Trading to hedge: Static hedging

Trading scenarios

- Trading as a dealer (TRN)
  - Strategy: post bids and asks, try to make money on the turn. *Maximize profits*
- Trading on information (F1, PD0)
  - Strategy: forecast price trend (possibly by reading the order flow). *Maximize profits*
- Hedging (H1, H3)
  - Goal is risk reduction. *Minimize tracking error*

Hedging: risk reduction

- We have some risk exposure that can't be directly mitigated (reduced).
- Example: A bank portfolio of loans might be exposed to risk from unexpected interest rate changes.
  - The bank can't simply sell the loans because
    - The loans are earning returns that the bank can't get elsewhere.
    - There might be no market for the loans.

- Example: An airline is exposed to risk arising from changes in the price of fuel.
  - It might enter into long-term fixed-price contracts, but if the airline's projected fuel needs change, it will be difficult to modify the contracts.
- Example: A pension fund with a large portfolio of stocks has a negative market outlook in the short run (weeks or months).
  - Selling the stocks and repurchasing them will lead to substantial trading costs.

We won't try to eliminate *all* risks.

- Hedging is expensive.
  - Most hedges will incur trading costs.
- The securities that we need may not exist.
- There are some risk exposures that we (or our investors) might want us to keep.
  - A bond fund with expertise in credit scoring might want to hedge interest rate risk, but not credit risk.
  - Investors in gold mining stocks usually want some exposure to the price of gold. They don't want the firm to eliminate this exposure.
- We want to be thoughtful and selective about the risks we hedge and the risks we keep.

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The basic hedging principle

- Reduce risk by establishing a position in a security that is negatively correlated with the risk exposure.
- Negative correlation: the *value of the hedge* moves against or opposite to the risk exposure.
  - The ideal hedging security is cheap to buy, easy to trade, and very highly correlated with the risk exposure.
  - If we can go long or short the hedging security, it doesn't matter of the correlation is positive or negative.

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Static hedging

- When we buy/sell the hedging security, we need to trade quickly.
  - Until the hedging position is established we have risk.
  - But if we trade too quickly we'll incur high trading costs.
- The trade-off is risk vs. cost
- If the hedge just needs to be set up initially, and doesn't have to be modified, it is a *static hedge*.
  - The hedging in the first RIT case is static.

Dynamic hedging

- In some situations the hedge position must be adjusted after the initial set-up. This is a *dynamic hedge*.
- The need for dynamic hedging typically arises in
  - Stock portfolios that have put and call options.
  - Bond portfolios that try to match the duration of some liability.
- The second RIT case involves a dynamic hedge.

Sample situation 1: Removing the market *return* in CAT

- CAT is the ticker symbol for Caterpillar (a manufacturer of heavy equipment)
- Portfolio manager Beth has $10 Million to invest.
- If she thinks that Caterpillar is undervalued, she simply buys CAT.
- Suppose that Beth thinks that Caterpillar is undervalued *relative to the market*.
  - She's analyzed the heavy equipment industry, but has no opinion on interest rates, commodity prices, consumer spending or any of the many other things that drive the market.
  - She wants to invest in the difference between the return on CAT and the return on the market.

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Betting on the return difference, $r_{CAT} - r_M$

- If the return on the market is $r_M = 5\%$ and $r_{CAT} = 7\%$, she wants a return of 2%.
  - If $r_M = -11\%$ and $r_{CAT} = -8\%$, she wants a return of 3%
- She wants to be long CAT and short the market.
- She'll use the Standard and Poors Composite Index to approximate "the market".
- To mirror the market *"M,"* there are two candidate hedge securities.
  - She can go long or short the SPDR (ticker symbol "SPY")
  - She can go long or short the S&P Composite E-mini futures contract.

