# Eurodollar Futures

## Concepts

- Eurodollar Futures (EDF)
- Futures rate
- Convexity adjustment

## Reading

- Veronesi, Chapter 6
- Tuckman, Chapter 17
- Sundaresan, Chapter 15

## Eurodollar Futures (EDF)

- Eurodollar futures are cash-settled futures contracts with final futures price based on three-month LIBOR at the expiration date: $G(T) = 100(1 - {}_{T}L_{T+0.25})$
- For example, if 3-month LIBOR is 1% on the futures expiration date, the EDF price is 99.00.
- Contracts are based on $1,000,000 par, but marked to market based on the change in the unannualized rate. I.e., each one basis point change in the EDF price induces a mark-to-market of $25 = 1,000,000 x 0.0001/4.

## The Eurodollar Futures Market

- EDFs are traded on the Chicago Mercantile Exchange.
- For quotes and contract specifications, see http://www.cmegroup.com/trading/interest-rates/stir/eurodollar_quotes_globex.html
- Contracts with expiration dates every month for nearest 6 months, and then every quarter March, June, September, December, out ten years.
- Contracts with expiration up to three years are very liquid. These futures prices form the basis for calibrating the short end of the LIBOR term-structure for LIBOR-based derivative pricing models. LIBOR swap rates are used for the long end of the LIBOR term structure.

## Example

- Let's consider a stylized example of a EDF based on the 0.5-year riskless rate ${}_T r_{T+0.5}$ in our model.
- Suppose the contract expires at time 0.5.

| Time 0 | Time 0.5 |
|---|---|
| | 93.996 = 100-6.004 |
| 94.6375 =0.5(93.996+95.279) | |
| | 95.279 = 100-4.721 |

## The Futures Rate

- Define the *futures rate* as

  $g$ = (100-EDF price)/100.
- The time 0 futures rate for this contract is

  $g_{0.5}^{1}$=(100-94.6375)/100 = 5.3625%

- **Class Problem**: What is the forward rate $f_{0.5}^{1}$?

## The Convexity Adjustment (I)

- The futures rate is higher than the corresponding forward rate. Thus, to extract forward rates from EDF rates, it is necessary to make an adjustment commonly called the "convexity adjustment."
- The difference arises for two reasons. Here is one:
- The futures rate is the risk-neutral expected future rate: $G_T^{T+0.25} = E\{100(1-{}_TL_{T+0.25}\} \Rightarrow g_T^{T+0.25} = E\{{}_TL_{T+0.25}\}$ and similarly, in our stylized example, $g_T^{T+0.5} = E\{{}_Tr_{T+0.5}\}$.
- But for $T = 0.5$, which is one period out in our model,

  $1/(1+f_{0.5}^1/2) = F_{0.5}^1 = d_1/d_{0.5} = E\{{}_{0.5}d_1\} = E\{1/(1+{}_{0.5}r_1/2)\}$

  $> 1/(1+E\{{}_{0.5}r_1\}/2)$ because $1/(1+{}_{0.5}r_1/2)$ is convex in ${}_{0.5}r_1$

  $\Rightarrow f_{0.5}^1 < E\{{}_{0.5}r_1\} = g_{0.5}^1$.

## The Convexity Adjustment (II)

- For expiration dates farther out, there is the additional effect of marking to market.
- For example, in a FRA, all the "marking-to-market" $f$-$r$ comes at the end.
- In the EDF, the sum of marks-to-market are

  $g(0)-g(1) + g(1)-g(2) + \ldots + g(T-1)-r = g(0)-r$

  but negative marks are reinvested at higher rates, while positive marks are reinvested at lower rates, so the futures rate $g$ must be higher than the forward rate $f$ to compensate for this adverse effect.
- This is the same as the way that the marking to market in a bond futures contract makes the futures price lower than the forward price of the underlying bond.

## Class Problem

- Consider again a stylized example of a EDF based on the 0.5-year riskless rate ${}_{1}r_{1.5}$ in our model.
- Suppose the contract expires at time 1 and the contract is marked to market every 0.5 years.
- Fill in the tree of EDF prices below:

Time 0 Time 0.5 Time 1

## Class Problems…

- What is the time 0 futures rate $g_1^{1.5}$?
- What is the time 0 forward rate $f_1^{1.5}$?