# High Frequency Quoting: Short-Term Volatility in Bids and Offers

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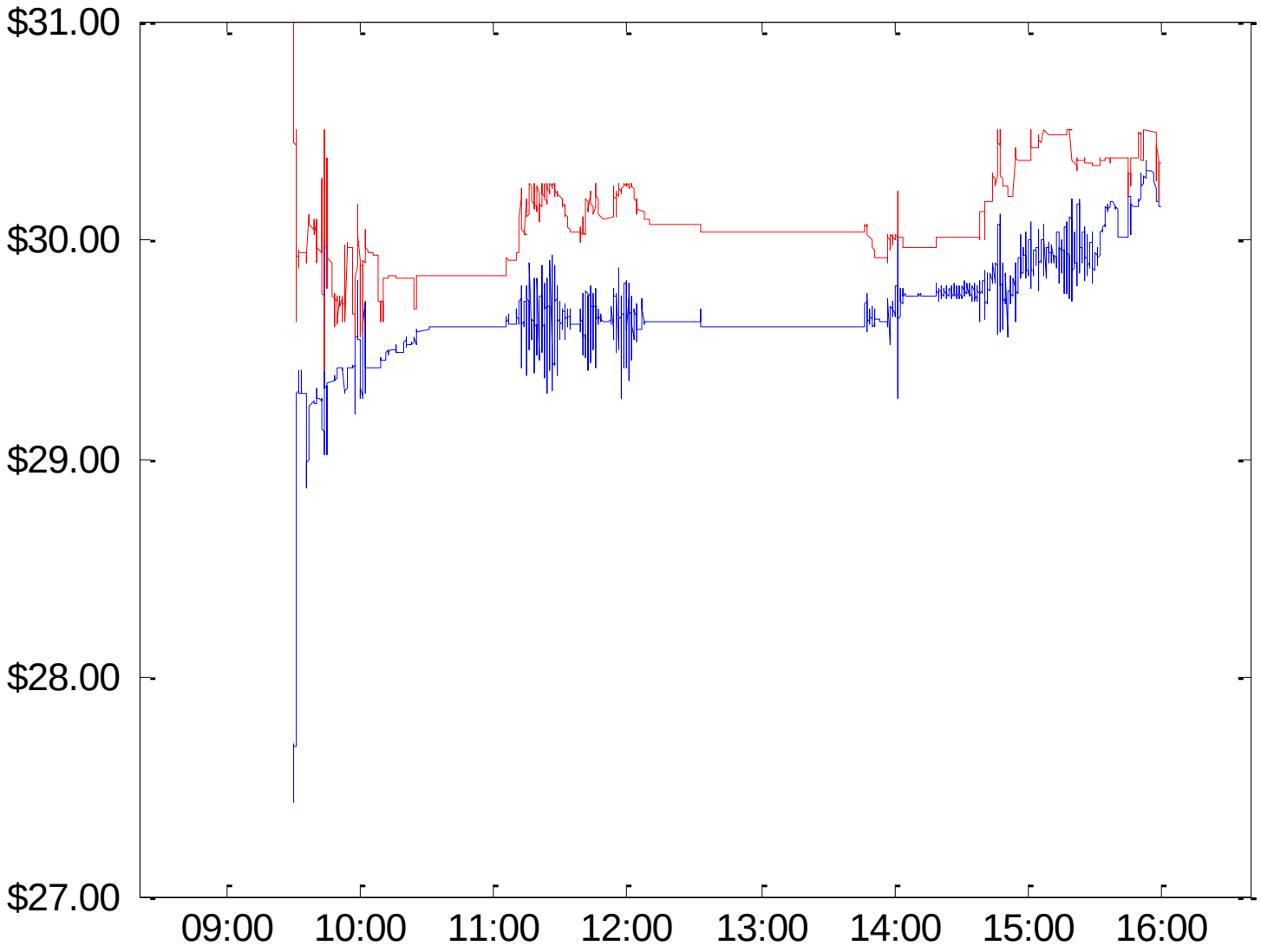
# Disclaimers

- I teach in an entry-level training program at a large financial firm that is generally thought to engage in high frequency trading.
- I serve on a CFTC advisory committee that discusses issues related to high frequency trading.
- I accept honoraria for presentations at events sponsored by financial firms.

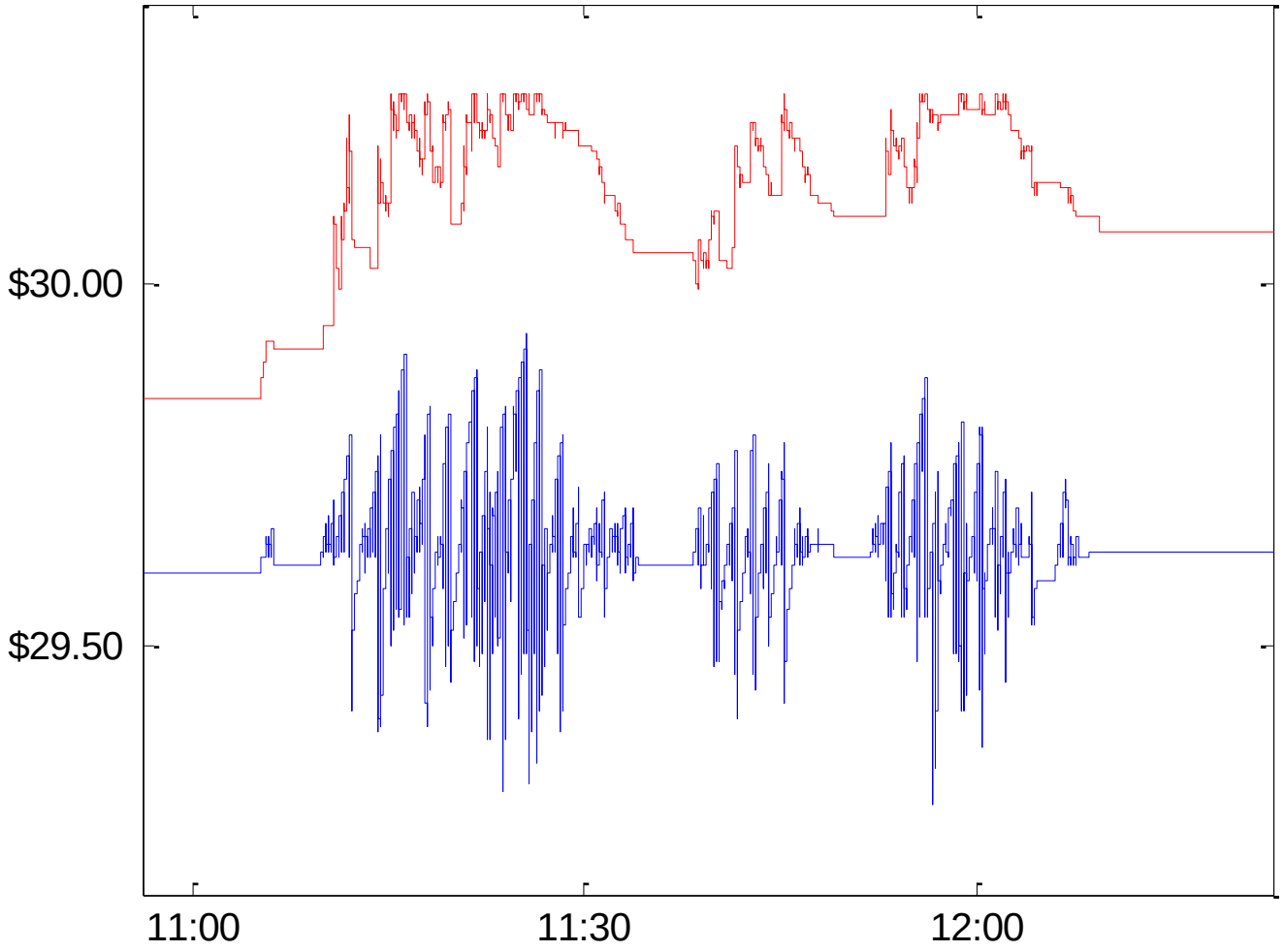
# What does quote volatility look like?

- In US equity markets, bids and offers from all trading venues are consolidated and disseminated in real time.
  - The highest bid is the National Best Bid (NBB)
  - The lowest offer is the National Best Offer (NBO)
- Next slide: the NBBO for AEPI on April 29, 2011

# Figure 1. AEPI bid and offer, April 29, 2011

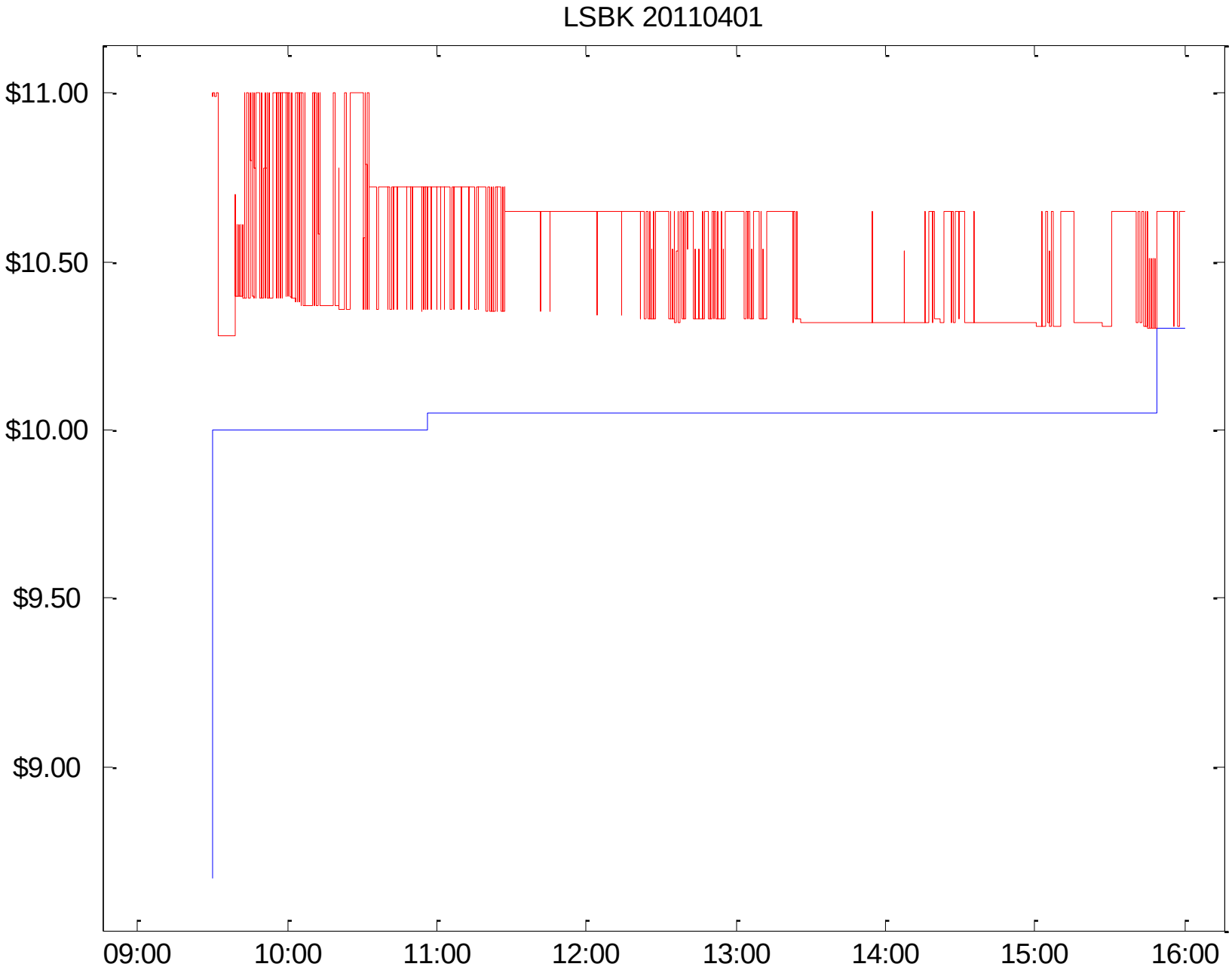


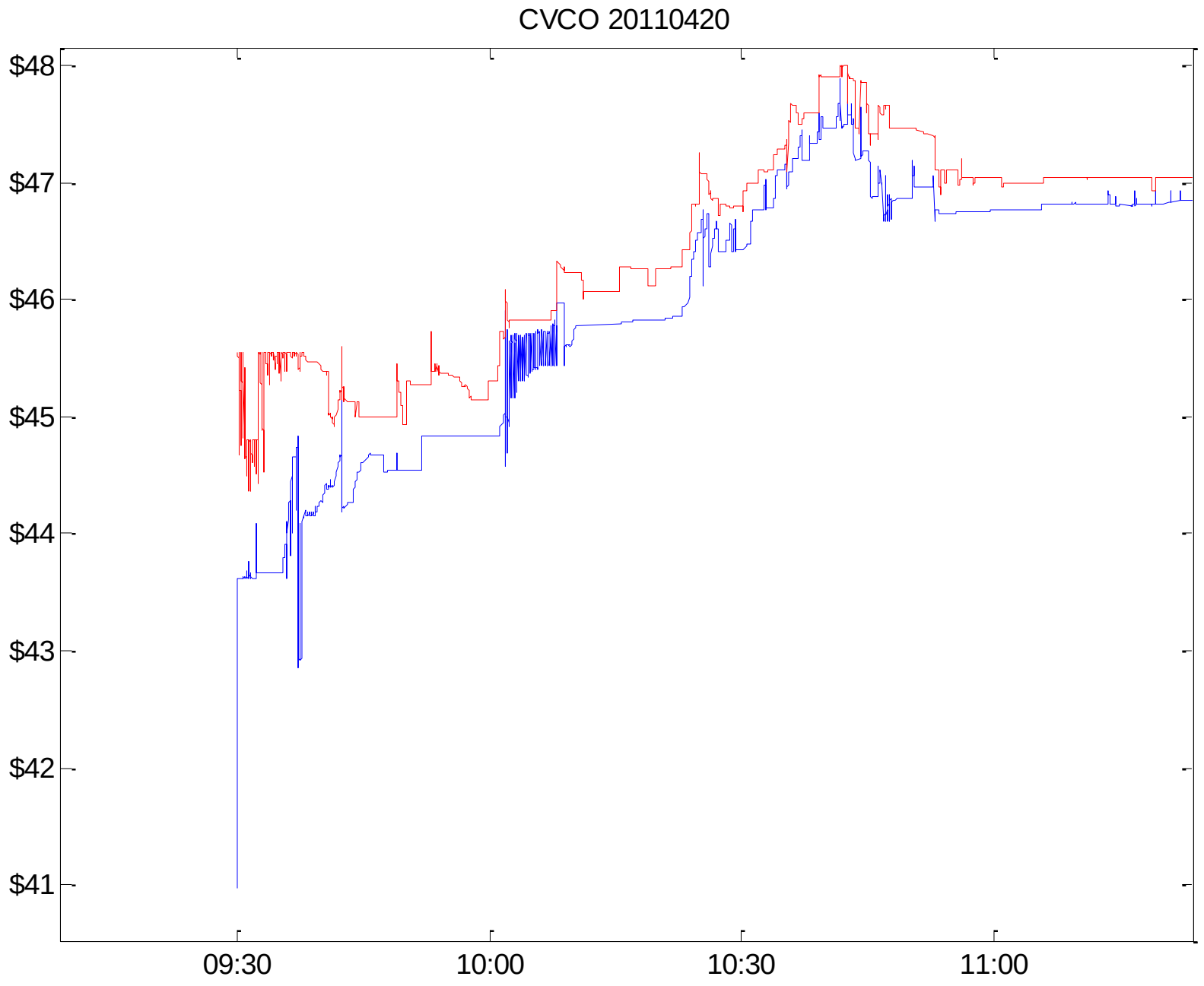
# Figure 1. AEPI bid and offer on April 29, 2011 (detail)



# Features of the AEPI episodes

- Extremely rapid *oscillations* in the bid.
- Start and stop abruptly
- Mostly one-sided
  - activity on the ask side is much smaller
- Episodes don't coincide with large long-term changes in the stock price.


