HowToReadMathematica

Empirical Market Microstructure

(2006, Oxford University Press)

Companion *Mathematica* notebook

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This notebook is a primer for reading *Mathematica* notebooks.

```
Text @ Style["Notebook evaluated " <> DateString["DateTime"], "Subtitle"]
```

Notebook evaluated Wednesday 6 June 2007 17:41:37

Rules

Most *Mathematica* notation mimics normal usage. The major difference is the equals sign. Suppose that we encounter the following two equations in text:

$$m_t = m_{t-1} + u_t \qquad (1)$$

$$p_t = m_t + c\, q_t \qquad (2)$$

Intuitively, we know that these are describing dynamics of some time-subscripted variables. These equations are really "rules" for generating successive realizations of m_t and p_t. In *Mathematica*, equation (1) would be written as;

```
m_{t_} :→ m_{t-1} + u_t
```

$m_{t\_} :\to m_{t-1} + u_t$

The blue background signals that this line is input to *Mathematica*, rather than simply text. The beige background identifies *Mathematica* output. The notation :→ (or sometimes →) means "replace the left-hand side with the right-hand side". The `t_` on the left-hand side of the rule is a pattern (template) that allows use to apply the rule to m_{t+5}, m_{xyz} or $m_{whatever}$, not just m_t. To apply the rule, we use the notation "`/.`". For example:

```
m_t /. m_{t_} :→ m_{t-1} + u_t
```

$m_{-1+t} + u_t$

Note that *Mathematica* writes expressions with numbers before variables: $-1+t$ rather than $t-1$. Similarly:

```
m_{t+5} /. m_{t_} :→ m_{t-1} + u_t
```

```
m_{4+t} + u_{5+t}
```

Mathematica doesn't really know that t indexes time, of course:

```
m_{whatever} /. m_{t_} :→ m_{t-1} + u_t
```

```
m_{-1+whatever} + u_{whatever}
```

The text expression for m_t has a label "(1)" that allows us to subsequently refer to the rule. In *Mathematica*, we assign the rule to a variable:

```
m_{Rule} = m_{t_} :→ m_{t-1} + u_t;
```

The semicolon at the end of the line suppresses output. Now a reference to m_{Rule} invokes the rule we just defined. E.g.:

```
m_t /. m_{Rule}
```

```
m_{-1+t} + u_t
```

We can apply the rule repeatedly:

```
m_t /. m_{Rule} /. m_{Rule} /. m_{Rule}
```

```
m_{-3+t} + u_{-2+t} + u_{-1+t} + u_t
```

And so forth. Here's another rule

```
p_{Rule} = p_{t_} :→ m_t + c q_t;
```

Applying the rules in succession:

```
p_t /. p_{Rule} /. m_{Rule}
```

```
m_{-1+t} + c q_t + u_t
```

We can use rules in defining other rules. Here's an expression for the first difference:

```
Δp_{Rule} = Δp_{t_} :→ (p_t /. p_{Rule} /. m_{Rule}) - (p_{t-1} /. p_{Rule});
```

For example:

```
Δp_t /. Δp_{Rule}
```

```
-c q_{-1+t} + c q_t + u_t
```

Equations and "=="

We also use equal signs to set up equations. *Mathematica* uses the double-equal sign in this context. For example:

```
Solve[0 == a x^2 + b x + c, x]
```

$$\left\{\left\{x \to \frac{-b + \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\}, \left\{x \to -\frac{b + \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\}\right\}$$