Dynamic Corporate Finance

1. Overview of the literature

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3. Black-Cox model of a defaultable perpetuity

4. Leland model of optimal capital structure with taxes and bankruptcy costs

5. Acharya-Carpenter model of defaultable callables with stochastic interest rates

Selected Readings and References

Merton, R. C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance*, 29, 449-470.

Black, F., and J. C. Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance*, 31, 351-367.

Brennan, M. J., and E. S. Schwartz, 1977a, Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion, *Journal of Finance*, 32, 1699-1715.

Geske, R., 1977, The valuation of corporate liabilities as compound options, *Journal of Financial and Quantitative Economics*, 12, 541-552.

Fischer, E. O., R. Heinkel, and J. Zechner, 1989, Dynamic capital structure choice: Theory and tests, *Journal of Finance*, 44, 19-40.

Leland, H. E., 1994, Risky debt, bond covenants, and optimal capital structure, *Journal of Finance*, 49, 1213-1252.

Longstaff, Francis A., and Eduardo S. Schwartz, 1995, A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance*, 50, 789-819.

Jarrow, Robert A., and Stuart Turnbull, 1995, Pricing derivatives on financial securities subject to default risk, *Journal of Finance*, 50, 53-86.

Leland, H. E., and K. Toft, 1996, Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads, *Journal of Finance*, 51, 987-1019.

Anderson, Ronald W., and Suresh Sundaresan, 1996, Design and valuation of debt contracts, *Review of Financial Studies*, 9, 37-68.

Jarrow, Robert A., David Lando, and Stuart Turnbull, 1997, A Markov model for the term structure of credit spreads, *Review of Financial Studies* 10, 481-523.

Duffie, Darrell, and Kenneth Singleton, 1999, Modeling term structures of defaultable bonds, *Review of Financial Studies*, 12, 687-720.

Acharya, V. V., and J. N. Carpenter 2002, Corporate bond valuation and hedging with stochastic interest rates and endogenous bankruptcy, *Review of Financial Studies* 15, 1355-1383.

Bhamra, H., L. Kuehn, and I. Strebulaev, 2010, The levered equity risk premium and credit spreads: A unified framework, *Review of Financial Studies* 23, 645-703.

Chen, H., 2010, Macroeconomic conditions and the puzzles of credit spreads and capital structure, *Journal of Finance* 65, 2171-2212.

Bolton, P., H. Chen, and N. Wang, 2011, A unified theory of Tobin's q, corporate investment, financing and risk management, *Journal of Finance* 66, 1545-1578.

Gryglewicz, S., 2011, A theory of corporate financial decisions with liquidity and solvency concerns, *Journal of Financial Economics* 99, 365-384.

He, Z., and W. Xiong, 2012, Rollover risk and credit risk *Journal of Finance* 67, 391-430.

Demarzo, P., M. Fishman, Z. He, and N. Wang, 2012, Dynamic agency and the q theory of investment, *Journal of Finance* 67, 2295-2340.

Bolton, P., H. Chen, and N. Wang, 2013, Debt, taxes, and liquidity, NBER working paper.

Diamond, D., and Z. He, 2014. A theory of debt maturity: The long and short of debt overhang, *Journal of Finance* 69, 719-762.

He, Z., and K. Milbradt, 2014, Endogenous liquidity and defaultable bonds, *Econometrica* 82, 1443-1508.

Hugonnier, J., S. Malamud, and E. Morellec, 2015, Capital supply uncertainty, cash holdings and investment, *Review of Financial Studies* 28, 391-445.

Gomes, J., and L. Schmid, 2016, Equilibrium asset pricing with leverage and default, Wharton School Research Paper 92.

Mcquade, T., 2016, Stochastic volatility and asset pricing puzzles, Stanford University working paper.

