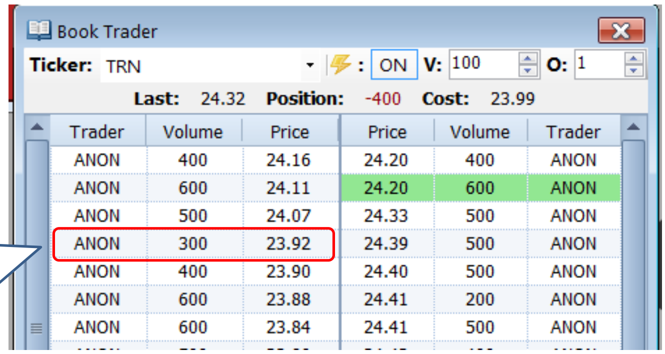
F1 Results

News vs. no-news

- With news visible, the median trading profits were about $130,000 (485 player-sessions)
- With the news screen turned off, median trading profits were about $165,000 (283 player-sessions)
- Trading strategies that worked.
- Market vs. limit order strategies.

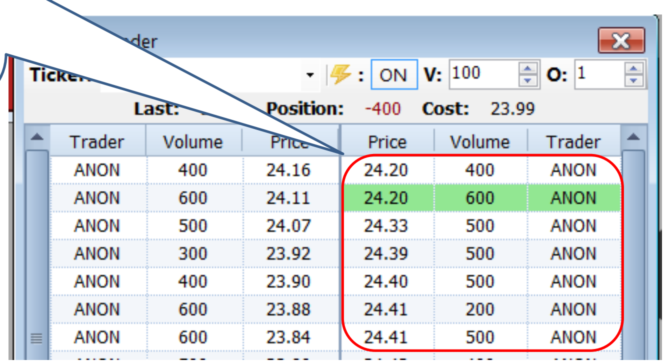
Entering limit and orders to buy



With V=100 and O(offset)=1, a left-click in this region will submit a limit order to *buy* 100 shares priced at 23.92 + 0.01 = 23.93.
The click inserts a new limit buy order at the indicated position.

To buy with a market order, you have to lift offers on the other side of the book.

You have to right click on the offer side to generate a market buy order.



Entering limit and market orders to sell

To submit a *market sell order*, point anywhere in the bid side of the book and right-click.

To insert a new *limit sell order*, point to a row and left-click

Trading to hedge: dynamic hedging

The RIT H1 portfolio hedge case: some key features

- The risk is market risk in a known portfolio.
- The hedging security is a stock index futures contract.
- The relation between the portfolio return and futures return is linear, but partially random.
- *Partially random*: we need to estimate beta from a statistical model and a data sample.
- *Linear*: once we implement the hedge, we don't have to adjust it. The hedge is *static*.

The RIT H3 case: an option hedge

- We are short a risky security (a call option).
- We will try to hedge by going long the stock.
- The value of the call option is an exact, known function of the stock price (the Black-Scholes equation).
  - *We don't need to estimate a statistical model.*
- The Black-Scholes value of a call is nonlinear.
  - *When the stock price changes, we need to adjust our hedge.*
  - The hedge is *dynamic*.