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The index, the ETF, and the futures contract

- The S&P composite index is a weighted average of the prices of 500 stocks. It is computed every fifteen seconds.
  - Many market data systems use "SPX" to denote the index.
    - But since it is not a traded security "SPX" is not a real ticker symbol.
  - As of November, 2014, $SPX \approx 2{,}000$.
- Ticker symbol SPY refers to the exchange-traded-fund (ETF) based on the index.
  - It actually is traded. SPY *is* a real ticker symbol.
  - It is constructed to have a value of one-tenth the index.
    - As of November, 2014 its price is $SPY \approx \$200$.
  - The SPY tracks the SPX closely, but not perfectly.
    - Discrepancies arise due to dividends, management fees, and so on.

- The E-mini S&P futures contract
  - Ticker symbols for futures contracts have a two-character product code ("SP") followed by a month/year code that denotes the maturity of the contract.
    - We'll use "SP" to denote the nearest maturity.
  - The SP price quotes are reported in index points.
  - The size of the contract is $\$50 \times SPX$.
  - The contract is *cash settled*.
    - Suppose I go long the contract today (time 0) at a price of $SP_0 = 2{,}000$.
    - Suppose at maturity (time T) the index is at $SPX_T = 2{,}100$.
    - I receive (from the short side) $(SPX_T - SP_0) \times \$50 = (2{,}100 - 2{,}000) \times \$50 = \$5{,}000$
  - Note: this discussion is somewhat simplified. It ignores margin and daily resettlement.

Method I: Buying CAT and shorting the SPY

- Suppose that CAT is about \$100 per share, and that the $SPX \approx 2{,}000$
- Buy \$10,000,000/\$100 = 100,000 sh of CAT
- The SPY represents one-tenth of the S&P index. $SPY \approx \$200$
  - Beth goes short \$10,000,000/\$200=50,000 sh of SPY.
    - She borrows 50,000 sh of SPY and sells them.
- She's long 100,000 sh of CAT and short 50,000 sh of SPY

Suppose that $r_{CAT} = 7\%$ and $r_M = 5\%$

- CAT stock goes from \$100 to \$107.
  - Beth's 100,000 shares are now worth \$10,700,000.
- The SPY is initially at \$200.
  - A 5% return corresponds to a price of \$210.
  - The value of Beth's short position is $50{,}000 \times \$210 = \$10{,}500{,}000$.
- The net value of Beth's overall position (CAT + SPY) has gone up by \$200,000
- This is a 2% return on the \$10 Million initial investment.

Suppose that $r_{CAT} = -8\%$ and $r_M = -11\%$

- CAT stock goes from \$100 to \$92.
  - Beth's 100,000 shares are now worth \$9,200,000.
- The SPY is initially at \$200.
  - $r_{SPY} = -11\%$ return corresponds to a price of \$178.
  - The value of Beth's short position is $50{,}000 \times \$178 =$ \$8,900,000.
- The net value of the overall position (CAT + SPY) has gone up by \$300,000
- This is a 3% return on the \$10 Million initial investment.

Problem: suppose that $r_{CAT} = -10\%$ and $r_M = -6\%$. Work out the return on Beth's \$10 Million investment.

- Answer in online copy of handout.
- The price of CAT goes from \$100 to \$90.
  - Beth's shares are worth \$9,000,000
- The SPY goes from \$200 to \$188.
- The value of Beth's short position is $50{,}000 \times \$188 =$ \$9,400,000.
- The net change is $-\$400{,}000$, a -4% return.

Method II: Buying CAT and shorting the futures contract

- As in method I, Beth buys 100,000 sh of CAT
- As of November, 2014 (time "0"), the level of the S&P index is about $SPX_0 = 2{,}000$.
- An E-Mini S&P index futures contract has a notional value of $\$50 \times SPX = \$50 \times 2{,}000 = \$100{,}000/contract$.
- She goes short $\frac{\$10{,}000{,}000}{\$100{,}000} = 100\ contracts$ at 2,000

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Suppose that $r_{CAT} = 7\%$ and $r_M = 5\%$

- CAT stock goes from $100 to $107.
  - Beth's 100,000 shares are now worth $10,700,000.
- "$r_M = 5\%$":The SPX goes from 2,000 to 2,100
  - To settle her 100 short contracts, Beth pays
    $(2{,}100 - 2{,}000) \times \$50 \times 100 = \$500{,}000$
- The net gain is $200,000 (a 2% return on the $10 Million initial investment).

