American Options, Callable Bonds, and Swaptions

Outline

- American Options and Early Exercise
- Intuition from Put-Call Parity
- Optimal Exercise Policy
- Valuing an American Call
- Valuing a Callable Bond
- Option-Adjusted Spread
- Interest Rate Sensitivity of a Callable Bond
- Negative Convexity
- Swaptions and Cancelable Swaps

Reading

- Tuckman and Serrat, Chapter 18

American Options

- The options we have considered so far have been "European" options, meaning that they can only be exercised on a single date, the "expiration date."
- However, the options embedded in callable bonds and fixed rate mortgages are "American" options, meaning that they can be exercised any time across a range of possible exercise dates, typically payment dates.
- The exercise time is chosen by the option holder, i.e., the issuer of the bond or loan. Valuing these securities requires determining the exercise policy of the option holder.
- We sometimes assume the option holder exercises (calls the bonds) to maximize the value the option, as in the case of an agency bond.
- With mortgages, borrowers often exercise "suboptimally," e.g., prepaying when rates are high because they have to move house. This reduces the option value and increases the MBS value.

Put-Call Parity for European Options

- To illustrate the trade-offs of exercising an American option sooner rather than later, it's useful to review put-call parity.
- Consider a European call and put on the same underlying asset with value V, with the same strike price K, and the same expiration date T.
- The payoff of the call is $Max(V_T – K, 0)$.
- The payoff of the put is $Max(K - V_T, 0)$.
- Note that $Max(V_T – K, 0) = Max(K - V_T, 0) + V_T – K$.
- I.e., Call payoff = Put payoff + Payoff of underlying excluding any cash flows prior to time T, - Payoff of K par of T-year zeroes.
- Thus, by the L.O.O.P, any time prior to T, Call price = Put price + PV of the underlying ex interim cash flows, - the PV of the strike price:

$$C_t = P_t + V_t^{ex} - {}_t d_T K$$

Example

- Recall the 1-year European call embedded in the 1.5-year 5.5%-coupon bond that we saw before.
- Its price was $C_0 = 0.157$.
- The underlying asset, ex interim coupons, was 102.75 par of zeroes mat. at 1.5, worth $V_0^{ex} = 102.75 \times 0.922242 = 94.7604$.
- The PV of the 100 strike price is $100 \times 0.947649 = 94.7649$.
- Therefore, by put-call parity, the put price must be

$$P_t = C_t - V_t^{ex} + {}_t d_T K = 0.157 - 94.7604 + 94.7649 = 0.162$$

- If we price the put in the tree we get the same answer:

Put = 0.9730 x 0.5 x (0.33 + 0) =0.162

Put = 0.9709 x 0.5 x (0.68 + 0) =0.33

Put 0

Underlying bond 99.32
Put 0.68

Bond 100.03
Put 0

Bond 100.60
Put 0

No Early Exercise of Calls on Assets that Have No Payments Prior to Expiration:

- Prior to expiration, an American call on a “non-paying" asset, such as a zero, is worth more than its exercise value:

  American call value

  ≥ European call value

  = European put value + $V - d_T K$

  > $V - d_T K$

  > $V - K$

  = exercise value.

- Therefore, it’s better to sell the call than to exercise it early (assuming the strike price is constant or decreasing over time).

Early Exercise of American Calls on Assets with Intervening Payments

- If the underlying asset makes payments prior to option expiration —i.e., ”interim payments” such as coupons or dividends—then early exercise can be optimal if the call gets deep enough in the money and the interim payment to be captured with early exercise is high:

