Mortgage-Backed Securities

Outline

- Fixed-Rate Mortgages
- Prepayment Models and Valuation Effects
- Pass-Throughs
- CMOs

Reading

- Tuckman and Serrat, Chapter 20

Basic Fixed Rate Mortgage

- With a basic fixed rate mortgage, the borrower is scheduled to make **level monthly payments** consisting of

  interest on the amount of the loan outstanding, at the pre-determined fixed mortgage rate, and

  principal payments which reduce the outstanding loan balance.
- The size of the monthly payment is set so that the original loan is paid off after a prespecified amount of time, typically 30 years.
- In other words, the fixed monthly payment makes the present value of the 30-year stream, discounted at the **mortgage rate**, equal to the principal amount of the loan.

Monthly Payment

- By convention, the quoted mortgage rate is *annualized with monthly compounding.*
- Using the annuity formula from the yield lecture, we can get a closed-form expression for the monthly payment:

$$\text{prin} = \sum_{n=1}^{360} \frac{\text{pmt}}{(1 + r_m / 12)^n} = \frac{\text{pmt}}{r_m / 12}\left(1 - (1 + r_m / 12)^{-360}\right)$$

$$\Rightarrow \text{pmt} = \frac{\text{prin} \times r_m}{12\left(1 - (1 + r_m / 12)^{-360}\right)}$$

- Example: If the original balance is \$100,000 and the mortgage rate is 7.25%, then the monthly payment is \$682.18.

Example: Amortization Schedule for 30-Year, Monthly 7.25% Mortgage

Month	Beginning Principal Balance	Monthly Payment	Monthly Interest	Scheduled Principal Repayment	Ending Principal Balance
1	100,000.00	682.18	604.17	78.01	99,922
2	99,921.99	682.18	603.70	78.48	99,844
3	99,843.51	682.18	603.22	78.96	99,765
4	99.764.55	682.18	602.74	79.43	99,685
360	678.08	682.18	4.10	678.08	0

Note that on any month, the present value of the remaining stream of payments, discounted at the fixed mortgage rate equals the remaining principal balance.

Semi-Annual Payment Formula

- We'll assume semi-annual payments so we don't have to rebuild our binomial tree.
- For a T-year fixed-rate, level-pay mortgage with semi-annual mortgage (coupon) rate *c*, the formulas become

$$\text{prin} = \sum_{n=1}^{2T} \frac{\text{pmt}}{(1+c/2)^n} = \frac{\text{pmt}}{c/2}\left(1-(1+c/2)^{-2T}\right)$$

$$\Rightarrow \text{pmt} = \frac{\text{prin} \times c/2}{1-(1+c/2)^{-2T}}$$

- Example: For a 1.5-year, 5.5% mortgage with semi-annual payments and $100 principal, the semi-annual payment is $35.18.

Amortization Schedule for 1.5-Year 5.5% Semi-Annual Mortgage

Period Ending	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.5	100.00	35.18	2.75	32.43	67.57
1.0	67.57	35.18	1.86	33.33	34.24
1.5	34.24	35.18	0.94	34.24	0

- We can think of this as
  - a single mortgage,
  - a pool of identical mortgages, or
  - a *pass-through* security that receives a fixed fraction of all cash flows that flow through the pool.

Benchmark Prepayment Models

To illustrate various features of fixed rate mortgages and their impact on MBS valuation and risk management, let's consider three benchmark models:

1) No prepayment: Mortgage = noncallable bond with amortizing principal.
2) Value-minimizing prepayment: Mortgage = callable bond with amortizing principal.
3) Deterministic prepayment: Sometimes mortgagors are forced to prepay when rates are high because they have to move. There is a baseline average rate of prepayment that is not interest rate dependent. Mortgage = shorter-term bond.
4) Mixture: Actual mortgages contain elements of all these features.

