











	US Zero Excess Returns 1976-2018 Annualized Means, Vols, Sharpe Ratios in Percent Excess return = Raw Return minus Riskless Return									
- (	Zero maturity	Mean raw return %	Mean excess return %	Excess return volatility %	Sharpe ratio	Excess return variance bps				
	. 1	5.46	1.00	1.52	0.66	2				
	2	6.02	1.56	2.99	0.52	9				
	3	6.42	1.97	4.22	0.47	18				
	4	6.96	2.50	5.50	0.45	30				
	5	7.39	2.93	6.74	0.43	45	Sum of the			
	6	7.87	3.41	7.83	0.44	61	zero return			
	7	7.99	3.53	8.86	0.40	78	variances			
	8	8.24	3.78	9.80	0.39	96	= 585			
	9	8.41	3.95	10.65	0.37	113				
	10	8.42	3.96	11.47	0.35	132				
* *	Evideno Volatilit	ce sugge y does i	ests longer ncrease lik	zeroes do ce duration,	have , but r	higher expe nean returns	cted returns. s do not.			
۲	Sharpe	ratios d	ecrease w	ith maturity	/					

Annua	lized №	leans, Vol	s, Sharpe	Ratio	os in Perce	ent
Zero maturity	Mean raw return	Mean excess return	Excess return volatility	Sharpe I ratio	Excess return variance	
1	2.83	0.36	1.02	0.35	1.04	
2	3.23	0.76	1.82	0.42	3.31	
3	3.36	0.88	2.45	0.36	5.98	
4	3.55	1.08	3.15	0.34	9.89	
5	3.64	1.17	3.87	0.30	15.00	Sum of the
6	3.86	1.39	4.43	0.31	19.59	variances
7	3.78	1.31	5.04	0.26	25.37	= 192
8	3.91	1.43	5.58	0.26	31.09	
9	3.99	1.52	6.09	0.25	37.10	
10	4.00	1.53	6.63	0.23	44.01	/
l on	ner zer	oes have h	nigher mea	n reti	irns	

	1	2	3	4	5	6	7	8	9	1(
	1.00	0.96	0.91	0.88	0.84	0.80	0.77	0.75	0.73	0.7
2	0.96	1.00	0.98	0.96	0.93	0.90	0.87	0.85	0.83	0.8
3	0.91	0.98	1.00	0.99	0.97	0.95	0.92	0.90	0.89	0.8
4	0.88	0.96	0.99	1.00	0.99	0.98	0.95	0.94	0.92	0.9
5	0.84	0.93	0.97	0.99	1.00	0.99	0.97	0.96	0.95	0.9
6	0.80	0.90	0.95	0.98	0.99	1.00	0.99	0.99	0.97	0.9
7	0.77	0.87	0.92	0.95	0.97	0.99	1.00	1.00	0.99	0.9
8	0.75	0.85	0.90	0.94	0.96	0.99	1.00	1.00	1.00	0.9
9	0.73	0.83	0.89	0.92	0.95	0.97	0.99	1.00	1.00	0.9
10	0.71	0.81	0.87	0.90	0.93	0.95	0.96	0.98	0.99	1.0

2011	elal	1011	Mai	.rix						
	1	2	3	4	5	6	7	8	9	1
1	1.00	0.94	0.82	0.75	0.67	0.61	0.55	0.54	0.53	0.
2	0.94	1.00	0.95	0.89	0.82	0.76	0.70	0.68	0.67	0.
3	0.82	0.95	1.00	0.97	0.90	0.86	0.82	0.80	0.78	0.
4	0.75	0.89	0.97	1.00	0.98	0.94	0.89	0.87	0.85	0.
5	0.67	0.82	0.90	0.98	1.00	0.98	0.93	0.91	0.89	0.
6	0.61	0.76	0.86	0.94	0.98	1.00	0.98	0.97	0.95	0.
7	0.55	0.70	0.82	0.89	0.93	0.98	1.00	1.00	0.98	0.
8	0.54	0.68	0.80	0.87	0.91	0.97	1.00	1.00	0.99	0.
9	0.53	0.67	0.78	0.85	0.89	0.95	0.98	0.99	1.00	0.
10	0.52	0.66	0.76	0.83	0.86	0.91	0.94	0.96	0.99	1.