American call value ≥ European call value

= European put value + $V^{\text{ex interim payments}} - d_T K$

>?=?<? $V^{\text{including interim payments}} - K$

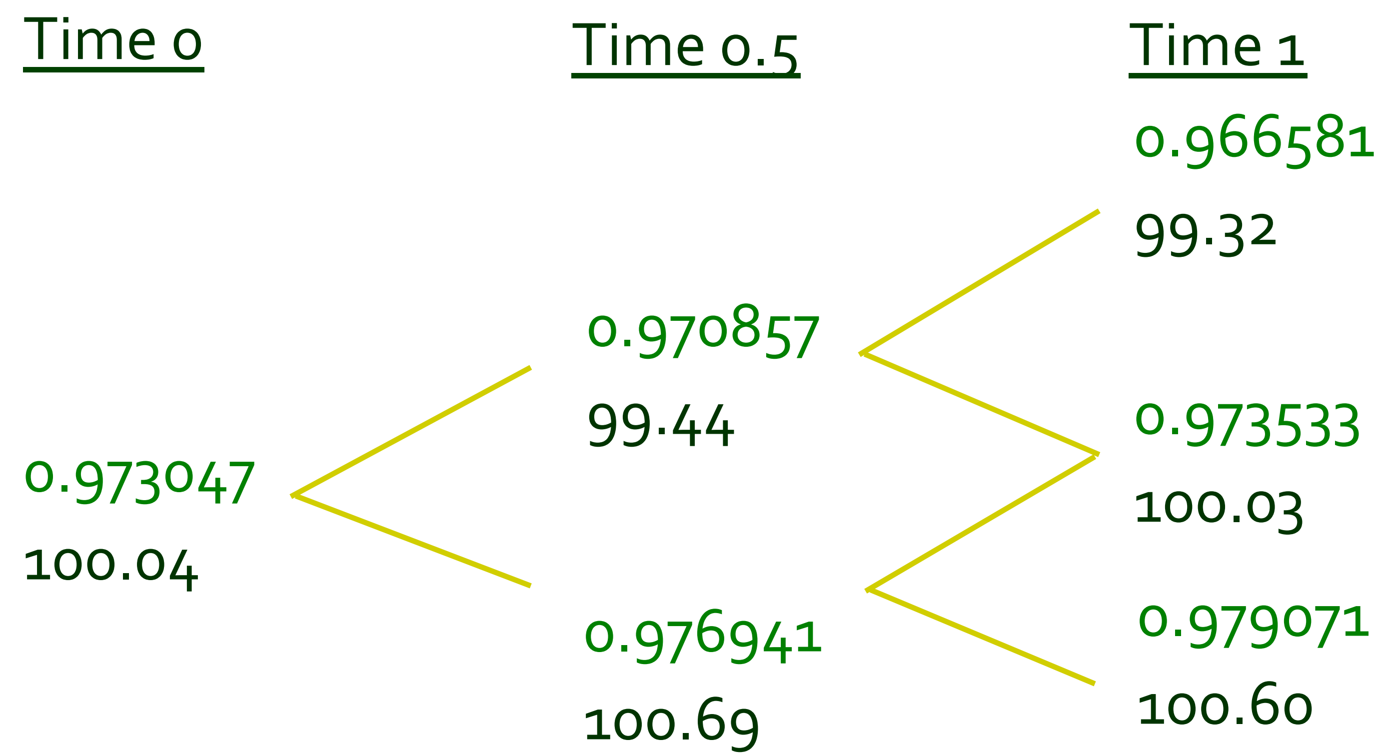
- Exercise or wait?
- Exercise: capture PV of interim payments – good if they're large
- Wait: keep the interest on the strike price – good if rates are high
- Wait: save the “put value” – good if volatility is high

American Call on the 1.5-Year 5.5%-Coupon Bond

- The issuer of a callable bond typically has the option to call the bond any time across a range of dates.
- Suppose the issuer of the 1.5-year 5.5%-coupon bond we saw before can call the bond for par on any coupon date, time 0, 0.5, or 1, immediately after the coupon is paid.
- The issuer's option is an American call on the noncallable 5.5%-coupon bond maturing at time 1.5, with strike equal to par, exercisable on any coupon date, ex-coupon.

Noncallable 1.5-Year 5.5%-Coupon Bond

- Each node in the tree below lists
  - the current ex-coupon price of the 5.5%-coupon bond, and
  - the current price of a zero with 6 months to maturity.
- At each node, the bond is priced as a package of zeroes .
- The American call gives the holder the right to buy the NC bond for 100 on any coupon date.
- When should the call be exercised?

