

Caps, Floors, and Collars

Concepts and Buzzwords

- Caps
- Capped Floaters
- Floors
- Floaters with Floors
- Collars
- Floaters with Collars
- Strike rate, settlement frequency, index, notional amount, calls on yields, puts on yields, portfolio of options

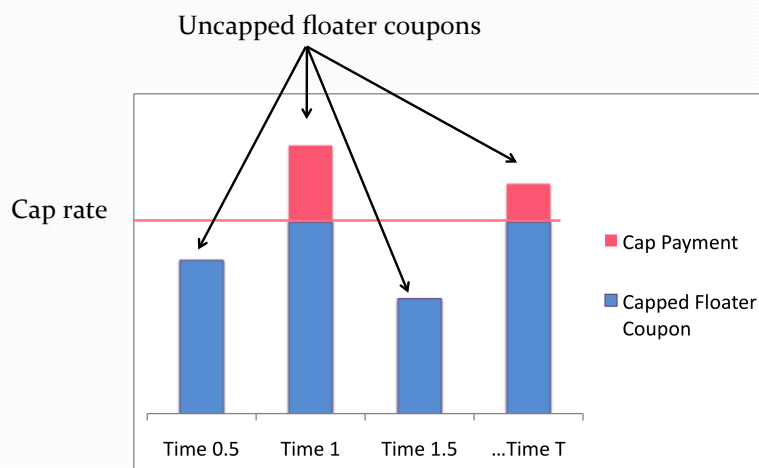
Reading

- Tuckman and Serrat, Chapter 18
- Veronesi, pp.387-392

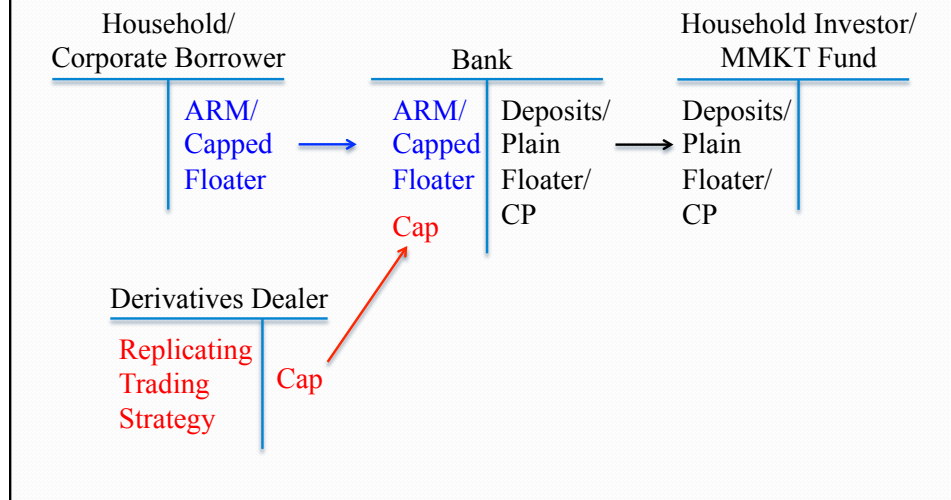
Interest Rate Caps

- A cap provides a guarantee to the issuer of a floating or variable rate note or adjustable rate mortgage that the coupon payment each period will be no higher than a certain amount.
- In other words, the coupon rate will be *capped* at a certain *ceiling* or *cap rate* or *strike rate*.
- Caps are either offered over-the-counter by dealers or embedded in a security.