US Ze	ero R	etu	rns	Facto	or St	ruct	ure 1	1976	-201	18
Fac	tor λ's (va	riances)	in basis   ivial risk	points, vo	's in %. λ Sum c	's, vol's, a of the fac	nd SR's are	e annualiz n variano	ed. ces = 58	5
Factor:	<b>1</b>	2	3	4	5	6	7	8	9	10
Factor $\lambda$ :	567	14	3	1	0	0	0	0	0	0
% of total:	96.8%	2.4%	0.6%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Factor vol.:	23.81	3.74	1.85	0.91	0.50	0.29	0.25	0.08	0.06	0.04
Factor mean:	9.51	1.12	0.13	0.25	0.22	0.09	0.02	0.07	0.09	0.01
Factor SR:	0.40	0.30	0.07	0.27	0.44	0.32	0.07	0.81	1.51	0.29
1-yr zero q:	0.05	0.19	0.22	0.42	0.53	0.66	0.14	0.04	0.00	-0.01
2-yr zero q:	0.11	0.33	0.30	0.48	0.25	-0.67	-0.13	-0.15	0.08	0.01
3-yr zero q:	0.16	0.39	0.30	0.17	-0.58	0.08	0.14	0.49	-0.32	-0.02
4-yr zero q:	0.22	0.41	0.20	-0.21	-0.33	0.24	-0.08	-0.54	0.48	0.06
5-yr zero q:	0.28	0.37	0.02	-0.52	0.35	-0.01	-0.31	-0.06	-0.52	-0.15
6-yr zero q:	0.33	0.24	-0.27	-0.25	0.25	-0.13	0.23	0.53	0.50	0.20
7-yr zero q:	0.37	0.07	-0.46	0.19	-0.07	-0.06	0.55	-0.31	-0.17	-0.41
8-yr zero q:	0.41	-0.09	-0.34	0.27	-0.10	0.09	-0.27	-0.12	-0.22	0.69
9-yr zero q:	0.44	-0.29	-0.03	0.19	-0.08	0.10	-0.55	0.21	0.23	-0.52
10-yr zero q:	0.47	-0.50	0.57	-0.20	0.07	-0.09	0.34	-0.07	-0.06	0.13
										Q matrix



US Ze	ero F	Retu	rns	Factor Structure
Factor:	1	2	3	<ul> <li>The signs of factors</li> <li>make factor Sharps</li> </ul>
Factor variance:	567	14	3	<ul> <li>Factor 1 explains 97</li> </ul>
% of total:	96.8%	2.4%	0.6%	market risk.
Factor vol.:	23.81	3.74	1.85	Factor 1 loadings and a second sec
Factor mean:	9.51	1.12	0.13	duration.
Factor Sharpe Ratio	0.40	0.30	0.07	Positive shocks to f
1-yr zero q:	0.05	0.19	0.22	downward parallel
2-yr zero q:	0.11	0.33	0.30	curve.
3-yr zero q:	0.16	0.39	0.30	This is why durat
4-yr zero q:	0.22	0.41	0.20	hedging against
5-yr zero q:	0.28	0.37	0.02	curve shifts work
6-yr zero q:	0.33	0.24	-0.27	Factor 1 has an anr
7-yr zero q:	0.37	0.07	-0.46	of 40 basis points.
8-yr zero q:	0.41	-0.09	-0.34	This is what gives t
9-yr zero q:	0.44	-0.29	-0.03	higher expected rel
10-yr zero q:	0.47	-0.50	0.57	have higher loading

JS Zero Returns Factor Structure – Factor	1	
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۲	The signs of fac	tors are	arbitrary,	set to
	make factor Sha	arpe ratio	os positive	e.

- 7% of total bond
- re proportional to
- actor 1 look like shifts to the yield
- ion-based parallel yield s so well!
- ualized Sharpe ratio
- he longer zeroes urns, because they s on this factor.