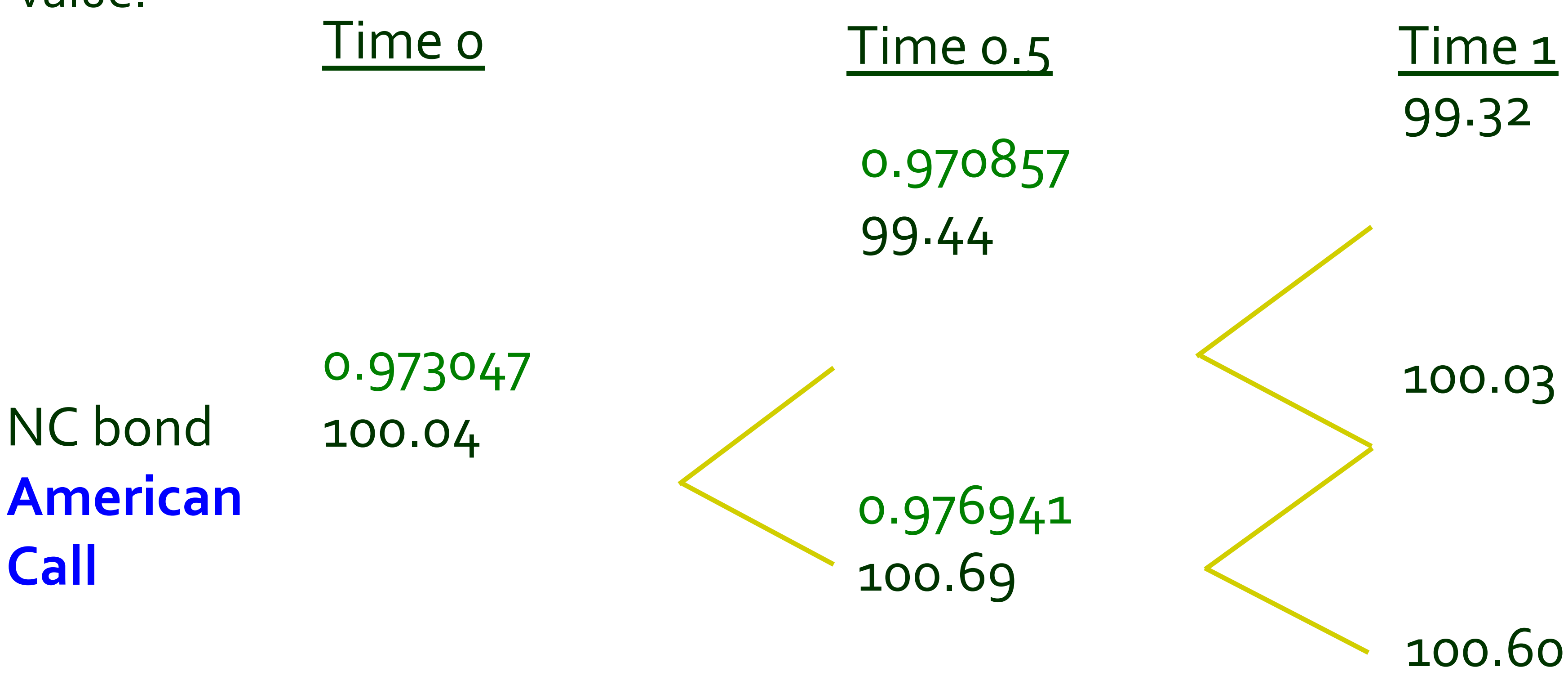


Exercise Policy for the American Call

- To value the call and assess its risk properties, we need to know how it will be exercised.
- At each time and state the option holder must decide whether or not to exercise the option. The complete set of decisions is called the option holder's *exercise policy*.
- As a first pass, let's assume that the option holder chooses the exercise policy to *maximize the option value.*
- This is appropriate if the option holder is unconstrained and can freely sell or hedge the option. This is a good assumption for a callable bond issuer such as FNMA or FHLMC.
- As we'll see later, this is not as good an assumption for the borrower in a fixed rate mortgage.

Class Problem: American Call

- Working backwards in time, decide what the call holder would do if he got to a given point with the call still alive.
- At time 1, the last call date, if the option were still alive, the holder would just exercise it if it were in the money or else let it expire worthless.
- At earlier dates, the holder can either exercise, or else wait one period, pay the coupon, and proceed optimally from there, whichever gives the option greater value.

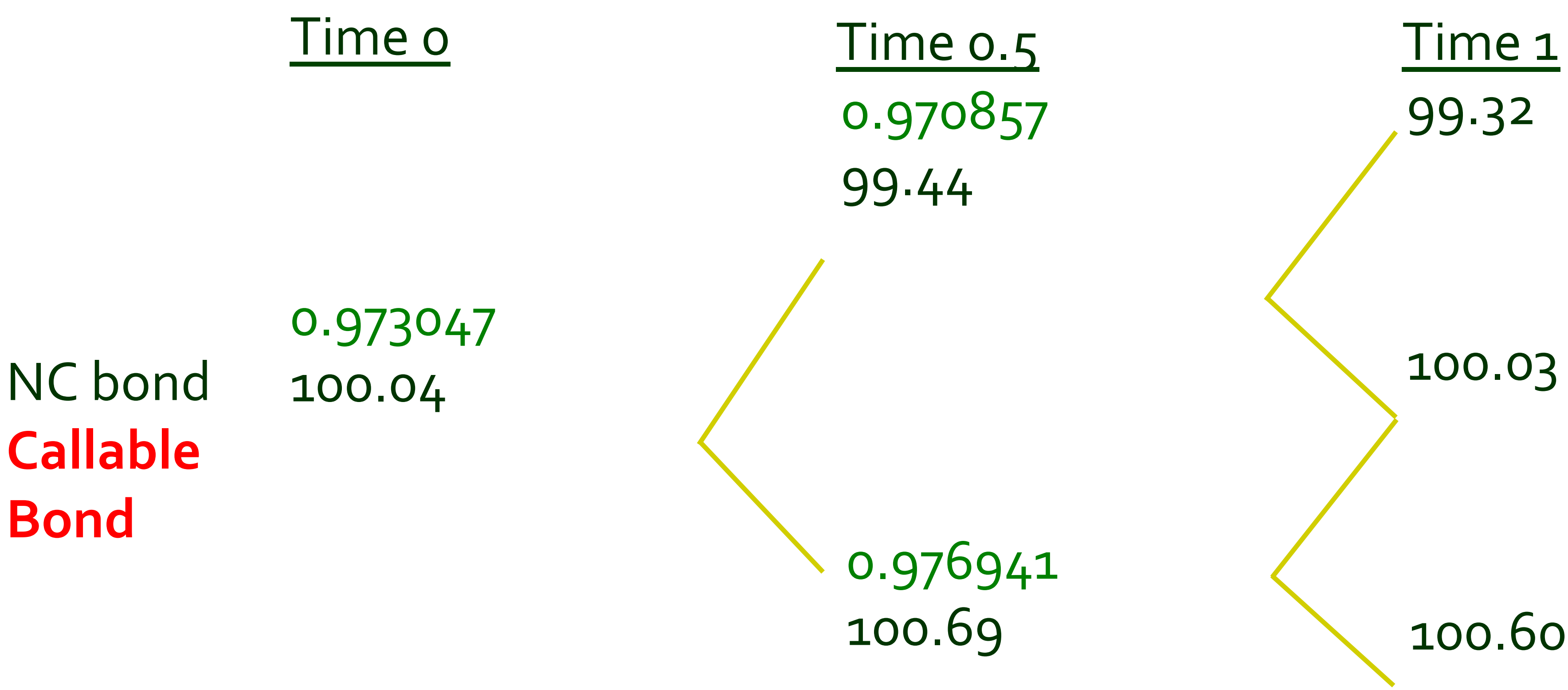


Valuing the Callable Bond

- Callable Bond = Noncallable Bond – American Call Option
  - Issuer (borrower): short the bond, long the option
  - Investor (lender): long the bond, short the option
- To value the callable bond, assume that the issuer follows a strategy for exercising the call that
  - *maximizes the value of the call*, or equivalently,
  - *minimizes the cost of the callable bond*.
- Since we have already valued the noncallable bond and the option, we know the value of the callable bond at each point. At time 0, callable bond = 100.04 – 0.34 = 99.70
- The next slides calculate the callable bond value from scratch and verify that Callable = Noncallable – Call always holds.