LSBK 20110401
$11.00
$10.50
$10.00
$9.50
$9.00
09:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00


CVCO 20110420
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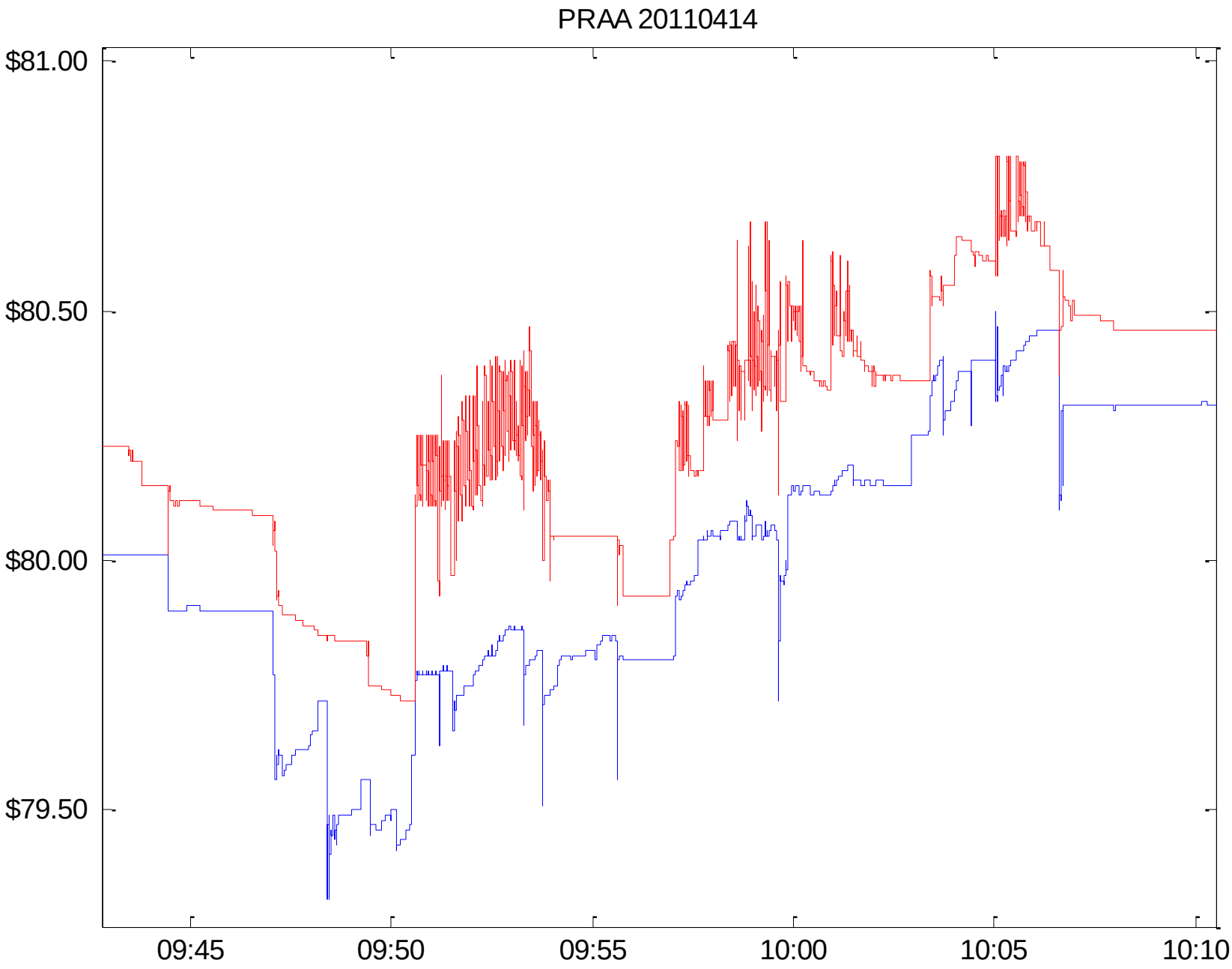
$47
$46
$45
$44
$43
$42
$41
09:30
10:00
10:30
11:00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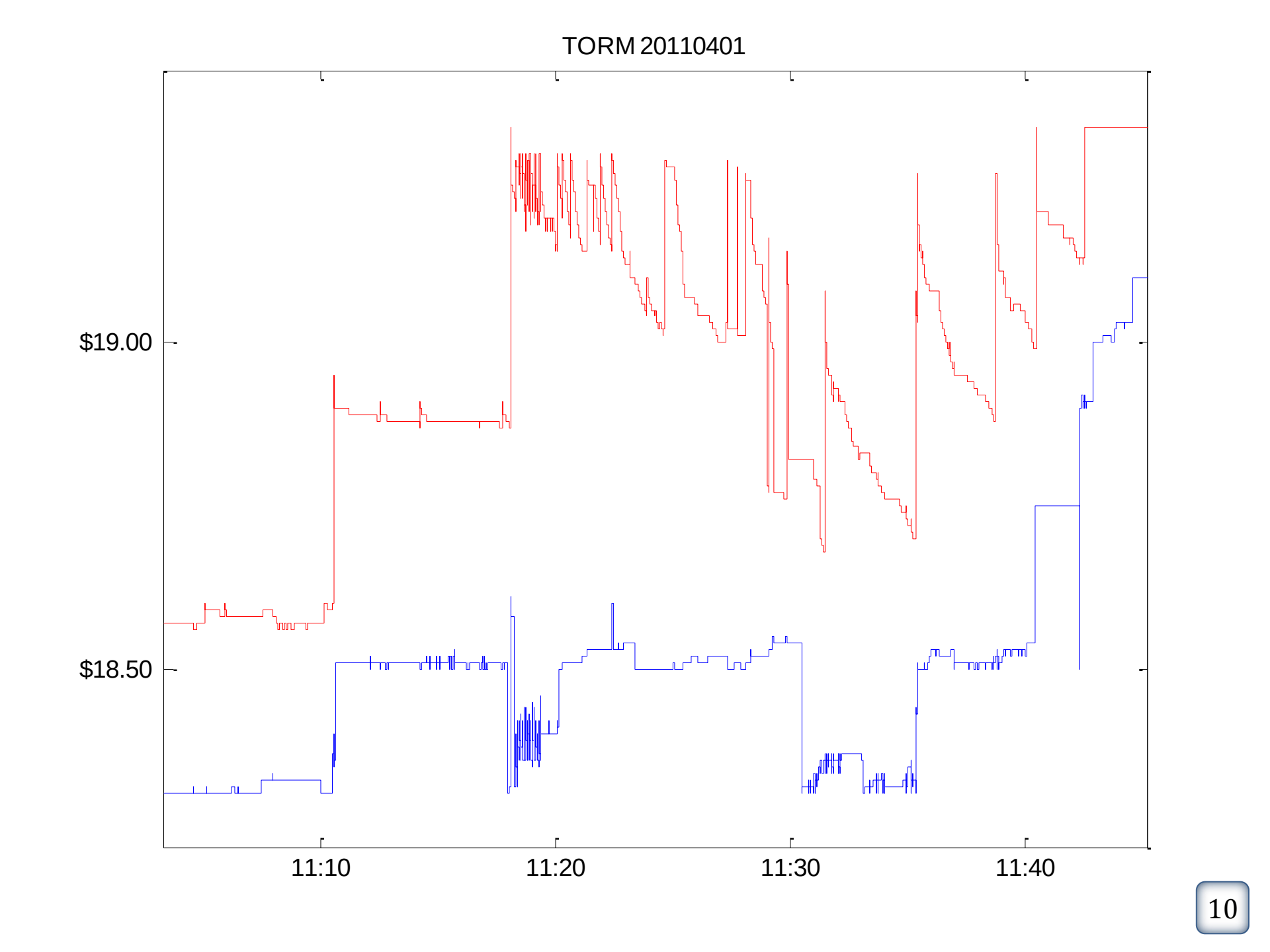
$80.50
$80.00
$79.50
09:45
09:50
09:55
10:00
10:05
10:10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$18.50
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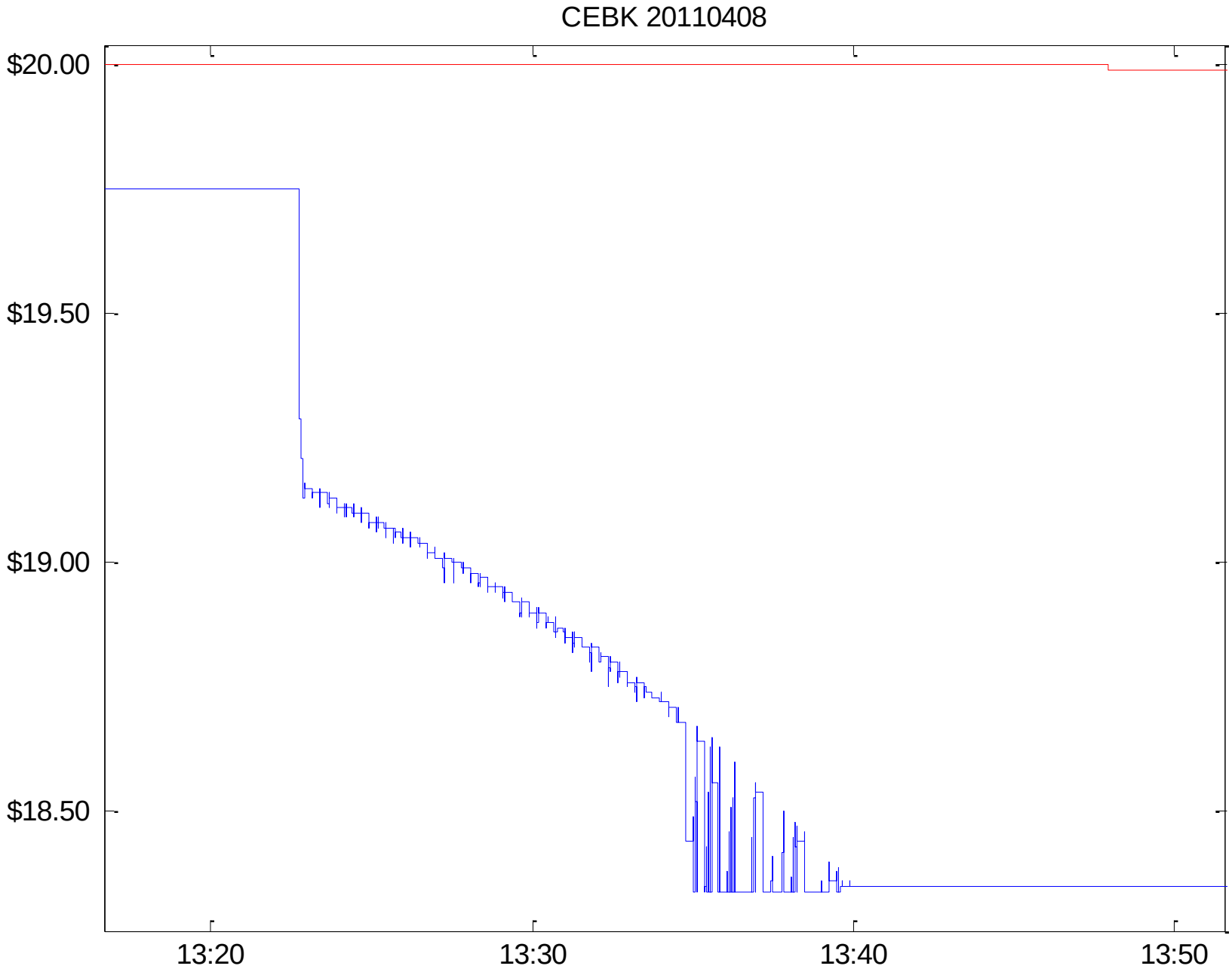
11:20
11:30
11:40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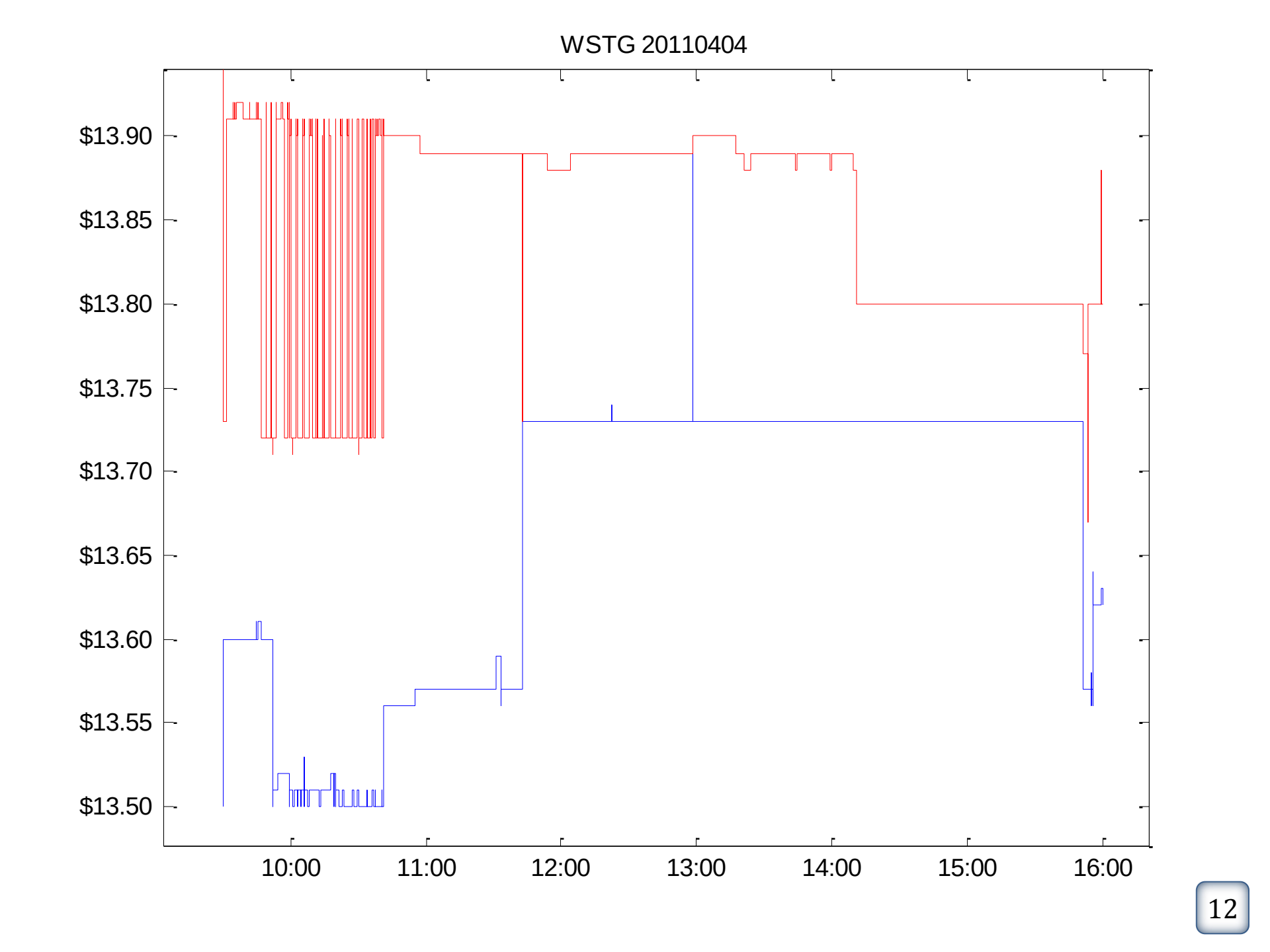
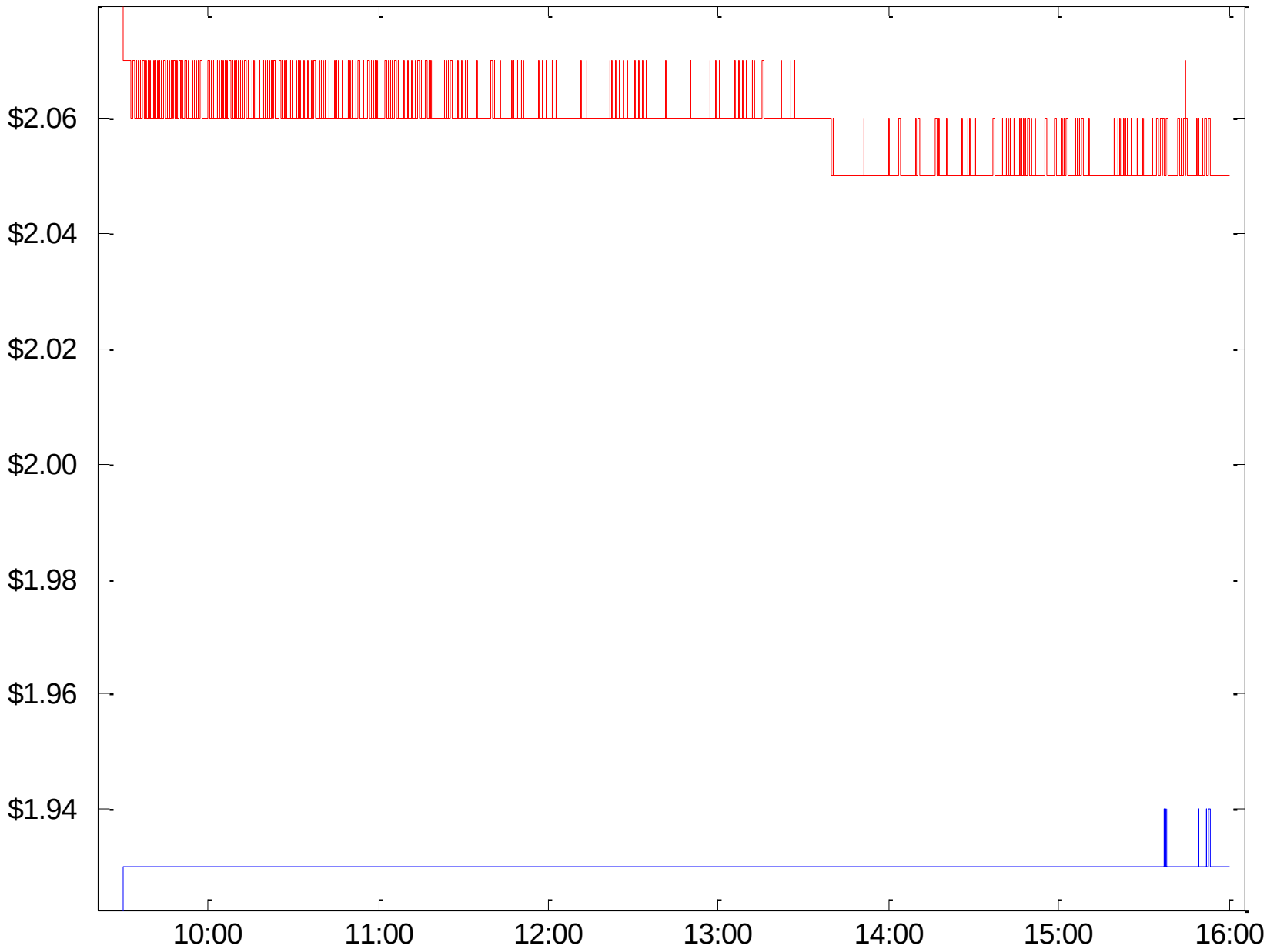
$20.00
$19.50
$19.00
$18.50
13:20
13:30
13:40
13:50