Cap and Capped Floater Coupons



Macro Finance Picture: Caps in the Financial System



Payoff Rule for a Typical Cap

- Each payment date, the cap pays the difference, if positive, between a floating *index* rate and the cap rate, multiplied by a prespecified *notional amount* of principle or par value, divided by the annual payment *frequency*.
- For example, a T -year, semi-annual cap, indexed to the 6-month rate, with \$100 notional principle, and cap rate k pays

$$100 \max(r_{t-0.5} - k, 0) / 2 \text{ at time } t, t = 0.5, 1, 1.5, \dots, T$$

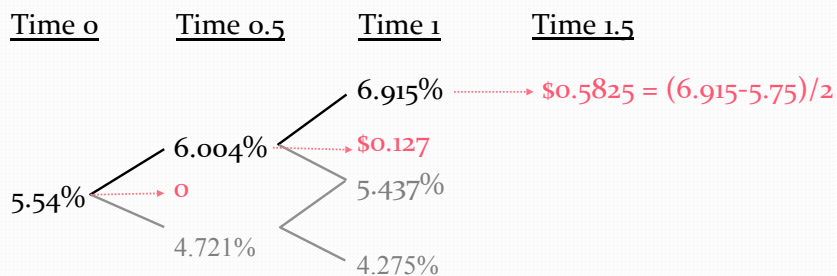
Payoff Rule for Capped Floater

- Consider the net position of the issuer of \$100 par of a floating rate note who either buys a matching cap from a dealer or else embeds the cap in the note at issue:
- Capped Floater = Floater minus Cap
- Time t Coupon Payment of Capped Floater

$$100 \min({}_{t-0.5}r, k) / 2 = 100 \cdot {}_{t-0.5}r / 2 - 100 \max({}_{t-0.5}r - k, 0) / 2$$

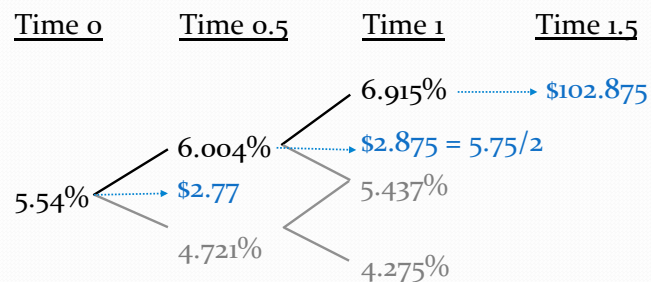
Example: Cap Payments

- Consider a \$100 notional of 1.5-year semi-annual cap with strike rate $k = 5.75\%$, indexed to the 6-month rate.
- At time 0, the 6-month rate is 5.54%, so the cap is out-of-the-money, and pays 0 at time 0.5.
- The later cap payments depend on the path of interest rates. Suppose rates follow the up-up path below.



Example: Capped Floater Payments

- Consider a \$100 par of 1.5-year semi-annual floater with coupon capped at $k = 5.75\%$.
- Here are the capped floater payments along the up-up interest rate path below:



Caplet (Call on Yield)

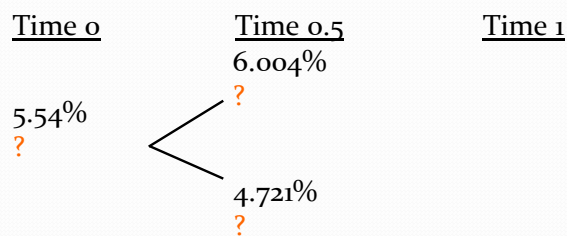
- The individual cap payments are called caplets.
- Each caplet is a kind of call on the 0.5-year rate set 0.5-years before the payment date
- The payoff of the caplet on the time $t-0.5$ -year rate, for \$100 notional amount and strike rate k , is

$$100 \max(r_{t-0.5} - k, 0) / 2$$

- We can value each caplet individually, and then sum their values to get the total cap value.
- For example, the 1.5-year cap is the sum of
 - a caplet on $r_{0.5}$ -- payoff known at time 0, paid at time 0.5
 - a caplet on ${}_{0.5}r_1$ -- payoff known at time 0.5, paid at time 1
 - a caplet on ${}_1r_{1.5}$ -- payoff known at time 1, paid at time 1.5

