

# High Frequency Quoting: Short-Term Volatility in Bids and Offers

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# Disclaimers

- ❑ I teach in an entry-level training program at a large financial firm that is generally thought to engage in high frequency trading.
- ❑ I serve on a CFTC advisory committee that discusses issues related to high frequency trading.
- ❑ I accept honoraria for presentations at events sponsored by financial firms.

# What does quote volatility look like?

- ❑ In US equity markets, bids and offers from all trading venues are consolidated and disseminated in real time.
  - The highest bid is the National Best Bid (NBB)
  - The lowest offer is the National Best Offer (NBO)
- ❑ Next slide: the NBBO for AEPI on April 29, 2011

# Figure 1. AEPI bid and offer, April 29, 2011

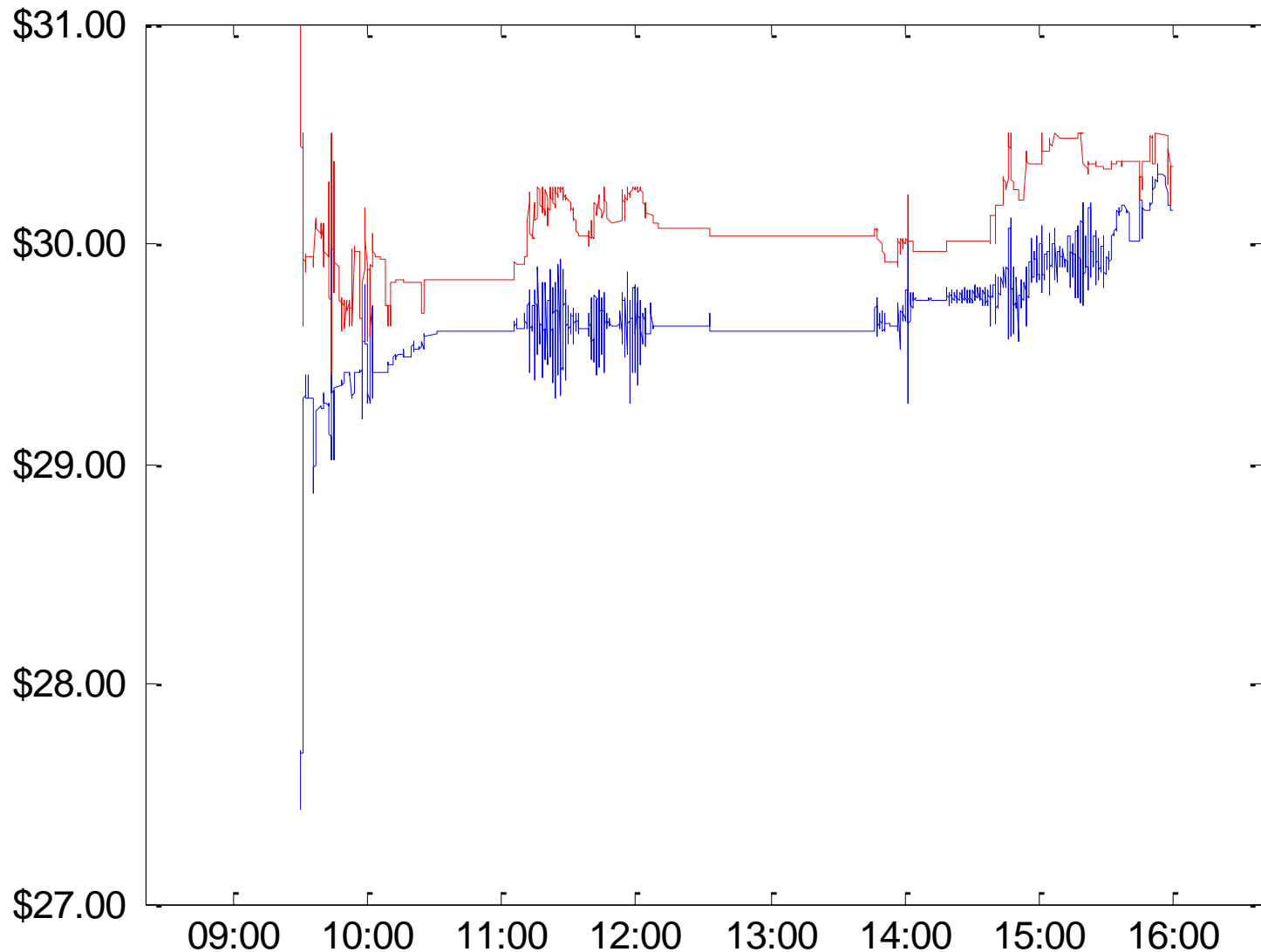
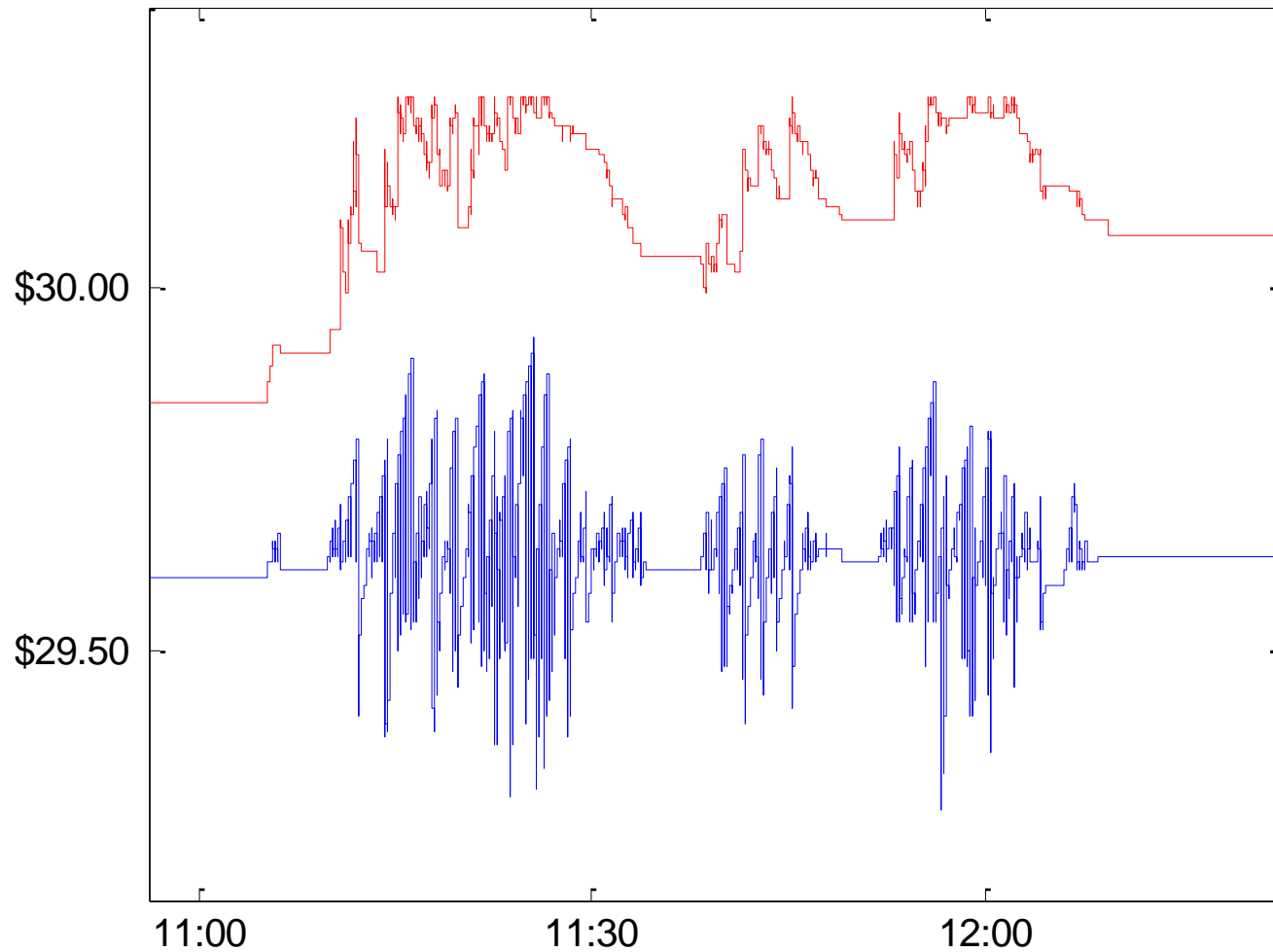
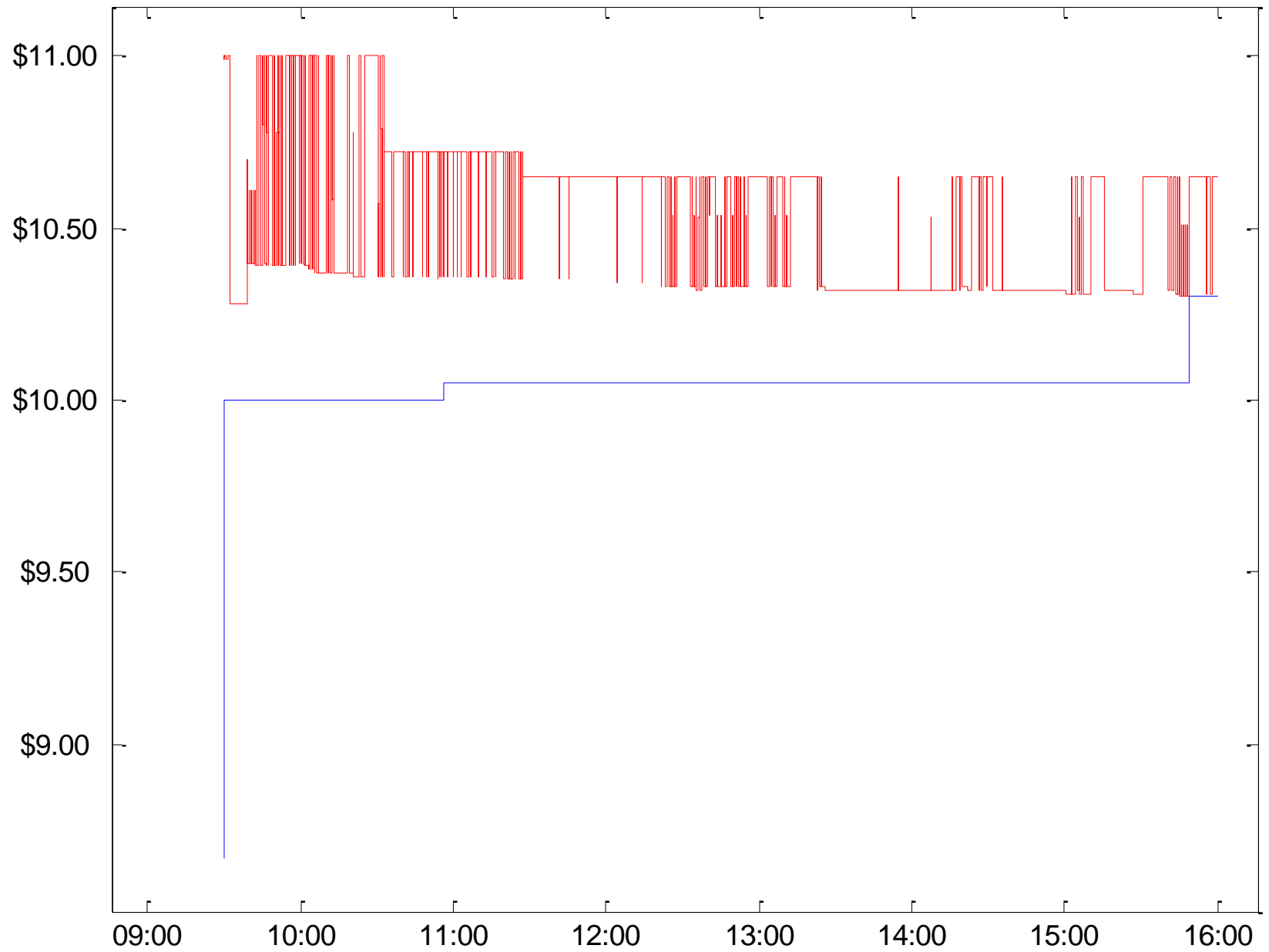