Overview of the Literature

- Much of the theory treats interest rates as constant to focus on the problems of optimal or strategic behavior of competing corporate claimants.
  - Merton (1974) analyzes a risky zero-coupon bond and characterizes the optimal call policy for a callable coupon bond.
  - Brennan and Schwartz (1977) model callable convertible debt.
  - Black and Cox (1976) and Geske (1977) value coupon-paying debt when asset sales are restricted and solve for the equity holders' optimal default policy.
  - Fischer, Heinkel, and Zechner (1989a,b), Leland (1994), Leland and Toft (1996), Leland (1998), and Goldstein, Leland and Ju (2000) embed the optimal default and call policy in the problem of optimal capital structure.
  - Anderson and Sundaresan (1996), Huang (1997), Mella-Barral and Perraudin (1997), Acharya, Huang, Subrahmanyam, and Sundaram (1999), and Fan and Sundaresan (2000) introduce costly liquidations and treat bankruptcy as a bargaining game.

- Other models allow for stochastic interest rates and take a different approach to the treatment of bankruptcy.
  - Some impose exogenous bankruptcy triggers in the form of critical asset values or payout levels. These include the models of Brennan and Schwartz (1980), Kim, Ramaswamy, and Sundaresan (1993), Neilsen, Saá-Requejo, and Santa-Clara (1993), and Longstaff and Schwartz (1995), Briys and de Varenne (1997), and Collin-Dufresne and Goldstein (2001). Cooper and Mello (1991) and Abken (1993) model defaultable swaps assuming that equity holders can sell assets to make swap or bond payments. Shimko, et al. (1993) model a zero-coupon bond.
  - Other papers model default risk with a hazard rate or stochastic credit spread. See, for example, Ramaswamy and Sundaresan (1986), Jarrow, Lando, and Turnbull (1993), Madan and Unal (1993), Jarrow and Turnbull (1995), Duffie and Huang (1996), Duffie and Singleton (1999), and Das and Sundaram (1999).
- Acharya and Carpenter (2002) incorporate stochastic interest rates in the model of optimal default and call.
- More recently, Bhamra, Kuehn, and Strebulaev (2010) combine a corporate finance model of the firm with a consumption based asset pricing model to explain credit spreads and default policies in general equilibrium.

- Chen (2010) embeds a structural model of the firm into a consumption based asset pricing model to explain credit spreads.
- Bolton, Chen, and Wang (2011) study how firm investment policies are affected by the need to hold cash due to costly external financing, finds a wedge between marginal Q and average Q resulting from the friction of costly external financing.
- Gryglewicz (2011) characterizes the liquidity (cash) policy of a levered firm and shows how this interacts with default policy (credit risk).
- He and Xiong (2012) analyze how a fall in debt market liquidity affects debt rollover strategies and then credit risk of firms.
- Demarzo, Fishman, He and Wang (2012) study how firm investment policy is affected by an agency problem as in Demarzo, Sannikov (2006).
- Bolton, Chen and Wang (2013) aim to explain the underinvestment puzzle by introducing the friction of costly external financing and seeing how firm leverage policies are affected by the need to hold cash.
- Diamond and He (2014) show how debt maturity affects the debt overhang problem in the corporate investment decision.

- He and Milbradt (2014) build an explicit search model for OTC trading and show how secondary market liquidity and credit risk can affect and exacerbate each other.
- Hugonnier, Malamud, and Morellec (2015) study how a firm's choice in exercising a real option is affected by capital market frictions where the firm needs to search for outside investors. The firm will hold cash as a response and the model is about the interplay between the cash and investment policies.
- Gomes and Schmid (2016) build a general equilibrium model to understand how firm default and investment policies are affected by macroeconomic aggregates.
- Mcquade (2016) incorporates stochastic volatility into a standard structural model of the firm to try and explain the credit spread puzzle.