US Ze	ero R	letu	rns	Factor Structure – Factor 2
Factor:	1	2	3	Eactor 2 explains most of the rest of
Factor variance λ:	567	14	3	the risk.
% of total:	96.8%	2.4%	0.6%	Factor 2 is long short-term zeroes
Factor vol.:	23.81	3.74	1.85	and short long-term zeroes.
Factor mean:	9.51	1.12	0.13	Positive shocks to factor 2 look like
Factor Sharpe Ratio	0.40	0.30	0.07	steepening of the yield curve.
1-yr zero q:	0.05	0.19	0.22	<ul> <li>Factor 2 delivers an annualized</li> </ul>
2-yr zero q:	0.11	0.33	0.30	Sharpe ratio of 30 basis points.
3-yr zero q:	0.16	0.39	0.30	Short zeroes are long this factor and
4-yr zero q:	0.22	0.41	0.20	long zeroes are short this factor. This
5-yr zero q:	0.28	0.37	0.02	helps explain why longer zeroes have
6-yr zero q:	0.33	0.24	-0.27	lower Sharpe ratios.

0.07

-0.09

-0.29

-0.50

0.37

0.41

0.44

0.47

-0.46

-0.34

-0.03

0.57

7-yr zero q:

8-yr zero q:

9-yr zero q:

10-yr zero q:

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					/Sum o	f the fac	tor return	n varianc	es = 192	2
Factor:	1	2	3	4	5	6	7	8	9	10
Factor $\lambda$ :	180.2	8.1	2.6	1.0	0.3	0.0	0.0	0.0	0.0	0.0
% of total:	93.7%	4.2%	1.3%	0.5%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%
Factor vol.:	13.42	2.85	1.61	1.02	0.58	0.21	0.18	0.09	0.05	0.02
Factor SR:	0.28	0.24	0.08	0.11	-0.10	0.21	-0.18	-0.93	1.66	0.54
1-yr zero q:	0.05	0.19	0.26	0.25	0.59	-0.58	-0.37	-0.07	0.08	0.0
2-yr zero q:	0.10	0.35	0.37	0.33	0.38	0.59	0.36	-0.02	0.03	0.0
3-yr zero q:	0.15	0.40	0.29	0.31	-0.54	-0.12	-0.17	0.41	-0.37	-0.04
4-yr zero q:	0.21	0.44	0.12	-0.13	-0.38	-0.14	0.08	-0.48	0.57	0.0
5-yr zero q:	0.27	0.40	-0.12	-0.59	0.22	-0.13	0.21	-0.12	-0.52	-0.0
6-yr zero q:	0.32	0.23	-0.33	-0.21	0.17	0.18	-0.25	0.61	0.42	0.0
7-yr zero q:	0.37	0.01	-0.42	0.33	-0.01	0.27	-0.43	-0.41	-0.17	-0.3
8-yr zero q:	0.41	-0.13	-0.27	0.31	0.00	-0.15	0.26	-0.06	-0.13	0.7
9-yr zero q:	0.45	-0.29	0.03	0.15	0.00	-0.30	0.49	0.19	0.16	-0.5
10-yr zero q:	0.48	-0.42	0.57	-0.32	-0.03	0.19	-0.31	-0.07	-0.04	0.1

				• Factor 1 is still by fay the most
Factor:	1	2	3	<ul> <li>Factor 1 is still by far the most important dimension of bond market</li> </ul>
Factor var:	180.2	8.1	2.6	risk (explains 94%).
% of total:	93.7%	4.2%	1.3%	<ul> <li>Factor 1 loadings still look proportional</li> </ul>
Factor vol.:	13.42	2.85	1.61	to duration.
Factor SR:	0.28	0.24	0.08	<ul> <li>So positive shocks to factor 1 still look</li> </ul>
1-yr zero q:	0.05	0.19	0.26	like downward parallel shifts to the vie
2-yr zero q:	0.10	0.35	0.37	curve and duration-based hedging
3-yr zero q:	0.15	0.40	0.29	against parallel yield curve shifts shoul
4-yr zero q:	0.21	0.44	0.12	still work well.
5-yr zero q:	0.27	0.40	-0.12	<ul> <li>Factor 1 has an annualized Sharpe rati</li> </ul>
6-yr zero q:	0.32	0.23	-0.33	of 28 basis points.
7-yr zero q:	0.37	0.01	-0.42	This is what gives the longer zeroes
8-yr zero q:	0.41	-0.13	-0.27	higher expected returns, but not so
9-yr zero q:	0.45	-0.29	0.03	pronounced as in the US.
10-yr zero q:	0.48	-0.42	0.57	

