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Does Option Compensation Increase Managerial Risk Appetite?

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ABSTRACT

This paper solves the dynamic investment problem of a risk averse manager compensated with a call option on the assets he controls. Under the manager's optimal policy, the option ends up either deep in or deep out of the money. As the asset value goes to zero, volatility goes to infinity. However, the option compensation does not strictly lead to greater risk seeking. Sometimes, the manager's optimal volatility is less with the option than it would be if he were trading his own account. Furthermore, giving the manager more options causes him to reduce volatility.

MANAGERS WITH CONVEX COMPENSATION SCHEMES play an important role in financial markets. This paper solves for the optimal dynamic investment policy for a risk averse manager paid with a call option on the assets he controls. The paper focuses on how the option compensation impacts the manager's appetite for risk when he cannot hedge the option position.

On one hand, the convexity of the option makes the manager shun payoffs that are likely to be near the money. Under the optimal policy, the manager either significantly outperforms his benchmark or else incurs severe losses. Furthermore, in examples of optimal trading strategies, asset volatility goes to infinity as asset value goes to zero.

Yet option compensation does not strictly lead to greater risk seeking. As asset value grows large, or if the evaluation date is far away, the manager moderates asset risk. For example, if the manager has constant relative risk aversion (CRRA), asset volatility converges to the Merton constant as asset value goes to infinity. In some situations, the manager actually chooses a lower asset volatility than he would if he were investing on his own, because the leverage inherent in his option magnifies his exposure to the asset volatility.

In addition, with all constant or decreasing absolute risk averse utility functions from the hyperbolic absolute risk averse (HARA) class, giving the manager more options causes him to reduce asset volatility. In the CRRA case, for example, the manager targets a fixed volatility for his personal

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portfolio of options and outside wealth. Giving the manager more options increases the volatility of his personal portfolio. To offset this, he reduces the volatility of the underlying asset portfolio.

An explicit example of the investment problem solved here is that of a portfolio manager paid with a share of the positive profits of the fund, net of a benchmark, like the incentive fee of a hedge fund. However, the essence of the problem solved here appears in many other contexts. For instance, a corporate manager who controls firm leverage or asset volatility and holds executive stock options that he cannot hedge faces a similar investment problem. The investment problem of shareholders of a levered firm resembles the problem solved here if the firm is privately held. Although the complete, continuous-time market modeled here is less appropriate for a corporate manager than for a fund manager, corporate managers do have the ability to manage firm risk dynamically by using forward contracts, swaps, and other derivatives.

In some situations, an explicit option contract does not exist but compensation is still convex in performance. For example, one might argue that the compensation of a trader at a securities firm is a convex function of his profits because he has limited liability. Another possible example is that of mutual fund managers. They typically receive a simple percentage of initial asset value, but the asymmetry in the relation between performance and subsequent new money flows found empirically by Sirri and Tufano (1998) and Chevalier and Ellison (1997) may produce a convex compensation function.

Several lines of research consider the impact of options on the owner's risk preferences in a static framework. Much of this literature assumes that the manager is risk neutral or that the manager can hedge the option, so that his objective is to maximize its value. This will generally lead the manager to increase asset risk as much as possible. For example, Jensen and Meckling (1976) and the corporate finance literature that followed show that value-maximizing levered equity holders will prefer more asset volatility, even at the expense of firm value. Grinblatt and Titman (1989) show that a fund manager who can hedge his incentive fee will try to maximize the value of the fee by increasing fund leverage as much as possible. Other papers addressing this issue examine the risk incentives of a utility-maximizing manager paid with an option he cannot hedge. Smith and Stulz (1985) show that a risk averse corporate manager with a convex compensation function will want to hedge some, though not all, of firm risk. Starks (1987) models the utility-maximizing market beta for a fund manager with an incentive fee when the manager can choose only once.

This paper is the first to solve the dynamic portfolio choice problem of a manager paid with an option he cannot hedge. Under general assumptions about the continuous-time financial market, the benchmark payoff, and the manager's preferences, the optimal random terminal portfolio value exceeds the benchmark by some margin when both the state price and the benchmark value are low and is zero otherwise. For cases in which security prices are geometric Brownian motions and the manager has decreasing absolute risk aversion, the paper provides explicit expressions for the manager's optimal trading strategy. Because he is risk averse the manager does not strictly prefer to increase asset volatility. Rather, he dynamically adjusts volatility in response to changes in asset value. When the manager is close to the evaluation date and near the money, small changes in market conditions lead to large shifts in portfolio holdings as the manager attempts to get in the money while remaining solvent.

The extreme nature of the manager's optimal payoff with the option may seem somewhat unrealistic. In the frictionless, continuous-time, complete market modeled here, the manager has the ability to implement payoffs that have zero probability of being near the money. The markets in which real managers operate are not so flexible. Incomplete markets, short-selling and borrowing constraints, transaction costs, and the discreteness of trading all hamper the manager's ability to customize his random payoff. For this reason, the distributions of asset returns we see in practice are likely to be less extreme.

In addition, the model illustrates the risk incentives created by a specific contract with a single evaluation period. In practice, managers face multiple evaluation periods, and compensation contracts can be complex. For example, multiperiod hedge fund contracts typically contain a high-water-mark provision that sets the strike price of each period's incentive fee equal to the all time high of fund value. Corporate compensation schemes can contain a variety of incentive components, including options, restricted stock, and performance bonuses. The behavior of the manager clearly depends on the whole compensation package. Nevertheless, the model provides a useful example of the kind of payoff the manager seeks in the presence of an option and the nature of the trading strategy that implements that payoff.[1]

A number of features of the manager's optimal investment policy are consistent with empirical findings. For example, the potential for severe losses or bankruptcies may help explain the high attrition rates for hedge fund managers documented by Brown, Goetzmann, and Ibbotson (1999). The large and rapid adjustments to portfolio holdings that the manager makes in response to market moves when he is near the money are consistent with the evidence of Fung and Hsieh (1997) that the portfolio weights of hedge funds fluctuate in a wide range and change quickly. The tendency for the manager to increase volatility if the fund is doing poorly is like the pattern of risk shifting found for mutual fund managers in Brown, Harlow, and Starks (1996) and Chevalier and Ellison (1997). Although these empirical findings could be consistent with many models, the presence of convex compensation functions provides at least one explanation.