Defaultable Zero-Coupon Bonds (Black, Scholes, 1973, Merton 1973, 1974)

- Black and Scholes (1973) and Merton (1973) recognized that corporate equity could be viewed as a call option on the firm assets, while corporate debt could be viewed as nondefaultable debt minus a put on firm value.
- Letting V represent the value of firm assets, E equity, D debt, O equity's default option, and P nondefaultable debt with the same cash flows as the firm debt, then $E = V - P + O$ and $D = P - O$. The default option O is equity's option to exchange the firm for the promised debt cash flows. This is typically viewed as a put on the firm assets, in which light, equity is a call on the firm assets.
- If debt is a zero with face value K maturing at time T, then the payoffs to equity and debt are $E_T = (V_T - K)^+$ and $D_T = K - (K - V_T)^+ = K \wedge V_T$.
- If r and firm asset volatility σ are constant and firm payout is zero, then the values of equity and debt are given by the Black-Scholes formula with V replacing S:

$$E_0 = V_0 N(d_1) - e^{-rT} K N(d_2) \text{ and } D_0 = V_0 - E_0 = e^{-rT} K N(d_2) + V_0 N(-d_1) .$$

- This approximate structural model of debt is analyzed in Merton (1974) and is the basis of the KMV model used in practice to estimate default probabilities, "distance to default" and other measures of credit risk.

General Structural Model of Corporate Liabilities

In general, structural models of corporate liabilities take firm assets to be a diffusion

$$dV_t = (r_t V_t - a_t V_t)\, dt + \sigma_t V_t \, dB_t^* \ , \tag{1}$$

where a_t is the asset payout rate, covering both equity and debt: $a_t V_t = \delta_t E_t + C_t$. If bankruptcy occurs at time τ, the values of debt and equity can be represented as

$$D_t = \mathrm{E}_t^* \{ \int_t^\tau \frac{\beta_u}{\beta_t} C_t \, dt + \frac{\beta_\tau}{\beta_t} V_\tau \} \text{ and } E_t = V_t - D_t \ . \tag{2}$$

- Some models take debt to be a collection of bonds with different priority on the residual firm value, with separate valuations for senior and junior debt.
- Others incorporate taxes in the payout rate and model the value of debt tax shields when coupon expense is tax-deductible.
- Others incorporate bankruptcy costs or liquidation costs as a deadweight loss to firm value at the time of bankruptcy.
- The issue of whether firm assets can be treated as tradeable for the purpose of contingent claims pricing may be resolved if both nondefaultable bonds and equity are tradeable, or one can appeal to general market completeness.

Defaultable Perpetual Coupon Bonds (Black and Cox, 1976)

- Suppose equity holders control a firm that has issued a perpetuity/consol bond with continuous coupon c, and the equity holders service the debt with equity infusions unless they declare bankruptcy and surrender the firm to the debt holders. Suppose debt covenants prohibit dividend payments and risk shifting. Suppose r and firm volatility σ are constant. Firm value follows

$$dV = rV\,dt + \sigma V\,dB^* \ . \tag{3}$$

- The equity value-maximizing default policy will be to default iff firm value falls to a critical level $\bar{V}$. Under this policy, the value of the perpetuity is a function of firm value, $f(V)$ and in the continuation region, f satisfies the fundamental o.d.e.

$$\frac{1}{2}\sigma^2 V^2 f_{vv} + rVf_v - rf + c = 0 \ . \tag{4}$$

- The solution is of the form $f(V) = \frac{c}{r} + K_1 V + K_2 V^{-\gamma}$ where $\gamma = \frac{2r}{\sigma^2}$.
- The boundary condition $f(V) \to \frac{c}{r}$ as $V \to \infty$ implies $K_1 = 0$.

- The boundary condition $f(V) \to V$ as $V \to \bar{V}$ implies $K_2 = \bar{V}^{\gamma+1} - \frac{c}{r}\bar{V}^{\gamma}$.
- The smooth-pasting condition for optimization over $\bar{V}$ is $f_v(\bar{V}) = 1$, which implies $\bar{V} = \frac{\gamma}{\gamma+1}\frac{c}{r} = \frac{c}{r+\sigma^2/2}$, proportional to c and decreasing in r and σ^2.
- Therefore, the value of the perpetuity is

$$f(V) = \frac{c}{r} - [\frac{c}{r} - \bar{V}](V/\bar{V})^{-\gamma} = \frac{c}{r} - [(\frac{\gamma}{\gamma+1})^{\gamma} - (\frac{\gamma}{\gamma+1})^{\gamma+1}](\frac{c}{r})^{\gamma+1}V^{-\gamma},$$

which is the value of the nondefaultable perpetuity minus the value of equity's default option. The value of the equity is

$$e(V) = V - f(V) = V - \frac{c}{r} + [\frac{c}{r} - \bar{V}](V/\bar{V})^{-\gamma} \ . \tag{5}$$