Benchmark 1: No Prepayment

Period Ending	Beginning Balance	Scheduled Payment (formula)	Interest (BBx5.5%/2)	Principal (SP-Int)	Ending Balance (BB-Prin)
0.5	100.00	35.18	2.75	32.43	67.57
1.0	67.57	35.18	1.86	33.33	34.24
1.5	34.24	35.18	0.94	34.24	0

- With no prepayment, the mortgage would just be a stream of three fixed cash flows, each equal to 35.18.
- It could be valued as a package of zeroes:

  35.18 x (0.973047 + 0.947649 + 0.922242) = 100.02

Mortgagor's Prepayment Option

Period Ending	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.5	100.00	35.18	2.75	32.43	67.57
1.0	67.57	35.18	1.86	33.33	34.24
1.5	34.24	35.18	0.94	34.24	0

- The mortgagor has the option to pay off the mortgage at any time without penalty by paying the remaining principal balance.
- For example, with the mortgage above, the mortgagor can pre-pay an additional 67.57 at time 0.5 (on top of the scheduled payment of 35.18) and remove the obligation to pay the remaining two payments.
- Or the mortgagor could pay 34.24 at time 1 and get out of the last payment.

Mortgagor's Prepayment Option

- Think of paying off the mortgage as buying back the remaining stream of payments.
- Then the prepayment option is an American call option where
  - the underlying asset is the remaining stream of payments
  - the strike price is the remaining principal balance.
- The option is
  - at the money when the market yield on the remaining (nonprepayable) monthly payments is equal to the original mortgage rate,
  - in the money when the market rate is below the mortgage rate
  - out of the money when the market rate is above the mortgage rate.

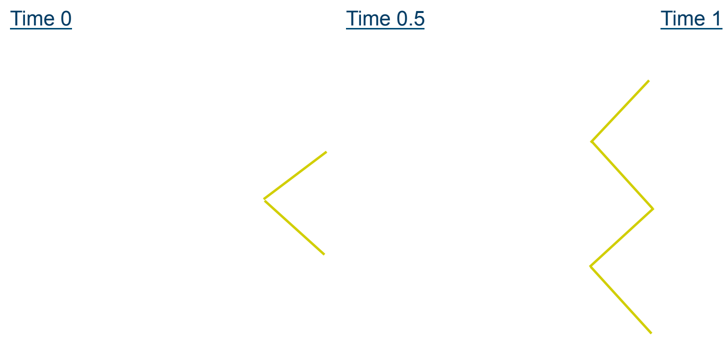
Benchmark 2: Value-Minimizing Prepayment Policy

- If the borrower prepays in a way to minimize the cost of the mortgage, then mortgage is like a callable bond.
- Each period, the borrower chooses to prepay or wait, according which action minimizes the mortgage value.
- We'll ignore the possibility of partial prepayment.
  - If partial prepayments were applied to reduce the level of each remaining payment equally, then value-minimizing would dictate prepaying all or nothing.
  - In practice, partial payments apply to the latest payments first. In an upward-sloping yield curve, this means that the options that are the least in the money, or most out of the money, must be exercised first.

Mortgage with Value-Minimizing Prepayment

- At each state, the borrower can leave the loan outstanding, or else pay off the loan by paying the remaining principal balance in addition to the currently scheduled payment.
- **Class Problem**: Assume the borrower chooses the action that minimizes the mortgage value and fill in the tree of decisions and values.

At each node, the mortgage value is the minimum of remaining principal and wait value.



The number at each node represents the value of the remaining cash flows from the mortgage, excluding the currently *scheduled* payment.

Non-Market-Based Prepayment

- If all borrowers prepaid according to a strategy that minimized the mortgage value (maximized the option value), mortgage cash flows would be a function of interest rates and could be valued by replication and no arbitrage, just as we valued callable bonds.
- However, some prepayment occurs because mortgagors sell the house and pay off the loan, even when rates are high.
- Agency MBS are guaranteed against defaults. In case of a default, the remaining principal balance is passed through to the MBS like a prepayment.
- Non-market-based prepayments that are uncorrelated across borrowers average out in a large enough pool (by the law of large numbers) and a well-diversified pool just experiences the average non-market-based prepayment rate across borrowers.
- With a large enough pool, this becomes deterministic.