The paper proceeds as follows. Section I describes the manager's preferences and opportunity set, and Section II states the manager's investment problem. Section III reviews the standard investment problem. Section IV

[1]In a multiperiod model available from the author, a CRRA manager receives in each period a fraction of asset value plus an at-the-money option. Prior to the final period, the manager does not risk bankruptcy, but the payoff he seeks still either significantly outperforms or significantly underperforms the benchmark. Thus, career concerns reduce the manager's incentive to take all-or-nothing gambles, but the option still makes him shun near-the-money payoffs.

solves for the manager's optimal random payoff. Section V gives examples of the solution with specific benchmarks, and Section VI gives examples of optimal trading strategies. Section VII concludes. Proofs of the main results are in the Appendix.

I. Assumptions

The manager controls assets with initial value X_0. His wealth at time T is the payoff of a call option on the assets plus a constant, $K > 0$, that includes fixed compensation and personal wealth. Letting X_T represent the value of the assets at time T, $\alpha > 0$ represent the number of options or the percentage of positive profits, and B_T represent the option strike price or benchmark payoff, the manager's terminal wealth is

$$\alpha(X_T - B_T)^+ + K. \tag{1}$$

A. Manager Preferences

The manager chooses an investment policy to maximize his expected utility of terminal wealth. His utility function U is strictly increasing, strictly concave, at least twice continuously differentiable, and defined on a domain containing $[K,\infty)$. U'' is nondecreasing, and $U'(W)$ approaches zero as W approaches infinity. Consequently, the inverse marginal utility function $I \equiv U'^{-1}$ is a well-defined, strictly decreasing, convex, continuously differentiable function from $(0,\infty)$ onto a range containing $[K,\infty)$. For example, the HARA utility functions with constant or decreasing absolute risk aversion (CARA or DARA),

$$U(W) = \frac{1-\gamma}{\gamma}\left(\frac{A(W-w)}{1-\gamma}\right)^{\gamma}, \tag{2}$$

with $\gamma < 1$, $w < K$, and $A > 0$, satisfy these hypotheses.

B. Asset Prices

The manager operates in a complete, arbitrage-free, continuous-time financial market consisting of a riskless asset with instantaneous interest rate r and n risky assets. The risky asset prices, $P_i, i = 1, \ldots, n$, are stochastic processes driven by a standard n-dimensional Brownian motion, W, which is defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. The risky asset prices are governed by the equations

$$\frac{dP_{i,t}}{P_{i,t}} = (r_t + \mu_{i,t})\,dt + \sigma'_{i,t}\,dWt, \tag{3}$$

where the interest rate r, the excess appreciation rates μ_i, and the volatility vectors σ_i' are stochastic processes that satisfy standard regularity conditions (see, e.g., Karatzas and Shreve (1991)). In particular, the interest rate process r, the vector process $\mu = (\mu_1, \ldots, \mu_n)$, and the volatility matrix process σ, whose ith row is σ_i', are bounded and progressively measurable with respect to $\{\mathcal{F}_t\}$, the $\mathcal{P}$-augmentation of the filtration generated by the Brownian motion. In addition, the covariance matrix $\Sigma = \sigma\sigma'$ is strongly nondegenerate.

C. Trading Strategies

A trading strategy for the manager is an n-dimensional process $\{\pi_t : 0 \leq t \leq T\}$ whose ith component, $\pi_{i,t}$, is the value of the holdings of risky asset i in the asset portfolio at time t. An admissible trading strategy, π, must be progressively measurable with respect to $\{\mathcal{F}_t\}$, must prevent portfolio value from falling below zero, and must satisfy $\int_0^T \|\pi_t\|^2\, dt < \infty$, a.s. Under an admissible trading strategy π, the value of the asset portfolio evolves according to

$$dX_t = (r_t X_t + \pi_t' \mu_t)\, dt + \pi_t' \sigma_t\, dW_t. \tag{4}$$

D. Benchmark

The option strike price or benchmark payoff, B_T, can be either fixed or stochastic. In general, B_T is a strictly positive, $\mathcal{F}_T$-measurable random variable that satisfies

$$\mathrm{E}[\zeta_T B_T] < \infty, \tag{5}$$

where ζ_t is the "state price density" or "stochastic discount factor" defined by

$$\zeta_t \equiv e^{-\int_0^t (r_u + \|\theta_u\|^2/2)\, du - \int_0^t \theta_u'\, dW_u}, \tag{6}$$

and

$$\theta_t \equiv \sigma_t^{-1} \mu_t. \tag{7}$$

The most typical benchmark for hedge fund managers is a constant, such as $B_T = X_0 e^{yT}$, where y is zero or a Treasury yield (see Brown et al. (1999)). Other possible benchmarks might be the payoffs of index portfolios.

II. The Manager's Investment Problem

The manager's dynamic problem is to choose an admissible trading strategy π to maximize his expected utility of terminal wealth:

$$\max_{\pi} \mathrm{E}[U(\alpha(X_T - B_T)^+ + K)]$$

subject to

$$dX_t = (r_t X_t + \pi_t' \mu_t)\, dt + \pi_t' \sigma_t \, dW_t$$

and

$$X_t \geq 0 \;\; \forall t \in [0, T]. \tag{8}$$

The manager cannot synthetically sell the option through trading in his personal account. If he could, he would invest the sale proceeds in his optimal portfolio. His objective would then be to maximize the value of the option by maximizing volatility, and the problem would have no solution.

The result of the martingale pricing theory is that the set of random terminal payoffs that can be generated by feasible trading strategies corresponds to the set of nonnegative $\mathcal{F}_T$-measurable random payoffs X_T that satisfy the budget constraint $E\zeta_T X_T \leq X_0$. Therefore, the dynamic problem (equation (8)) of choosing an optimal trading strategy π is equivalent to the static problem of choosing an optimal payoff X_T:[2]

$$\max_{X_T} \mathrm{E}[U(\alpha (X_T - B_T)^+ + K)]$$

subject to

$$\mathrm{E}[\zeta_T X_T] \leq X_0 \tag{9}$$

and

$$X_T \geq 0.$$

In the solution to the static problem above, the random variable ζ_T in the budget constraint plays an important role. The random variable $\zeta_T(\omega)$ has an economic interpretation as the price of one unit of payoff to be received in state ω at time T, relative to its probability of occurrence. The variable ζ_T is also the reciprocal of the terminal value of the mean-variance efficient (MVE) portfolio process

$$M_t \equiv 1/\zeta_t \tag{10}$$

that is generated by the trading strategy $\pi_t = M_t \Sigma_t^{-1} \mu_t$, starting from an initial value of one.