WSTG 20110404
$13.90
$13.85
$13.80
$13.75
$13.70
$13.65
$13.60
$13.55
$13.50
10:00
11:00
12:00
13:00
14:00
15:00
16:00


AAME 20110418
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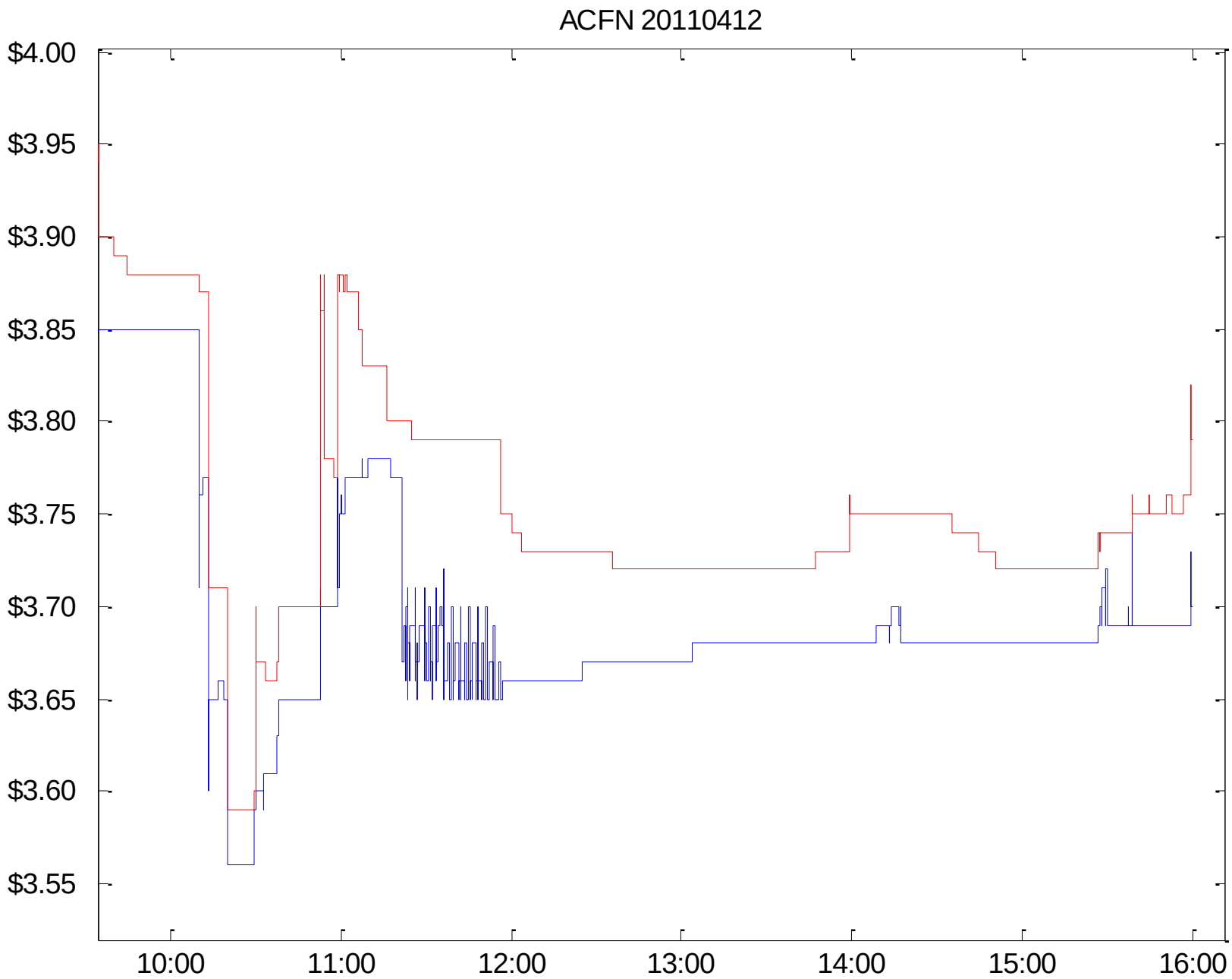
$2.04
$2.02
$2.00
$1.98
$1.96
$1.94
10:00
11:00
12:00
13:00
14:00
15:00
16:00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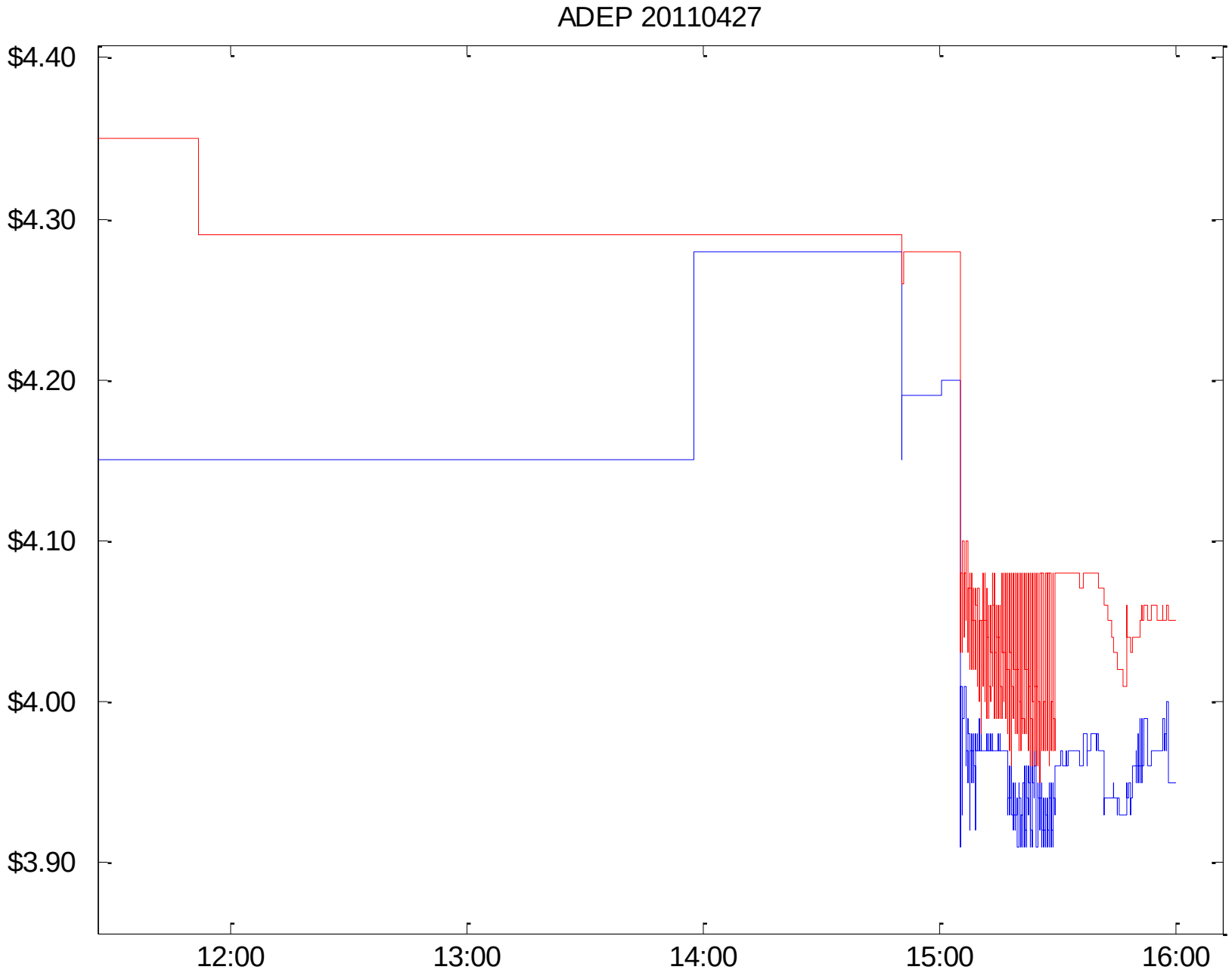
$3.95
$3.90
$3.85
$3.80
$3.75
$3.70
$3.65
$3.60
$3.55
10:00
11:00
12:00
13:00
14:00
15:00
16:00


ADEP 20110427
$4.40
$4.30
$4.20
$4.10
$4.00
$3.90
12:00
13:00
14:00
15:00
16:00

# Quote volatility: the questions

- What is its economic meaning and importance?
- How should we measure it?
- Is it elevated? Relative to what?
- Has it increased along with wider adoption of high-speed trading technology?