- Using results on first passage times of Brownian motion, it can be shown that $(V/\bar{V})^{-\gamma} = \mathrm{E}^*\{e^{-r\bar{\tau}}\}$, the present value of unit payoff to be received at the time of bankruptcy, $\bar{\tau} = \inf\{t : V_t = \bar{V}\}$.
- Thus, the default option is seen clearly as equity's option to exchange the firm for the nondefaultable perpetuity when firm value reaches $\bar{V}$.

Optimal Capital Structure with Taxes and Bankruptcy Costs (Leland, 1994)

- Now suppose that in bankruptcy, a fraction α of firm value is lost to lawyers.
- If bankruptcy occurs when firm asset value V falls to $\bar{V}$, then debt value is

$$f(V) = \frac{c}{r} - [\frac{c}{r} - (1-\alpha)\bar{V}](V/\bar{V})^{-\gamma}\ , \tag{6}$$

and the present value of the bankruptcy costs, net firm value, and equity value are

$$bc(V) = \alpha\bar{V}(V/\bar{V})^{-\gamma}, \tag{7}$$

$$v(V) = V - \alpha\bar{V}(V/\bar{V})^{-\gamma},\ \text{and} \tag{8}$$

$$e(V) = v(V) - f(V) = V - \frac{c}{r} + [\frac{c}{r} - \bar{V}](V/\bar{V})^{-\gamma}\ . \tag{9}$$

- Ex post, the bankruptcy costs come out of debt holders' pockets, so the equity-value maximizing value of default boundary $\bar{V}$ is the same as before.
- Ex ante, the unlevered equity holders would pay the bankruptcy costs in the form of lower debt sale price, and without a tax benefit, leverage would be suboptimal.

- Now interpret V as the after-tax value of the firm assets, and suppose that coupon expense is fully tax-deductible, delivering a flow tax benefit to the firm of xc, where x is the corporate tax rate. The present value of the tax benefit is

$$tb(V) = \frac{xc}{r}[1 - (V/\bar{V})^{-\gamma}]\ , \tag{10}$$

and the values of the firm, debt, and equity are

$$v(V) = V - bc(V) + tb(V) = V - \alpha\bar{V}(V/\bar{V})^{-\gamma} + \frac{xc}{r}[1 - (V/\bar{V})^{-\gamma}]\ ,$$

$$f(V) = \frac{c}{r} - [\frac{c}{r} - (1-\alpha)\bar{V}](V/\bar{V})^{-\gamma}\ ,\ \text{and}$$

$$e(V) = v(V) - f(V) = V - (1-x)\frac{c}{r} + [(1-x)\frac{c}{r} - \bar{V}](V/\bar{V})^{-\gamma}\ .$$

- If coupon expense were always fully tax-deductible, then the ex-ante firm-value-maximizing policy would be never to go bankrupt, $\bar{V} = 0$, which would violate limited liability of equity–equity value could go negative.
- The ex-post equity-value-maximizing default boundary is $\bar{V} = \frac{(1-x)c}{r+\sigma^2/2}$.
- The risky interest rate paid by the debt, $R = c/f(V)$ and the credit spread, $R - r$, are increasing in c and α, decreasing in x, and non-monotonic in σ^2.

- **Debt Capacity** Debt value is a hump-shaped function of the coupon c, reaching a maximum at

$$c_{max}(V) = V(1+\gamma-(1-\alpha)(1-x)\gamma)^{-1/\gamma}/((1-x)\gamma/r(1+\gamma)) \;, \quad (11)$$

at which point debt value is

$$f_{max}(V) = \frac{V\gamma(1+\gamma)^{-(1+1/\gamma)}[(1+\gamma)^{1/\gamma}(1+\gamma-(1-\alpha)(1-x)\gamma)^{-1/\gamma}]}{r[(1-x)\gamma/r(1+\gamma)]} \quad (12)$$

which is the debt capacity of the firm.