[2] See, for example, Cox and Ross (1976), Harrison and Kreps (1979), Karatzas, Lehoczky, and Shreve (1987), and Cox and Huang (1989) for the development of these methods and their application to optimal portfolio choice.

III. Review of the Standard Investment Problem

The optimal policy for the standard investment problem of maximizing expected utility of terminal wealth is characterized by Merton (1969, 1971) using dynamic programming and by Karatzas et al. (1987) and Cox and Huang (1989) using martingale methods. In the standard problem, the investor solves

$$\max_{X_T} \mathrm{E}[U(X_T)]$$

subject to

$$\mathrm{E}[\zeta_T X_T] \leq X_0 \tag{11}$$

and

$$X_T \geq 0.$$

The first-order conditions require that the assignment of different payoffs to different states makes the manager's marginal utility proportional to the level of the state price ζ_T whenever that leads to a nonnegative payoff, and further that the budget constraint is satisfied with equality. These conditions uniquely determine the optimal random terminal payoff:

$$X_T^* = I(\lambda^* \zeta_T)^+, \tag{12}$$

where $I = U'^{-1}$ and λ^* solves $\mathrm{E}[\zeta_T I(\lambda \zeta_T)^+] = X_0$. For example, if $U(W) = \log W$, then $I(y) = 1/y$, and $X_T^* = X_0/\zeta_T = X_0 M_T$.

The investor's optimal wealth or portfolio value is the stochastic process

$$X_t^* = \mathrm{E}[(\zeta_T/\zeta_t) X_T^* | \mathcal{F}_t]. \tag{13}$$

The optimal trading strategy, which generates the wealth process X^* and leads to the terminal payoff X_T^*, exists, but in general we do not obtain an explicit expression for it.

In some special cases, the martingale method does deliver an explicit expression for the trading strategy. One case is that in which the investor has log utility. Then $X_t^* = X_0/\zeta_t = X_0 M_t$. A comparison of the stochastic differential equation for $X_0 M_t$ resulting from Itô's lemma with the wealth evolution equation (equation (4)) shows that the optimal portfolio weights at each time t are $\pi_t^*/X_t^* = \Sigma_t^{-1} \mu_t$.

Another special case is that in which the security prices are geometric Brownian motions; that is, the market coefficients r, μ, and σ are constant. If we specify a particular functional form for the utility function U, then we may be able to compute intermediate portfolio value X_t^* explicitly from equation (13). In that case, X_t^* is equal to $x^*(t, \zeta_t)$ for some real-valued function

x^*, because ζ is a Markov process. If the function x^* is sufficiently smooth, then Itô's lemma yields a diffusion coefficient for dx^* that we can identify with $\pi_t'\sigma$ in the wealth evolution equation (equation (4)) to obtain the following equation for the optimal trading strategy:

$$\pi_t^* = -\zeta_t x_\zeta^*(t,\zeta_t)\Sigma^{-1}\mu, \tag{14}$$

where x_ζ^* is the partial derivative of x^* with respect to its second argument.

IV. The Optimal Payoff with the Option

With the option in the investment problem (equation (9)), the manager's objective function $U(\alpha(X_T - B_T)^+ + K)$ is not concave in the variable X_T. First-order conditions dictate that when $X_T > B_T$, the manager's marginal utility should be proportional to the state price ζ_T and otherwise $X_T = 0$. In addition, the budget constraint should hold with equality. However, these conditions do not uniquely determine the optimal policy. In particular, they do not specify the states at time T in which the option should be in the money.

To identify the states in which the option should be in the money and thus solve problem (9), I concavify the objective function. The concavification of a function u, if it exists, is the smallest concave function that dominates u (see Aumann and Perles (1965) for a formal definition). I solve problem (9) with the concavified objective function using standard methods. In the process I verify that the policy that is optimal for the concavified objective function is also optimal for the true objective function because it never takes on values where the two functions disagree.

Define $u : \mathcal{R} \times (0,\infty) \to \mathcal{R}$ by

$$u(x,b) = \begin{cases} U(\alpha(x-b)^+ + K) & \text{for } x \geq 0, \\ -\infty & \text{otherwise.} \end{cases} \tag{15}$$

In terms of u, the manager's problem is

$$\max_{X_T} \mathrm{E}[u(X_T,B_T)] \quad \text{subject to } \mathrm{E}[\zeta_T X_T] \leq X_0. \tag{16}$$

The dotted line in Figure 1 plots u as a function of x. The objective function u is not concave in the choice variable x. However, for each b, $u(\cdot,b)$ has a concavification $\tilde{u}(\cdot,b)$, illustrated by the dashed line in Figure 1. The concavified objective function replaces part of the original function with a chord between $x = 0$ and another point, $x = \hat{x} > b$, at which the slope of the chord equals the slope of u at $\hat{x}$, so that the resulting function is concave. Lemma 1 establishes that for each value of $b > 0$, such a point $\hat{x}(b)$ exists.

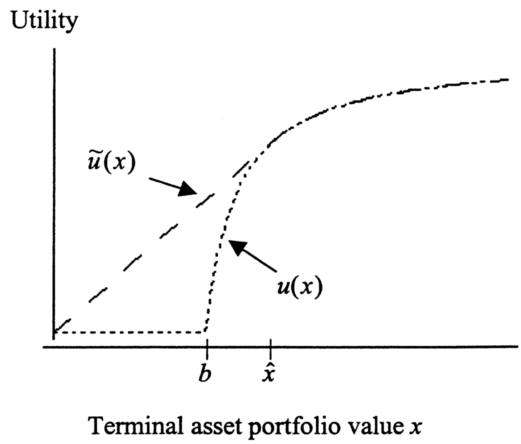

Figure 1. Manager's original and concavified objective functions. The dotted line represents the manager's objective function $u(x) = U(\alpha(x - b)^+ + K)$, where U is the manager's concave utility function and α, B_T, and K are positive constants. The dashed line represents the concavification of $u(x)$, $\tilde{u}(x)$.