Class Problem: Callable Bond

- Working backwards in time, decide what the issuer would do to minimize the cost of debt if he got to a given point with the bond still outstanding.
- The issuer's choices are to either
  - call the bond and pay 100, or
  - leave the bond outstanding for another period, pay another coupon, and then proceed with optimal debt service from there.



Another Way to See Callable = Noncallable – Call: See Refunding Profit as Option Exercise Value

- Suppose the way the firm finances the call is by selling new noncallable bonds with the same coupon and maturity as the old debt--a "refunding."
- The profit from refunding

  = proceeds of sale of new debt - call price

  = value of the noncallable -100

  = option exercise value
- With this plan in place, a firm that has issued a callable bond pays the noncallable cash flows until maturity one way or another, but also gets to do a refunding along the way if rates fall far enough.
- The firm's net position = short the callable bond

  = short the noncallable bond, long the call option on the NC bond.

Class Problems

What are the SR dollar durations and SR durations of

1) the noncallable bond?

2) the call?

3) the callable bond?

Option-Adjusted Spread

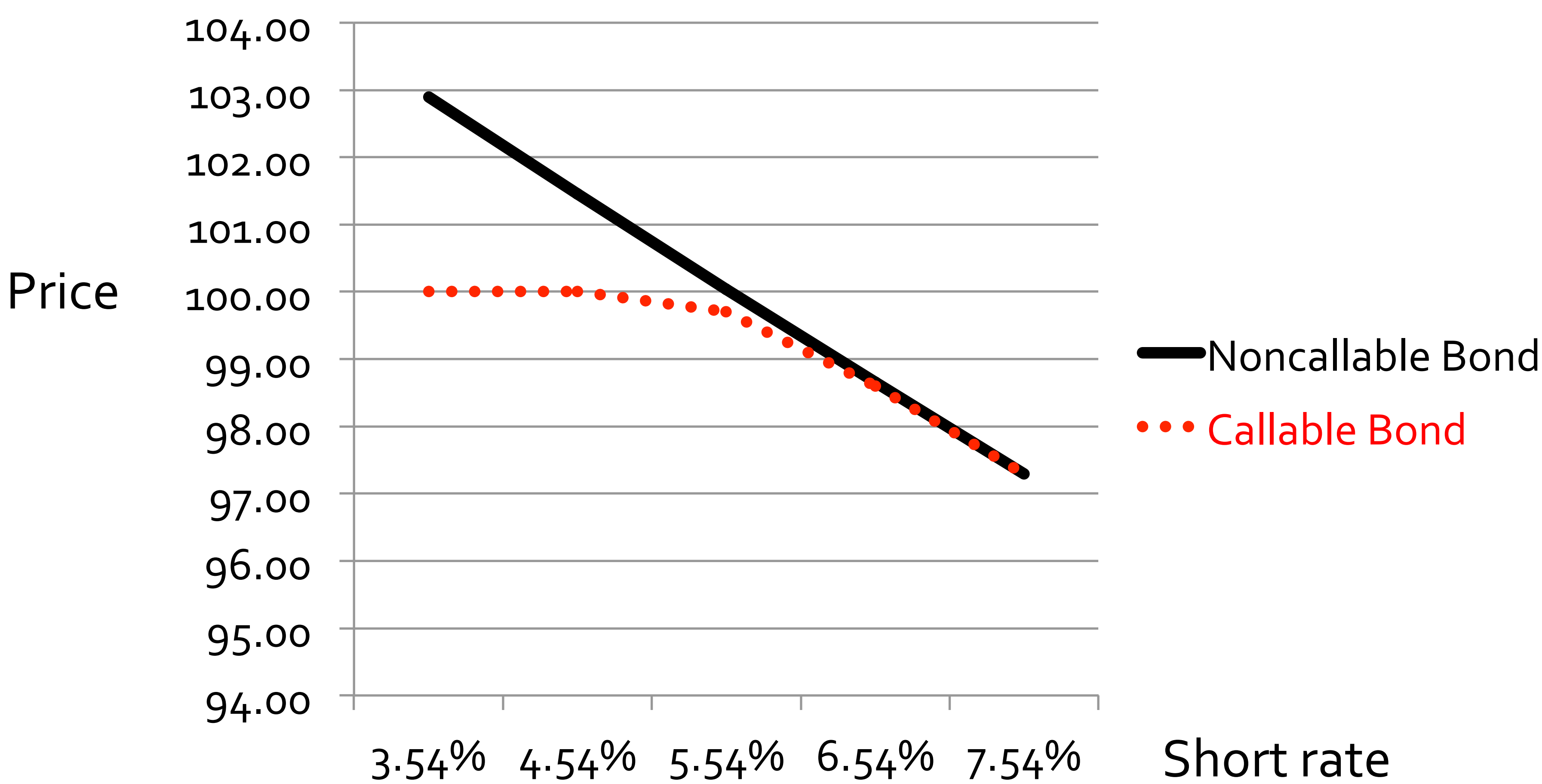
- A corporate bond pays less than promised in default.
- This credit risk makes it worth less than a nondefaultable bond with same coupon and maturity.
- This lower price implies a higher yield, that is, a "credit spread" in the corporate bond yield over the yield of a similar Treasury.
- If the bond is callable, the price is lower because of the embedded call, which also results in a higher yield to maturity.
- In fact, the call and default risks interact in a complicated way.
- In practice, people try to calculate the component of the spread of a callable defaultable bond over a noncallable Treasury that is due to just to credit risk, and not call risk.
- This is the so-called option-adjusted spread.