China	Zero	Reti	urns	Factor Structure – Factors 2 & 3
				♦ As in the US returns, factor 2 is long
Factor:	1	2	3	short-term zeroes and short long-
Factor var:	180.2	8.1	2.6	term zeroes.
% of total:	93.7%	4.2%	1.3%	Positive shocks to factor 2 look like
Factor vol.:	13.42	2.85	1.61	steepening twists to the yield curve.
Factor SR:	0.28	0.24	0.08	<ul> <li>Factor 2 delivers an annualized</li> </ul>
1-yr zero q:	0.05	0.19	0.26	Sharpe ratio of 24 basis points.
2-yr zero q:	0.10	0.35	0.37	Short zeroes are long this factor and
3-yr zero q:	0.15	0.40	0.29	long zeroes are short this factor,
4-yr zero q:	0.21	0.44	0.12	which again helps explain why longer
5-yr zero q:	0.27	0.40	-0.12	zeroes have lower Sharpe ratios.
6-yr zero q:	0.32	0.23	-0.33	<ul> <li>As in the US, factor 3 is long the</li> </ul>
7-yr zero q:	0.37	0.01	-0.42	ends, short the middle. A positive
8-yr zero q:	0.41	-0.13	-0.27	shock to factor 3 makes the yield
9-yr zero q:	0.45	-0.29	0.03	curve more humped, or curved.
10-yr zero q:	0.48	-0.42	0.57	These 3 factors are sometimes called
				level, slope, and curvature.

<b>US-China</b> Bo 2007-2015	ond Facto	r Correlat	tions
<ul> <li>The US and China structure, describe curvature factors.</li> </ul>	a zero returns h ed by so-called	nave a remarkal I level, steepnes	bly similar ss, and
Why? What are the	ne macro econo	omic factors bel	nind them?
<ul> <li>However, the corr countries are quit segmentation.</li> </ul>	<ul> <li>However, the correlations of these factors ac countries are quite low, consistent with man segmentation.</li> </ul>		s the two
<ul> <li>This suggests opp bond investors.</li> </ul>	oortunities for a	liversification fo	or global
Lev	/el Slop	oe Cur	vature
Level	-0.24	-0.05	0.09
Slope	0.02	0.19	-0.05
Curvature	0.09	-0.05	0.05





Example	Factor:	1 (Level)	2 (Slope)
Suppose an investor has a short position of	Factor λ:	567	14
Using the factor loadings to the right, what	Factor vol.:	23.81	3.74
present value of the 1-year zero must the	Factor SR:	0.40	0.30
investor buy to hedge factor-1 risk?	1-yr zero q: 2-yr zero q:	0.05	0.19
$q_{PL,1} = v_1 * 0.05 - 100 * 0.47 = 0$	3-yr zero q:	0.16	0.39
With this factor-1 hedge on the investor is	4-yr zero q:	0.22	0.41
hedging the "level" risk of the short 10-year	6-yr zero q:	0.28	0.37
position by buying almost 10 times as much	7-yr zero q:	0.37	0.07
hedge.	8-yr zero q:	0.41	-0.09
But this leaves the investor very exposed to	10-yr zero q:	0.44	-0.29
"slope" risk (also called "curve" risk).		q	<sub>i1's</sub> <b>q</b> <sub>i2's</sub>