LEMMA 1: *Let $u'(x,b) = \partial u(x,b)/\partial x$, for $x > b$. For every $b > 0$, there exists a unique point $\hat{x}(b) > b$ such that*

$$\frac{u(\hat{x}(b),b) - u(0,b)}{\hat{x}(b)} = u'(\hat{x}(b),b). \tag{17}$$

It follows that $\tilde{u}: \mathcal{R} \times (0,\infty) \to \mathcal{R}$ defined by

$$\tilde{u}(x,b) = \begin{cases} -\infty & \text{for } x < 0, \\ u(0) + u'(\hat{x}(b),b)x & \text{for } 0 \le x \le \hat{x}(b), \\ u(x,b) & \text{for } x > \hat{x}(b) \end{cases} \tag{18}$$

is concave in x. Furthermore, $\tilde{u}(x,b) \ge u(x,b)$ for all $(x,b) \in \mathcal{R} \times (0,\infty)$, and $\tilde{u}(x,b) = u(x,b)$ for $x = 0$ and for all $x \ge \hat{x}(b)$.

It turns out that the optimal random payoff X_T for the manager's problem never takes on values between zero and $\hat{x}(B_T)$ where the true and the concavified objective functions differ. That the manager would never want $X_T \in (0, B_T]$ with positive probability is fairly clear given his budget constraint and objective function in problem (9). Such an event would use up part of the manager's budget without adding utility. Less obvious is that the manager would never want $X_T \in (0, \hat{x}(B_T))$ with positive probability. To gain intuition for this, note that because the chord between these points lies above the manager's true objective function, the average utility of the endpoint payoffs, zero and $\hat{x}(B_T)$, exceeds the utility of the average of those endpoints. Thus, a payoff that takes on values in between those endpoints on a set of states with positive probability could be dominated by a payoff that instead takes on the value zero on part of that set and $\hat{x}(B_T)$ on the other part.

Solving the problem with the concave objective function $\tilde{u}$ is similar to solving the standard investment problem, with $\tilde{u}'$ appropriately defined. The solution to the concavified problem is also optimal for the original problem (equation (9)). The theorem below states this solution formally.

THEOREM 1: *Let*

$$h(y,b) = (I(y/\alpha) - K)/\alpha + b \quad \text{for all } y > 0, b > 0. \tag{19}$$

Assume that

$$\mathcal{X}(\lambda) \equiv \mathrm{E}[\zeta_T h(\lambda\zeta_T, B_T) 1_{\{h(\lambda\zeta_T, B_T) > \hat{x}(B_T)\}}] < \infty \quad \text{for all } \lambda > 0. \tag{20}$$

Then there exists a unique $\lambda^* > 0$ *such that* $\mathcal{X}(\lambda^*) = X_0$ *and the unique optimal payoff for the manager with the option is*

$$X_T^* = h(\lambda^*\zeta_T, B_T) 1_{\{h(\lambda^*\zeta_T, B_T) > \hat{x}(B_T)\}}, \tag{21}$$

where $1_A = 1$ *whenever* A *occurs and* $1_A = 0$ *otherwise.*

The optimal payoff has an all-or-nothing quality. Either the option is as far out of the money as possible, or it is in the money by at least $\hat{x}(B_T) - B_T$. It does not pay for the manager to be just marginally in the money, because he must expend substantial resources to bring asset value into the money at all.

When the option is in the money, the asset value sets the manager's marginal utility proportional to the state price ζ_T. The states in which the option is in the money are $\{\omega \in \Omega : \zeta_T(\omega) < u'(\hat{x}(B_T), B_T)/\lambda^*\}$. It is not hard to show that $u'(\hat{x}(B_T), B_T)$ is decreasing in B_T. Thus, the in-the-money states correspond to those in which the values of both the state price and the benchmark are low. In other words, the manager is in the money when payoffs are cheap and the benchmark is easy to beat.

V. Special Benchmarks

To provide some concrete examples, this section illustrates the optimal payoff for the manager's problem with two specific benchmark payoffs. These two benchmark payoffs serve in the examples of optimal trading strategies in the next section.

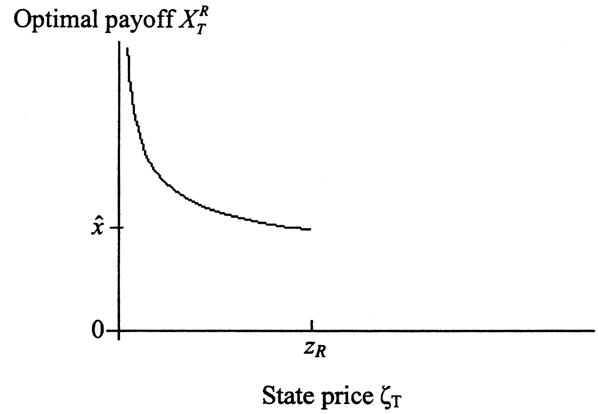

Figure 2. Optimal random payoff. The random variable X_T^R maximizes $E[U(\alpha(X_T - B_T)^+ + K)]$ subject to the constraints that $E[\zeta_T X_T] \le X_0$, $X_T \ge 0$, and X_T is measurable with respect to the filtration generated by asset prices up to time T. U is the manager's concave utility function, α, B_T, and K are positive constants, and ζ_T is the state price.

First, if the manager is measured against a constant or riskless benchmark, the manager's optimal terminal fund value is

$$X_T^R = [(I(\lambda_R \zeta_T/\alpha) - K)/\alpha + B_T]1_{\{\zeta_T < z_R\}}, \tag{22}$$

where λ_R solves $E\zeta_T i(\lambda\zeta_T, B_T) = X_0$ and $z_R = \alpha U'(\alpha(\hat{x}(B_T) - B_T) + K)/\lambda_R$. In particular, the set of states in which the manager's option is in the money is simply the set of states in which the relative state price falls below a certain critical value. A plot of the optimal terminal wealth X_T^R as a function of the state price density ζ_T appears in Figure 2. Optimal terminal wealth X_T^R is greater than $\hat{x}$ and decreasing in ζ_T until ζ_T hits z_R. Then X_T^R jumps from $\hat{x}$ down to zero.