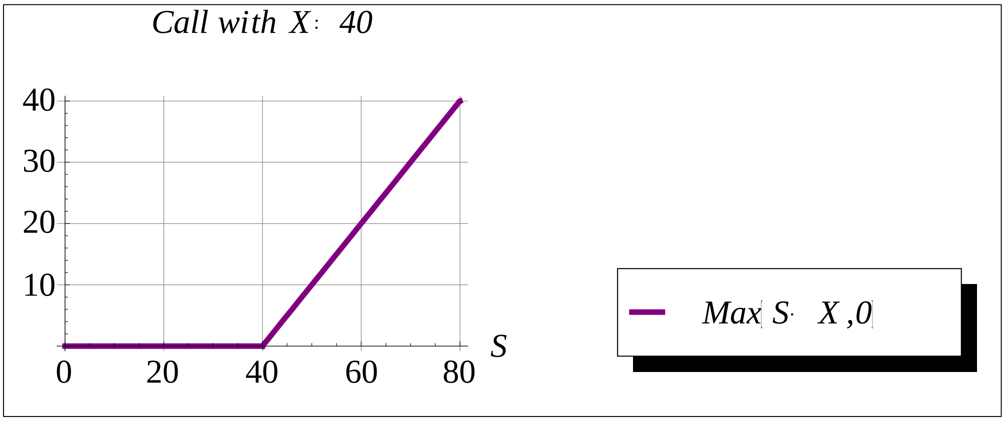
Review

- An *American call* gives the holder the right to buy the underlying stock (S) at *exercise/strike price* X up to and including *maturity date* T.
  - An *American put* gives the holder the right to *sell* the underlying …
- A *European option* can only be exercised at maturity.
- An option comes into existence when it is traded.
  - The person who sells a call has *written* the call, and is *short* the call

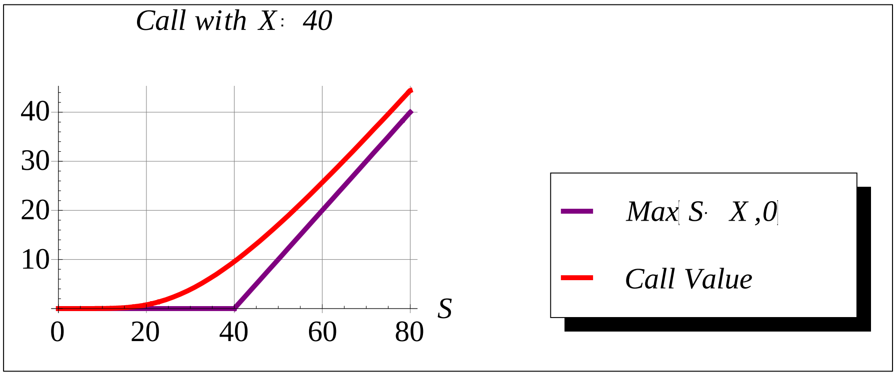
Intrinsic value of a call with exercise price X.

- If the current stock price is S, the gain from immediate exercise is

$$Intrinsic\ value = Max(S - X, 0)$$



C, the value of the call



- The call value is computed from the Black-Scholes equation.
- The vertical difference between the two lines is the time value of the call.
- Example: If $X = 40, S = 48$ and $C = 15$, then the intrinsic value is

$$Max[48 - 40,0] = 8$$

  - The time value is $15 - 8 = 7$.

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Call valuation: the Black-Scholes assumptions

- The stock (and risk-free bonds) may be bought and sold at any time without cost.
  - This allows us to construct and maintain perfectly hedged (risk-free) portfolios of stocks, bonds and calls.
  - In fact, real-world options traders use approximate (low risk) hedges.
- The stock price moves as the accumulation of small (infinitesimal) random changes that have constant volatility.
  - There are no sudden announcements or surprises.
  - In fact, the possibility of "jumps" can lead to discrepancies between Black-Scholes valuations and actual market prices.

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Black-Scholes: the inputs

- S the current stock price ($ per share)
- X the exercise price of the call option ($ per share)
- T time to maturity (years)
- r the risk-free interest rate (annual)
  - A 4% rate would be entered as $r = 0.04$
- σ the volatility of the underlying.
  - The standard deviation of the stock's annual return.
  - Example: "Over time the S&P index average annual return is about 10%, with a standard deviation of about 20%"
  - This would be entered as $\sigma = 0.20$

The Black-Scholes equation for C, the value of a call

- $C = \underbrace{S}_{\substack{Current\\stock\\price}} \times N(d_1) - \underbrace{X}_{\substack{Exercise\\price}} \times \underbrace{e^{-r\times T}}_{\substack{Present\\value\\factor}} \times N(d_2)$
  - r is the (risk-free) interest rate for borrowing and lending.
  - T is the time remaining to maturity.
  - d_1, d_2, and $N(\cdot)$ are given on the next slide.
  - This variant of the equation is correct for a European call on a non-dividend paying stock.

$$d_1 = \frac{ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

- $N(d)$ is the cumulative distribution for the standard normal density evaluated at d.
- $N(d)$ is given by the Excel function NORM.S.DIST(d,TRUE)