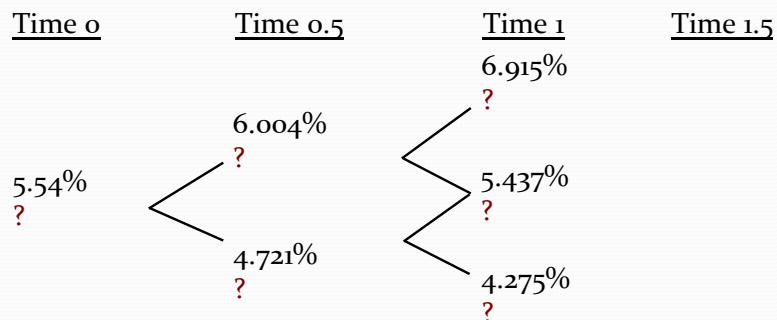
Class Problem: Caplet on ${}_{0.5}r_1$

- Recall the \$100 notional of 1.5-year semi-annual cap with strike rate $k = 5.75\%$.
- Consider its time 1 payment, **the caplet on ${}_{0.5}r_1$**
- Fill in the tree of values of this caplet below.



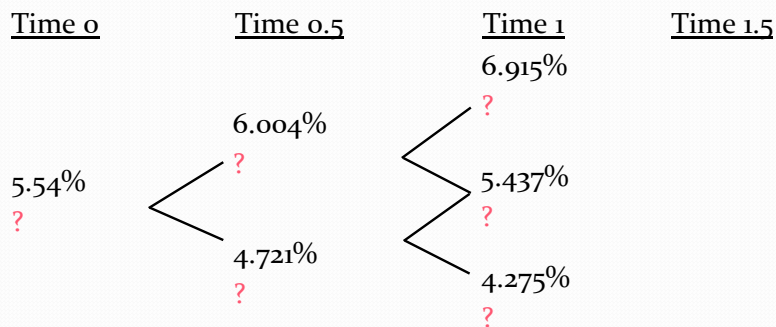
Class Problem: Caplet on ${}_1r_{1.5}$

- Consider the time 1.5 payment of the cap, **the caplet on ${}_1r_{1.5}$** .
- Fill in the tree of values of this caplet below.



Class Problem: The Whole Cap

Fill in the tree of values of **the whole cap** below. At each node, indicate the value excluding the currently scheduled payment.



2) What is the SR dollar duration of the cap?

3) What is the SR duration of the cap?

Class Problems: The Capped Floater

- 1) What is the value of a \$100 par of a 1.5-year floater with coupon rate capped at 5.75%?
- 2) What is the SR dollar duration of \$100 par of a plain vanilla, uncapped floater?
- 3) What is the SR duration of a plain vanilla floater?
- 4) What is the SR dollar duration of the capped floater?
- 5) What is the SR duration of the capped floater?

Interest Rate Floors

- A floor provides a guarantee to the *owner* of a floating note that the coupon payment each period will be no *less* than a certain floor rate or strike rate.
- Floors are generally embedded in a floating rate note, but could also be purchased separately from a dealer.

Payoff Rule for Typical Floor

- Each payment date, the floor pays the difference, if positive, between the floor rate and the floating rate multiplied by the notional amount of principle or par value, divided by the annual payment frequency.
- For example, a T -year, semi-annual floor, indexed to the 6-month rate, with \$100 notional principle, and floor rate k pays

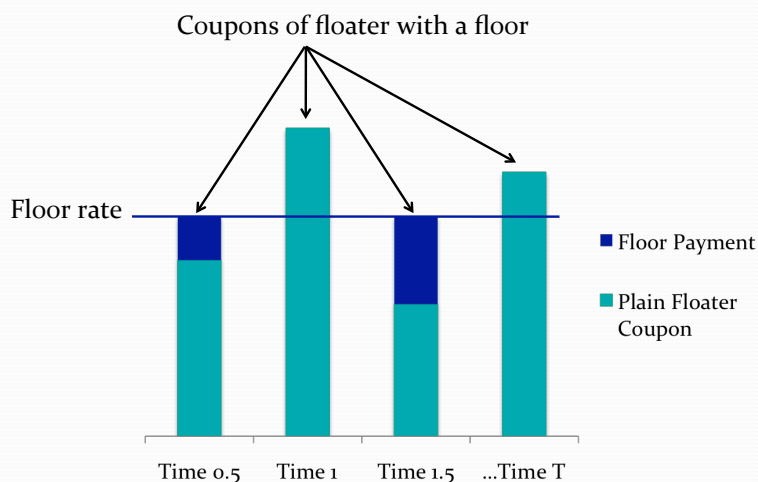
$$100 \max(k - r_{t-0.5}, 0) / 2$$
 at time t , $t = 0.5, 1, 1.5, \dots, T$