Another benchmark payoff of potential interest is $B_T = B_0 M_T$, where B_0 is a constant and $M_T = 1/\zeta_T$, the terminal value of the MVE portfolio described in equation (10). With this benchmark, the manager's optimal terminal fund value is still a simple function of the state price ζ_T. In particular, the in-the-money states for the manager again correspond to the set of states in which the state price falls below a critical value, z_M. The corollary below states this formally.

COROLLARY 1: *The optimal asset payoff with benchmark* $B_T \equiv B_0/\zeta_T$ *is*

$$X_T^M = [(I(\lambda_M \zeta_T/\alpha) - K)/\alpha + B_0/\zeta_T]1_{\{\zeta_T < z_M\}}, \tag{23}$$

where λ_M *solves* $\mathrm{E}[\zeta_T h(\lambda\zeta_T, B_0/\zeta_T) 1_{\{h(\lambda^*\zeta_T, B_0/\zeta_T) > \hat{x}(B_0/\zeta_T)\}}] = X_0$ *and* z_M *is the unique zero of*

$$g(\zeta_T) = u'(\hat{x}(B_0/\zeta_T), B_0/\zeta_T) - \lambda_M \zeta_T. \tag{24}$$

VI. Optimal Trading Strategies

This section presents the manager's optimal dynamic trading strategy in the special case that the coefficients of the price processes r, μ, and σ are constant, the benchmark payoff is either a constant or else equal to the payoff of the MVE portfolio in equation (10), and the manager has a DARA utility function as defined in equation (2). The derivation of the optimal trading strategy uses the standard procedure described in equations (13) and (14) of Section III. Note that the optimal solution to the manager's problem with a DARA utility function and outside wealth K is the same as the solution with a CRRA utility function and outside wealth equal to $k = K - w$.

A. Riskless Benchmark

For this section, assume that $B_T = B_0 e^{rT}$, where B_0 is a constant. Then the portfolio value is the process

$$X_t^R = e^{-r(T-t)}\left[\hat{x}N(d_{1,t}) + (\hat{x} - B_T + k/\alpha)\left(N(d_{2,t})\frac{N'(d_{1,t})}{N'(d_{2,t})} - N(d_{1,t})\right)\right], \tag{25}$$

and the manager's optimal trading strategy is

$$\pi_t^R = \left\{\frac{X_t^R}{1-\gamma} + e^{-r(T-t)}\left[\frac{\hat{x}N'(d_{1,t})}{\|\theta\|\sqrt{T-t}} - \frac{B_T - k/\alpha}{1-\gamma}N(d_{1,t})\right]\right\}\Sigma^{-1}\mu, \tag{26}$$

where $\hat{x} = \hat{x}(B_T)$, $d_{1,t} = [\ln(z_R/\zeta_t) + (r - \|\theta\|^2/2)(T-t)]/\|\theta\|\sqrt{T-t}$, $d_{2,t} = d_{1,t} + \|\theta\|\sqrt{T-t}/(1-\gamma)$, and N is the standard cumulative normal distribution function. These formulas lead to a number of results.

Proposition 1: *With the riskless benchmark and DARA utility,*

(i) *as* $\zeta_t \to 0$,

$$X_t^R \to +\infty \quad \textit{and} \quad \frac{\pi_t^R}{X_t^R} \to \frac{\Sigma^{-1}\mu}{1-\gamma},$$

(ii) *as* $\zeta_t \to +\infty$,

$$X_t^R \to 0,\ \pi_t^R \to 0, \quad and \quad \left\| \frac{\pi_t^R}{X_t^R} \right\| \to \infty,$$

(iii) *as* $t \to T$,

$$\begin{aligned} &\frac{\pi_t^R}{X_t^R} \to \frac{X_T^R - B_T + k/\alpha}{X_T^R} \frac{\Sigma^{-1}\mu}{1-\gamma}, \quad if\ \zeta_T < z_R, \\ &\pi_t^R \to 0 \quad and \quad \left\| \frac{\pi_t^R}{X_t^R} \right\| \to \infty, \quad if\ \zeta_T > z_R. \end{aligned} \tag{27}$$

Merton (1969, 1971) shows that in the standard investment problem (equation (11)), when security prices are geometric Brownian motions, the CRRA investor holds risky assets in the constant proportions $\pi_t^*/X_t^* = \Sigma^{-1}\mu/(1-\gamma)$, which implies that his portfolio has constant volatility. Part (i) of Proposition 1 says that when underlying asset value is high, the manager with the option follows the same constant volatility trading strategy he would follow if he were solving the standard problem, or if he were paid a linear share of profits. By contrast, part (ii) of the proposition indicates that when the asset portfolio is performing poorly, the value of the risky asset holdings goes to zero to meet the solvency constraint, but that value goes to zero more slowly than the total portfolio value, so that the proportional risky asset holdings, and thus portfolio volatility, converge to infinity as bankruptcy approaches. To illustrate, Figure 3 plots the optimal proportional holdings of risky assets as a function of total asset value for a CRRA manager with a riskless benchmark.

Part (iii) of Proposition 1 examines the trading strategy as the evaluation date draws near. In states in which the manager finishes in the money, $\zeta_T < z_R$, the proportional risky asset holdings converge to $[(X_T^R - B_T + k/\alpha)/X_T^R][\Sigma^{-1}\mu/(1-\gamma)]$. The intuition for this is as follows. The CRRA investor seeks proportional holdings $\Sigma^{-1}\mu/(1-\gamma)$. When the option is very likely to be in the money, the manager's personal share of the value of the risky assets that the managed portfolio holds is effectively $\alpha\pi^R$. The manager's wealth is effectively $\alpha(X_T^R - B_T) + k$. Thus, his personal risky asset holdings, in proportion to his wealth, are effectively $\alpha\pi^R/[\alpha(X_T^R - B_T) + k] = \Sigma^{-1}\mu/(1-\gamma)$, the standard CRRA proportions.