OAS

- First consider that in the absence of the call option, the credit spread of the bond is the amount by which the yield curve must be shifted up to correctly price the bond if it were nondefaultable.
- In the same spirit, the OAS is the amount that the interest rates in the model must be shifted upward to make the modeled callable bond price match the market price.
- The idea is that the model adjusts for the option, and then the OAS captures the additional price discount attributable to credit risk.
- In fact, OAS is calculated and quoted for all kinds of bonds: illiquid Treasuries, mortgage-backed securities, etc.

Negative Convexity of Callable Bonds

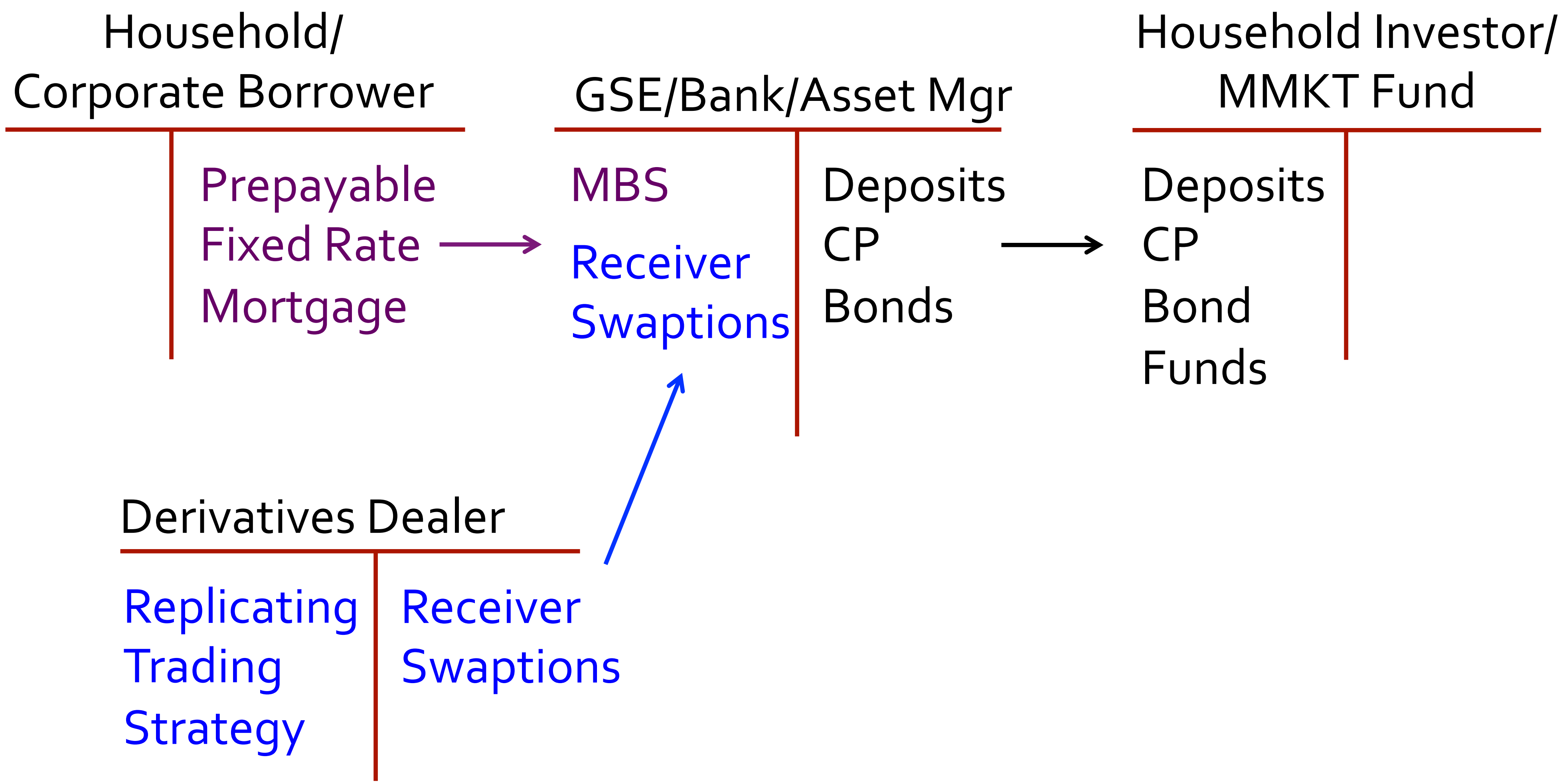
- The value of the embedded call option is a highly convex function of interest rates.
- So the short call position in the callable bond not only reduces its duration, it also gives it "negative convexity."