We can quantify the investor's exposure to slope risk.Factor $\lambda$ :567What is the investor's dollar loading on factor 2? Expected P&L? Variance of P&L? Volatility of P&L? Sharpe Ratio of P&L?Factor vol.:23.8196.8%2What is the investor's dollar loading on factor of P&L? Sharpe Ratio of P&L?Factor vol.:23.8196.8%296.8%296.8%296.8%296.8%296.8%296.8%296.8%296.8%296.8%296.8%296.8%297.8094.998.994.999.994.999.994.990.994.990.994.990.994.990.994.990.994.990.994.990.994.994.994.994.994.995.994.996.994.996.994.996.994.994.994.995.994.995.994.996.994.996.994.996.994.996.994.996.994.996.994.996.994.996.994.996.994.996.994.996.994.996.994.996.994.997.995.9 <th>Example, cont d</th> <th>Factor:</th> <th>1</th> <th></th>	Example, cont d	Factor:	1	
slope risk.% of total:96.8%2What is the investor's dollar loading on factor 2? Expected P&L? Variance of P&L? Volatility of P&L? Sharpe Ratio of P&L?Factor vol.:23.81 $q_{PL,2} = 940*0.19 - 100*(-0.50) = 228.6 =>$ $-yr$ zero q:0.05 $* E{PL} = 228.6*3.74\%*0.30 = $2.56$ $-yr$ zero q:0.16 $Var{PL} = 228.6^{2*}14$ bp = 73.1 $-yr$ zero q:0.28 $Vol{PL} = $8.55$ $-yr$ zero q:0.33 $* SR = E{PL}/Vol{PL} = 2.56/8.55 = 0.30$ $-yr$ zero q:0.37The investor's position is a pure play on slope. $9-yr$ zero q:0.41	We can quantify the investor's exposure to	Factor λ:	567	
What is the investor's dollar loading on factor 2? Expected P&L? Variance of P&L? Volatility of P&L? Sharpe Ratio of P&L?Factor vol.:23.81 Factor SR:0.40 01-yr zero q:0.05 2-yr zero q:1-yr zero q:0.05 2-yr zero q:0.11 0 $$q_{PL,2} = 940*0.19 - 100*(-0.50) = 228.6 =>$ 3-yr zero q:0.16 0• E{PL} = 228.6*3.74%*0.30 = \$2.564-yr zero q:0.22 0.28• Var{PL} = 228.62*14 bp = 73.1 05-yr zero q:0.28 0.33• Vol{PL} = \$8.557-yr zero q:0.37 0.37• SR = E{PL}/Vol{PL} = 2.56/8.55 = 0.308-yr zero q:0.41 0.41	slope risk.	% of total:	96.8%	2.
2? Expected P&L? Variance of P&L? Volatility of P&L? Sharpe Ratio of P&L?Factor SR: $1-yr$ zero q:0.40 $9P_{L,2} = 940*0.19 - 100*(-0.50) = 228.6 =>$ $2-yr$ zero q: $2-yr$ zero q:0.11 $$q_{PL,2} = 940*0.19 - 100*(-0.50) = 228.6 =>$ $3-yr$ zero q: $4-yr$ zero q:0.22 $\bullet E{PL} = 228.6*3.74\%*0.30 = $2.56$ $4-yr$ zero q: $2-yr$ zero q:0.22 $\bullet Var{PL} = 228.6^{2*}14$ bp = 73.1 $6-yr$ zero q: $2-yr$ zero q:0.33 $\bullet Vol{PL} = $8.55$ $7-yr$ zero q: $2-yr$ zero q:0.37 $\bullet SR = E{PL}/Vol{PL} = 2.56/8.55 = 0.30$ $8-yr$ zero q: $9-yr$ zero q:0.41	What is the investor's dollar loading on factor	Factor vol.:	23.81	З
of P&L? Sharpe Ratio of P&L?1-yr zero q: 2-yr zero q:0.05 0.11 $\$q_{PL,2} = 940*0.19 -100*(-0.50) = 228.6 =>$ 3-yr zero q: 0.160.11• E{PL} = 228.6*3.74%*0.30 = \$2.564-yr zero q: 0.220.22• Var{PL} = 228.6*3.74%*0.30 = \$2.565-yr zero q: 0.280.28• Var{PL} = 228.6*3.74%*0.30 = \$2.565-yr zero q: 0.280.28• Var{PL} = \$8.555-yr zero q: 0.330.37• Vol{PL} = \$8.557-yr zero q: 0.370.37• SR = E{PL}/Vol{PL} = 2.56/8.55 = 0.308-yr zero q: 0.410.41	2? Expected P&L? Variance of P&L? Volatility	Factor SR:	0.40	C