Although the optimal trading strategy has the potential for unboundedly high volatility, equation (27) in part (iii) of Proposition 1 implies that in some states the volatility of the managed assets can actually be less than the Merton constant volatility that a CRRA investor solving the standard investment problem would choose. In particular, this will be the case if

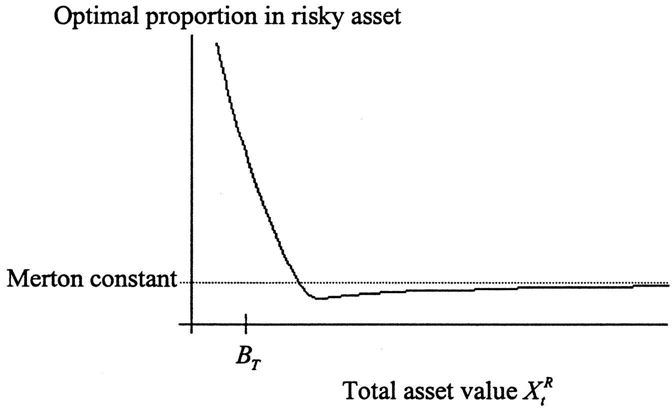

Figure 3. Optimal trading strategy for a CRRA manager. Optimal proportion of portfolio value invested in the risky asset as a function of portfolio value, X_t^R, one year prior to the evaluation date T. Terminal portfolio value X_T^R maximizes $\mathrm{E}[U(\alpha(X_T - B_T)^+ + K)]$ subject to the constraints that $\mathrm{E}[\zeta_T X_T] \le X_0$, $X_T \ge 0$, and X_T is measurable with respect to the filtration generated by asset prices up to time T. $\alpha = 0.15$, $B_T = 1$, $K = 0.03$, and the utility function U is CRRA with coefficient $1 - \gamma = 2$. Intermediate fund value is $X_t^R = \mathrm{E}_t\{(\zeta_T/\zeta_t)X_T^R\}$ where the state price density process is $\zeta_t = e^{-rt-\theta W_t-\theta^2 t/2}$ with $r = 0$, $\theta = 0.4$, and W_t Brownian motion at time t. The Sharpe ratio θ on the risky asset is μ/σ where the risky asset's excess expected return is $\mu = 0.08$ and its volatility is $\sigma = 0.2$. The Merton constant is $\mu/\sigma^2(1-\gamma)$, the optimal proportion for a CRRA investor solving the standard terminal wealth problem.

$-B_T + k/\alpha < 0$, t is near T and the option is in the money. This is visible in Figure 3. The reason is that the leverage inherent in the option magnifies the manager's exposure to the asset volatility. If the option is a large component of his compensation, he reduces asset volatility to offset the option's leverage effect.

In states in which the manager finishes out of the money, $\zeta_T > z_R$, asset value goes to zero. Just as in the bankruptcy states examined in part (ii) of Proposition 1, although the value of risky asset holdings goes to zero to meet the solvency constraint, portfolio volatility goes to infinity. Finally, note that when the manager is near the money (ζ_t near z_R) as the evaluation date approaches, small changes in the value of the mean-variance efficient portfolio lead to large trades as the manager alternates between the desire to get in the money and the need to remain solvent.

PROPOSITION 2: *With the riskless benchmark, CRRA utility, and outside wealth* $K = K_T \equiv K_0 e^{rT}$,

$$\lim_{T\to\infty} \frac{\pi_0^R}{X_0^R} = \frac{\Sigma^{-1}\mu}{1-\gamma}. \tag{28}$$

Under the assumptions of the proposition, the solution is essentially invariant to the interest rate. In particular, $e^{-rt}X_t^R$ and $e^{-rt}\pi_t^R$ are invariant to r. It follows that increasing the evaluation period T is essentially equivalent to increasing the Sharpe ratio $\|\theta\|$ and thus improving the manager's opportunity set. The result suggests that improving the manager's opportunity set has the same effect as putting him deeper in the money—he begins to act as if he is trading for his own account.

Finally, I close this section with a result that holds for all HARA utility functions described in equation (2).

PROPOSITION 3: *With the riskless benchmark and DARA or CARA utility, increasing the number of options, α, holding asset value constant, causes the manager to reduce asset volatility.*

Lemma 2 in the Appendix shows that as α increases, the minimal nonzero payoff, $\hat{x}$, decreases, making the optimal payoff smoother. As for the impact of an increase in α on the manager's choice of asset volatility, the intuition is as follows. The manager's personal portfolio is the package of options and cash with payoff $\alpha(X_T - B_T)^+ + K$. In the CRRA case, the manager is trying to keep the volatility of his personal portfolio fixed at the Merton constant. The volatility of his personal portfolio is, by Itô's lemma, equal to (the proportion of his personal portfolio value in the option) $\times$ (the option elasticity) $\times$ (the volatility of the underlying assets). Increasing α increases the first factor, causing the manager to decrease the third factor. Finally, consider a CARA manager. He essentially targets a certain risky asset value in his personal portfolio. Increasing α increases his exposure to the managed portfolio's risky assets. He offsets this by decreasing the managed portfolio's volatility.[3]

B. MVE Portfolio as Benchmark

Now set $B_t \equiv B_0/\zeta_t$. Let $\hat{b} = B_0/z_M$ and let $\hat{x} = \hat{x}(\hat{b})$. Let $\pi^B = B_t\Sigma^{-1}\mu$, the trading strategy that generates the benchmark portfolio. Then the optimal portfolio process is

$$X_t^M = B_t N(d_{5,t}) + e^{-r(T-t)}\left[(\hat{x} - \hat{b} + k/\alpha)N(d_{4,t})\frac{N'(d_{3,t})}{N'(d_{4,t})} - (k/\alpha)N(d_{3,t})\right], \tag{29}$$

[3] In a study of hedge fund performance, Ackermann, McEnally, and Ravenscraft (1998) regress fund volatility on a variety of fund characteristics and find no relation between fund volatility and the incentive fee percentage. However, their regression does not control for cross-sectional variation in other variables that would affect the volatility choice, such as the size of the fund and the extent to which the incentive fee is in the money.

