:
3.85 = (2.09 + 0.5(2.03+1.71)) *0.9709
0

Time 1:
2.03 = (1.41+ 0.5(0.689+0.694))*0.9661
1.71 = (1.41 + 0.5(0.694+0))*0.9735
0

Time 1.5:
0.689 =0.9622*0.72
0.694 =0.9700*0.72
0
0

PO from Mortgage with Value-Minimizing Prepayment Policy

Time 0 | Time 0.5 | Time 1 | Time 1.5

At each node, the PO value is the value of the remaining cash flows, excluding the current principal payment.

Time 0:
95.21 = (23.99+ 0.5(71.7+76.01)) *0.9730

Time 0.5:
71.70 = (24.65 + 0.5(48.79+49.62)) *0.9709
76.01

Time 1:
48.79 = (25.33+ 0.5(25.04+25.25))*0.9661
49.62 = (25.33 + 0.5(25.25+26.03))*0.9735
51.36

Time 1.5:
25.04 =0.9622*26.03
25.25 =0.9700*26.03
26.03
26.03

Note: IO + PO = Mortgage
4.55 + 95.21 = 99.76

Interest Rate Sensitivity of Mortgage, IO, and PO with No Prepayments

Time 0.5

Mortgage
IO
PO

75.57
4.01
71.56

76.52
4.06
72.47

Interest Rate Deltas

- Mortgage:
  - -(75.57-76.52)/(0.06004-0.04721)= 75
- IO:
  - -(4.01-4.06)/(0.06004-0.04721) = 3
- PO:
  - -(71.56-72.47)/(0.06004-0.04721)= 71

With no prepayments, each security is just a stream of fixed cash flows.
4.06 = 2.09*0.9769 + 1.41*0.9538 + 0.72*0.9309
72.47 = 24.65*0.9769 + 25.33*0.9538 + 26.03*0.9309

Interest Rate Sensitivity of Mortgage, IO, and PO with Value-Minimizing Prepayment Policy

	Time 0.5
	75.55
	3.85
Mortgage	71.70
IO	
PO	76.01
	0
	76.01

Interest Rate Deltas

- Mortgage:
  - -(75.55-76.01)/(0.06004-0.04721)= 36
- IO:
  - -(3.85-0)/(0.06004-0.04721) = -300
- PO:
  - -(71.70-76.01)/(0.06004-0.04721)= 336

With optimal prepayments

- the delta of the mortgage is lower,
- the delta of the IO is negative.

Price-Rate Curves for IO and PO