# Economic consequences of quote volatility

- Noise
- Execution price risk
  - For marketable orders
  - For dark trades
- Intermediaries' look-back options
- Quote-stuffing
- Spoofing

# Quote volatility and noise: "flickering quotes"

- Noise degrades the informational value of a price signal.
- "The improvements in market structure have also created new challenges, one of which is the well-known phenomenon of "ephemeral" or "flickering" quotes.
- *Flickering quotes create problems like bandwidth consumption and decreased price transparency."*
  - CIBC World Markets, comment letter to SEC, Feb. 4, 2005.

# Execution price risk for marketable orders

- A marketable order is one that is priced to be executed immediately.
  - "Buy 100 shares at the market" instructs the broker to buy, paying the current market asking price (no matter how high).
- All orders face arrival time uncertainty.
- Time uncertainty → price uncertainty

# Execution price risk for marketable orders

# Execution price risk for dark trades

- A dark trading venue does not display a bid or offer.
  - Roughly 30% of total volume is dark.
- In a dark market the execution price of a trade is set *by reference* to the bid and offer displayed by a lit market.
- Volatility in these reference prices induces execution price risk for the dark trades.

# Quote volatility and look-back options

- Many market rules and practices reference “the current NBBO”
  - Due to network latencies, “current” is a fuzzy term.
  - In practice, “current” means “at any time in the past few seconds”
- One dominant party might enjoy the flexibility to pick a price within this window.
- “Lookback option” Stoll and Schenzler (2002)

# “Spoofing” manipulations

- A dark pool buyer enter a spurious sell order in a visible market.
- The sell order drives down the NBBO midpoint.
- The buyer pays a lower price in the dark pool.

What are the connections
between high-frequency quoting
and high-frequency *trading*?

# The first studies: HFT is beneficial/benign

- "HF traders are the new market makers."
- Empirical studies
  - Hendershott, Jones and Menkveld (2011)
  - Hasbrouck and Saar (2012)
  - Brogaard, Hendershott and Riordan (2012)
- Studies generally find that HFT activity is associated with (causes?) higher market quality.

# Follow-on: "Maybe HF traders are predatory."

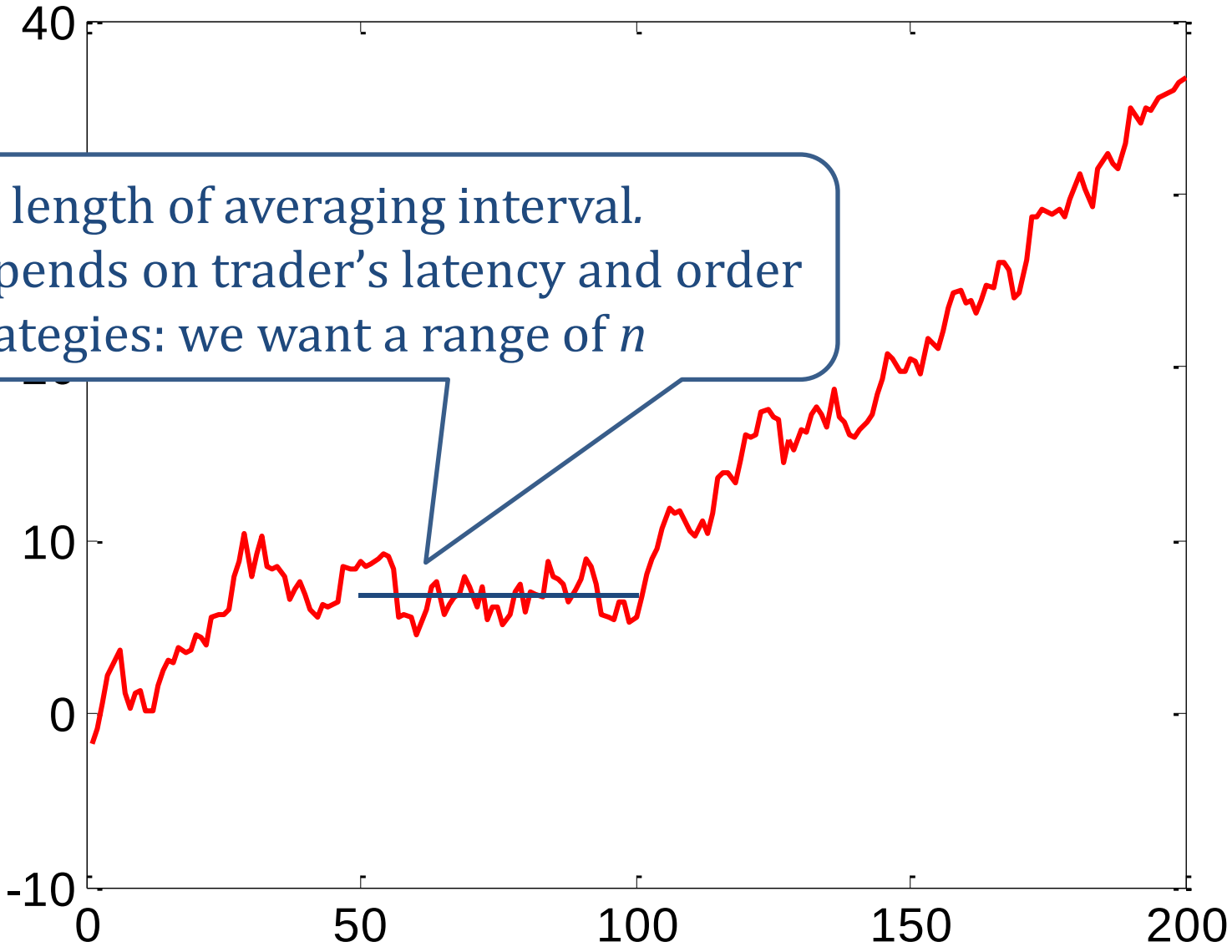
- They profit from HF information asymmetries at the expense of natural liquidity seekers.
- Theory: Jarrow and Protter (2011); Foucault, Hombert and Rosu (2012); Biais, Declerk and Moinas (2013)
- Empirical: Baron, Brogaard and Kirilenko (2012); Weller (2012); Clark-Joseph (2012)

# The spillover questions

- Quote volatility is an inverse measure of market quality.
  - It measures a kind of liquidity risk.
- HFT is perceived to have grown over the past decade.
  - Quoting and trading use the same technology.
  - Certainly some market participants perceive a rise in quote volatility.
  - What do the data say?

# Descriptive statistics: computation and interpretation

# Local variances about local means




*n* = length of averaging interval.
Depends on trader's latency and order strategies: we want a range of *n*

# Rough variances

- For computational efficiency, let $n$ increase as a dyadic ("powers of two") sequence
- Let $\sigma_j^2 =$ variance over interval $n_j = n_0 2^j$
  - Here, $n_0 = 50\ ms$
- In a signal processing context, $\sigma_j^2$ is a *rough* variance.
  - Averaging is a smoothing process. Deviations from the mean are "rough".

# The incremental ("wavelet") variances

- Let $\nu_j^2 = \sigma_j^2 - \sigma_{j-1}^2$ be the incremental variance change when the averaging period doubles from $n_{j-1}$ to $n_j$.
- Using the language of signal processing, $\nu_j^2$ is a *wavelet* variance.
  - "$\nu_j^2$ is associated with variation on a time scale of $\tau_j = n_0 2^{j-1}$"
- For computational efficiency, it is calculated using wavelet transforms (a relative of Fourier transforms).
  - It can be defined, interpreted and computed without invoking wavelets.

# Interpretation

- To assess economic importance, I present the (wavelet and rough) variance estimates in three ways.
  - In mils per share
  - In basis points
  - As a short-term/long-term ratio

# Mils per share

- Variances are computed on bid and offer price levels.
- Reported volatilities are scaled to $mils/share$.
  - One mil = $0.001
- Most trading charges are assessed per share.
  - Someone sending a marketable order to a US exchange typically pays an "access fee" of about three mils/share.
  - An executed limit order receives a "liquidity rebate" of about two mils/share.

# Basis points (One bp = 0.01%)

- Volatilities are first normalized by price (bid-ask average)
- The rough volatility in basis points:
  - $\frac{\sigma_j}{Price} \times 10{,}000$
- "One bp is a one cent bid-offer spread on a $100 stock."

# The short/long variance ratio

- For a random walk with per period variance $\sigma^2$, the variance of the $n$-period difference is $n\sigma^2$.
- An conventional variance ratio might be something like
  - $V = \frac{60 \times one\ minute\ return\ variance}{one\ hour\ return\ variance}$
- For a random walk, $V = 1$.
  - Due to microstructure effects we usually find $V > 1$.
- Extensively used in microstructure studies: Barnea (1974); Amihud and Mendelson (1987); etc.

# Variance ratios (cont'd)

- The wavelet variance ratio is constructed from the incremental (wavelet) variances
  - $V_{j,J} = 2^{J-j} \times \frac{\nu_j^2}{\nu_J^2}$
  - $J$ is the highest level (longest time scale) in the analysis (27 minutes).
- The rough variance ratio is
  - $VR_{j,J} = 2^{J-j-1} \times \frac{\sigma_j^2}{\nu_J^2}$
- Like traditional variance ratios, any excess above unity indicates inflation of short-term volatility relative to fundamental volatility.

# The empirical analysis

CRSP Universe 2001-2011. (Share code = 10 or 11; average price $2 to $1,000; listing NYSE, Amex or NASDAQ)

In each year, chose 150 firms in a random sample stratified by dollar trading volume

**2011 April TAQ with one-*millisecond* time stamps**

**High-resolution analysis**

2001-2011 April TAQ data with one-second time stamps

Lower-resolution analysis

# Table 1. Summary Statistics, 2011

| | | Dollar trading volume quintile | | | | |
|---|---|---|---|---|---|---|
| | Full sample | 1 (low) | 2 | 3 | 4 | 5 (high) |
| No. of firms | 150 | 30 | 30 | 30 | 30 | 30 |
| NYSE | 47 | 0 | 5 | 7 | 16 | 19 |
| Amex | 6 | 2 | 2 | 0 | 1 | 1 |
| NASDAQ | 97 | 28 | 23 | 23 | 13 | 10 |
| Avg. daily trades | 1,331 | 31 | 431 | 1,126 | 3,478 | 16,987 |
| Avg. daily quotes | 23,928 | 967 | 7,706 | 24,026 | 53,080 | 181,457 |
| Avg. daily NBBO records | 7,138 | 328 | 3,029 | 7,543 | 16,026 | 46,050 |
| Avg. daily NBB changes | 1,245 | 120 | 511 | 1,351 | 2,415 | 4,124 |
| Avg. daily NBO changes | 1,164 | 103 | 460 | 1,361 | 2,421 | 4,214 |
| Avg. price | $15.62 | $4.87 | $5.46 | $17.86 | $27.76 | $51.60 |
| Market capitalization of equity, $ Million | $683 | $41 | $202 | $747 | $1,502 | $8,739 |

## Table 2. Time scale variance estimates, 2011

| | Rough volatilities, $\sigma_j$ | | | Wavelet variances, $\nu_j^2$ | | | |
|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Time scale | $\sigma_j$, mils | $\sigma_j$, bp | Variance ratio | $\sqrt{\nu_j^2}$, mils | $\sqrt{\nu_j^2}$, bp | Variance ratio | Bid-Offer Corr |
| < 50 ms | 0.28 | 0.16 | 4.22 | | | | |
| 50 ms | 0.39 | 0.22 | 3.99 | 0.2 | | | |
| 100 ms | 0.55 | 0.31 | 3.79 | 0.3 | | | |
| 200 ms | 0.76 | 0.43 | 3.53 | | | | |
| 400 ms | 1.05 | 0.59 | 3.21 | | | | |
| 800 ms | 1.46 | 0.83 | 2.90 | | | | |
| 1,600 ms | 2.02 | 1.14 | 2.64 | 1. | | | |
| 3.2 sec | 2.80 | 1.58 | 2.40 | 1.9 | | | |
| 6.4 sec | 3.90 | 2.18 | 2.12 | 2.7 | | | |
| 12.8 sec | 5.43 | 2.99 | 1.88 | 3.7 | | | |
| 25.6 sec | 7.54 | 4.10 | 1.70 | 5.2 | | | |
| 51.2 sec | 10.48 | 5.61 | 1.54 | 7.2 | | | |
| 102.4 sec | 14.53 | 7.68 | 1.42 | 10.0 | | | |
| 3.4 min | 20.12 | 10.51 | 1.32 | 13.8 | | | |
| 6.8 min | 27.88 | 14.40 | 1.23 | 19.22 | | | |
| 13.7 min | 38.55 | 19.70 | 1.16 | 26.45 | 13.33 | 1.08 | 0.88 |
| 27.3 min | 52.84 | 26.79 | 1.08 | 35.73 | 17.91 | 1.00 | 0.90 |

A trader who faces time uncertainty of 400 ms incurs price risk of 1.05 *mils/share* or 0.59 *basis points*.

At a time scale of 400 ms., the rough variance is 3.21 times the value implied by a random walk with variance calibrated to 27.3 minutes.