Benchmark 3: Deterministic Prepayment

- Let's use a simple stylized example.
- Suppose that with certainty 50% of the pool prepays at time 0.5, regardless of the level of interest rates, and another 50% of the remaining mortgages prepay at time 1 (leaving 25% continuing to time 1.5.).
- Then the cash flows to the pool are riskless and described as follows:

Period Ending	**Beg. Bal.**	**Scheduled Payment** SPxrem.frac.	**Interest** BBx5.5%/2	**Principal** SP-Int	**Principal Prepayment** pp.rate x EB	**End. Bal.** BB-SP-PP
0.5	100.00	35.18	2.75	32.43	33.78	33.78
1.0	33.78	17.59	0.93	16.66	8.56	8.56
1.5	8.56	8.80	0.24	8.56	0	0

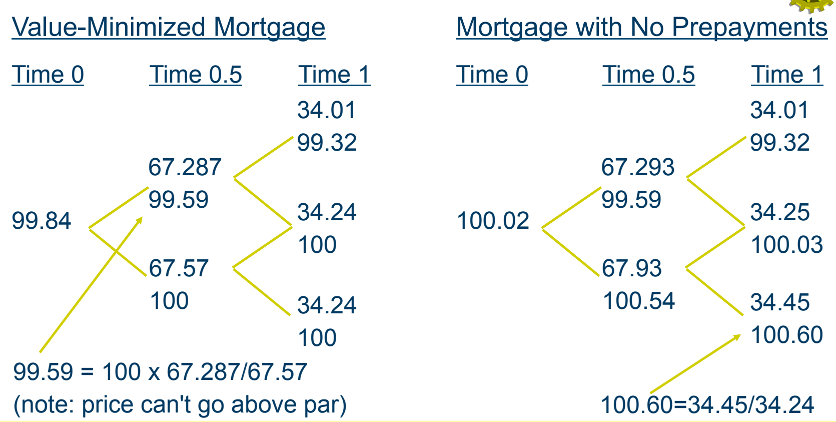
- **Class Problem:** What would the mortgage be worth in this case?

Some Prepayment Measures and Deterministic Prepayment Scenarios Used in Practice

- SMM – Single Monthly Mortality rate: proportion of remaining pool that prepays over the month
- CPR – Conditional Prepayment Rate: annualized prepayment rate (SMMx12)
- 12-Year Average Life scenario: assume no prepayment until year 12, then all at once
- FHA experience: schedule of prepayments based on data
- PSA (Public Securities Association) convention for 30-year mortgages: 0.2% CPR in month 1, 0.4% CPR in month 2, …, 6% CPR in month 30, then 6% CPR in months 31-360.

Practitioners sometimes quote prepayment scenarios as a percent of this PSA schedule.

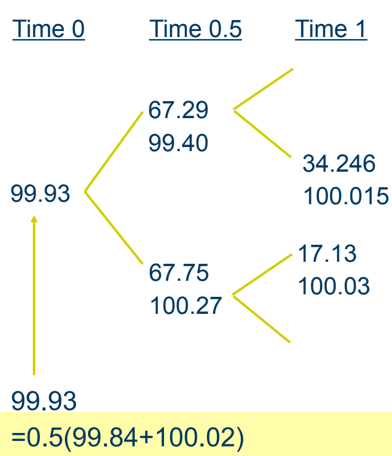
Effects of Pooling Different Types of Borrowers



At each node above, the first number is the total pool value, and the second is the value as a percent of remaining principal.

Mortgage Pool with Both Types

Now consider a pool consisting half of mortgages that never prepay and half of mortgages that optimally prepay.



- In the beginning, the value of the pool is just the average of the value of the two types.
- However, over time, if the optimal prepayers do prepay, then the composition of the pool changes. The remaining pool consists of non-prepayers.
- Often, older mortgages prepay slower because the faster prepayers have dropped out. This slow down in prepayment speeds is called *burnout*.
- If prepayments depend on the level of interest rates, then the mortgage pool value is also *path-dependent*. For example, at time 1 in the middle state, the mortgage price is higher if the path was down-up than up-down, because the optimal prepayers are gone.

Benchmark 4: Mortgage Prepayment Rates with Both Value-Minimizing and Noise

Value-Minimizing PPMT Rates		Pure Noise PPMT Rates		Mixture PPMT Rates	
Time 0	Time 0.5 …	Time 0	Time 0.5 …	Time 0	Time 0.5 …
	0%		50%		20% e.g., 20% of borrowers move house
	100%		50%		90% e.g., 10% of borrowers can't get refinancing

Valuation Approach in Practice

Rather than develop "structural" models of prepayment behavior that detail the decision-making process of mortgagors, we estimate a reduced-form prepayment model:

1) Estimate an empirical model of prepayments as a function of current and past interest rates, pool age and size, seasonality, and other variables.

