F1 Results

News vs. no-news

- With news visible, the median trading profits were about $130,000 (485 player-sessions)
- With the news screen turned off, median trading profits were about $165,000 (283 player-sessions)
- Trading strategies that worked.
- Market vs. limit order strategies.
**Entering limit and orders to buy**

With $V=100$ and $O$(offset)=$1$, a left-click in this region will submit a limit order to *buy* 100 shares priced at $23.92 + 0.01 = 23.93$.  
*The click inserts a new limit buy order at the indicated position.*

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To *buy* with a market order, you have to lift offers on the other side of the book.  

You have to right click on the offer side to generate a market buy order.
Entering limit and market orders to sell

To submit a market sell order, point anywhere in the bid side of the book and right-click.

To insert a new limit sell order, point to a row and left-click.

Trading to hedge: dynamic hedging
The RIT H1 portfolio hedge case: some key features

- The risk is market risk in a known portfolio.
- The hedging security is a stock index futures contract.
- The relation between the portfolio return and futures return is linear, but partially random.
- Partially random: we need to estimate beta from a statistical model and a data sample.
- Linear: once we implement the hedge, we don’t have to adjust it. The hedge is static.

The RIT H3 case: an option hedge

- We are short a risky security (a call option).
- We will try to hedge by going long the stock.
- The value of the call option is an exact, known function of the stock price (the Black-Scholes equation).
  - We don’t need to estimate a statistical model.
- The Black-Scholes value of a call is nonlinear.
  - When the stock price changes, we need to adjust our hedge.
  - The hedge is dynamic.
Review

- An American call gives the holder the right to buy the underlying stock \( (S) \) at exercise/strike price \( X \) up to and including maturity date \( T \).
  - An American put gives the holder the right to sell the underlying ...
- A European option can only be exercised at maturity.
- An option comes into existence when it is traded.
  - The person who sells a call has written the call, and is short the call

Intrinsic value of a call with exercise price \( X \).

- If the current stock price is \( S \), the gain from immediate exercise is
  \[
  \text{Intrinsic value} = \max(S - X, 0)
  \]
The call value is computed from the Black-Scholes equation.

The vertical difference between the two lines is the time value of the call.

Example: If $X = 40$, $S = 48$ and $C = 15$, then the intrinsic value is 

$$Max[48 - 40, 0] = 8$$

- The time value is $15 - 8 = 7$.

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**Call valuation: the Black-Scholes assumptions**

- The stock (and risk-free bonds) may be bought and sold at any time without cost.
  - This allows us to construct and maintain perfectly hedged (risk-free) portfolios of stocks, bonds and calls.
  - In fact, real-world options traders use approximate (low risk) hedges.
- The stock price moves as the accumulation of small (infinitesimal) random changes that have constant volatility.
  - There are no sudden announcements or surprises.
  - In fact, the possibility of “jumps” can lead to discrepancies between Black-Scholes valuations and actual market prices.
Black-Scholes: the inputs

- $S$ the current stock price ($ per share)
- $X$ the exercise price of the call option ($ per share)
- $T$ time to maturity (years)
- $r$ the risk-free interest rate (annual)
  - A 4% rate would be entered as $r = 0.04$
- $\sigma$ the volatility of the underlying.
  - The standard deviation of the stock’s annual return.
  - Example: “Over time the S&P index average annual return is about 10%, with a standard deviation of about 20%”
  - This would be entered as $\sigma = 0.20$

The Black-Scholes equation for $C$, the value of a call

$$
C = S \times N(d_1) - X \times e^{-rT} \times N(d_2)
$$

- $r$ is the (risk-free) interest rate for borrowing and lending.
- $T$ is the time remaining to maturity.
- $d_1, d_2,$ and $N(\cdot)$ are given on the next slide.
- This variant of the equation is correct for a European call on a non-dividend paying stock.
\[ d_1 = \frac{\ln \left( \frac{S}{X} \right) + (r + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]

- \( N(d) \) is the cumulative distribution for the standard normal density evaluated at \( d \).
- \( N(d) \) is given by the Excel function NORM.S.DIST(d,TRUE)

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Black-Scholes.xlsx

Values for the SAC call in the RIT H3 case.
Hedging

- The stock and the call move in the same direction.
  - They are very highly correlated.
  - Over short time intervals $\rho \approx 1$.
- A short position in the call and a long position in the stock move in opposite directions.
  - They are very negatively correlated.
  - Over short time intervals $\rho \approx -1$.
- Can we find a risk-free portfolio combination?

The H3 Case

- We’re in a bank equity derivatives group.
- A customer wants to buy a call option on SAC.
  - The customer would prefer to buy an exchange-traded call if one were available.
  - This isn’t possible, so the customer comes to us.
- Acting as dealer, we sell to the customer.
- In analyzing the trade, we concentrate on pricing and hedging.
Pricing of the call

- $X = 50$, $r = 0$, $\sigma = 0.15$ ("15% per year"), $T = 20$ trading days.
- Annualize using trading days: $T = \frac{20}{252} = 0.079365$
- $C = \$0.843/\text{share}$
- We need to factor in a profit for ourselves so we quote a price of $\$1.41$ to the customer.
  - $\$1.41$: "We price the call using $\sigma = 0.25$.”
  - The customer can do the calculations: they know the mark-up they’re paying.