# Figure 2. Wavelet variance ratios across time scale and dollar volume quintiles

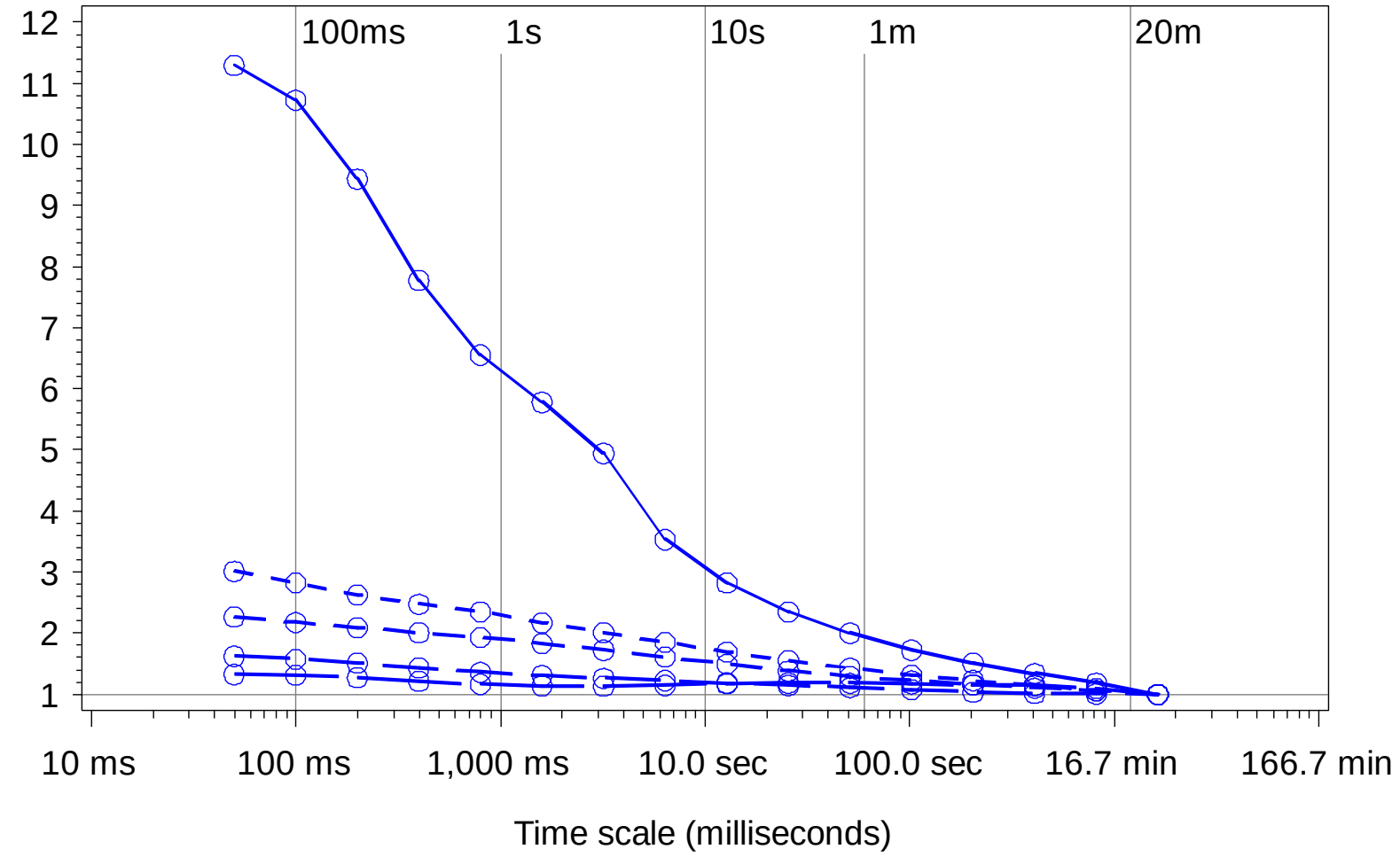
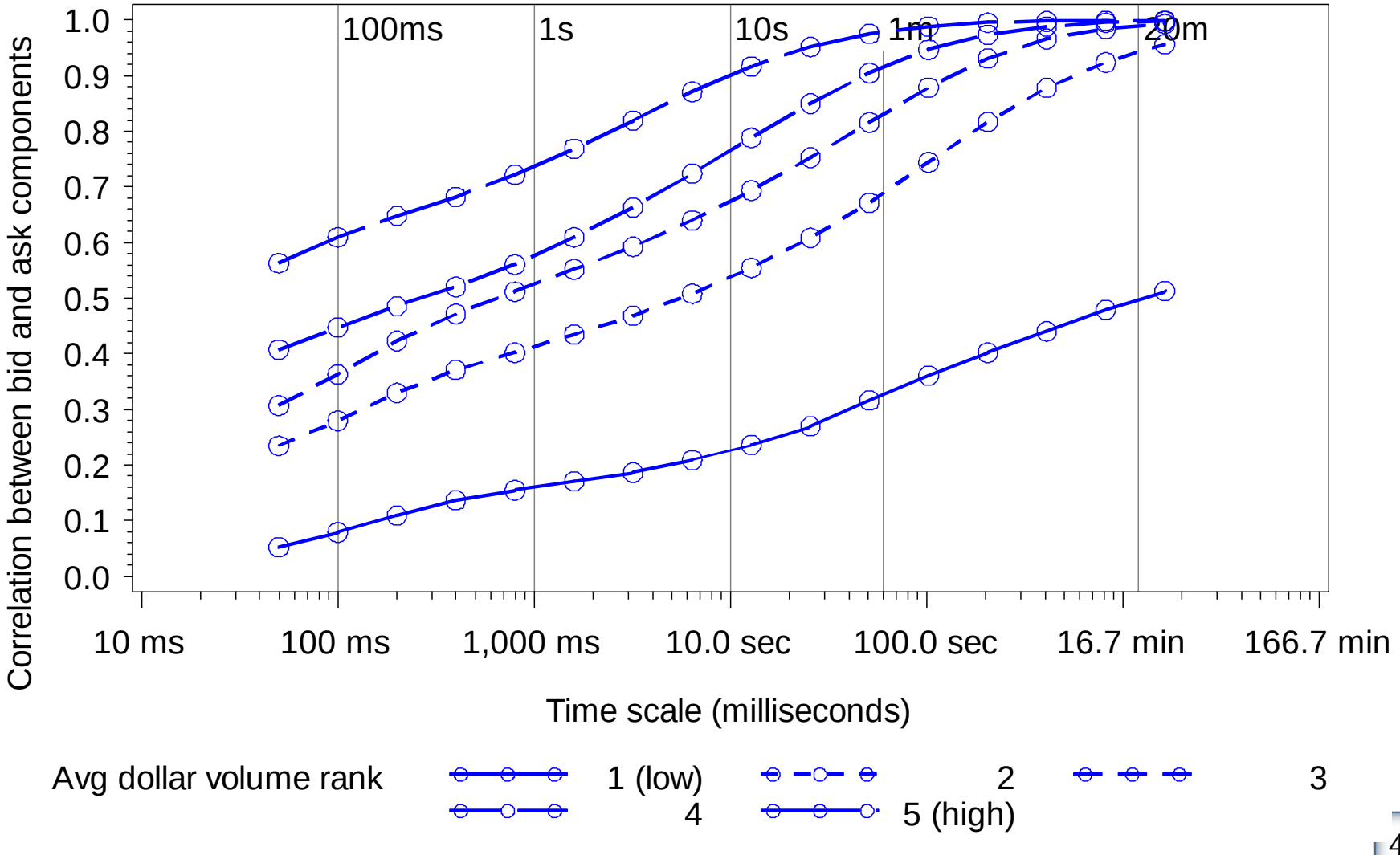


Table 2. Time scale variance estimates, 2011

How closely do the bid and offer track at the indicated time scale?

| Time scale | (1) $\sqrt{\sigma_j^2}$, mils | (2) $\sqrt{\sigma_j^2}$, bp | (3) | (4) | (5) | (6) | (7) Bid-Offer Corr |
|---|---|---|---|---|---|---|---|
| < 50 ms | 0.28 | 0.16 | 4.22 | | | | |
| 50 ms | 0.39 | 0.22 | 3.99 | 0.27 | 0.15 | 3.76 | 0.32 |
| 100 ms | 0.55 | 0.31 | 3.79 | 0.38 | 0.21 | 3.58 | 0.36 |
| 200 ms | 0.76 | 0.43 | 3.53 | 0.53 | 0.30 | 3.27 | 0.41 |
| 400 ms | 1.05 | 0.59 | 3.21 | 0.73 | 0.41 | 2.88 | 0.44 |
| 800 ms | 1.46 | 0.83 | 2.90 | 1.01 | 0.57 | 2.59 | 0.47 |
| 1,600 ms | 2.02 | 1.14 | 2.64 | 1.40 | 0.79 | 2.38 | 0.51 |
| 3.2 sec | 2.80 | 1.58 | 2.40 | 1.94 | 1.09 | 2.16 | 0.55 |
| 6.4 sec | 3.90 | 2.18 | 2.12 | 2.71 | 1.49 | 1.84 | 0.60 |
| 12.8 sec | 5.43 | 2.99 | 1.88 | 3.77 | 2.04 | 1.65 | 0.64 |
| 25.6 sec | 7.54 | 4.10 | 1.70 | 5.23 | 2.79 | 1.51 | 0.69 |
| 51.2 sec | 10.48 | 5.61 | 1.54 | 7.25 | 3.82 | 1.39 | 0.74 |
| 102.4 sec | 14.53 | 7.68 | 1.42 | 10.04 | 5.22 | 1.29 | 0.79 |
| 3.4 min | 20.12 | 10.51 | 1.32 | 13.87 | 7.14 | 1.21 | 0.83 |
| 6.8 min | 27.88 | 14.40 | 1.23 | 19.22 | 9.78 | 1.15 | 0.86 |
| 13.7 min | 38.55 | 19.70 | 1.16 | 26.45 | 13.33 | 1.08 | 0.88 |
| 27.3 min | 52.84 | 26.79 | 1.08 | 35.73 | 17.91 | 1.00 | 0.90 |

# Figure 3. Wavelet correlations between the National Best Bid and National Best Offer



# The 2011 results: a summary

- Variance ratios: short term volatility is much higher than we'd expect relative to a random-walk.
- In mils per share or basis points, *average* short term volatility is economically meaningful, but small.

# Historical analysis

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In each year, chose 150 firms in a random sample stratified by dollar trading volume

2011 April TAQ with one-*millisecond* time stamps

High-resolution analysis

**2001-2011 April TAQ data with one-second time stamps**

**Lower-resolution analysis**

# High-resolution analysis …
# … with low resolution data

- TAQ with millisecond time stamps only available from 2006 onwards
- TAQ with one second time stamps available back to 1993.
- Can we draw inferences about *subsecond* variation from second-stamped data?

# The problem

| | |
|---|---|
| Quote A | 10:01:35 |
| Quote B | 10:01:35 |
| Quote C | 10:01:35 |

- Where within the second did these quotes actually occur?
- With a few simple assumptions, we know how they are distributed and how they may be simulated.

## Recall the constant intensity Poisson process …

- $N(t) =$ no. of events in an interval $(0, t)$
- $s_i =$ arrival time of event $i$
- If $N(t) = n$, then $s_1, s_2, \ldots, s_n$ have the same distribution as the order statistics in a sample of $n$ independent $U(0, t)$ random variables.
- This suggests a simple procedure…

| | |
|---|---|
| Quote A | 10:01:35 |
| Quote B | 10:01:35 |
| Quote C | 10:01:35 |

- Draw three U(0,1) random numbers
- Sort them
- Assign them as the millisecond remainders

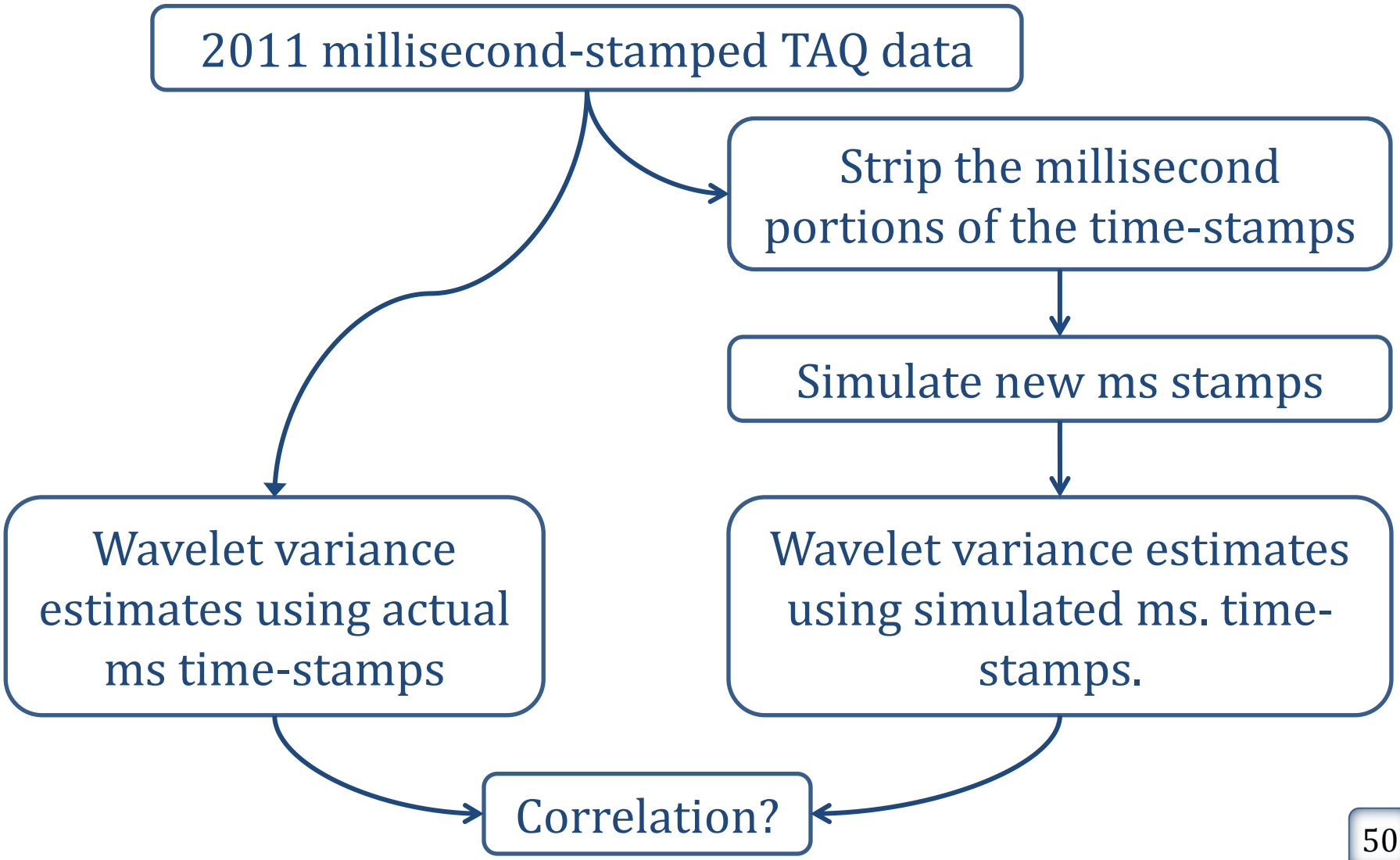
| | |
|---|---|
| Quote A | 10:01:35***.243*** |
| Quote B | 10:01:35***.347*** |
| Quote C | 10:01:35***.912*** |

- Compute variance estimates using the simulated time stamps.

# Formalities

- Assume that
  - The quotes are correctly sequenced.
  - Arrivals within the second are Poisson with (unknown) constant intensity.
  - The bid and offer process is independent of the within-second arrival times.
- Then each calculated statistic constitutes a draw from the corresponding Bayesian posterior.

# Does this really work?



# Back to the 2001-2011 historical sample

- Variance estimations will be based on simulated millisecond time-stamps.