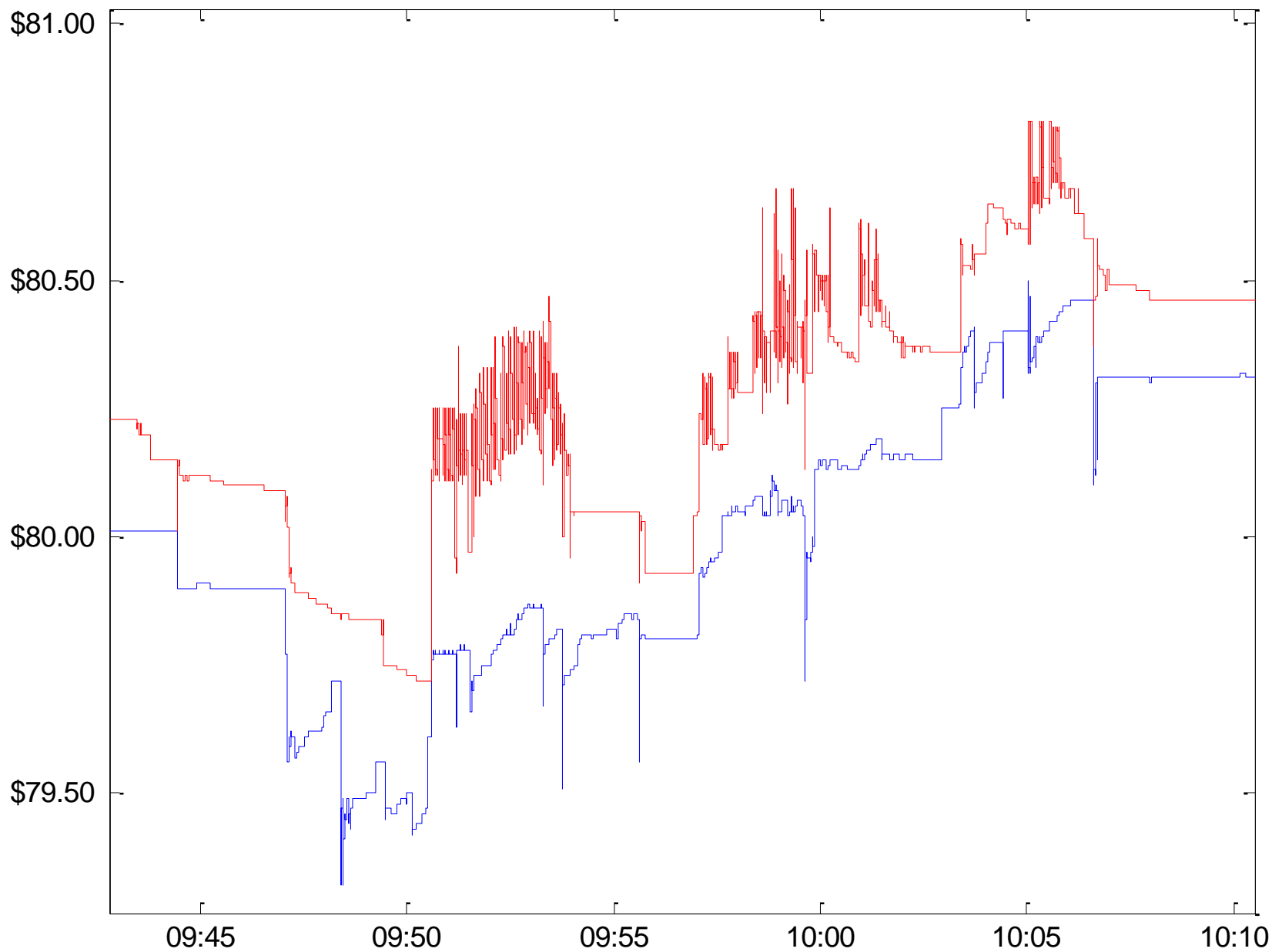
Figure 1. AEPI bid and offer on April 29, 2011 (detail)