   For example, prepayment rate this period = α*exp(β1*(coupon minus rate) + β2*(lagged rates) + β3*(percent of pool outstanding) + β4*(1 if summer) + ...

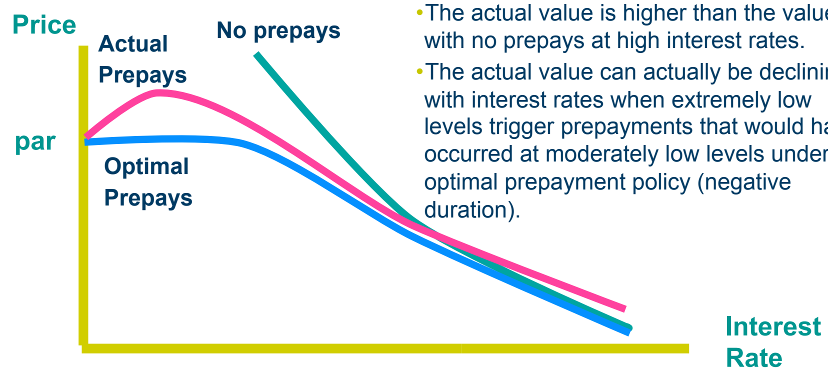
Valuation Approach in Practice…

2) Calculate the value of this empirically estimated payoff function as its risk-neutral expected discounted value as follows:
   a) Simulate mortgage cash flows along thousands of different paths in the interest rate tree using the estimated prepayment function to determine prepayments along each path.
   b) Discount the cash flows along each path back to time 0, using the short rates along the path.
   c) Average the discounted payoff value across the different paths, weighting by the risk-neutral probability of each path.
   d) The simulation is necessary because there are typically too many paths to do the calculation exactly. For example, in a 30-year monthly tree, there are 2^{360} paths.

Option-Adjusted Spread (OAS)

- The option-adjusted spread on a given mortgage-backed security implied by its market price is the spread one would need to add to each of the short rates on the interest rate tree to make the model price equal the market price.
- This gives a measure of the cheapness of the mortgage-backed security, after accounting for the presence of the borrower's prepayment option.

Price-Rate Curve for Pool



- The optimal value is always the lowest.
- The actual value is higher than the value with no prepays at high interest rates.
- The actual value can actually be declining with interest rates when extremely low levels trigger prepayments that would have occurred at moderately low levels under the optimal prepayment policy (negative duration).

CMOs

- The simplest way to create securities from a pool of mortgages is to create pass-throughs, each receiving a fixed share of the cash flows that pass through the pool.
- In practice, issuers of MBS have found it profitable to carve up the cash flows from a mortgage pool in more exotic ways, for example selling simpler short-term securities to money market finds and complex residuals to mortgage and hedge funds.
- The resulting securities are called *collateralized mortgage obligations* (CMOs).
  - IOs and POs are examples of CMOs, also called strips.
  - Another scheme is to form *sequential tranches*.

Basic Tranche CMOs

- Each tranche is a claim to a certain amount of principal, plus interest on that principal. The tranches are ordered.
- At first, Tranche A receives all principal payments and prepayments to the pool, plus interest on that principal, until its principal is paid off.
- All other tranches receive only interest on their outstanding principal.
- After tranche A is paid off, then tranche B receives all principal payments, plus interest. Later tranches receive only interest.
- After tranche B is paid off, tranche C gets principal, etc.

Basic Tranche CMOs

- For example, consider the 1.5-year, 5.5% semi-annual mortgage from the previous example.
- Divide this into three tranches.
  - Tranche A gets the first $30 of principal.
  - Tranche B gets the next $30 of principal.
  - Tranche C gets the last $40 of principal.