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Suppose that $r_{CAT} = -8\%$ and $r_M = -11\%$

- CAT stock goes from \$100 to \$92.
  - Beth's 100,000 shares are now worth \$9,200,000.
- "$r_M = -11\%$":The SPX goes from 2,000 to 1,780
  - To settle her 100 short contracts, Beth pays
    $$(1{,}780 - 2{,}000) \times \$50 \times 100 = -\$1{,}100{,}000$$
  - Beth receives \$1,100,000
- Her positions are now worth \$10,300,000: (a 3% return on the \$10 Million initial investment).

Problem: suppose that $r_{CAT} = -10\%$ and $r_M = -6\%$. Work through the numbers for method II. (How much to settle the futures contracts? What is the net percentage return?)

- Answer in online copy of handout
- Beth's shares are worth \$9,000,000
- "$r_M = -6\%$":The SPX goes from 2,000 to 1,880
  - To settle her 100 short contracts, Beth pays
    $$(1{,}880 - 2{,}000) \times \$50 \times 100 = -\$600{,}000$$
  - Beth *receives* \$600,000
- Her net position is now worth \$9,600,000.
- This is a loss of \$400,000, a -4% return.

Situation 2: Removing the market *risk* from CAT

- Beth owns \$10 Million worth of CAT
- She likes CAT, but would like to eliminate the market *risk* in CAT.
  - Market risk: randomness in CAT's return that is *driven by the market*.
- We need a model of the joint randomness in CAT and the market.
  - We'll use a simple linear regression.
  - Regress the returns on CAT vs the returns on M.

- $\underbrace{r_{CAT,t}}_{\substack{return\ on\ CAT \\ in\ month\ t}} = \underbrace{\alpha_{CAT}}_{\substack{CAT's \\ alpha}} + \underbrace{\beta_{CAT}}_{\substack{CAT's \\ beta}} \times \underbrace{r_{SPY,t}}_{\substack{return\ on\ SPY \\ in\ month\ t}} + \underbrace{e_{CAT,t}}_{\substack{regression \\ error}}$
- Simple linear regression in Excel
  - Use in-cell formulas, SLOPE and INTERCEPT
  - Use array formula, LINEST. *Can also be used for multiple regression.*
  - Using Excel's charting menus: plot data on an XY scatterplot; add an estimated trend line; display the equation of the trend line.
  - Use Excel's data analysis menu to run the regression and display the output. *Can also be used for multiple regression.*
    - This method computes more diagnostic statistics.
    - But the results do not automatically update if the data change. You need to rerun the regression.

Approach

- Download prices for CAT stock and the SPY (or the S&P index)
- We'll use month-end prices from 2009-2013.
- Construct monthly returns for CAT and the SPY.
- Plot them and find the best fit linear regression line.
  - A linear regression takes two variables "*x* and *y*" and relates them as a straight line plus an error.
  - For data point i, $y_i = \alpha + \beta \times x_i + e_i$
- The data and details are in workbook *H1.xlsx*, worksheet *CATSPY*, posted to the web.
- You'll be doing similar calculations for the stocks in the RIT hedging case.