Floater with a Floor

- Consider the net position of an investor who holds \$100 par of a floating rate note with an embedded floor or else holds \$100 par of a floater and has purchased a floor separately.
- Floater with a Floor = Floater plus Floor
- Time t Coupon Payment of Floater with a Floor

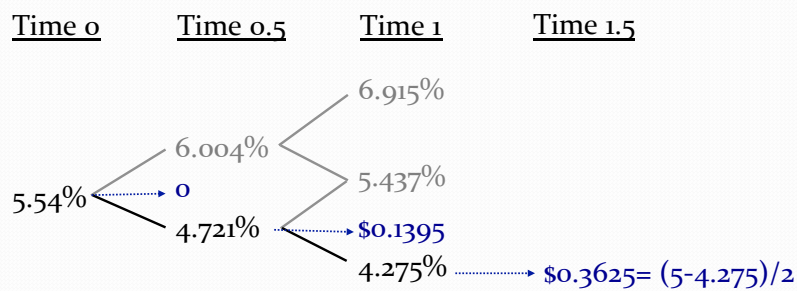
$$100 \max(r_{t-0.5}, k)/2 = 100 r_{t-0.5}/2 + 100 \max(k - r_{t-0.5}, 0)/2$$

Floor and Floater Coupons



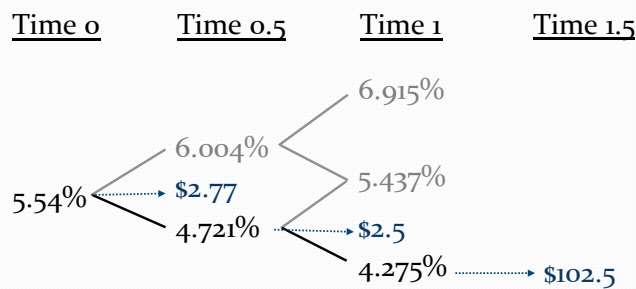
Example: Floor Payments

- Consider a \$100 notional of 1.5-year semi-annual floor with strike rate $k = 5\%$, indexed to the 6-month rate.
- At time 0, the 6-month rate is 5.54%, so the floor is out-of-the-money, and pays 0 at time 0.5.
- The later floor payments depend on the path of interest rates. Suppose rates follow the down-down path below.



Example: Floater with a Floor

- Consider a \$100 par of 1.5-year semi-annual floater with a 5% floor.
- Here are the capped floater payments along the down-down interest rate path below:



Floorlet (Put on Yield)

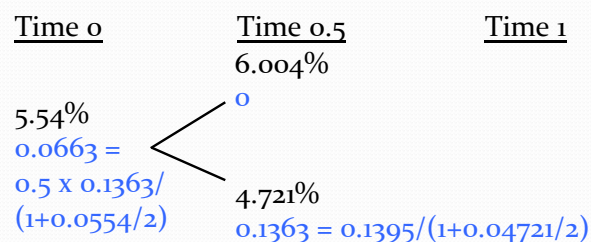
- The individual floor payments are called floorlets.
- Each floorlet is a kind of put on the 0.5-year rate set 0.5-years before the payment date
- The payoff of the floorlet on the time $t-0.5$ -year rate, for \$100 notional amount and strike rate k , is

$$100 \max(k - r_{t-0.5}, 0)/2$$

- We can value each floorlet individually, and then sum their values to get the total floor value.
- For example, the 1.5-year floor is the sum of
 - a floorlet on $r_{0.5}$ -- payoff known at time 0, paid at time 0.5
 - a floorlet on ${}_{0.5}r_1$ -- payoff known at time 0.5, paid at time 1
 - a floorlet on ${}_{1.5}r_1$ -- payoff known at time 1, paid at time 1.5