# Table 5. Summary statistics, historical sample, 2001-2011 (*only odd numbered years are shown)*

| | 2001 | 2003 | 2005 | 2007 | 2009 | 2011 |
|---|---|---|---|---|---|---|
| No. firms | 146 | 150 | 150 | 150 | 150 | 150 |
| NYSE | 108 | 51 | 48 | 55 | 56 | 47 |
| Amex | 22 | 11 | 8 | 14 | 5 | 6 |
| NASDAQ | 16 | 88 | 94 | 81 | 89 | 97 |
| Avg. daily trades | 142 | 187 | 425 | 970 | 1,790 | 1,331 |
| Avg. daily quotes | 1,078 | 1,299 | 5,828 | 12,521 | 39,378 | 23,928 |
| Avg. daily NBB changes | 103 | 203 | 596 | 772 | 1,618 | 1,210 |
| Avg. daily NBO changes | 103 | 213 | 729 | 789 | 1,731 | 1,126 |
| Avg. price | $18.85 | $14.83 | $16.10 | $15.81 | $10.72 | $15.62 |
| Market equity cap $ Million | $745 | $189 | $325 | $480 | $316 | $683 |

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| Avg. daily trades | 142 | 187 | 25% CAGR | | 1,790 | 1,331 |
| Avg. daily quotes | 1,078 | 1,299 | 36% CAGR | | 29,279 | 23,928 |
| Avg. daily NBB changes | 103 | 203 | 596 | 772 | 1,618 | 1,210 |
| Avg. daily NBO changes | 103 | 213 | 729 | 789 | 1,731 | 1,126 |
| Avg. price | $18.85 | $14.83 | $16.10 | $15.81 | $10.72 | $15.62 |
| Market equity cap $ Million | $745 | $189 | $325 | $480 | $316 | $683 |

# What statistics to consider?

- Long-term volatilities changed dramatically over the sample period.
- Variance ratios (normalized to long-term volatility) are the most reliable indicators of trends.

## Table 6. Wavelet variance ratios for bids and offers, 2001-2011

Panel A: Computed from *unadjusted* bids and offers

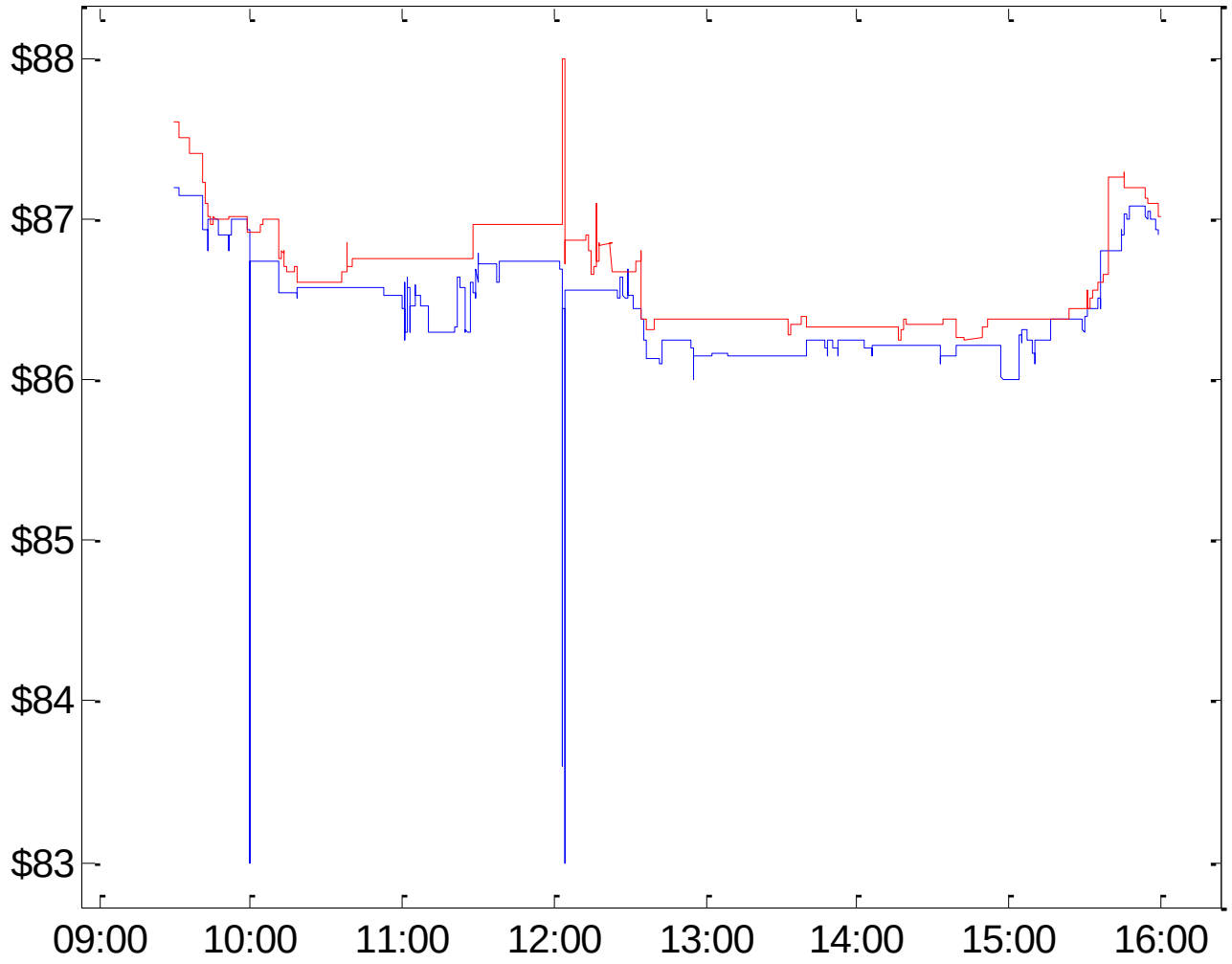
| Time scale | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 ms | 5.22 | 7.16 | 6.03 | 10.28 | 6.69 | 8.57 | 6.96 | 6.06 | 4.52 | 7.08 | 4.70 |
| 100 ms | 5.44 | 6.58 | 5.28 | 9.69 | 6.51 | 8.07 | 6.27 | 5.38 | 4.12 | 6.26 | 4.32 |
| 200 ms | 5.28 | 6.28 | 5.13 | 9.03 | 6.22 | 7.34 | 5.33 | 4.64 | 3.68 | 5.40 | 3.74 |
| 400 ms | 4.59 | 5.23 | 5.00 | 8.16 | 5.75 | 6.30 | 4.25 | 3.84 | 3.21 | 4.53 | 3.07 |
| 800 ms | 3.12 | 4.04 | 3.93 | 5.57 | 5.03 | 5.10 | 3.41 | 3.11 | 2.76 | 3.71 | 2.56 |
| 1,600 ms | 2.11 | 2.55 | 3.25 | 4.11 | 4.14 | 4.05 | 2.89 | 2.59 | 2.43 | 3.04 | 2.23 |
| 3.2 sec | 1.98 | 2.24 | 2.93 | 3.38 | 3.48 | 3.37 | 2.56 | 2.29 | 2.17 | 2.53 | 2.01 |
| 6.4 sec | 1.94 | 2.11 | 2.62 | 2.91 | 2.93 | 2.92 | 2.35 | 2.08 | 1.95 | 2.16 | 1.82 |

# No trend in quote volatilities?

- Maybe …
  - "Flickering quotes" aren't new.
  - Recent concerns about high frequency trading are all media hype.
  - The good old days weren't really so great after all.

- What *did* quote volatility look like circa 2001?

**Figure 4 Panel A. Bid and offer for PRK, April 6, 2001.**

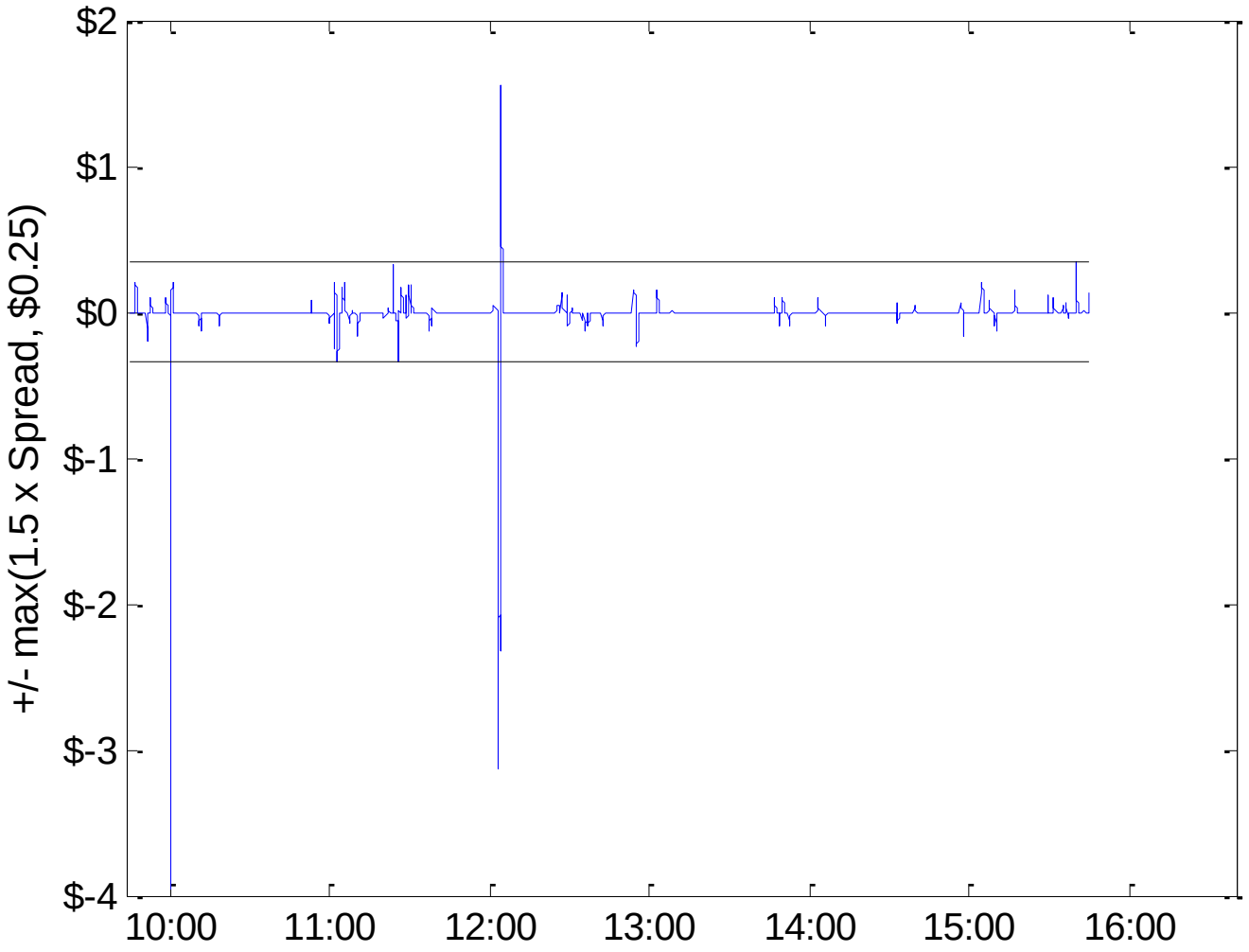


# Compare

- PRK in 2001 vs. AEPI in 2011
  - AEPI: low amplitude, intense oscillation.
  - PRK: large amplitude, no oscillation.
- PRK-type noise is called “pop” noise
- It can be filtered out by clipping
  - I clip the short-run noise to

$$Max(1.5 \times spread, \$0.25)$$

Figure 4 Panel B. PRK, April 6, 2001, Rough component of the bid



# Table 6. Wavelet variance ratios for bids and offers, 2001-2011, *Detail*

Panel A: Computed from *unadjusted* bids and offers

| Time scale | 2001 | ... | 2011 |
|---|---|---|---|
| 50 ms | 5.22 | | 4.70 |
| 100 ms | 5.44 | | 4.32 |
| 200 ms | 5.28 | | 3.74 |
| 400 ms | 4.59 | | 3.07 |
| 800 ms | 3.12 | | 2.56 |
| 1,600 ms | 2.11 | | 2.23 |
| 3.2 sec | 1.98 | | 2.01 |
| 6.4 sec | 1.94 | | 1.82 |

Panel B: Computed from *denoised* bids and offers

| Time scale | 2001 | ... | 2011 |
|---|---|---|---|
| 50 ms | 1.60 | | 4.46 |
| 100 ms | 1.57 | | 4.07 |
| 200 ms | 1.56 | | 3.57 |
| 400 ms | 1.55 | | 3.00 |
| 800 ms | 1.57 | | 2.52 |
| 1,600 ms | 1.64 | | 2.20 |
| 3.2 sec | 1.81 | | 2.00 |
| 6.4 sec | 2.11 | | 1.82 |

## Table 6. Wavelet variance ratios for bids and offers, 2001-2011

Panel B. Computed from *denoised* bids and offers

| Time scale | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 ms | 1.60 | 2.37 | 3.15 | 7.02 | 6.09 | 8.24 | 6.56 | 5.83 | 4.20 | 6.79 | 4.46 |
| 100 ms | 1.57 | 2.32 | 3.09 | 6.82 | 5.89 | 7.76 | 5.89 | 5.17 | 3.83 | 6.00 | 4.07 |
| 200 ms | 1.56 | 2.27 | 3.03 | 6.48 | 5.61 | 7.04 | 4.99 | 4.45 | 3.41 | 5.18 | 3.57 |
| 400 ms | 1.55 | 2.23 | 2.94 | 5.90 | 5.16 | 6.02 | 3.96 | 3.68 | 2.97 | 4.36 | 3.00 |
| 800 ms | 1.57 | 2.19 | 2.83 | 5.00 | 4.47 | 4.82 | 3.13 | 2.98 | 2.56 | 3.58 | 2.52 |
| 1,600 ms | 1.64 | 2.20 | 2.71 | 3.99 | 3.60 | 3.79 | 2.63 | 2.51 | 2.27 | 2.94 | 2.20 |
| 3.2 sec | 1.81 | 2.30 | 2.62 | 3.44 | 3.02 | 3.16 | 2.33 | 2.23 | 2.04 | 2.46 | 2.00 |
| 6.4 sec | 2.11 | 2.51 | 2.59 | 3.20 | 2.65 | 2.75 | 2.15 | 2.04 | 1.86 | 2.11 | 1.82 |

# Summary of the variance ratio evidence

- Without filtering: no trend in quote volatility.
- With filtering
  - Volatility in earlier years is lower
  - Maybe a trend
    - But highest values are mostly in 2004-2006
- The effects of filtering suggest that
  - Early years: volatility due to spikes
  - Later years: volatility reflects oscillations
- What changed?

# SEC's Reg NMS ("National Market System")

- Proposed in 2004; adopted 2005; implemented in 2006.
- Defined the framework for competition among equity markets.
- Enhanced protection against trade-throughs
  - Example: market A is bidding $10 and market B executes a trade at $9.
- For a market's bid and offer to be protected, they have to accessible instantly (electronically)
  - This requirement essentially forced all markets to become electronic.