Going from prices to returns

=C4/C3-1

	A	B	C	D	E	F	G
1			Price	Price		Return	Return
2	Year	Date	CAT	SPY		CAT	SPY
3	2009	20090130	$30.85	$82.83			
4	2009	20090227	$24.61	$73.93		-0.2023	-0.1074
5	2009	20090331	$27.96	$79.52		0.1361	0.0756
6	2009	20090430	$35.58	$87.42		0.2725	0.0993
7	2009	20090529	$35.46	$92.53		-0.0034	0.0585

Method 1: Use the SLOPE and INTERCEPT functions

"y" r_{CAT} "x", "r_M", r_{SPY}

β_{CAT}

Beta and intercept from SLOPE and INTERCEPT functions:

1.8589 =SLOPE(F4:F62,G4:G62)

-0.0037 =INTERCEPT(F4:F62,G4:G62)

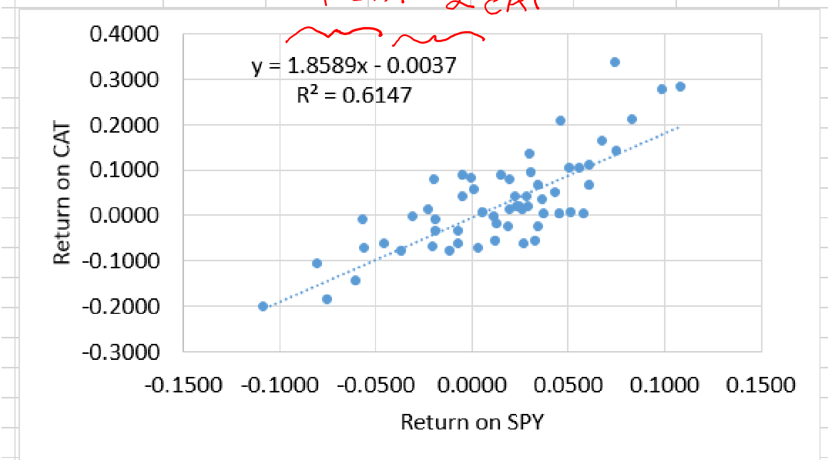
α_{CAT}

Beta with zero intercept

1.8336 =SUMPRODUCT(F4:F62,G4:G62)/SUMPRODUCT(G4:G62,G4:G62)

Method 2: Make a scatterplot with a trendline

β_{CAT} α_{CAT}



Method 3: Use the LINEST array function

β_{CAT} α_{CAT}

	1.8589	-0.0037	{=LINEST(F4:F62,G4:G62,TRUE,TRUE)}
	0.1949	0.0090	Std. Errors.
R^2	0.6147	0.0654	Mean Sq. Error?
	90.9345	57	# observations
	0.3886	0.2436	

Method 4: Use the DATA > Analysis > Regression menu

If you don't see "Data Analysis" on the DATA menu, you may need to enable the Analysis-ToolPak add-in. You can reach this menu from FILE→Options→Add-Ins

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.7840
R Square	0.6147
Adjusted R Squa	0.6079
Standard Error	0.0654
Observations	59

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.3886	0.3886	90.9345	2.08E-13
Residual	57	0.2436	0.0043		
Total	58	0.6322			

	Coefficient	*Standard E*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0*	*Upper 95.0%*
Intercept	-0.0037	0.0090	-0.4077	0.6850	-0.0216	0.0143	-0.0216	0.0143
X Variable 1	1.8589	0.1949	9.5360	0.0000	1.4685	2.2492	1.4685	2.2492

Interpretation of one observation

- $r_{CAT,t} = \alpha_{CAT} + \beta_{CAT} \times r_{SPY,t} + e_{SPY,t}$
- In $t = June, 2009,\ r_{CAT,t} = 0.334\ (33.4\%)$ and $r_{SPY,t} = 0.075\ (7.5\%)$
- Statistical interpretation:
  - $0.334 = \underbrace{-0.004 + 1.859 \times 0.075}_{\substack{0.135\ (13.5\%) \\ Predicted\ value\ of\ r_{CAT,t}}} + \underbrace{0.199}_{\substack{Regression \\ error}}$
- Economic interpretation:
  - "In June, 2009, factors in the broader market caused CAT to go up by 13.5%. An additional return of 19.9% came from factors unrelated to the market."
  - These unrelated factors would be due to industry- and company-specific effects.