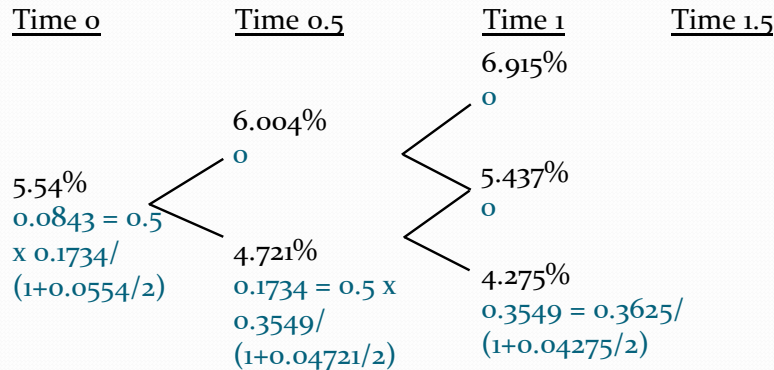
Example: Floorlet on ${}_{0.5}r_1$

- Recall the \$100 notional of the 1.5-year 5% semi-annual floor.
- Consider its time 1 payment, the floorlet on ${}_{0.5}r_1$.
- Fill in the tree of values of this floorlet below.



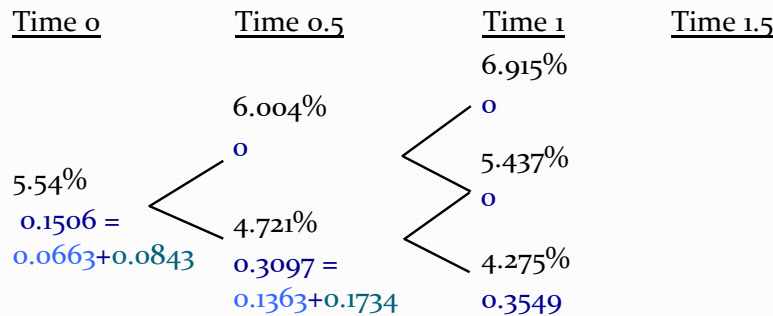
Example: Floorlet on ${}_1r_{1.5}$

- Consider its time 1.5 payment of the floor, the floorlet on ${}_1r_{1.5}$.
- Fill in the tree of values of this floorlet below.



Example: The Whole Floor

Fill in the tree of values of the whole floor. At each node, indicate the value excluding the currently scheduled payment.



The SR dollar duration of the floor is $-(0 - 0.3097) / (6.004\% - 4.721\%) = 24.14$.

The SR duration of the floor is $24.14 / 0.1506 = 160.3$.

The Floater with a Floor

- 1) What is the value of a \$100 par of a 1.5-year floater with 5% floor?

$$100 + 0.1506 = 100.1506$$

- 2) What is the SR dollar duration of this floater with a floor?

$$0 + 24.14 = 24.14$$

- 3) What is the SR duration of this floater with a floor?

$$24.14/100.1506 = 0.24$$

Interest Rate Collars

- A collar is a long position in a cap and a short position in a floor.
- The issuer of a floating rate note might use this to cap the upside of his debt service, and pay for the cap with a floor.
- Collars are generally embedded in a floating rate note, but could also be purchased separately from a dealer.

Example of Collared Floater

- 1) What is the value of \$100 par of a 1.5-year floater with a 5%-5.75% collar?

$$100 + 0.1506 - 0.193 = 99.9576$$

- 2) What is the SR dollar duration of this collared floater?

$$0 + 24.14 - (-30.91) = 55.05$$

- 3) What is its SR duration?

$$55.05/99.9576 = 0.55$$