Size

- The customer wants to buy 200 calls.
- The standard contract size is 100 shares.
- All prices are quoted on a per-share basis, but when the customer buys, they pay us $\$1.41 \times 100 \times 200 = \$28,200$. 
Hedging

- Immediately after the customer agrees to the trade, we are short 200 calls.
  - If the price of SAC falls, we’ll be okay. The call will expire unexercised.
  - If the price of SAC rises, the call will be exercised against us, costing us money.
- We don’t want to make a directional bet on SAC. We want to hedge.

Can we hedge by buying $200 \times 100 = 20,000$ sh SAC?

- “Worst case: SAC goes from $50$ to, say, $100$. At that point, the customer will exercise. They buy 20,000 from us at $50$.”
- “Shouldn’t we lock in a $50$ purchase price for SAC by buying all the shares we might need right now?”
△ (“Delta”) = Slope = 0.508
To hedge that we’ve written, we need to be long 0.508 shares of SAC

The recipe

- Sell 200 100-share call options at $1.41
  - Receive cash of $28,200.
- Buy $200 \times 100 \times 0.508 = 10,169$ shares of stock
  - Pay with “borrowed” money
- Are we really hedged?
  - What happens if $S$ changes by $\pm$ $0.01$?
The Black-Scholes calculations

Next: the mark-to-market position statements

<table>
<thead>
<tr>
<th>S</th>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.00</td>
<td>Cash received</td>
<td>28,200</td>
</tr>
<tr>
<td></td>
<td>(20,000 ×</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.41)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stock (10,169sh × $50)</td>
<td>508,429</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49.99</td>
<td>Cash received</td>
<td>28,200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stock (10,169sh × $49.99)</td>
<td>508,327</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.01</td>
<td>Cash received</td>
<td>28,200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stock (10,169sh × $50.01)</td>
<td>508,530</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To design the hedge, prepare a table that gives hedge ratios and the number of shares you should be long for SAC prices between $46 and $54 in $0.20 increments.

- As the stock price changes, this is the target amount you should be trying to hold.
- But adjusting the hedge too often, or using market orders will boost the trading costs.
- Remember: the size of the position is 20,000 shares.

We’ve concentrated on hedging short term changes in the stock price \( S \).

- This is called delta hedging.

The hedge ratio also depends on \( T, r \) and \( \sigma \).

- \( \sigma \) depends on market conditions and new information.
- Even if \( r \) and \( \sigma \) are constant, \( T \) will change with the passage of time.
- Adjusting the hedge requires us to trade in the direction of the market.
  - We sell when $S$ falls, buy when $S$ rises.
  - Do our trades push the price against us?
- Are there other traders who are also trying to delta-hedge?
- What if there is a significant news announcement when the market is closed?

Hedging and *jumps*

- Delta hedging works best when successive price movements are small.
  - Slow accumulations of low-intensity information
- Example SPY, April 15, 2011
- Delta hedging does not work well when prices move due to large information shocks.
At 10AM, a customer wants a one-year ACOR call with \( X = \$20 \).

\( T = 1 \); assume \( \sigma = 0.5, r = 0 \).

At 10AM, the stock price is \( S \approx \$21 \).

From Black-Scholes \( C = \$2.976 \) and \( \Delta = N(d_1) = 0.623 \).

We sell ten 100-share call options to a customer at \$6.

We hedge by buying 623 shares of ACOR.

At about 13:20 the stock price goes to \$27.

At \( S = \$27 \), \( C = \$7.576 \) and \( \Delta = 0.875 \).
Mark-to-market positions

<table>
<thead>
<tr>
<th>Time</th>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:00</td>
<td>Cash received</td>
<td>2,976 Calls, mark-to-mkt</td>
</tr>
<tr>
<td></td>
<td>from sale of calls</td>
<td>($2.976 \times 100 \times 10)</td>
</tr>
<tr>
<td></td>
<td>Stock (623 sh @ $21)</td>
<td>13,083 Loan / charge to capital</td>
</tr>
<tr>
<td></td>
<td>13,083</td>
<td>3,024 Net worth</td>
</tr>
<tr>
<td>14:00</td>
<td>Cash</td>
<td>7,576 Calls ($7.576 \times 100 \times 10)</td>
</tr>
<tr>
<td></td>
<td>6,000</td>
<td>13,083 Charge to capital</td>
</tr>
<tr>
<td></td>
<td>Stock (623 sh @ $27)</td>
<td>16,821</td>
</tr>
</tbody>
</table>

Replication

- An alternative direction for the H3 case.
- The customer says, “The bank wants $1.41 for a call that Black-Scholes says is worth only $0.84? That’s crazy. Can we ‘manufacture’ the option ourselves?”
  - How does the customer do this?