Decomposition of CAT's risk

- $r_{CAT,t} = \alpha_{CAT} + \beta_{CAT} \times r_{SPY,t} + e_{CAT,t}$
- $Var\left(r_{CAT,t}\right) = \sigma^2_{CAT} = \beta^2_{CAT} \times \sigma^2_{SPY} + \sigma^2_{e,CAT}$
- Note: α_{CAT} is constant and doesn't contribute any risk.
- Interpretation: $\underbrace{\sigma^2_{CAT}}_{\substack{Total\ risk \\ of\ CAT}} = \underbrace{\beta^2_{CAT} \times \sigma^2_{SPY}}_{\substack{CAT's\ market \\ risk}} + \underbrace{\sigma^2_{e,CAT}}_{\substack{CAT's\ firm-specific \\ risk}}$

Implications for hedging

- $r_{CAT,t} = \alpha_{CAT} + \beta_{CAT} \times r_{SPY,t} + e_{CAT,t}$
- $\beta_{CAT} \approx 1.86$ is a multiplier
  - If the market is up 1%, then all else equal, we expect CAT to be up 1.86%
- If we are long \$1 in CAT, we should be short $\beta_{CAT} \times \$1 \approx \$1.86$ of the SPY.
- To eliminate the market risk in \$10 Million worth of CAT we can
  - Short \$18.6 Million worth of SPY
    - $\frac{\$18.6 Million}{\$200} \approx 93{,}000\ shares\ of\ SPY$
  - Or, short \$18.6 Million notional of the index futures contract
  - $\frac{\$18.6 Million}{2{,}000 \times \$50} \approx 186\ Contracts$

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Example

- If $r_{SPY} = 0.01 (= 1\%)$, then we expect (all else equal, ignoring α_{CAT}) that $r_{CAT} = 0.0186$.
- Our \$10 Million position in CAT goes up by \$186,000.
- A 1% gain on SPY corresponds to the S&P going from 2,000 to 2,020.
  - We settle our 186 futures contracts by *paying*
    $$186 \times (2{,}020 - 2{,}000) \times \$50 = \$186{,}000$$
  - This is a total offset.

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The RIT H1 hedging case

- We have a \$100 Million portfolio and we need to hedge the market risk with a stock index futures contract for one month.
  - We need to design and implement the hedge.
  - Figure out how many contracts to short, and trade to reach that position.
- The market index is the RTX. The current value of the RTX is 1,050.
- The RTX futures contract has a notional value of $RTX \times \$250$. At present, this is $1{,}050 \times \$250 = \$262{,}500$.
- The contract is cash settled. At maturity the long side receives $(RTX_{maturity} - 1{,}050) \times \$250$.
  - Example. If the RTX in one month is 1,045, then the long side receives $(1{,}045 - 1{,}050) \times \$250 = -\$1{,}250$
  - Since the RTX has declined, the long side pays the short side \$1,250.

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Materials (workbook *H1.xlsx*, posted to web)

- Worksheet *CATSPY* (already used earlier)
- Worksheet *Portfolio* contains the composition of the portfolio.
- Worksheet *Securities* has the price history for the portfolio's ten securities.

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Worksheet *Portfolio*

	A	B	C	D	E
1	Stock	Price	Shares held	Value	Weight
2	GD	$50.00	176,000	$8,800,000	0.088
3	POP	$80.00	156,250	$12,500,000	
4	RMS	$25.00	580,000	$14,500,000	
5	BBL	$16.00	281,250	$4,500,000	
6	TC	$84.00	100,000	$8,400,000	
7	GEB	$52.00	200,000	$10,400,000	
8	PKR	$154.00	100,000	$15,400,000	
9	TTW	$62.00	200,000	$12,400,000	
10	NWL	$8.00	1,112,500	$8,900,000	
11	GCS	$21.00	200,000	$4,200,000	
12	Total			$100,000,000	