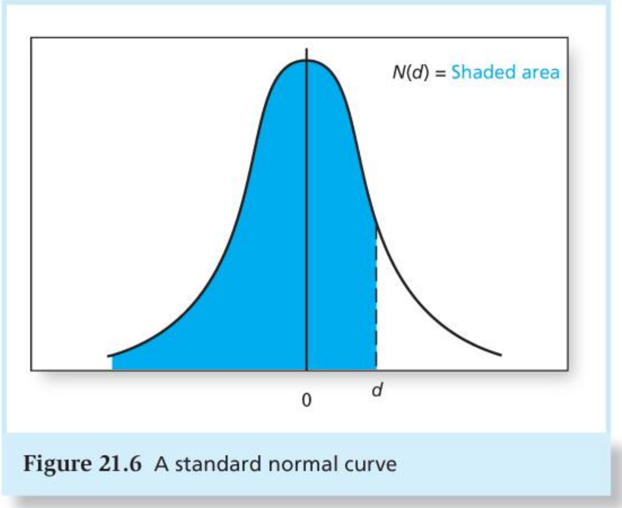

Figure 21.6 A standard normal curve

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Black-Scholes.xlsx

Values for the SAC call in the RIT H3 case.

	A	B	C
1	Black-Scholes calculations		
2			
3	Inputs		
4	Stock price, S	50	
5	Exercise price of call, X	50	
6	Maturity, years	0.079365079	=20/252
7	Risk-free rate (annual)	0	
8	Standard Deviation (annualized s)	0.15	
9			
10	Outputs		
11	Present value of X	50.000	
12	s*sqrt(t)	0.042	
13	d1	0.021	
14	d2	-0.021	
15	**Delta (=N(d1))**	0.508	
16	N(d2)*PV of X	24.579	
17			
18	**Value of Call**	0.843	
19	Value of Put	0.843	

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Hedging

- The stock and the call move in the same direction.
  - They are *very* highly correlated.
  - Over short time intervals $\rho \approx 1$.
- A short position in the call and a long position in the stock move in opposite directions.
  - They are very negatively correlated.
  - Over short time intervals $\rho \approx -1$.
- Can we find a risk-free portfolio combination?

The H3 Case

- We're in a bank equity derivatives group.
- A customer wants to buy a call option on SAC.
  - The customer would prefer to buy an exchange-traded call if one were available.
  - This isn't possible, so the customer comes to us.
- Acting as dealer, we sell to the customer.
- In analyzing the trade, we concentrate on pricing and hedging.

Pricing of the call

- $X = 50, r = 0, \sigma = 0.15$ ("15% per year"), $T = 20\ trading\ days$.
- Annualize using trading days: $T = \frac{20}{252} = 0.079365$
- $C = \$0.843/share$
- We need to factor in a profit for ourselves so we quote a price of $1.41 to the customer.
  - $1.41: "We price the call using $\sigma = 0.25$."
  - The customer can do the calculations: they know the mark-up they're paying.