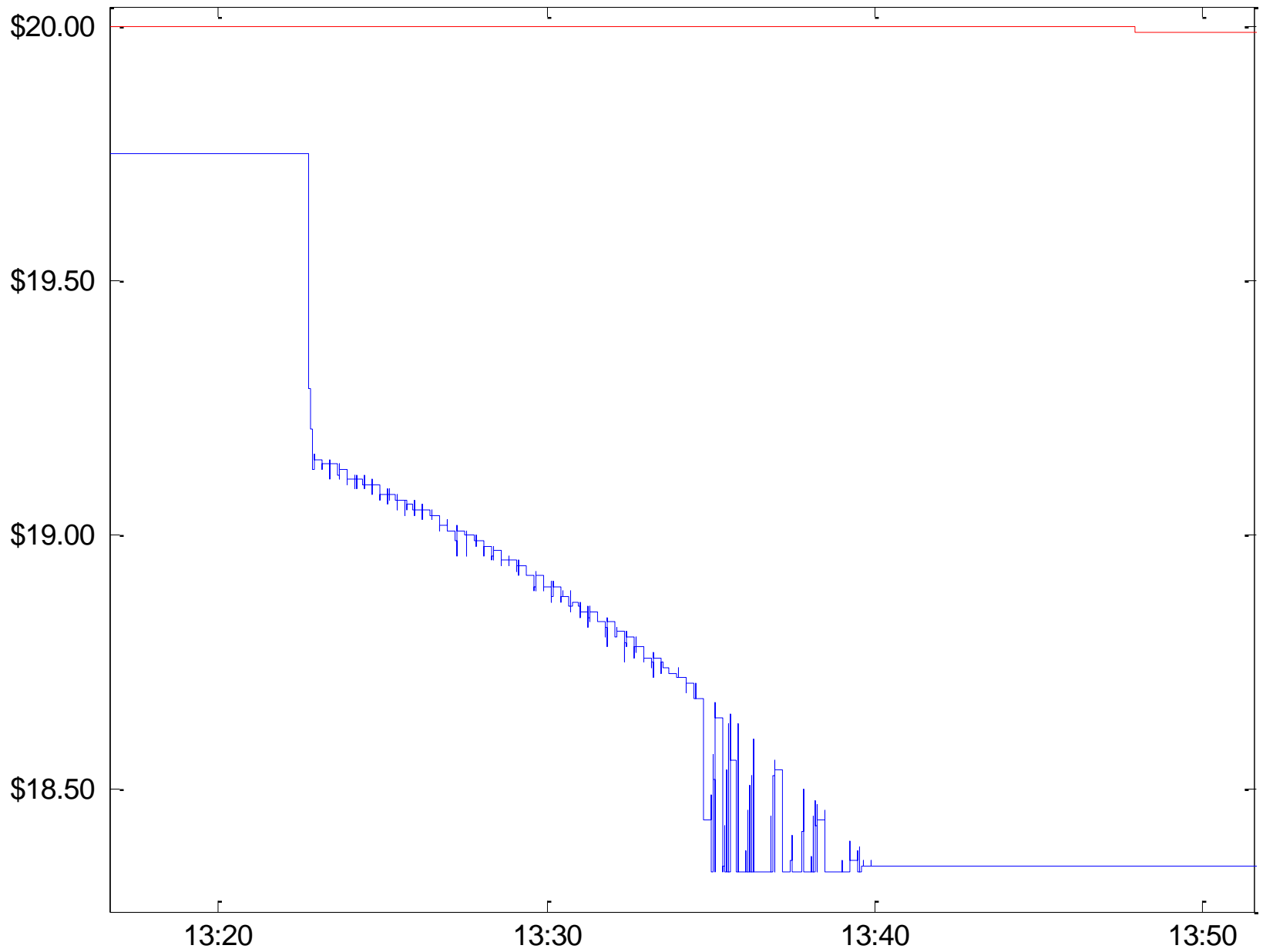
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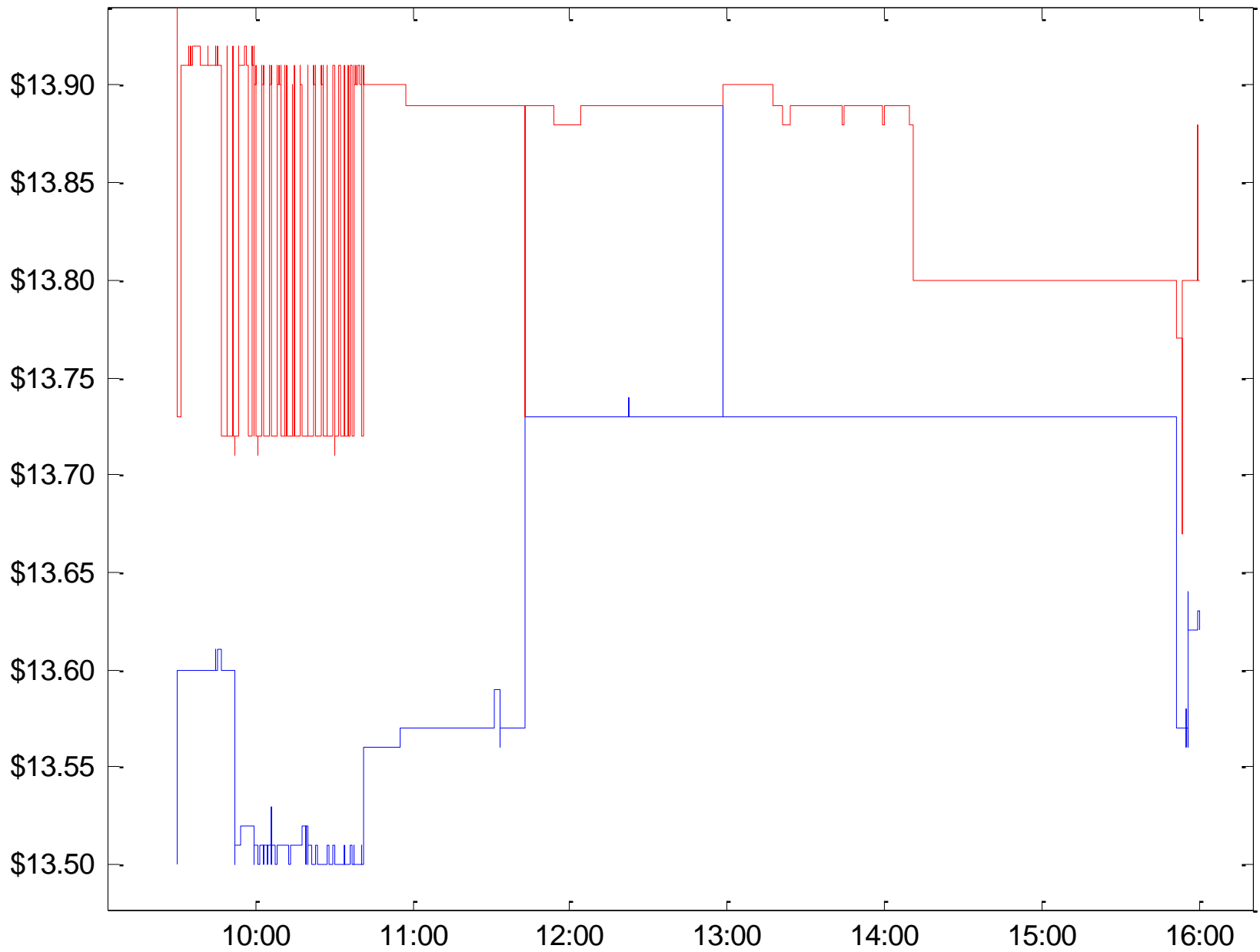
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