# Before and after

- Prior to Reg NMS
  - Trading dominated by slow, manual floor markets
  - Weak protection against trade-throughs
- Post Reg NMS
  - Bids and offers are firm and accessible.
  - Strong trade-through protection

# So has quote volatility increased?

- Apples vs. oranges
  - The nature of quotes has changed.
- Quote volatility has changed
  - From infrequent large changes to frequent (and oscillatory) small changes.
- Possibly a overall small increase,
  - But nothing as strong as the trend implied by the growth in quote messaging rates.

# Follow-up questions

- What strategies give rise to the episodic oscillations?
- Are the HFQ episodes unstable algos?
- Are they sensible strategies to detect and access liquidity?

# Extra overheads

# Dark trades: internalized execution

- A broker receives a retail buy order.
  - The order is not sent to an exchange or any other venue.
  - The broker sells directly to the customer at the National Best Offer (NBO)
- Volatility in the NBO → volatility in execution price.

# Dark trading

- "Dark" the market executing the order did not previously post a visible bid or offer at the execution price.
  - The trade itself is promptly reported.
- Dark mechanisms
  - Hidden (undisplayed) limit orders
  - Internalized executions
  - Dark pools

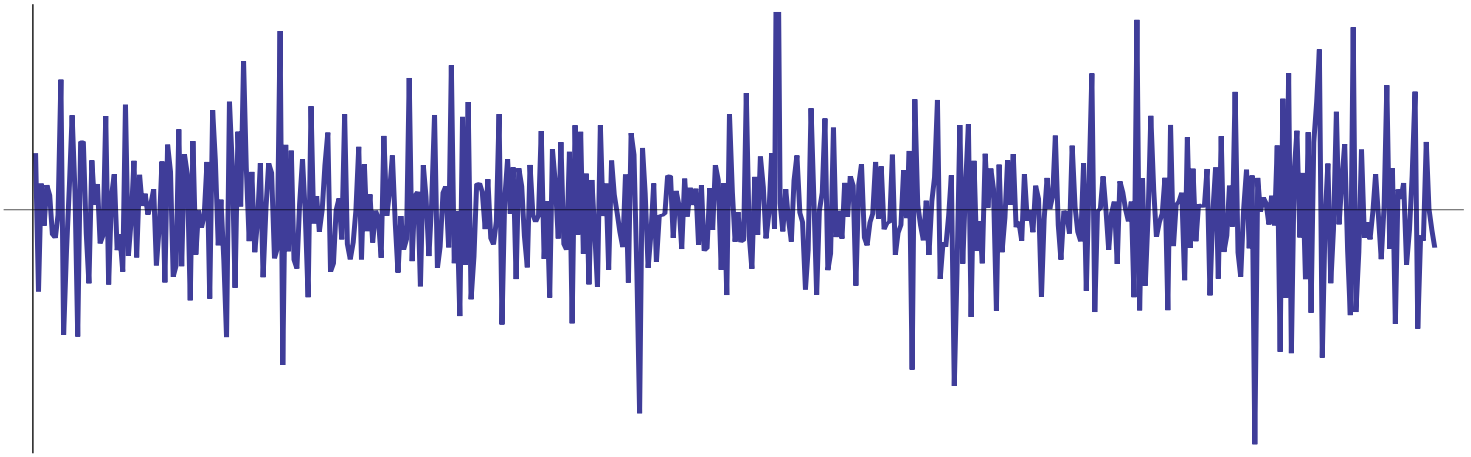
# Dark trades: dark pools

- Mechanism
  - Traders send buy and sell orders to a computer.
  - The orders are not displayed.
  - If the computer finds a feasible match, a trade occurs.
- The trade is priced at the midpoint of the National Best Bid and Offer (NBBO)
- Volatility in the NBBO causes volatility in the execution price.

# Look-back options

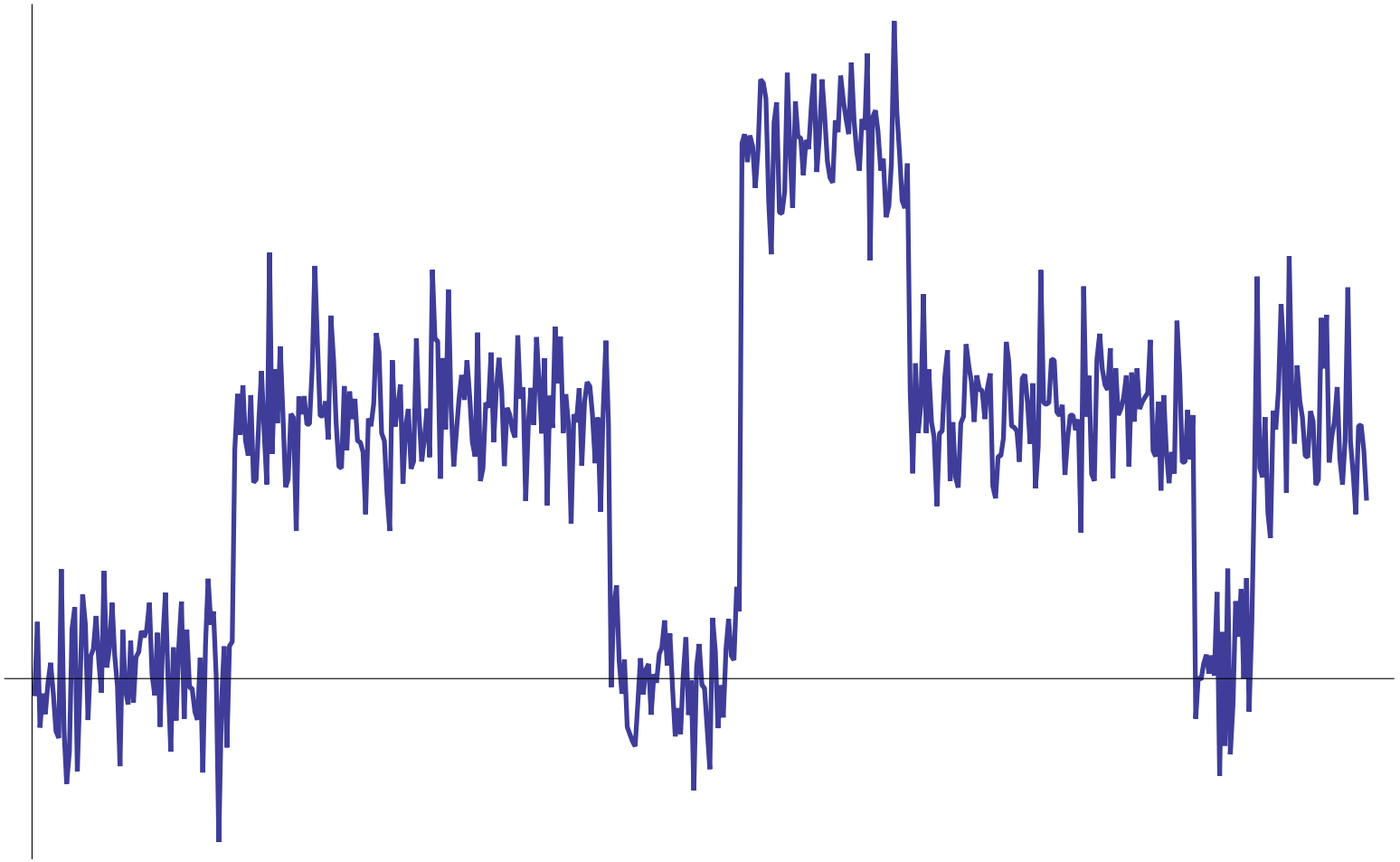
- Internalization: a broker receives a retail buy order and executes the order at the NBO.
- Problem: how does the customer know what the NBO is or was?
- Might the dealer take the highest price in the interval of indeterminacy?
  - Stoll and Schenzler (2002)

# What's lost by first-differencing?



- First difference plot of a simulated series.

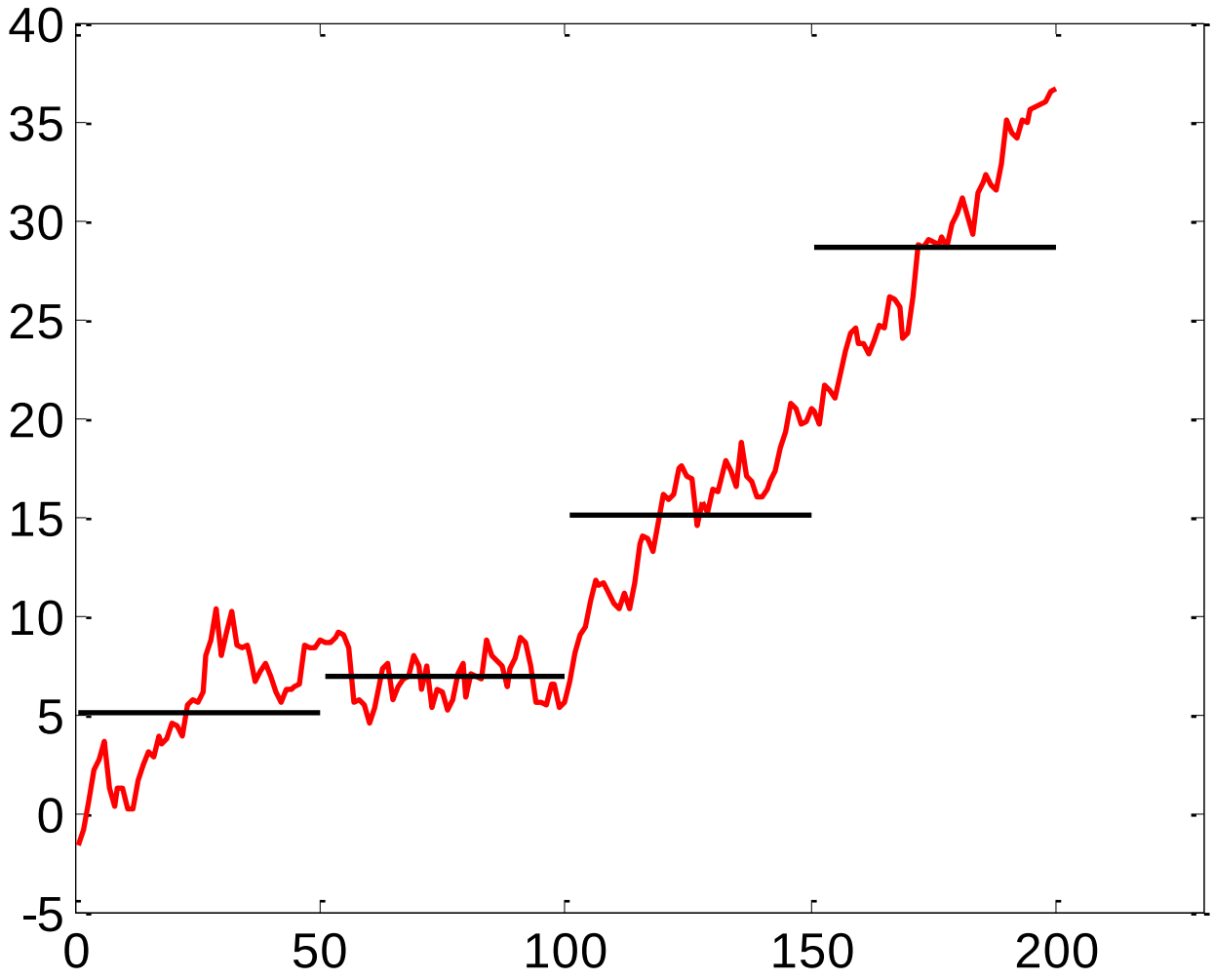
# … and the integrated series



# Analyzing quote volatility

- Usual approach
  - *parametric model* for variance of price *changes* (ARCH, GARCH, …)
- This study
  - *Non-parametric* analysis of variances of price *levels*

# Variance about a local mean of a random walk



# Table 8. Wavelet bid and offer variances, 2001-2011

Panel A. Rough variances, mils per share

| time scale | 2001 | 2003 | 2005 | 2007 | 2009 | 2011 |
|---|---|---|---|---|---|---|
| 50 ms | 0.05 | 0.07 | 0.23 | 0.08 | 0.25 | 0.07 |
| | (<0.01) | (<0.01) | (0.03) | (<0.01) | (0.03) | (0.01) |
| 200 ms | 0.34 | 0.45 | 1.52 | 0.45 | 1.50 | 0.40 |
| | (0.01) | (0.02) | (0.17) | (0.02) | (0.19) | (0.08) |
| 800 ms | 1.50 | 1.85 | 5.56 | 1.42 | 5.16 | 1.42 |
| | (0.05) | (0.07) | (0.53) | (0.07) | (0.42) | (0.22) |
| 3.2 sec | 6.82 | 6.77 | 16.03 | 4.19 | 16.45 | 4.74 |
| | (0.52) | (0.22) | (1.25) | (0.16) | (0.96) | (0.49) |
| 25.6 sec | 80.46 | 47.03 | 84.18 | 25.18 | 109.42 | 30.76 |
| | (16.17) | (2.57) | (5.75) | (0.98) | (13.16) | (3.22) |
| 6.8 min | 735.03 | 489.43 | 862.96 | 302.16 | 1,638.73 | 333.50 |
| | (30.23) | (12.26) | (73.94) | (23.79) | 492.40) | (12.05) |
| 27.3 min | 2,511.15 | 1,554.80 | 2,872.45 | 1,046.55 | 4,623.58 | 1,164.98 |
| | (80.99) | (39.18) | 335.55) | 101.74) | 849.95) | (41.65) |

## Table 8. Wavelet bid and offer variances, 2001-2011

Panel B. Rough variance ratios

| Time scale | 2001 | 2003 | 2005 | 2007 | 2009 | 2011 |
|---|---|---|---|---|---|---|
| 50 ms | 1.60 | 3.15 | 6.09 | 6.56 | 4.20 | 4.46 |
| | (0.02) | (0.11) | (0.39) | (0.40) | (0.31) | (1.42) |
| 200 ms | 1.57 | 3.06 | 5.76 | 5.47 | 3.65 | 3.84 |
| | (0.02) | (0.10) | (0.36) | (0.30) | (0.26) | (1.16) |
| 800 ms | 1.56 | 2.91 | 4.94 | 3.87 | 2.91 | 2.94 |
| | (0.03) | (0.09) | (0.27) | (0.17) | (0.16) | (0.69) |
| 3.2 sec | 1.71 | 2.71 | 3.64 | 2.78 | 2.31 | 2.28 |
| | (0.09) | (0.10) | (0.15) | (0.09) | (0.09) | (0.35) |
| 25.6 sec | 2.36 | 2.60 | 2.42 | 2.03 | 1.74 | 1.70 |
| | (0.37) | (0.29) | (0.07) | (0.05) | (0.04) | (0.12) |
| 6.8 min | 1.37 | 1.50 | 1.49 | 1.37 | 1.32 | 1.23 |
| | (0.04) | (0.03) | (0.02) | (0.02) | (0.03) | (0.02) |
| 27.3 min | 1.12 | 1.16 | 1.16 | 1.12 | 1.11 | 1.08 |
| | (0.02) | (0.02) | (0.02) | (0.02) | (0.02) | (0.01) |