and the manager's optimal trading strategy is

$$\pi_t^M = \pi_t^B N(d_{5,t}) + e^{-r(T-t)}\left[\frac{\hat{x}N'(d_{3,t})}{\|\theta\|\sqrt{T-t}} + \frac{(\hat{x}-\hat{b}+k/\alpha)}{1-\gamma}N(d_{4,t})\frac{N'(d_{3,t})}{N'(d_{4,t})}\right]\Sigma^{-1}\mu, \tag{30}$$

where $d_{3,t} = [\ln(z_M/\zeta_t) + (r - \|\theta\|^2/2)(T-t)]/\|\theta\|\sqrt{T-t}$, $d_{4,t} = d_{3,t} + \|\theta\|\sqrt{T-t}/(1-\gamma)$, and $d_{5,t} = d_{3,t} + \|\theta\|\sqrt{T-t}$. Note that d_3 and d_4 are just d_1 and d_2 with the critical state price z_R replaced by z_M. For the purpose of comparing the trading strategies with the riskless and MVE benchmarks, I rewrite the riskless benchmark trading strategy in equation (26) as

$$\pi_t^R = e^{-r(T-t)}\left[\frac{\hat{x}N'(d_{1,t})}{\|\theta\|\sqrt{T-t}} + \frac{(\hat{x}-B_T+k/\alpha)}{1-\gamma}N(d_{2,t})\frac{N'(d_{1,t})}{N'(d_{2,t})}\right]\Sigma^{-1}\mu. \tag{31}$$

Comparing the two trading strategies in equations (30) and (31) shows that the trading strategy with the MVE benchmark consists of a component that tracks the benchmark, weighted by the factor $N(d_{5,t})$, and a component that behaves like the riskless benchmark trading strategy. As the portfolio looks more and more likely to finish in the money, $N(d_{5,t})$ approaches one, so the manager essentially undoes the effect of the benchmark and then invests in the optimal portfolio for a riskless benchmark, a result similar to that of Admati and Pfleiderer (1997).

In terms of the proportional holdings of the risky assets, the component that dominates depends on whether the manager's personal volatility target exceeds the volatility of the benchmark portfolio. Roughly speaking, if the utility function is more risk averse than log utility, the component tracking the MVE benchmark dominates. Otherwise, the component that behaves like the riskless benchmark strategy dominates. Part (i) of the proposition below states this idea precisely.

Proposition 4: *With the MVE benchmark and DARA utility,*

(i) *as* $\zeta_t \to 0$,

$$X_t^M \to +\infty$$

and

$$\frac{\pi_t^M}{X_t^M} \to \frac{\Sigma^{-1}\mu}{1-\gamma}, \quad \text{if } 1-\gamma \le 1,$$

$$\frac{\pi_t^M}{X_t^M} \to \Sigma^{-1}\mu, \quad \text{if } 1-\gamma > 1,$$

(ii) *as* $\zeta_t \to +\infty$,

$$X_t^M \to 0,\ \pi_t^M \to 0,\ and \left\| \frac{\pi_t^M}{X_t^M} \right\| \to \infty,$$

(iii) *as* $t \to T$,

$$\begin{aligned} \frac{\pi_t^M}{X_t^M} &\to \left(\frac{B_T}{X_T} + \frac{X_T^M - B_T + k/\alpha}{X_T^M(1-\gamma)} \right) \Sigma^{-1}\mu, \quad && if\ \zeta_T < z_M, \\ \pi_t^M &\to 0\ and \left\| \frac{\pi_t^M}{X_t^M} \right\| \to \infty, && if\ \zeta_T > z_M. \end{aligned} \tag{32}$$

To understand the limiting portfolio holdings as $t \to T$ when the manager is in the money in equation (32) above, think of the manager's personal risky asset holdings as those that are necessary to generate his personal payoff, $\alpha(X^M - B)$. The trading strategy that generates that payoff is $\alpha(\pi^M - \pi^B)$. In addition, the manager's personal wealth is effectively $\alpha(X^M - B) + k$. Thus, his proportional holdings are effectively $[\alpha(\pi^M - B)/(\alpha(X^M - B) + k)]\Sigma^{-1}\mu = \Sigma^{-1}\mu/(1-\gamma)$, the standard CRRA proportions.

VII. Conclusion

This paper provides a rigorous description of the optimal dynamic investment policy for a risk averse fund manager compensated with a call option on the assets he controls. The setting is sufficiently general that the solution for the optimal payoff could serve in an equilibrium model in which option-compensated managers control some of the assets in the economy. The solution technique, concavifying the objective function, applies to other problems in which option payoffs appear in the objective function.

In general, the effects of option compensation on the manager's appetite for risk are more complex than simple intuition about option pricing might suggest. The convexity of the option makes the manager seek payoffs that are likely to be "away from the money" and can lead to dramatic increases in volatility. Yet examples of the optimal trading strategy for DARA utility show that the option does not simply cause the manager to increase asset volatility. Explicit expressions show how the manager dynamically adjusts volatility as asset value changes. As asset value grows large, the manager moderates portfolio risk.

Somewhat surprisingly, the manager with the option can in some situations set the volatility of the asset portfolio below the level he would choose if he were trading his own account. Furthermore, giving the manager more options makes him seek less risk. However, options that are deep out of the money seem to provide incentives for excessive risk taking. This may be one reason why firms sometimes reset the strike prices of compensatory options after poor stock price performance has put them out of the money.

Appendix

Proof of Theorem 1: First, although the concavification $\tilde{u}$ is not differentiable at $x = 0$, we can define a set-valued function $\tilde{u}'$ on $[0,\infty) \times (0,\infty)$ by

$$\tilde{u}'(x,b) = \begin{cases} [u'(\hat{x}(b),b),\infty) & \text{for } x = 0, \\ \{u'(\hat{x}(b),b)\} & \text{for } 0 < x \leq \hat{x}(b), \\ \{u'(x,b)\} & \text{for } x > \hat{x}(b). \end{cases} \tag{A1}$$

The function $\tilde{u}'(x,b)$ is essentially the derivative of $\tilde{u}$ with respect to x.[4] In particular, for every $y \in \mathcal{R}$ and $x \geq 0$, and for every $m \in \tilde{u}'(x,b)$,

$$\tilde{u}(y,b) - \tilde{u}(x,b) \leq m(y - x). \tag{A2}$$

Furthermore, strict inequality holds whenever $x > \hat{x}(b)$ and $y \neq x$.