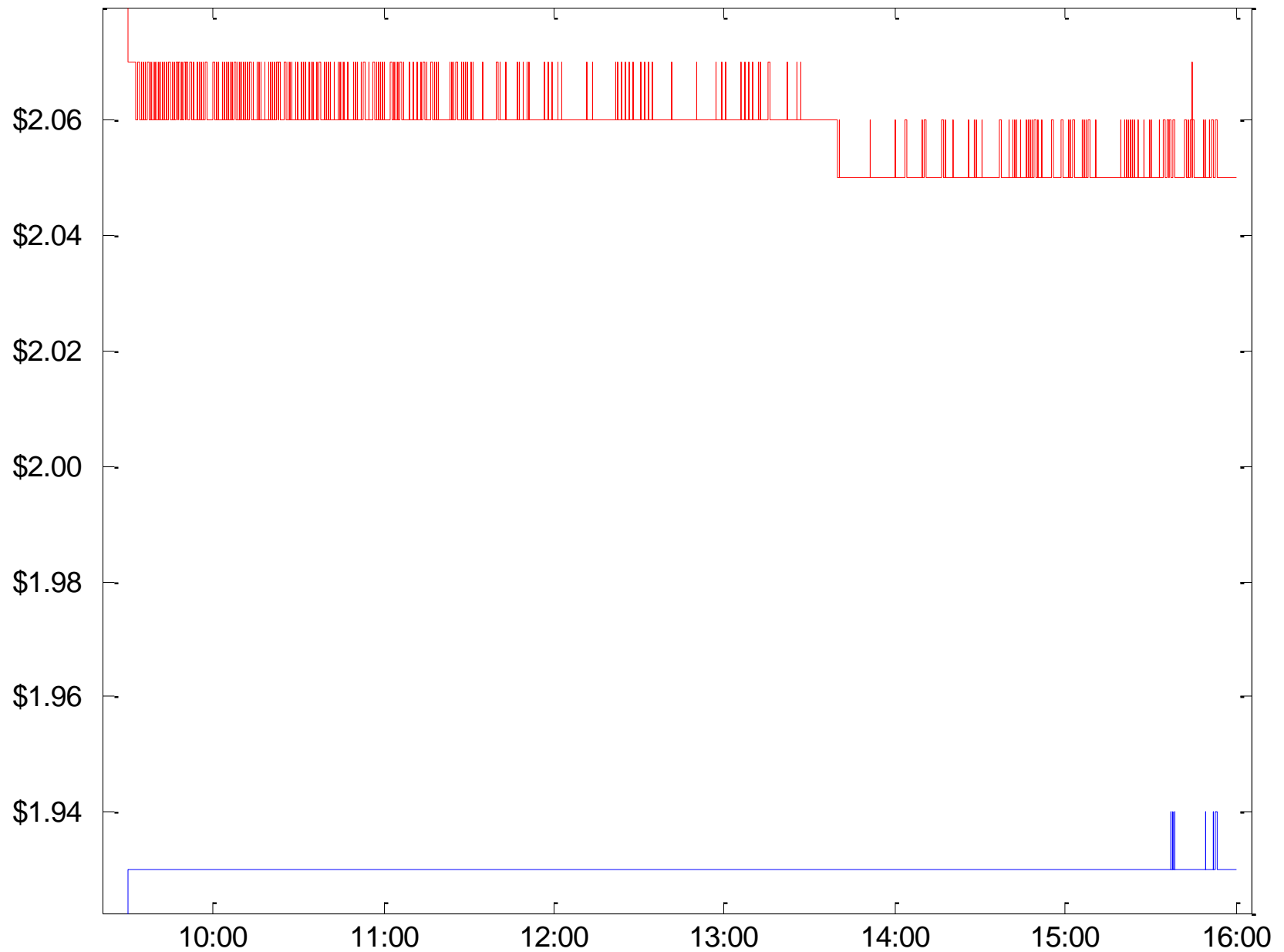
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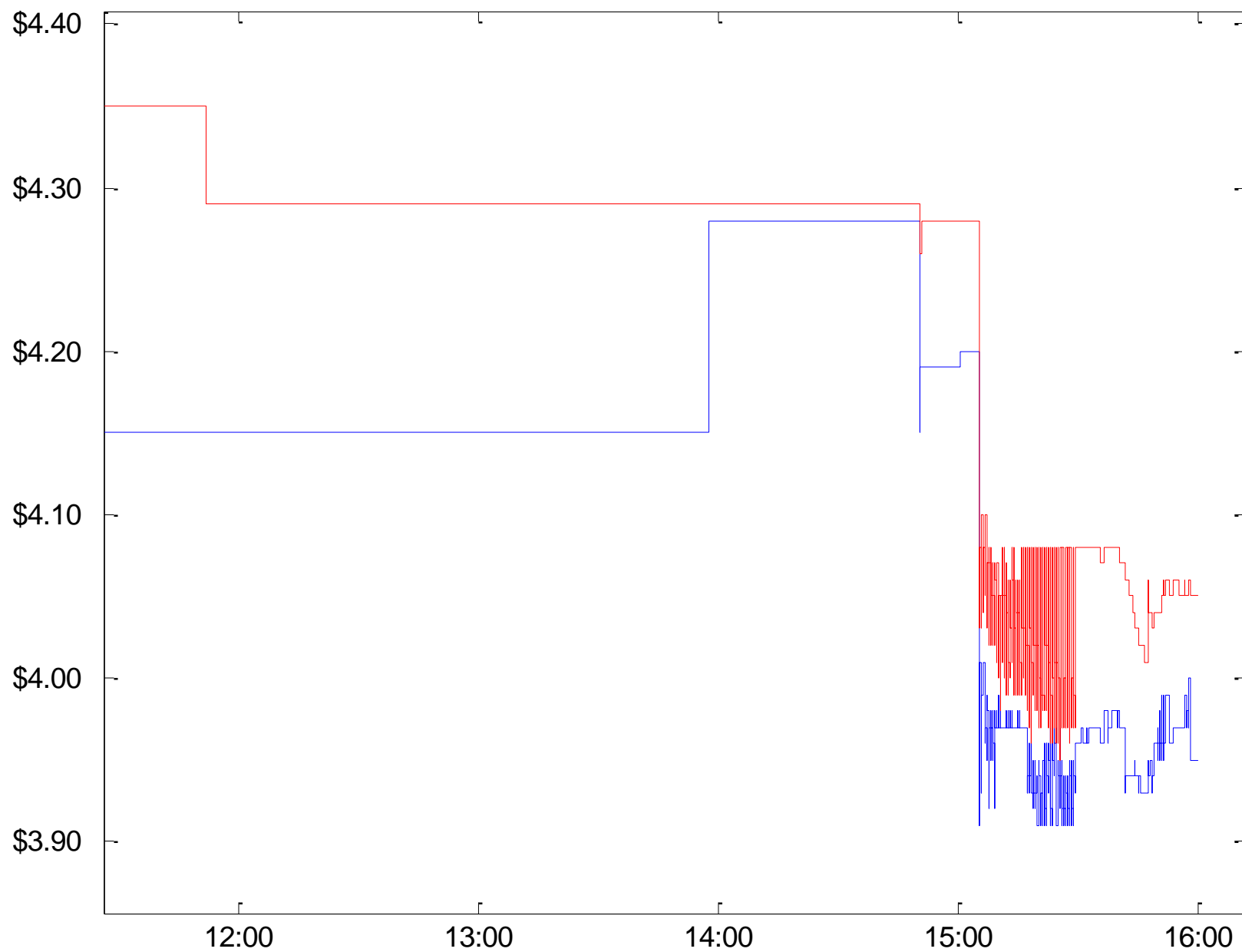
# WSTG 20110404