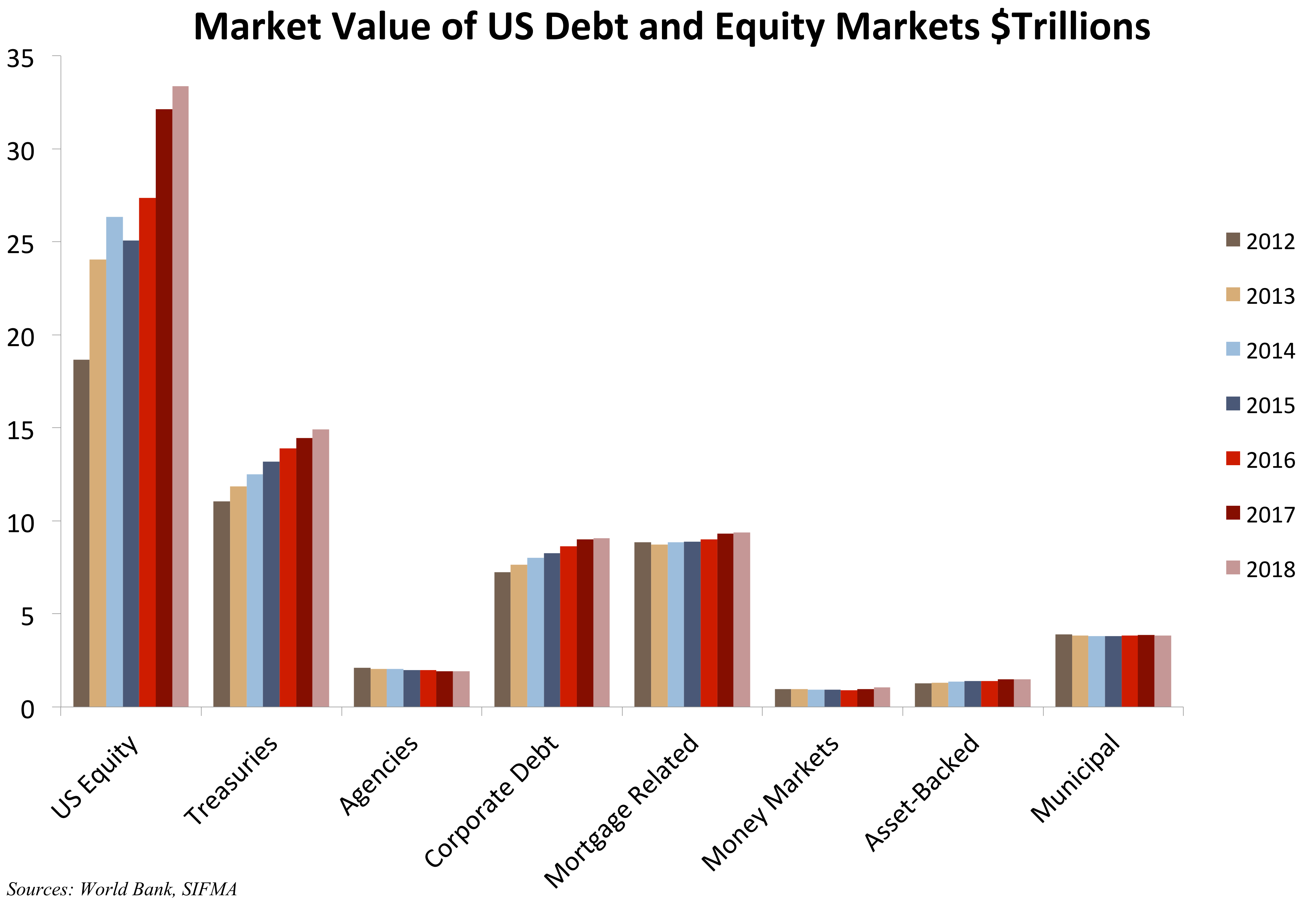


The Market for Swaptions

- The embedded options in callable bonds and fixed-rate mortgages mean that portfolios with agency bonds, corporate bonds, and mortgage-backed securities are short options and thus have a lot of negative convexity.
- Even when these portfolios are duration-hedged against changes in interest rates, they are exposed to the risk of an increase in volatility.
- This creates a demand for markets where these options can be traded wholesale by banks, asset managers, and other financial institutions.
- Rather than trading options on bonds, dealers find it easier to make markets in options on interest rate swaps, called swaptions.
- As we'll see, an option on a swap with a strike of zero is equivalent to an option on a bond with a strike of par.