Second, we can define an inverse function for $\tilde{u}'(\cdot,b)$, $i:(0,\infty) \times (0,\infty) \to [0,\infty)$, by

$$i(y,b) = [(I(y/\alpha) - K)/\alpha + b]1_{\{y<u'(\hat{x}(b),b)\}}. \tag{A3}$$

The function i is the inverse of $\tilde{u}$ in the sense that

$$y \in \tilde{u}'(i(y,b),b) \quad \text{for all } b > 0. \tag{A4}$$

Third, under assumption (20) in the statement of the theorem, the function $\mathcal{X}(\lambda) = \mathrm{E}[\zeta_T i(\lambda\zeta_T, B_T)]$, for $\lambda > 0$, is continuous and strictly decreasing. Furthermore, $\mathcal{X}(\lambda) \to \infty$ as $\lambda \to 0$, and $\mathcal{X}(\lambda) \to 0$ as $\lambda \to \infty$. Therefore, there exists a unique $\lambda^* > 0$ such that $\mathcal{X}(\lambda^*) = X_0$.

Finally, note that $X_T^* = i(\lambda^*\zeta_T, B_T)$. Let Y be any other feasible payoff that is not almost surely equal to X_T^*. Then Y provides lower expected utility than X_T^*:

$$\mathrm{E}\{u(Y,B_T) - u(X_T^*,B_T)\} = \mathrm{E}\{u(Y,B_T) - \tilde{u}(X_T^*,B_T)\} \tag{A5}$$

$$\leq \mathrm{E}\{\tilde{u}(Y,B_T) - \tilde{u}(X_T^*,B_T)\} \tag{A6}$$

$$< \mathrm{E}\{\lambda^*\zeta_T(Y - X_T^*)\} \tag{A7}$$

$$\leq \lambda^*(\mathrm{E}[\zeta_T Y] - X_0) \leq 0. \tag{A8}$$

[4] Formally, for each b, $\tilde{u}'(\cdot,b)$ is the *subdifferential* of $\tilde{u}(\cdot,b)$. See Rockafellar (1970).

Equation (A5) follows from the fact that X_T^* never takes on values where u and $\tilde{u}$ disagree. Equation (A7) follows from equations (A2) and (A4). Q.E.D.

LEMMA 2: *For the HARA utility functions defined in equation (2), the critical point $\hat{x}$ in the concavified objective function $\tilde{u}$ decreases as α increases.*

Proof: By the definition in equation (17), $\hat{x}$ satisfies

$$U(\alpha(\hat{x}-b)+K) - U(K) = \alpha\hat{x}U'(\alpha(\hat{x}-b)+K), \tag{A9}$$

and

$$\frac{d\hat{x}}{d\alpha} = \frac{bU'(\alpha(\hat{x}-b)+K)}{-\alpha^2\hat{x}U''(\alpha(\hat{x}-b)+K)} - \frac{\hat{x}-b}{\alpha}, \tag{A10}$$

for each value of the benchmark b. By concavity,

$$U(\alpha(\hat{x}-b)+K) - U(K) < U'(K)\alpha(\hat{x}-b). \tag{A11}$$

Equations (A9) and (A11) imply

$$\frac{U'(K)}{U'(\alpha(\hat{x}-b)+K)} > \frac{\hat{x}}{\hat{x}-b}. \tag{A12}$$

It suffices to demonstrate the result for just the CRRA and CARA utility functions. I give the proof for the CRRA case only. Equation (A10) yields

$$\frac{d\hat{x}}{d\alpha} = \frac{bK - \alpha(\hat{x}-b)((1-\gamma)\hat{x}-b)}{\alpha^2(1-\gamma)\hat{x}}, \tag{A13}$$

and equation (A9) yields

$$\frac{\alpha((1-\gamma)\hat{x}-b)+K}{K} = \frac{U'(K)}{U'(\alpha(\hat{x}-b)+K)}. \tag{A14}$$

Equations (A12) and (A14) together imply that the numerator on the right-hand side of equation (A13) is less than zero. Q.E.D.

LEMMA 3: *Let $G(x) = N(x)/N'(x)$, where N is the cumulative normal distribution.*

(i) $G'(x) = 1 + xG(x) > 0$,
(ii) $G''(x) > 0$,
(iii) $(x_2 - x_1)G(x_2)G(x_1) - G(x_2) + G(x_1) > 0$ *for* $x_2 > x_1$,
(iv) $G(x)^2 - G'(x) \leq 0$.

Proof of Proposition 3: It suffices to show that $d\|\pi_0^R\|/d\alpha < 0$ for both the CRRA and CARA utility functions. In each case, $\pi_t^R = \rho(t,\zeta_t)\Sigma^{-1}\mu$, for a real-valued function ρ. It suffices to show that $\rho_0 \equiv \rho(0,1)$ is decreasing in α. I give the proof for the CRRA case only. Write the initial wealth and portfolio holdings as

$$X_0 = e^{-rT}N'(d_1)[\hat{x}G(d_1) + (\hat{x} - B_T + K/\alpha)(G(d_2) - G(d_1))], \tag{A15}$$

$$\rho_0 = \frac{e^{-rT}N'(d_1)}{\|\theta\|\sqrt{T}}[\hat{x} + (\hat{x} - B_T + K/\alpha)(d_2 - d_1)G(d_2)], \tag{A16}$$

where $d_2 = d_1 + \|\theta\|\sqrt{T}/(1-\gamma)$ and $\hat{x} = \hat{x}(\alpha)$ but the dependence is suppressed for brevity. Equation (A15) implicitly defines d_1 as a function of α.

$$\frac{d(d_1)}{d\alpha} = \frac{-\dfrac{d\hat{x}}{d\alpha}G(d_2) + \dfrac{K}{\alpha^2}(G(d_2) - G(d_1))}{\hat{x} + (\hat{x} - B_T + K/\alpha)(d_2 - d_1)G(d_2).} \tag{A17}$$

It follows that

$$\begin{aligned}\frac{d\rho_0}{d\alpha} = {} & \frac{e^{-rT}N'(d_1)}{\|\theta\|\sqrt{T}[\hat{x} + (\hat{x} - B_T + K/\alpha)(d_2 - d_1)G(d_2)]} \\ & \times \left\{\hat{x}\left[\frac{d\hat{x}}{d\alpha}G'(d_2) - \frac{K}{\alpha^2}(G'(d_2) - G'(d_1))\right]\right. \\ & \quad - (\hat{x} - B_T + K/\alpha)\frac{K}{\alpha^2}(d_2 - d_1) \\ & \quad \left. \times [(d_2 - d_1)G(d_2)G(d_1) - G(d_2) + G(d_1)]\right\} < 0,\end{aligned} \tag{A18}$$

where the last inequality follows from Lemmas 2 and 3. Q.E.D.

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