# AAME 20110418



ADEP 20110427



## Quote volatility: the questions

- ❑ What is its economic meaning and importance?
- ❑ How should we measure it?
- ❑ Is it elevated? Relative to what?
- ❑ Has it increased along with wider adoption of high-speed trading technology?

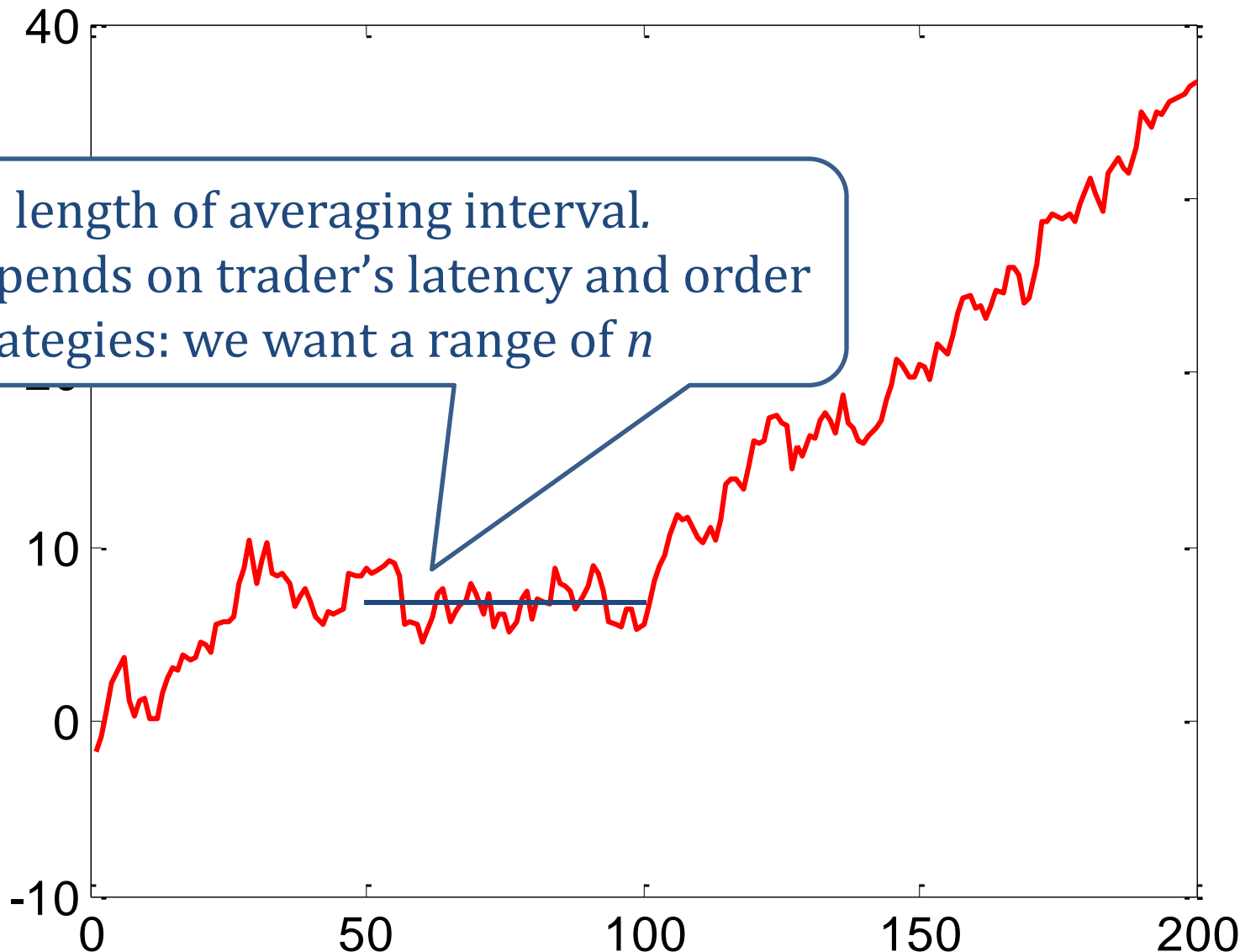
# Economic consequences of quote volatility

- ❑ Noise due to flickering quotes
- ❑ Execution price risk
  - For marketable orders
  - For dark trades
- ❑ Intermediaries' look-back options
- ❑ Quote-stuffing
- ❑ Spoofing

# Descriptive statistics: computation and interpretation

# Local variances about local means

$n$  = length of averaging interval.  
Depends on trader's latency and order  
strategies: we want a range of  $n$



# Interpretation

- ❑ To assess economic importance, I present the (wavelet and rough) variance estimates in three ways.
  - In mils per share
  - In basis points
  - As a short-term/long-term ratio

## Mils per share

- ❑ Variances are computed on bid and offer price levels.
- ❑ Reported volatilities are scaled to *mils/share*.
  - One mil = \$0.001
- ❑ Most trading charges are assessed per share.
  - Someone sending a marketable order to a US exchange typically pays an “access fee” of about three mils/share.
  - An executed limit order receives a “liquidity rebate” of about two mils/share.