Size

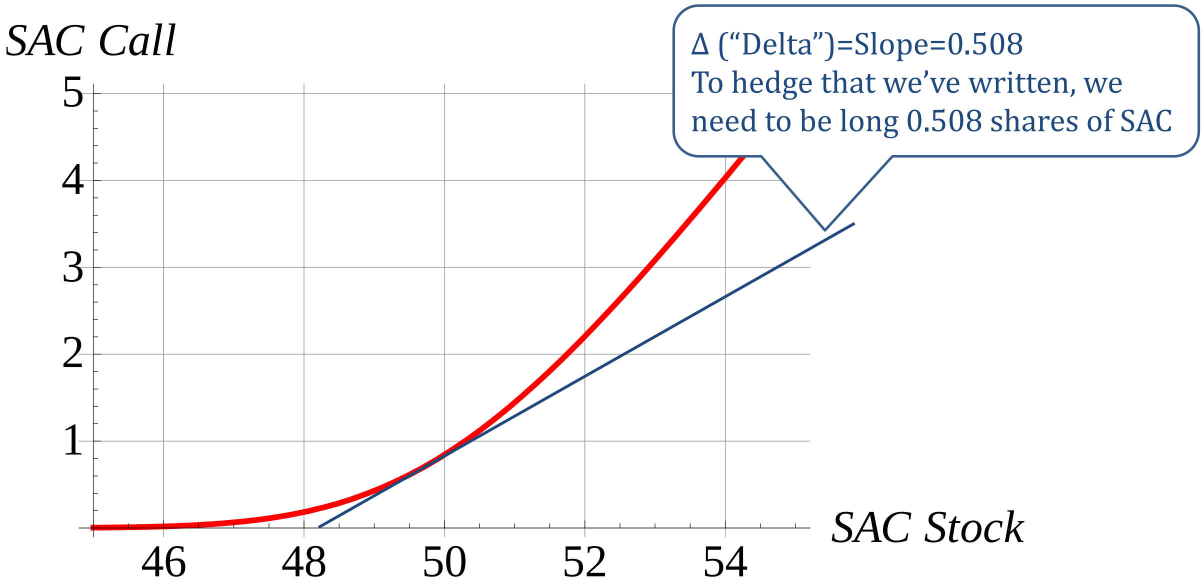
- The customer wants to buy 200 calls.
- The standard contract size is 100 shares.
- All prices are quoted on a per-share basis, but when the customer buys, they pay us $\$1.41 \times 100 \times 200 = \$28{,}200$.

Hedging

- Immediately after the customer agrees to the trade, we are short 200 calls.
  - If the price of SAC falls, we'll be okay. The call will expire unexercised.
  - If the price of SAC rises, the call will be exercised against us, costing us money.
- We don't want to make a directional bet on SAC. We want to hedge.

Can we hedge by buying $200 \times 100 = 20{,}000$ sh SAC?

- "Worst case: SAC goes from \$50 to, say, \$100. At that point, the customer will exercise. They buy 20,000 from us at \$50."
- "Shouldn't we lock in a \$50 purchase price for SAC by buying all the shares we might need right now?"



The recipe

- Sell 200 100-share call options at \$1.41
  - Receive cash of \$28,200.
- Buy $200 \times 100 \times 0.508 = 10{,}169$ shares of stock
  - Pay with "borrowed" money
- Are we really hedged?
  - What happens if S changes by $\pm$ \$0.01?

The Black-Scholes calculations

	A	B	C	H
1	Inputs			
2	Stock price, S	50	49.99	50.01
3	Exercise price of call, X	50	50	50
4	Maturity, years	0.079	0.079	0.079
5	Risk-free rate (annual)	0	0	0
6	Standard Deviation (annu	0.15	0.15	0.15
7				
8	Outputs			
9	Present value of X	50.000	50.000	50.000
10	s*sqrt(t)	0.042	0.042	0.042
11	d1	0.021	0.016	0.026
12	d2	-0.021	-0.026	-0.016
13	**Delta (=N(d1))**	0.508	0.507	0.510
14	N(d2)*PV of X	24.579	24.484	24.673
15				
16	**Value of Call**	0.843	0.838	0.848
17	Value of Put	0.843	0.848	0.838
18	ΔS		-0.0100	0.0100
19	ΔC		-0.0051	0.0051
20	ΔC/ΔS		0.5075	0.5094

- Next: the mark-to-market position statements

S		Assets	Liabilities	
50.00	Cash received (20,000 × $1.41)	28,200	16,857	Call, mark-to-mkt (~20,000 ×
	Stock (10,169*sh* × $50)	508,429	508,429	Loan / charge to capital
			11,343	Net worth
49.99	Cash received	28,200	16,756	Call, mark-to-mkt(~20,000 ×
	Stock (10,169*sh* × $49.99)	508,327	508,429	Loan / charge to capital
			11,343	Net worth
50.01	Cash received	28,200	16,959	Call, mark-to-mkt(~20,000 ×
	Stock (10,169*sh* × $50.01)	508,530	508,429	Loan / charge to capital
			11,343	Net worth

H3

- To design the hedge, prepare a table that gives hedge ratios and the *number of shares you should be long* for SAC prices between \$46 and \$54 in \$0.20 increments.
  - As the stock price changes, this is the target amount you should be trying to hold.
  - But adjusting the hedge too often, or using market orders will boost the trading costs.
  - Remember: the size of the position is **20,000** shares.