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	of P&L? Sharpe Ratio of P&L?	1-yr zero q:	0.05	C
$\begin{aligned} \$q_{PL,2} &= 940*0.19 - 100*(-0.50) = 228.6 => & 3-yr \ zero \ q: & 0.16 & 0 \\ \bullet \ E\{PL\} &= 228.6*3.74\%*0.30 = \$2.56 & 4-yr \ zero \ q: & 0.22 & 0 \\ \bullet \ Var\{PL\} &= 228.6^{2}*14 \ bp = 73.1 & 6-yr \ zero \ q: & 0.33 & 0 \\ \bullet \ Vol\{PL\} &= \$8.55 & 7-yr \ zero \ q: & 0.37 & 0 \\ \bullet \ SR &= \ E\{PL\}/Vol\{PL\} = 2.56/8.55 = 0.30 & 8-yr \ zero \ q: & 0.41 & -1 \\ The \ investor's \ position \ is \ a \ pure \ play \ on \ slope. & 9-yr \ zero \ q: & 0.44 & -1 \\ \hline\end{aligned}$		2-yr zero q:	0.11	C
• $E{PL} = 228.6*3.74\%*0.30 = $2.56$ 4-yr zero q:0.22• $Var{PL} = 228.6^{2}*14 bp = 73.1$ 5-yr zero q:0.28• $Vol{PL} = $8.55$ 6-yr zero q:0.33• $SR = E{PL}/Vol{PL} = 2.56/8.55 = 0.30$ 8-yr zero q:0.41The investor's position is a pure play on slope.9-yr zero q:0.44	\$q <sub>PL 2</sub> = 940*0.19 -100*(-0.50) = 228.6 =>	3-yr zero q:	0.16	C
• Var{PL} = $228.6^{2*}14$ bp = $73.1$ 5-yr zero q:       0.28         • Vol{PL} = $$8.55$ 6-yr zero q:       0.33         • SR = E{PL}/Vol{PL} = $2.56/8.55 = 0.30$ 8-yr zero q:       0.41         The investor's position is a pure play on slope.       9-yr zero q:       0.44	• $E{PL} = 228.6*3.74\%*0.30 = $2.56$	4-yr zero q:	0.22	C
• Vol{PL} = \$8.55       6-yr zero q:       0.33         • SR = E{PL}/Vol{PL} = 2.56/8.55 = 0.30       8-yr zero q:       0.41         The investor's position is a pure play on slope.       9-yr zero q:       0.44	• Var{PL} = $228 6^{2*}14 \text{ bn} = 73 1$	5-yr zero q:	0.28	C
• $VO[\{PL\}] = $8.55$ • $SR = E\{PL\}/Vo[\{PL\}] = 2.56/8.55 = 0.30$ The investor's position is a pure play on slope. 9-yr zero q: 0.41 9-yr zero q: 0.44	$\sqrt{2}$	6-yr zero q:	0.33	C
• SR = E{PL}/Vol{PL} = $2.56/8.55 = 0.30$ The investor's position is a pure play on slope. 9-yr zero q: 0.41 - 0.44	• VOI{PL} = \$8.55	7-yr zero q:	0.37	C
The investor's position is a pure play on slope. 9-yr zero q: 0.44 -	• SR = $E\{PL\}/Vol\{PL\} = 2.56/8.55 = 0.30$	8-yr zero q:	0.41	-0
	The investor's position is a pure play on slope.	9-yr zero q:	0.44	-0

Class Problem	Factor:	1 (Level)	2 (Slop
Suppose an investor has a short position of	Factor $\lambda$ :	567	
\$100 present value in the 5-year zero.	% of total:	96.8%	2
	Factor vol.:	23.81	3
Using the factor loadings to the right, what	Factor SR:	0.40	(
present values of the 1-year zero and 10-year	1-yr zero q:	0.05	(
zero, $v_1$ and $v_{10}$ must the investor buy to	2-yr zero q:	0.11	(
nedge <i>both</i> level and slope risk?	3-yr zero q:	0.16	(
Hint: 2 equations to set portfolio risk loadings	4-yr zero q:	0.22	(
$q_{PL,1}$ and $q_{PL,2}$ equal to zero, 2 unknown v's.	5-yr zero q:	0.28	
	6-yr zero q:	0.33	(
	7-yr zero q:	0.37	
	8-yr zero q:	0.41	-(
	9-yr zero q:	0.44	-(
	10-yr zero q:	0.47	-(
		<b>G</b> <sub>i1/c</sub>	a