## Basis points (One bp = 0.01%)

- Volatilities are first normalized by price (bid-ask average)
- The rough volatility in basis points:
  - $\frac{\sigma_j}{Price} \times 10,000$
- “One bp is a one cent bid-offer spread on a \$100 stock.”

# The short/long variance ratio

- ❑ For a random walk with per period variance  $\sigma^2$ , the variance of the  $n$ -period difference is  $n\sigma^2$ .
- ❑ An conventional variance ratio might be
  - $$V = \frac{60 \times \text{one minute return variance}}{\text{one hour return variance}}$$
- ❑ For a random walk,  $V = 1$ .
  - Microstructure: we usually find  $V > 1$ .
- ❑ Extensively used in microstructure studies: Barnea (1974); Amihud and Mendelson (1987); etc.

# The empirical analysis

CRSP Universe 2001-2011



In each year, chose 150 firms in a random sample stratified by dollar trading volume



**2011 April TAQ  
with one-  
*millisecond* time  
stamps**



**High-resolution  
analysis**

2001-2011  
April TAQ data  
with one-second  
time stamps



Lower-resolution  
analysis

# Table 1. Summary Statistics, 2011

		Dollar trading volume quintile				
	Full sample	1 (low)	2	3	4	5 (high)
No. of firms	150	30	30	30	30	30
NYSE	47	0	5	7	16	19
Amex	6	2	2	0	1	1
NASDAQ	97	28	23	23	13	10
Avg. daily trades	1,331	31	431	1,126	3,478	16,987
Avg. daily quotes	<b>23,928</b>	<b>967</b>	<b>7,706</b>	<b>24,026</b>	<b>53,080</b>	<b>181,457</b>
Avg. daily NBBO records	7,138	328	3,029	7,543	16,026	46,050
Avg. daily NBB changes	1,245	120	511	1,351	2,415	4,124
Avg. daily NBO changes	1,164	103	460	1,361	2,421	4,214
Avg. price	\$15.62	\$4.87	\$5.46	\$17.86	\$27.76	\$51.60
Market capitalization of equity, \$ Million	\$683	\$41	\$202	\$747	\$1,502	\$8,739

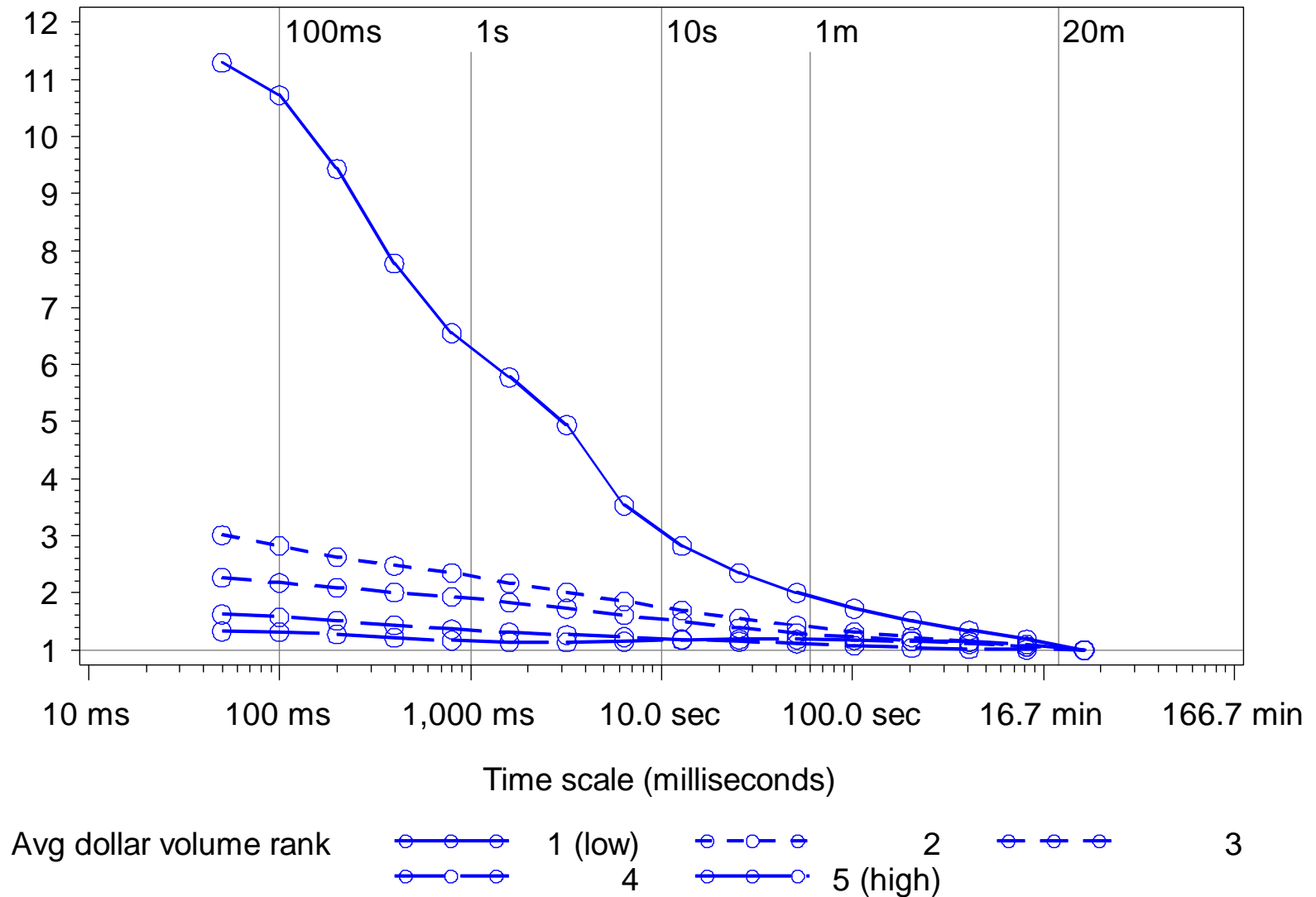
Table 2. Time scale variance estimates, 2011

	Rough volatilities, $\sigma_j$			Wavelet variances, $v_j^2$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Time scale	$\sigma_j$ , mils	$\sigma_j$ , bp	Variance ratio	$\sqrt{v_j^2}$ , mils	$\sqrt{v_j^2}$ , bp	Variance ratio	Bid-Offer Corr
< 50 ms	0.28	0.16	4.22				
50 ms	0.39	0.22	3.99	0.27	0.17		
100 ms	0.55	0.31	3.79	0.39	0.24		
200 ms	0.76	0.43	3.53	0.55	0.34		
400 ms	1.05	0.59	3.21	0.76	0.47		
800 ms	1.46	0.83	2.90	1.05	0.65		
1,600 ms	2.02	1.14	2.64	1.46	0.91		
3.2 sec	2.80	1.58	2.40	1.97	1.22		
6.4 sec	3.90	2.18	2.12	2.73	1.69		
12.8 sec	5.43	2.99	1.88	3.76	2.35		
25.6 sec	7.54	4.10	1.70	5.21	3.26		
51.2 sec	10.48	5.61	1.54	7.22	4.51		
102.4 sec	14.53	7.68	1.42	10.00	6.25		
3.4 min	20.12	10.51	1.32	13.81	8.56		
6.8 min	27.88	14.40	1.23	19.22	11.91		
13.7 min	38.55	19.70	1.16	26.45	16.48	1.08	0.88
27.3 min	52.84	26.79	1.08	35.73	22.28	1.00	0.90

A trader who faces time uncertainty of 400 ms incurs price risk of 1.05 *mils/share* or 0.59 *basis points*.