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Worksheet *Securities*

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Rotman In	Rotman In	GD Stock	POP Stock	RMS Stock	BBL Stock	TC Stock	GEB Stock	PKR Stock	TTW Stock	NWL Stoc	GGS Stock
2	Tick	RTX	RTF	GD	POP	RMS	BBL	TC	GEB	PKR	TTW	NWL	GGS
3	1	1,050.00	1,050.00	$ 50.00	$ 80.00	$ 25.00	$ 16.00	$ 84.00	$ 52.00	$ 154.00	$ 62.00	$ 8.00	$ 21.00
4	2	1,051.13	1,051.13	$ 49.99	$ 79.86	$ 24.95	$ 16.01	$ 84.12	$ 51.54	$ 153.15	$ 61.92	$ 8.07	$ 21.03
5	3	1,051.78	1,051.78	$ 50.07	$ 79.57	$ 25.09	$ 15.98	$ 83.65	$ 51.60	$ 153.70	$ 61.97	$ 8.06	$ 20.93
6	4	1,049.51	1,049.51	$ 50.12	$ 79.73	$ 24.97	$ 15.90	$ 83.62	$ 51.47	$ 153.54	$ 61.81	$ 8.00	$ 20.81
7	5	1,053.16	1,053.16	$ 50.23	$ 80.20	$ 25.02	$ 15.99	$ 83.47	$ 51.56	$ 153.99	$ 62.07	$ 8.06	$ 20.92
8	6	1,052.72	1,052.72	$ 50.27	$ 79.91	$ 24.98	$ 15.92	$ 83.45	$ 51.50	$ 154.98	$ 62.01	$ 8.10	$ 20.86
9	7	1,052.55	1,052.55	$ 50.14	$ 79.84	$ 24.96	$ 15.82	$ 83.87	$ 52.04	$ 154.99	$ 62.15	$ 8.07	$ 20.81
10	8	1,047.85	1,047.85	$ 49.72	$ 79.38	$ 24.85	$ 15.70	$ 83.48	$ 51.82	$ 153.15	$ 61.44	$ 8.04	$ 20.76
11	9	1,050.04	1,050.04	$ 49.72	$ 79.45	$ 24.92	$ 15.75	$ 83.65	$ 51.81	$ 153.57	$ 61.81	$ 8.03	$ 20.75
12	10	1,044.84	1,044.84	$ 49.44	$ 78.62	$ 24.99	$ 15.70	$ 83.31	$ 51.39	$ 151.82	$ 61.27	$ 7.96	$ 20.72
13	11	1,044.12	1,044.12	$ 49.49	$ 78.65	$ 24.98	$ 15.67	$ 82.82	$ 51.43	$ 151.55	$ 61.14	$ 7.93	$ 20.68

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Notes

- RTX is the index; RTF is the futures price.
  - In this case, they are the same; you can use either to represent "the market"

Two ways to estimate the portfolio beta, β_P

- Compute the portfolio weights $w_1, w_2, \dots, w_{10}$.
- Using the returns on the individual stocks, estimate the individual beta's: $\beta_1, \beta_2, \dots, \beta_{10}$.
- Compute the portfolio beta as the weighted average of the individual betas: $\beta_P = w_1\beta_1 + w_2\beta_2 + \cdots + w_{10}\beta_{10}$

- Compute the portfolio weights $w_1, w_2, \dots, w_{10}$.
- For each month t, compute the portfolio return as the wtd avg $r_{Pt} = w_1 r_{1t} + w_2 r_{2t} + \cdots + w_{10} r_{10t}$
- Using the portfolio returns, estimate β_P

Assignment

- Design the hedge.
  - Estimate the portfolio beta.
  - Figure out how many contracts to short.
- Implement the hedge
  - H1 is running on the server. In any given round, you need to actually establish the short position.
  - Play two rounds of H1.
  - Hints: Try to establish the hedge quickly. (This reduces your risk exposure.)
  - The important thing here is not overall profits: it is selling the correct number of futures contracts.
- Answer questions 1-3. (This submission will be online via NYU classes; you'll receive an email when the submission page is available.)
- Due date: Monday, April 20, 11:59 PM