- **Optimal Capital Structure** The value of the coupon that maximizes the firm value $v(V)$ is

$$c^*(V) = V(1+\gamma+\alpha(1-x)\gamma/x)^{-1/\gamma}/((1-x)\gamma/r(1+\gamma)) \quad (13)$$

- Optimal leverage, $f(V)/v(V)$ is decreasing in asset riskiness.

Table II

Comparative Statics of Financial Variables at the Optimal Leverage Ratio: Unprotected Debt

This table describes the behavior of the coupon, C^*, that maximizes firm value, and the debt, D^*, leverage, L^*, interest rate, R^*, yield spread, $R^* - r$, total firm value, v^*, equity value, E^*, and bankruptcy value, V_B^*, at the optimal coupon level, for unprotected debt. (V is the firm's asset value, σ^2 is the variance of the asset return, r is the risk-free interest rate, α is the fraction of asset value lost if bankruptcy occurs, and τ is the corporate tax rate.)

		Sign of Change in Variable for an Increase in:			
Variable	Shape	σ^2	r	α	τ
C^*	Linear in V	< 0, σ^2 small; > 0, σ^2 large	> 0	< 0	> 0
D^*	Linear in V	< 0	> 0	< 0	> 0
L^*	Invariant to V	< 0	> 0	< 0	> 0
R^*	Invariant to V	> 0	> 0	< 0	> 0
$R^* - r$	Invariant to V	> 0	< 0	< 0	> 0
v^*	Linear in V	< 0	> 0	< 0	> 0
E^*	Linear in V	> 0	< 0	> 0	< 0[a]
V_B^*	Linear in V	< 0	> 0	< 0	> 0[a]

[a]No effect if $\alpha = 0$.

Optimal Call/Default Policies and Hedging with Stochastic Interest Rates (Acharya and Carpenter, 2002)

- Suppose the interest rate r is a nonnegative Markov Itô process given by

$$dr_t = \mu(r_t, t)\, dt + \sigma_t(r_t, t)\, dB_t^* \tag{14}$$

  where μ and σ are continuous and satisfy Lipschitz and linear growth conditions.
- Suppose a firm has a single bond outstanding with flow coupon c, maturity T, and par value one, and interpret all other values as multiples of the bond par value. The value of the firm assets follow

$$\frac{dV_t}{V_t} = (r_t - \gamma_t)\, dt + \phi_t\, dW_t^* \tag{15}$$

  where W^* is Brownian motion under $\mathcal{P}^*$, $d\langle B^*, W^*\rangle_t = \rho_t$, and γ_t, ϕ_t, and ρ_t are deterministic.
- Consider three cases:
  1. the bond is defaultable, but noncallable;
  2. the bond is callable at deterministic call price k_t, but nondefaultable;
  3. the bond is both callable and defaultable.

- The issuer's optimal call or default policy is to maximize the value of the option:

$$\zeta_t = \sup_{t \le \tau \le T} \mathrm{E}_t^* \{ \frac{\beta_\tau}{\beta_t} (P_\tau - \kappa(V_\tau, \tau))^+ \} \tag{16}$$

  where P_t is the value of the noncallable, nondefaultable host bond with the same coupon and maturity:

$$P_t = \mathrm{E}_t^* \{ c \int_t^T \frac{\beta_u}{\beta_t} du + \frac{\beta_T}{\beta_t} \} \,, \tag{17}$$

  and $\kappa(v, t) = v, k_t$, or $k_t \wedge v$, depending on the bond in question.
- Because r is Markov, $P_t = p_H(r, t)$ for some function p_h s.t. $p_H(\cdot, t)$ is strictly increasing and continuous in r, and thus has a continuous inverse. Therefore, given $P_t = p$ and $V_t = v$, the value of issuer's option is

$$\zeta_t = f(p, v, t) \tag{18}$$

  for some continuous function f satisfying $f(p, v, t) \ge (p - \kappa(v, t))^+$. Moreover, the optimal stopping time is $\tau = \inf\{t \ge 0 : f(P_t, V_t, t) = (P_t - \kappa(V_t, t))^+\}$.