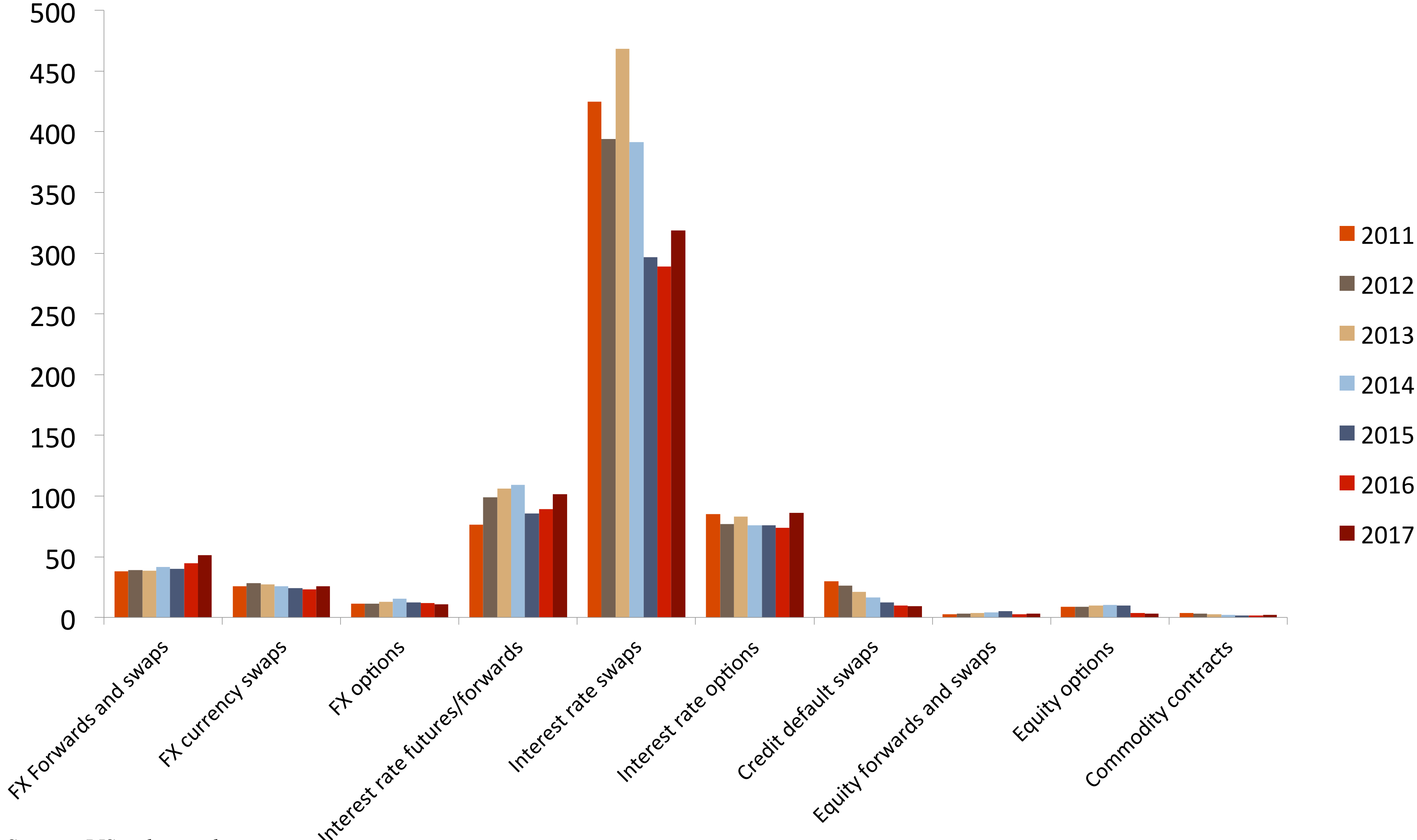
Macro Picture:
Swaptions in the Financial System