Cash Flows to Tranches Assuming No Prepayments

Pool

Period Ending	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.5	100.00	35.18	2.75	32.43	67.57
1.0	67.57	35.18	1.86	33.33	34.24
1.5	34.24	35.18	0.94	34.24	0

Tranches

Period Ending	A Int	A's BB = 30 A Prin	B Int	B's BB = 30 B Prin	C Int	C's BB = 40 C Prin
0.5	0.83	30.00	0.83	2.43	1.10	0.00
1.0			0.76	27.57	1.10	5.76
1.5					0.94	34.24

Benchmark 1: CMO Valuation Assuming No Prepayment

- With no prepayment, each tranche would just be a stream of four fixed cash flows, which could be valued as a package of zeroes:
- A: (0.83+30) x 0.973047 = 29.99
- B: (0.83 + 2.43) x 0.973047 + (0.76 + 27.57) x 0.947649 = 30.01
- C: 1.10 x 0.973047 + (1.10 + 5.76) x 0.947649 +
  (0.94 + 34.24) x 0.922242 = 40.02
- Note that the values of the tranches sum to the value of the pool: 29.99 + 30.01 + 40.02 = 100.02

Benchmark 3: CMOs with Deterministic Prepayment

Pool

Period Ending	Beg. Bal.	Sched. Payment SPxrem.frac.	Interest BBx5.5%/2	Principal SP-Int	Principal Prepaymt pp.ratexEB	End. Bal. BB-SP-PP
0.5	100.00	35.18	2.75	32.43	33.78	33.78
1.0	33.78	17.59	0.93	16.66	8.56	8.56
1.5	8.56	8.80	0.24	8.56	0	0

Tranches

Period Ending	A Int	A's BB = 30 A Prin	B Int	B's BB = 30 B Prin	C Int	C's BB = 40 C Prin
0.5	0.83	30.00	0.83	30.00	1.10	6.21
1.0					0.93	25.22
1.5					0.24	8.56

Benchmark 3: CMO Valuation Assuming Deterministic Prepayment

- With deterministic prepayments, each tranche would just be a stream of four fixed cash flows, which could be valued as a package of zeroes:
- A: (0.83 + 30.00) x 0.973047 = 29.99
- B: (0.83 + 30.00) x 0.973047 = 29.99
- C: (1.10 + 6.21) x 0.973047 + (0.93 +25.22) x 0.947649 + (0.24 + 8.56) x 0.922242 = 40.01
- Again, the values of the tranches sum to the value of the pool (allowing for rounding error):
  - 29.99 + 29.99 + 40.01 = 100.0025

Benchmark 2: CMOs with Pool-Value-Minimizing Prepayments

- Tranche A pays off in full at time 0.5 regardless of the level of interest rates, so A is still worth 29.99.
- If rates go up at time 0.5, tranche B pays out on schedule.
- If rates go down at time 0.5, tranche B pays off in full at time 0.5.
- Tranche C can be valued as the whole pool minus A minus B.

Time 0

A: 29.99 = 0.973047x(0.83 + 30.00)
B: 29.96 = 0.973047x[0.83 + 2.43 + 0.5(27.50 + 27.57)]
C: 39.89 = 99.84 – 29.99 – 29.96

Time 0.5

A: 0
B: 27.50 = 0.970857x(0.76 + 27.57)
C: 39.78 = 67.29 – 27.50

A: 0
B: 27.57
C: 40.00

Note that the value-minimization is done by the borrower at the level of the *whole mortgage*. For the derivatives, *there is no further optimization*. The derivative (CMO) cash flows are simply derived from those of the whole mortgage.

Benchmark 2: CMOs with Pool-Value-Minimizing Prepayments

- Tranche A pays off in full at time 0.5 regardless of the level of interest rates, so A is still worth 29.99.
- If rates go up at time 0.5, tranche B pays out on schedule.
- If rates go down at time 0.5, tranche B pays off in full at time 0.5.
- Tranche C can be valued as the whole pool minus A minus B.

Time 0

A: 29.99 = 0.973047x(0.83 + 30.00)
B: 29.96 = 0.973047x[0.83 + 2.43 + 0.5(27.50 + 27.57)]
C: 39.89 = 99.84 – 29.99 – 29.96

Time 0.5

A: 0
B: 27.50 = 0.970857x(0.76 + 27.57)
C: 39.78 = 67.29 – 27.50

A: 0
B: 27.57
C: 40.00

- The total cost of the embedded prepayment option is 100.02–99.84=0.18.
- Most of this, and the negative convexity, is borne by Tranche C: 40.02–39.89= 0.13.