- The value of the bond with one of these embedded options is the value of the noncallable, nondefaultable host bond minus the value of the embedded option:

$$p_X(p, v, t) = p - f_X(p, v, t) \ , \tag{19}$$

where $X = C$ for the pure callable bond, $X = D$ for the pure defaultable bond, and $X = CD$ for the callable defaultable bond.

Theorem 1 The following properties hold for all three embedded options.

1. $p_1 > p_2 \Rightarrow f(p_1, v, t) > f(p_2, v, t)$.
2. $v_1 < v_2 \Rightarrow f(p, v_1, t) \geq f(p, v_2, t)$.
3. $p_1 \neq p_2 \Rightarrow \frac{f(p_2,v,t)-f(p_1,v,t)}{p_2-p_1} \leq 1$ (call delta inequality).
4. $v_1 \neq v_2 \Rightarrow \frac{f(p,v_2,t)-f(p,v_1,t)}{v_2-v_1} \geq -1$ (put delta inequality).

- **Proposition 1** The values of the different embedded options relate as follows:

$$f_c(p, v, t) \vee f_D(p, v, t) \leq f_{CD}(p, v, t) \leq f_C(p, v, t) + f_D(p, v, t) \tag{20}$$

- The second inequality implies that the "option-adjusted" credit spread of a callable is less than the credit spread on the corresponding noncallable.

- Viewing the bond with the embedded option as a noncallable nondefaultable host bond minus a call on that bond with strike $\kappa(V_t, t)$ explains the empirical finding that corporate yield spreads on both callables and noncallables narrow as rates rise–because the embedded call falls out of the money.

- The optimal exercise policy is described by a critical level of the host bond price $b(v, t)$ above which the issuer calls or defaults.

Theorem 2 (Critical bond price boundary) Let $t \in [0, T)$ and $v > 0$. If there is any bond price p such that it is optimal to exercise the embedded option at time t given $P_t = p$ and $V_t = v$, then there exists a critical bond price $b(v, t) > \kappa(v, t)$ such that it is optimal to exercise the option if and only if $p \geq b(v, t)$.

- Earlier models with constant interest rates characterize the optimal exercise policy in terms of critical firm value below which to default and above which to call. This characterization is also valid here:

Theorem 3 (Critical firm value boundary) Let $t \in [0, T)$ and $p > 0$.

1. For the pure defaultable bond, there exists a critical firm value $v_D(p, t) < p$ such that, at time t, given $P_t = p$ and $V_t = v$, it is optimal to default if and only if $v \leq v_D(p, t)$.

2. For the callable defaultable bond, there exists a critical firm value $v_{CD}(p,t)$, satisfying $v_{CD}(p,t) \leq k_t$ and $v_{CD}(p,t) < p$, such that, at time t, given $P_t = p$ and $V_t = v$, it is optimal to default if and only if $v \leq v_{CD}(p,t)$. In addition, if there exists any firm value v at which it is optimal to call, then there exists a critical firm value $\bar{v}_{CD}(p,t) \geq k_t$ such that it is optimal to call if and only if $v \geq \bar{v}_{CD}(p,t)$.

- The final theorem describes the shape and relation of the boundaries, which turns out to explain corporate bond duration and hedging, and empirical patterns in yield spreads.

Theorem 4 For each $t \in [0,T)$,

1. $v_1 < v_2 \Rightarrow b_D(v_1,t) \leq b_D(v_2,t)$.
2. $p_1 < p_2 \Rightarrow v_D(p_1,t) \leq v_D(p_2,t)$.
3. $v_1 < v_2 \leq k_t \Rightarrow b_{CD}(v_1,t) \leq b_{CD}(v_2,t)$.
4. $k_t < v_1 < v_2 \Rightarrow b_{CD}(v_1,t) \geq b_{CD}(v_2,t)$.
5. $v \leq k_t \Rightarrow b_{CD}(v,t) \geq b_D(v,t)$.
6. $v > k_t \Rightarrow b_{CD}(v,t) \geq b_C(v,t)$.