Sources: World Bank, SIFMA

Notional Amounts of Derivatives Outstanding $Trillions

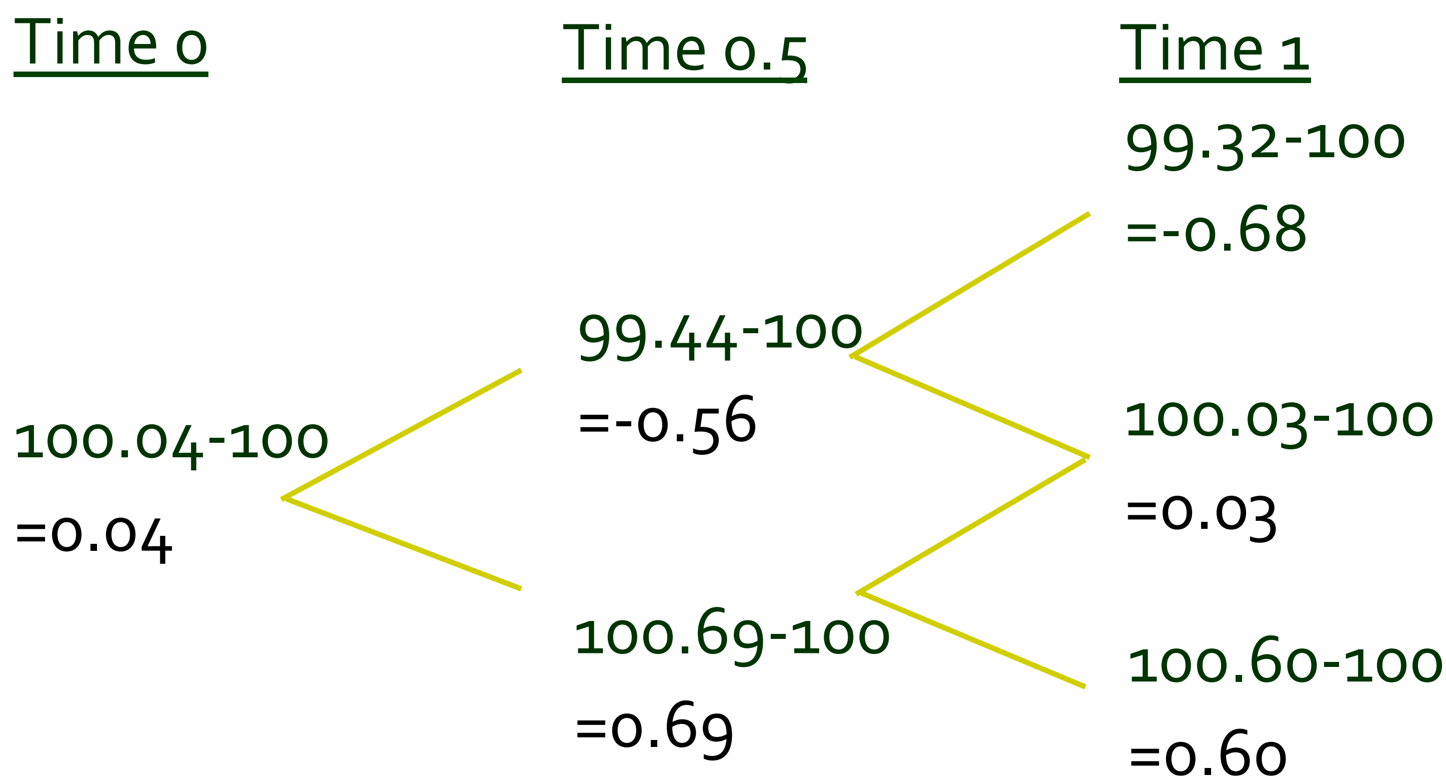


Sources: BIS and eurexchange

*Some equity forwards and options data points discontinued by BIS in 2016

1.5-Year 5.5% Interest Rate Swap

- Recall that a swap to receive fixed at 5.5% and pay floating is equivalent to a portfolio long a 5.5%-coupon bond, short a floater.
- Thus, the swap value is equal to the bond value minus par.
- So the value of 100 notional of a 1.5-year 5.5% swap equals the price of 100 par of the 1.5-year 5.5%- bond minus 100:



Receiver Swaption

- A receiver swaption is an option to enter a swap with a pre-specified fixed rate and pre-specified maturity at no cost, i.e., with a strike price of zero. Swaptions are typically American.
- We would value the American swaption using the value-maximizing methodology as before:

	Time 0	Time 0.5	Time 1
			-0.68 Max(-0.68-0, 0) =0
		-0.56 Max(-0.56, 0.9709x0.5x (0+0.03) =0.015	
Swap Receiver swaption	0.04 Max(0.04, 0.9730x0.5x (0.015+0.69) =0.34		0.03 Max(0.03-0, 0) =0.03
		0.69 Max(0.69, 0.9769x0.5x (0.03+0.60) =0.69	
			0.60 Max(0.60-0, 0) =0.60

Option on a Swap = Option on a Bond

- An call on a swap with a strike of zero is equivalent to an call on a bond with a strike of par.
- They have the same exercise value:
- Exercise value of call on swap with a strike equal to 0
  = swap value – 0
  = bond value – par – 0
  = exercise value of call on bond with strike equal to par.
- They have the same wait values, too, since wait values derive from future exercise values.
- Similarly, a put on a swap with a strike of zero is equivalent to an put on a bond with a strike of par.

Payer Swaption = Put on a Bond

- A put on a swap is called a payer swaption. It is the right to enter into a swap with a given fixed rate and maturity *paying fixed* and receiving floating.
- We would value it using the value-maximizing methodology as before:

Time 0

Swap
Payer swaption

0.04
Max(0-0.04, 0.9730x0.5x (0.56+0) =0.27

Time 0.5

-0.56
Max(0- -0.56, 0.9709x0.5x (0.68+0) =0.56

0.69
Max(0-0.69, 0) =0

Time 1

-0.68
Max(0 - -0.68, 0) =0.68

0.03
Max(0 -0.03, 0) =0

0.60
Max(0- 0.60, 0) =0

Swaptions and Cancelable Swaps

- Sometimes in a swap, one counterparty or the other has the option to cancel the swap.
- If the fixed *payer* has the option to cancel, it is a callable swap, which is a swap *minus* a receiver swaption.
- This is similar to a callable bond, in which the issuer, the fixed payer, has the option to call back the bonds.
- If the fixed *receiver* has the option to cancel, it is a putable swap, which is a swap *plus* a payer swaption.
- This is similar to a putable bond.
- A **putable bond** is a bond that can be put back to the issuer by the investor. A putable bond = a bond + a put.
- This is rare, but sometimes issuers use this feature to signal their credit quality, if they think they are undervalued by the market.

Options, Cancelable Bonds, and Swaps

Bonds			Swaps		
A Investor	← Fixed	B Issuer	A Fixed receiver	← Fixed	B Fixed payer

Call on bond w/strike par = Call on swap w/strike 0 = Receiver swaption
Example: the call on the 1.5-year 5.5% bond/swap is worth 0.34)

Put on bond w/strike par = Put on swap w/strike 0 = Payer swaption
Example: the put on the 1.5-year 5.5% bond/swap is worth 0.27)

Callable Bond = Bond – Call
99.70 = 100.04 – 0.34

Callable Swap = Swap – Call
-0.30 = 0.04 – 0.34

Putable Bond = Bond + Put
100.31 = 100.04 + 0.27

Putable Swap = Swap + Put
0.31 = 0.04 + 0.27