Table 3. Time scale variance across dollar volume quintiles, 2011
Panel A: Rough volatility, $\sigma_j$ in mils per share

| | | | Dollar trading volume quintiles | | | | |
|---|---|---|---|---|---|---|---|
| Level, j | Time scale | Full sample | 1 (low) | 2 | 3 | 4 | 5 (high) |
| 0 | < 50 ms | 0.28 | 0.15 | 0.19 | 0.29 | 0.37 | 0.40 |
| | | (<0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) |
| 1 | 50 ms | 0.39 | 0.21 | 0.27 | 0.40 | 0.51 | 0.56 |
| | | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) |
| 3 | 200 ms | 0.76 | 0.40 | 0.50 | 0.76 | 0.99 | 1.11 |
| | | (0.01) | (0.03) | (0.02) | (0.02) | (0.02) | (0.02) |
| 5 | 800 ms | 1.46 | 0.76 | 0.95 | 1.45 | 1.91 | 2.14 |
| | | (0.02) | (0.04) | (0.03) | (0.03) | (0.04) | (0.04) |
| 7 | 3.2 sec | 2.80 | 1.46 | 1.79 | 2.75 | 3.70 | 4.19 |
| | | (0.04) | (0.07) | (0.06) | (0.06) | (0.08) | (0.09) |
| 10 | 25.6 sec | 7.54 | 3.63 | 4.51 | 7.05 | 10.12 | 12.02 |
| | | (0.10) | (0.15) | (0.15) | (0.15) | (0.23) | (0.27) |
| 14 | 6.8 min | 27.88 | 11.83 | 15.29 | 24.76 | 38.35 | 47.61 |
| | | (0.39) | (0.47) | (0.50) | (0.52) | (0.94) | (1.14) |
| 16 | 27.3 min | 52.84 | 20.94 | 28.09 | 46.87 | 74.69 | 90.49 |
| | | (0.78) | (0.86) | (0.91) | (1.03) | (2.04) | (2.26) |

Table 3. Time scale variance across dollar volume quintiles, 2011
Panel B, Rough volatility, $\sigma_j$ in bp

| | | | Dollar trading volume quintiles | | | | |
|---|---|---|---|---|---|---|---|
| Level, j | Time scale | Full sample | 1 (low) | 2 | 3 | 4 | 5 (high) |
| 0 | < 50 ms | 0.16 | 0.26 | 0.21 | 0.14 | 0.11 | 0.08 |
| | | (<0.01) | (0.01) | (<0.01) | (<0.01) | (<0.01) | (<0.01) |
| 1 | 50 ms | 0.22 | 0.37 | 0.29 | 0.20 | 0.16 | 0.11 |
| | | (<0.01) | (0.02) | (0.01) | (<0.01) | (<0.01) | (<0.01) |
| 3 | 200 ms | 0.43 | 0.70 | 0.56 | 0.38 | 0.31 | 0.22 |
| | | (0.01) | (0.03) | (0.01) | (0.01) | (<0.01) | (<0.01) |
| 5 | 800 ms | 0.83 | 1.34 | 1.07 | 0.73 | 0.59 | 0.43 |
| | | (0.01) | (0.05) | (0.02) | (0.01) | (0.01) | (0.01) |
| 7 | 3.2 sec | 1.58 | 2.56 | 2.03 | 1.41 | 1.15 | 0.85 |
| | | (0.02) | (0.08) | (0.04) | (0.02) | (0.01) | (0.01) |
| 10 | 25.6 sec | 4.10 | 6.32 | 5.17 | 3.65 | 3.13 | 2.42 |
| | | (0.04) | (0.17) | (0.09) | (0.06) | (0.04) | (0.03) |
| 14 | 6.8 min | 14.40 | 20.19 | 17.85 | 12.96 | 11.92 | 9.64 |
| | | (0.14) | (0.52) | (0.35) | (0.22) | (0.16) | (0.15) |
| 16 | 27.3 min | 26.79 | 35.62 | 33.19 | 24.41 | 23.20 | 18.38 |
| | | (0.27) | (0.95) | (0.72) | (0.43) | (0.35) | (0.30) |

Table 3. Time scale variance estimates across $ volume quintiles, 2011
Panel C. Rough variance ratios

| | | | Dollar trading volume quintiles | | | | |
|---|---|---|---|---|---|---|---|
| Level, j | Time scale | Full sample | 1 (low) | 2 | 3 | 4 | 5 (high) |
| 0 | < 50 ms | 4.22 | 12.72 | 3.45 | 2.62 | 1.76 | 1.37 |
| | | (1.28) | (6.96) | (0.18) | (0.07) | (0.04) | (0.02) |
| 1 | 50 ms | 3.99 | 12.01 | 3.23 | 2.44 | 1.69 | 1.35 |
| | | (1.25) | (6.81) | (0.16) | (0.06) | (0.04) | (0.02) |
| 3 | 200 ms | 3.53 | 10.40 | 2.83 | 2.20 | 1.57 | 1.30 |
| | | (1.06) | (5.77) | (0.11) | (0.05) | (0.03) | (0.02) |
| 5 | 800 ms | 2.90 | 7.82 | 2.50 | 2.02 | 1.43 | 1.21 |
| | | (0.66) | (3.56) | (0.08) | (0.04) | (0.03) | (0.02) |
| 7 | 3.2 sec | 2.40 | 5.87 | 2.17 | 1.82 | 1.32 | 1.15 |
| | | (0.38) | (2.08) | (0.06) | (0.04) | (0.02) | (0.02) |
| 10 | 25.6 sec | 1.70 | 3.06 | 1.70 | 1.49 | 1.19 | 1.17 |
| | | (0.12) | (0.64) | (0.04) | (0.03) | (0.02) | (0.02) |
| 14 | 6.8 min | 1.23 | 1.58 | 1.24 | 1.17 | 1.04 | 1.16 |
| | | (0.02) | (0.12) | (0.03) | (0.03) | (0.02) | (0.02) |
| 16 | 27.3 min | 1.08 | 1.19 | 1.08 | 1.06 | 1.01 | 1.06 |
| | | (0.01) | (0.06) | (0.03) | (0.02) | (0.02) | (0.02) |

Table 3. Time scale variance estimates across $ volume quintiles, 2011
Panel D. Bid-offer correlations

| | | | Dollar trading volume quintiles | | | | |
|---|---|---|---|---|---|---|---|
| Level, j | Time scale | Full sample | 1 (low) | 2 | 3 | 4 | 5 (high) |
| 1 | 50 ms | 0.32 | 0.05 | 0.23 | 0.31 | 0.41 | 0.56 |
| | | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) |
| 3 | 200 ms | 0.41 | 0.11 | 0.33 | 0.42 | 0.49 | 0.65 |
| | | (0.01) | (0.01) | (0.02) | (0.02) | (0.01) | (0.02) |
| 5 | 800 ms | 0.48 | 0.15 | 0.40 | 0.51 | 0.56 | 0.72 |
| | | (0.01) | (0.01) | (0.02) | (0.02) | (0.02) | (0.02) |
| 7 | 3.2 sec | 0.55 | 0.19 | 0.47 | 0.59 | 0.66 | 0.82 |
| | | (0.01) | (0.02) | (0.02) | (0.02) | (0.02) | (0.02) |
| 10 | 25.6 sec | 0.70 | 0.27 | 0.61 | 0.75 | 0.85 | 0.95 |
| | | (0.01) | (0.02) | (0.03) | (0.02) | (0.02) | (0.03) |
| 14 | 6.8 min | 0.86 | 0.44 | 0.88 | 0.97 | 0.99 | 1.00 |
| | | (0.02) | (0.03) | (0.04) | (0.03) | (0.03) | (0.03) |
| 16 | 27.3 min | 0.90 | 0.51 | 0.96 | 0.99 | 1.00 | 1.00 |
| | | (0.02) | (0.04) | (0.05) | (0.03) | (0.04) | (0.03) |

# Context and connections

- Mainstream volatility modeling
- Analyses of high frequency trading
- Methodology: time scale resolution and variance estimation

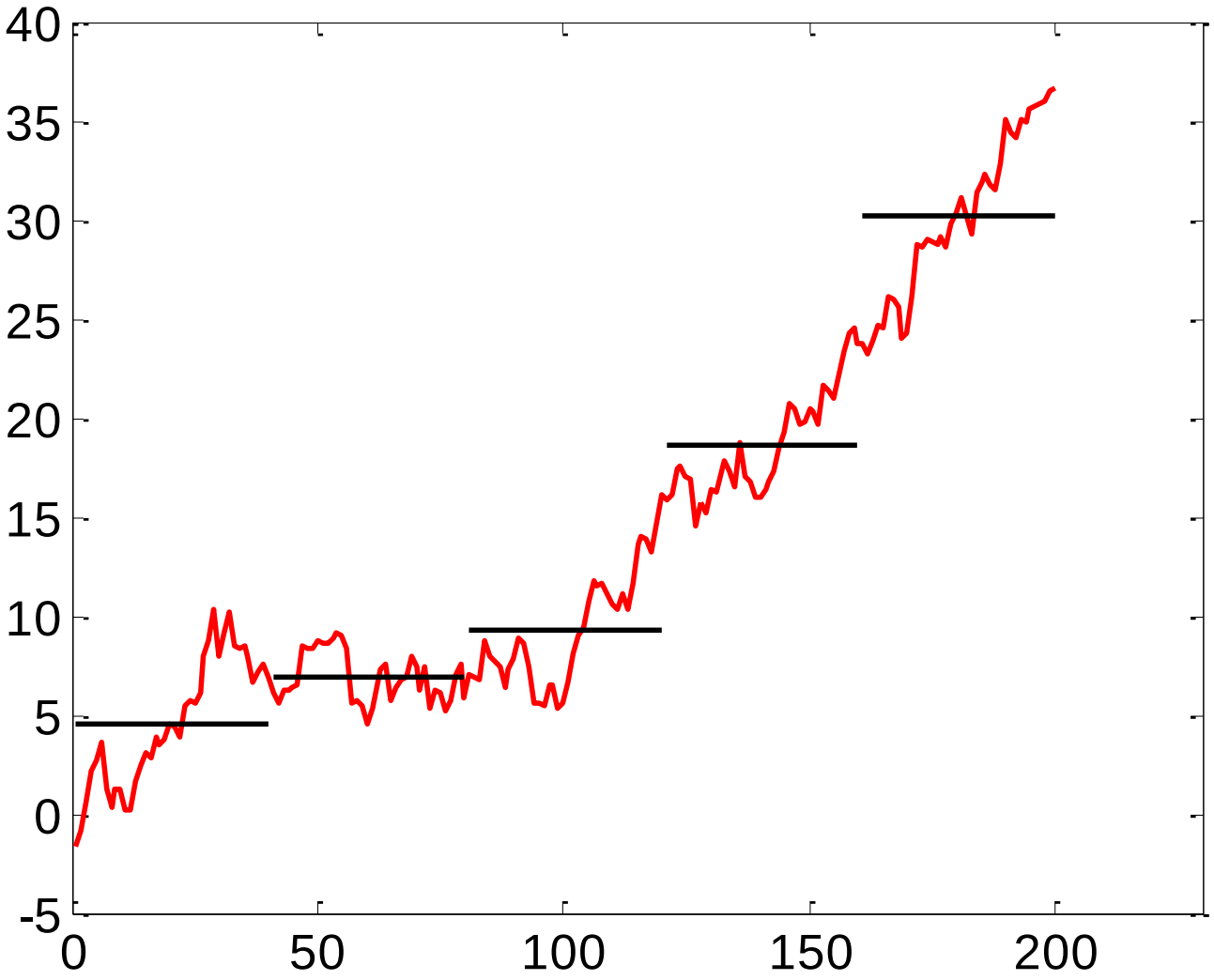
# Volatility Modeling

- Mainstream ARCH, GARCH, and similar models focus on fundamental/informational volatility.
  - Statistically: volatility in the unit-root component of prices.
  - Economically important for portfolio allocation, derivatives valuation and hedging.
- Quote volatility is non-informational
  - Statistically: short-term, stationary, transient volatility
  - Economically important for trading and market making.

# Realized volatility (RV)

- Volatility estimates formed from HF data.
  - RV = average (absolute/squared) *price changes*.
  - Andersen, Bollerslev, Diebold and Ebens (2001), and others
- At high frequencies, microstructure noise becomes the dominant component of RV.
- Hansen and Lunde (2006) advocate using local level averaging ("pre-averaging") to eliminate microstructure noise.

# Pre-averaging



# Quote volatility is microstructure noise

- Present study
  - Form local level averages
  - Examine volatility centered on these averages.
- Other contrasts with mainstream volatility modeling
  - Trade prices vs. *bid and offer quotes*
  - "Liquid" securities (indexes, Dow stocks, FX) vs. *mid- and low-cap issues*

# High-frequency trading (HFT)

- Institutional background of US equity markets
- HFT: a working definition
- The current debate

# US equity markets

- Fragmentation
  - There are multiple trading venues (market centers)
- Lit and dark markets
  - Lit markets display bid and offer quotes
  - Dark markets post no visible quotes.
- Low latency / high-frequency trading
  - The arms race for first-mover advantage
  - The segmentation of traders into the quick and the dead.

# HFT: Commodities Futures Trading Commission draft definition

- A form of automated trading that employs:
  - (a) *algorithms* for decision making, order initiation, generation, routing, or execution, for each individual transaction without human direction;
  - (b) low-latency *technology that is designed to minimize response times*, including proximity and co-location services;
  - (c) *high speed connections* to markets for order entry; and
  - (d) *high message rates* (orders, quotes or cancellations)

# The economic/regulatory debate

- "HF traders are the new market makers."
  - Like traditional dealers and specialists.
  - They provide valuable intermediation services.
- "HF traders profit by anticipating and front-running the orders of long-term investors."
  - They impose costs on these investors.
  - This will discourage the production and analysis of fundamental information.

# Is this risk zero-mean and diversifiable?

- For low-cap stocks, the volatility over three seconds averages 2.5 basis points (0.025%)
  - In a portfolio of 100 trades, the volatility is 0.25 basis points.
- What if, for particular agents, the risk is not zero-mean?