At a time scale of 400 ms., the rough variance is 3.21 times the value implied by a random walk with variance calibrated to 27.3 minutes.

**Figure 2. Wavelet variance ratios across time scale and dollar volume quintiles**



## The 2011 results: a summary

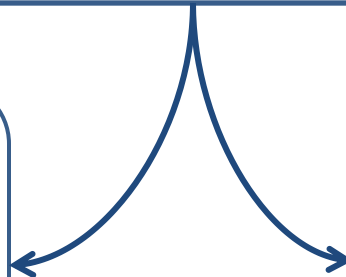
- ❑ Variance ratios: short term volatility is much higher than we'd expect relative to a random-walk.
- ❑ In mils per share or basis points, *average* short term volatility is economically meaningful, but small.

# Historical analysis

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2011 April TAQ  
with one-  
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High-resolution  
analysis

**2001-2011  
April TAQ data  
with one-second  
time stamps**



**Lower-resolution  
analysis**

**Table 5. Summary statistics, historical sample,  
2001-2011 (*only odd numbered years are shown*)**

	2001	2003	2005	2007	2009	2011
No. firms	146	150	150	150	150	150
NYSE	108	51	48	55	56	47
Amex	22	11	8	14	5	6
NASDAQ	16	88	94	81	89	97
Avg. daily trades	142	187	425	970	1,790	1,331
Avg. daily quotes	1,078	1,299	5,828	12,521	39,378	23,928
Avg. daily NBB changes	103	203	596	772	1,618	1,210
Avg. daily NBO changes	103	213	729	789	1,731	1,126
Avg. price	\$18.85	\$14.83	\$16.10	\$15.81	\$10.72	\$15.62
Market equity cap \$ Million	\$745	\$189	\$325	\$480	\$316	\$683

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NASDAQ	16	88	94	91	89	97
Avg. daily trades	142	187	200	1,790	1,790	1,331
Avg. daily quotes	1,078	1,209	1,209	1,209	1,209	23,928
Avg. daily NBB changes	103	203	596	772	1,618	1,210
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Market equity cap \$ Million	\$745	\$189	\$325	\$480	\$316	\$683

25% CAGR

36% CAGR

## What statistics to consider?

- ❑ Long-term volatilities changed dramatically over the sample period.
- ❑ Variance ratios (normalized to long-term volatility) are the most reliable indicators of trends.

**Table 6. Wavelet variance ratios for bids and offers, 2001-2011**

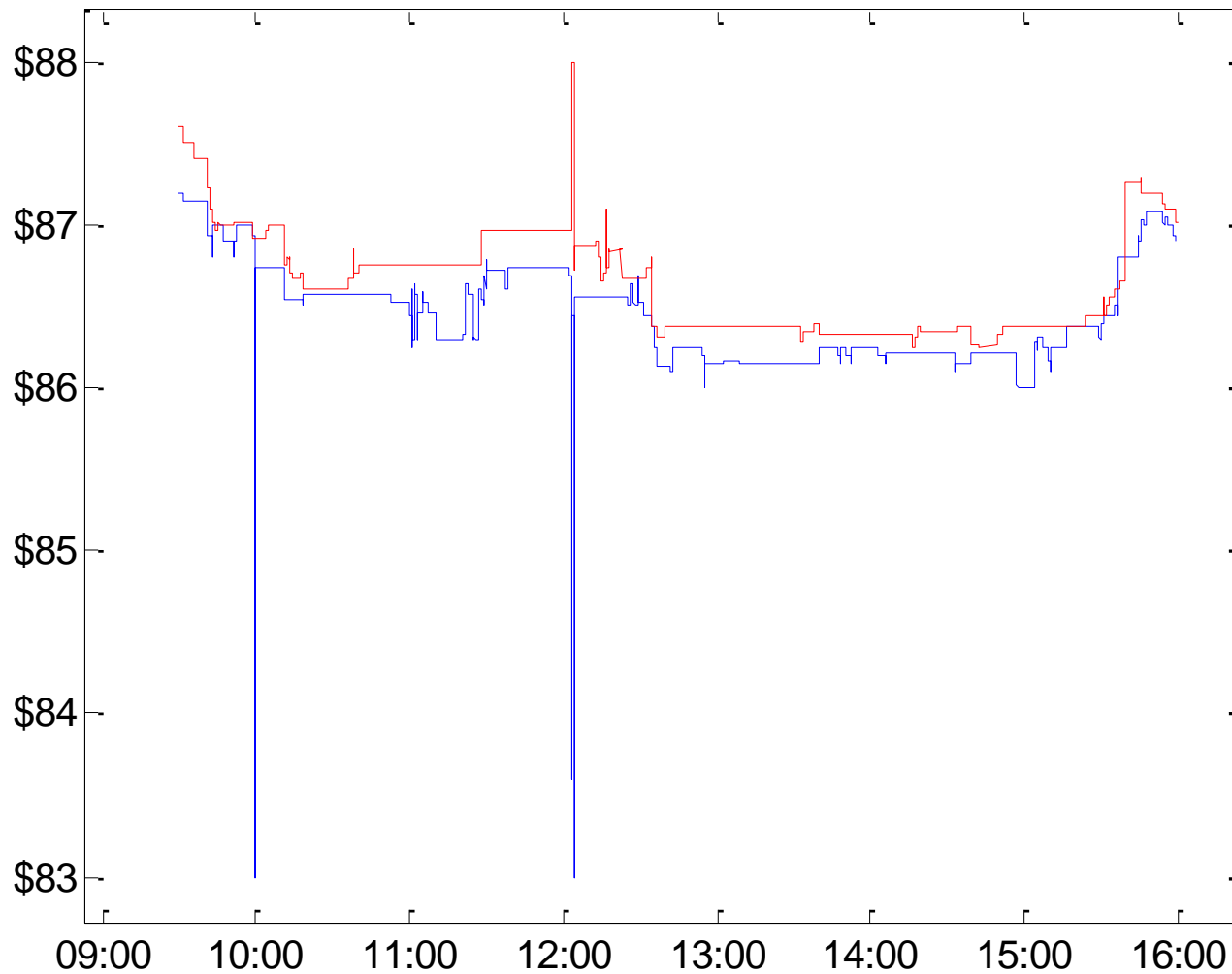
Panel A: Computed from *unadjusted* bids and offers