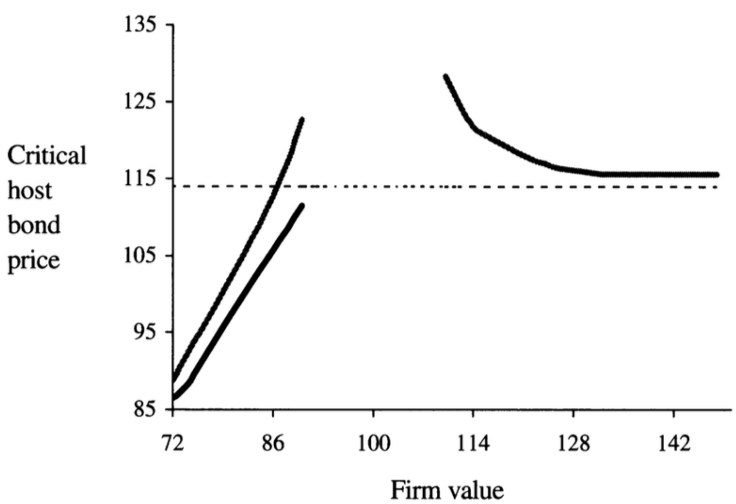

Figure 4
Optimal call and default boundaries
Critical host bond prices $b(v,t)$ for three 5-year, 10.25%-coupon bonds. The gray line corresponds to the callable defaultable, the black line corresponds to the pure defaultable, and the dotted line corresponds to the pure callable. For host bond prices below $b(v,t)$, it is optimal to continue, and for host bond prices above $b(v,t)$, it is optimal to default or call. Callable bonds are currently callable at par. The default payoff to bond holders is firm value. Call and default policies minimize bond values. The instantaneous interest rate follows $dr = \kappa(\mu - r)\,dt + \sigma\sqrt{r}\,d\tilde{Z}$; $\kappa = 0.5$, $\mu = 6.8\%$, $\sigma = 0.10$. Firm value follows $dV/V = (r-\gamma)\,dt + \phi\,d\tilde{W}$; $\gamma = 0.0$, $\phi = 0.20$. The instantaneous correlation between the interest rate and firm value processes is zero. Numerical approximations use a two-factor binomial lattice.

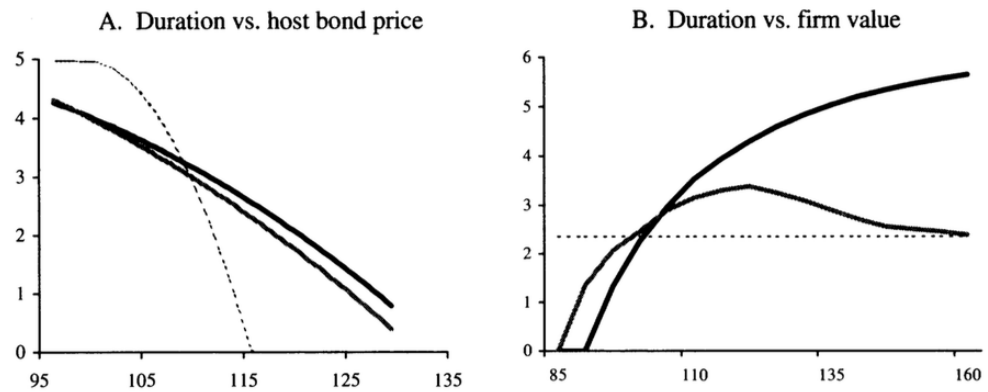

Figure 5
Dynamics of duration
Three 10-year, 11%-coupon bonds: the gray line represents the callable defaultable, the black line represents the pure defaultable, and the dotted line represents the pure callable. Duration is $-\frac{dp_X/p_X}{dy_H}$ where p_X is the price of the bond in question and y_H is the yield of its host bond. Callable bonds are currently callable at par. The default payoff to bond holders is firm value. Call and default policies minimize bond values. The instantaneous riskless rate follows $dr = \kappa(\mu - r)\,dt + \sigma\sqrt{r}\,d\tilde{Z}$; $\kappa = 0.5$, $\mu = 9\%$, $\sigma = 0.078$. Firm value follows $dV/V = (r - \gamma)\,dt + \phi d\tilde{W}$; $\gamma = 0.05$, $\phi = 0.15$. The instantaneous correlation between the interest rate and firm value processes is $\rho = -0.2$. Numerical approximations use a two-factor binomial lattice.