Practical complications

- We've concentrated on hedging short term changes in the stock price S.
  - This is called delta hedging.
- The hedge ratio also depends on T, r and σ.
  - σ depends on market conditions and new information.
  - Even if r and σ are constant, T will change with the passage of time.

- Adjusting the hedge requires us to trade in the direction of the market.
  - We sell when *S* falls, buy when *S* rises.
  - Do our trades push the price against us?
- Are there other traders who are also trying to delta-hedge?
- What if there is a significant news announcement when the market is closed?

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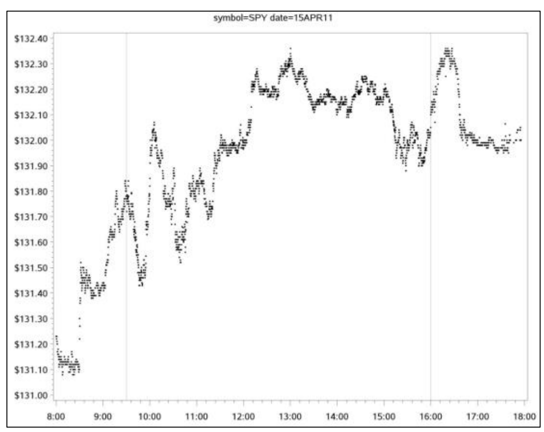
Hedging and *jumps*

- Delta hedging works best when successive price movements are small.
  - Slow accumulations of low-intensity information
- Example SPY, April 15, 2011
- Delta hedging does not work well when prices move due to large information shocks.



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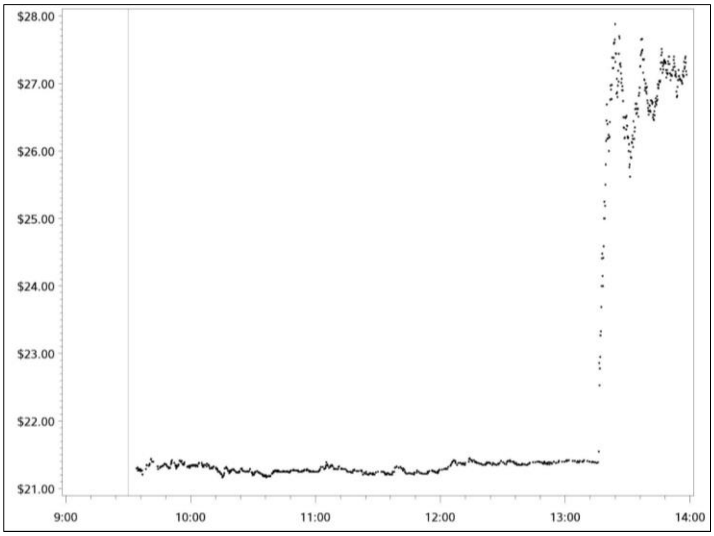
Selling a call on ACOR, 14 April 2011



- At 10AM, a customer wants a one-year ACOR call with $X = \$20$.
- $T = 1$; assume $\sigma = 0.5, r = 0$.
- At 10AM, the stock price is $S \approx \$21$.
- From Black-Scholes $C = \$2.976\ and\ \Delta = N(d_1) = 0.623$.
- We sell ten 100-share call options to a customer at $6
- We hedge by buying 623 shares of ACOR.
- At about 13:20 the stock price goes to $27.
- At $S = \$27, C = \$7.576$ and $\Delta = 0.875$

Mark-to-market positions

Time		Assets	Liabilities	
10:00	Cash received from sale of calls	6,000	2,976	Calls, mark-to-mkt ($2.976 × 100 × 10)
	Stock (623 sh @ $21)	13,083	13,083	Loan / charge to capital
			3,024	Net worth
14:00	Cash	6,000	7,576	Calls ($7.576 × 100 × 10)
	Stock (623 sh @ $27)	16,821	13,083	Charge to capital
			2,162	**Net worth**

Replication

- An alternative direction for the H3 case.
- The customer says, "The bank wants $1.41 for a call that Black-Scholes says is worth only $0.84? That's crazy. Can we 'manufacture' the option ourselves?"
- How does the customer do this?