<b>Time scale</b>	<b>2001</b>	<b>2002</b>	<b>2003</b>	<b>2004</b>	<b>2005</b>	<b>2006</b>	<b>2007</b>	<b>2008</b>	<b>2009</b>	<b>2010</b>	<b>2011</b>
50 ms	5.22	7.16	6.03	10.28	6.69	8.57	6.96	6.06	4.52	7.08	4.70
100 ms	5.44	6.58	5.28	9.69	6.51	8.07	6.27	5.38	4.12	6.26	4.32
200 ms	5.28	6.28	5.13	9.03	6.22	7.34	5.33	4.64	3.68	5.40	3.74
400 ms	4.59	5.23	5.00	8.16	5.75	6.30	4.25	3.84	3.21	4.53	3.07
800 ms	3.12	4.04	3.93	5.57	5.03	5.10	3.41	3.11	2.76	3.71	2.56
1,600 ms	2.11	2.55	3.25	4.11	4.14	4.05	2.89	2.59	2.43	3.04	2.23
3.2 sec	1.98	2.24	2.93	3.38	3.48	3.37	2.56	2.29	2.17	2.53	2.01
6.4 sec	1.94	2.11	2.62	2.91	2.93	2.92	2.35	2.08	1.95	2.16	1.82

## No trend in quote volatilities?

- Maybe ...
  - “Flickering quotes” aren’t new.
  - Recent concerns about high frequency trading are all media hype.
  - The good old days weren’t really so great after all.
  
- What *did* quote volatility look like circa 2001?

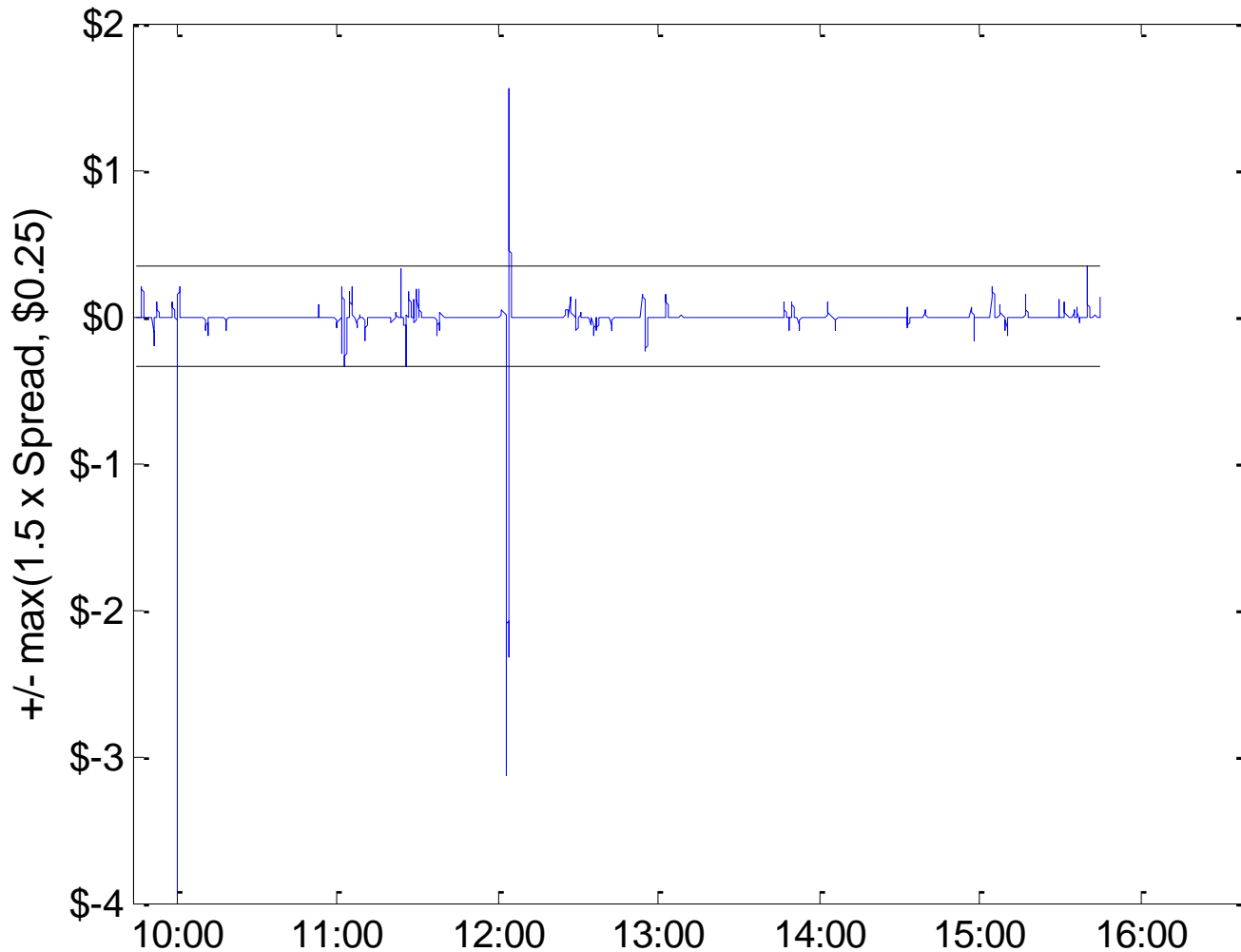
**Figure 4 Panel A. Bid and offer for PRK, April 6, 2001.**



# Compare

- ❑ PRK in 2001 vs. AEPI in 2011
  - AEPI: low amplitude, intense oscillation.
  - PRK: large amplitude, no oscillation.
- ❑ PRK-type noise is called “pop” noise
- ❑ It can be filtered out by clipping
  - I clip the short-run noise to  
 $\text{Max}(1.5 \times \text{spread}, \$0.25)$

Figure 4 Panel B. PRK, April 6, 2001, Rough component of the bid



**Table 6. Wavelet variance ratios for bids and offers, 2001-2011**

Panel B. Computed from *denoised* bids and offers

<b>Time scale</b>	<b>2001</b>	<b>2002</b>	<b>2003</b>	<b>2004</b>	<b>2005</b>	<b>2006</b>	<b>2007</b>	<b>2008</b>	<b>2009</b>	<b>2010</b>	<b>2011</b>
50 ms	1.60	2.37	3.15	7.02	6.09	8.24	6.56	5.83	4.20	6.79	4.46
100 ms	1.57	2.32	3.09	6.82	5.89	7.76	5.89	5.17	3.83	6.00	4.07
200 ms	1.56	2.27	3.03	6.48	5.61	7.04	4.99	4.45	3.41	5.18	3.57
400 ms	1.55	2.23	2.94	5.90	5.16	6.02	3.96	3.68	2.97	4.36	3.00
800 ms	1.57	2.19	2.83	5.00	4.47	4.82	3.13	2.98	2.56	3.58	2.52
1,600 ms	1.64	2.20	2.71	3.99	3.60	3.79	2.63	2.51	2.27	2.94	2.20
3.2 sec	1.81	2.30	2.62	3.44	3.02	3.16	2.33	2.23	2.04	2.46	2.00
6.4 sec	2.11	2.51	2.59	3.20	2.65	2.75	2.15	2.04	1.86	2.11	1.82

# So has quote volatility increased?

- ❑ Apples vs. oranges
  - The nature of quotes has changed.
- ❑ Quote volatility has changed
  - From infrequent large changes to frequent (and oscillatory) small changes.
- ❑ Possibly a overall small increase,
  - But nothing as strong as the trend implied by the growth in quote messaging rates.