$$\frac{ds_X}{dy_H} = \frac{\frac{dp_X/p_X}{dy_H}}{\frac{dp_X/p_X}{dy_X}} - 1 = \frac{\text{duration}_X}{\text{modified duration}_X} - 1.$$

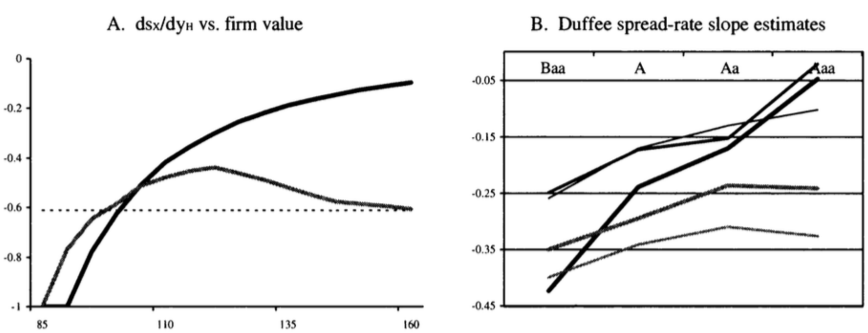

Figure 6
Theoretical and empirical slopes of the spread-rate relation
The theoretical slope in 6A is ds_X/dy_H, where s_X is the yield spread of the bond in question and y_H is the yield of its host bond. Callable bonds are currently callable at par. The default payoff to bond holders is firm value. Call and default policies minimize bond values. The instantaneous riskless rate follows $dr = \kappa(\mu - r)\,dt + \sigma\sqrt{r}\,d\tilde{Z}$; $\kappa = 0.5$, $\mu = 9\%$, $\sigma = 0.078$, $r_0 = 9\%$. Firm value follows $dV/V = (r - \gamma)\,dt + \phi d\tilde{W}$; $\gamma = 0.05$, $\phi = 0.15$. The instantaneous correlation between the interest rate and firm value processes is $\rho = -0.2$. Numerical approximations use a two-factor binomial lattice. The empirical slope in 6B is the estimate of b_1 in a regression of the form $\Delta\text{SPREAD}_t = b_0 + b_1 \Delta Y_{1/4,t} + b_2 \Delta\text{TERM}_t + \varepsilon_t$, where ***SPREAD*** is the mean spread of the yields of corporate bonds in a given sector over equivalent maturity Treasury bonds, $Y_{1/4}$ is the 3-month Treasury yield, and ***TERM*** is the difference between the 30-year constant-maturity Treasury yield and the 3-month Treasury bill yield, from Duffee (1998). Panel A: solid line, noncallable defaultable; light line, callable defaultable; broken line, callable. Panel B: thick solid line, noncallable–long; medium solid line, noncallable–medium; thin solid line, noncallable–short; thick light line, callable–long; medium light line, callable–intermediate.

Problem

Consider the firm with asset value V_t and debt equal to a perpetuity that pays flow c in the Black and Cox (1976) framework.

Suppose that the perpetuity is not only defaultable, as in the Black and Cox (1976) model detailed above, but also callable at a call price K where $\bar{V} < K < \frac{c}{r}$, where $\bar{V}$ is the optimal default boundary of the noncallable defaultable perpetuity.

Use the ODE approach detailed above to solve for the optimal critical lower and upper firm values V_d and V_c such that the firm optimally defaults if $V \leq V_d$, optimally calls if $V \geq V_c$, and continues servicing the debt of $V_d < V < V_c$.