Housing Prices and Growth

James A. Kahn[1]
Federal Reserve Bank of New York and New York University
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[1]The views expressed in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of New York or the Federal Reserve System. Please direct all correspondence to the author at: jkahn@stern.nyu.edu or Stern School of Business, New York University, 44 West 4th Street, New York, NY 10012.

Abstract

This paper develops a growth model with land, housing services, and other goods. Under certainty the model exhibits a balanced aggregate growth but with underlying sectoral change. The paper then introduces a Markov regime-switching specification for productivity growth in the non-housing sector, and shows that such regime switches are a plausible candidate for explaining–both qualitatively and quantitatively–the large low-frequency changes in housing price trends. The paper also uses micro data to calibrate a key cross-elasticity parameter that governs the relationship between productivity growth and home price appreciation. The calibrated model can explain most of the acceleration in housing prices that has occurred since the mid-1990s, and also suggests a contributing explanation for the recent downturn.

With the acceleration of housing prices since the mid-1990's in the United States, as well as the recent dramatic downturn, there has been increased attention given to the causes and effects of the enormous increase in housing wealth. From 1996 to 2006, the real quality-adjusted price of new houses has appreciated at an average rate of 2.2 percent annually. As Figure 1 indicates, similarly strong real appreciation took place in the 1960s and 1970s, averaging 2 percent growth, followed by nearly two decades of real depreciation, with average growth of -0.6 percent. Markets in particular geographical areas have exhibited still greater volatility (cf. Himmelberg et al, 2005). These sustained changes have been variously attributed to speculative bubbles (Shiller), expansions and contractions in monetary policy (Iacovielli et al), and developments in financial markets—innovations such as new types of mortgage instruments, or breakdowns such as the recent subprime mortgage boom and bust. Such explanations suggest that market irrationality has played an important role in housing market fluctuations.

This paper examines whether at least the medium term behavior of housing prices is consistent with a more conventional rational expectations framework. It develops a stochastic growth model in which land, capital, and labor are inputs to production, and housing and non-housing consumption provide utility to consumers. Technical progress in non-housing related output is presumed to have a Markov regime-switching component as in Kahn and Rich (2007). The paper shows that for plausible calibrations of the model, regime switches in productivity growth can explain the bulk of the large trend changes in housing prices. Specifically, the benchmark calibration predicts an acceleration in housing prices of 2.25 percent resulting from the jump in productivity growth that took place in the mid-1990s, compared with the 2.8 percent acceleration found in the data. Given that the model that has no mechanism to produce the declines in real interest rates that occurred in 2000-2005, the results leave little room for bubbles and other market failures as important contributors to persistent movements in housing prices.

A key parameter turns out to be the elasticity of substitution between housing and non-housing consumption. This parameter has been featured in many studies related to housing and household production (e.g. Benhabib, Rogerson and Wright, 1991, Greenwood and Hercowitz, 1991, Piazzesi and Schneider, 2007.). At the same time, many studies of housing have assumed, presumably for convenience, a value one for this elasticity. We provide evidence based on both aggregate and microeconomic data that this elasticity is considerably less than one. This low elasticity plays a crucial role in the model's ability to explain both qualitatively and quantitatively the magnitude of housing price fluctuations.

1 Background

The real value of housing wealth, as measured by flow of funds data, has grown an average of 4.6 percent since 1952. This compares with 3.4 percent growth of private net worth excluding real estate, and 3.5 percent growth of personal consumption expenditures over the same time period. Figure 2a plots the ratio of nominal housing wealth to nominal consumption expenditure. This ratio nearly doubles between 1952 and 2005. Figure 2b plots the much more volatile ratio of housing wealth to total net worth. While the enormous volatility of non-real estate wealth hinders precise inferences about relative trends, this disparity is robust to different time periods, and is not just the result of the runup of the last 5-10 years in real estate wealth. The bottom line is that real estate has gone from 27 percent of net worth in 1952 to 42 percent by the end of 2005. Heathcote and Morris (2005), however, argue that there are problems with the Flow of Funds data, particularly over long periods of time, and construct their own measures of housing wealth (though only going back to 1975) that exhibit a less clearcut trend.

What drives housing prices is a controversial topic. In one well-known study, Mankiw and Weil (1989) argued that population demographics were the prime determininant, and predicted (highly inaccurately, as it turned out) that prices would fall in the subsequent two decades with the maturation of the baby boom generation and resulting decline in the growth rate of the prime home-owning age group. More recently, Glaeser et al (2005) have argued that price increases since 1970 largely reflect artificial supply restrictions. We will take a different approach in examining the fundamental determinants of long-run trends in relative prices in a fully dynamic general equilibrium context. There it turns out that productivity growth (in particular, productivity growth in the non-housing sectors of the economy) is the most important driver of housing prices at low to medium frequencies. Productivity has exhibited periodic changes in trend that are well-represented by a regime-switching model (see Kahn and Rich, 2007). Future work will also examine other low-frequency movements, including labor supply, which has also exhibited low-frequency movements (see Figure 3), and demographic changes.

One possible explanation for the relative increase in housing prices is a simple income effect, or non-homogeneity in preferences. As people get wealthier, they may prefer to have more of their consumption coming from housing services, the price of which will tend to rise because of its being relatively intensive in land, a fixed factor. The (nominal) share of housing services in GDP has gone from 7.5 percent in 1952 to over 10 percent in 2005. The share of housing services in consumer expenditures has gone from 12.2 percent to 14.6 percent over the same period, but in fact has been without any meaningful trend since 1960 or so. In any case, evidence against this proposition will be presented below. Another driver of housing prices could be differing technical progress trends in housing services versus other goods, as in Baumol (1967). The relatively large share of land and structures, two inputs usually thought to be less amenable to embodied technical progress, in the value of housing makes this story plausible. This paper will argue that the timing of low-frequency changes in both housing prices and productivity suggests that this mechanism is important.

There is some evidence that in fact the increase in housing wealth does not stem from an increase in the value of houses per se, but rather from the increase in the value of the land upon which they are built. First, a price index that include the value of land, the Conventional Mortgage Home Price Index, has increased approximately 0.75% faster than indexes that do not, such as the Census's Composite Construction Cost index, on an annual basis. Davis and Heathcote (2004) compute a land price index based on this type of differential and find that land values have increased at an average annual rate of approximately 3.5% (inflation-adjusted) over the period 1975-2005. That price index may, however have an upward bias from not adequately adjusting for quality changes.[1] Land price series available from the Bureau of Labor Statistics (see Figure 5) suggest behavior that is closer to the behavior of new home prices in Figure 1. We focus on the Census Bureau's quality-adjusted price of new homes in part because of concern over this bias, but also because it goes back to 1963.

Finally, Figure 6 depicts the behavior of the HP-trend component of productivity growth (relative to a linear trend) over the postwar period. Clearly its pattern is similar to low-frequency movements in land and housing prices, though the downturn in productivity clearly precedes the downturn in housing and land prices by several years. Kahn and Rich (2007) provide more detailed econometric estimates of a regime-switching model of the sort incorporated below into this paper, and find significant regime changes corresponding to the shifts depicted in the figure.

[1] In addition, long-term evidence (from *Historical Statistics of the United States*) would seem to indicate that land prices have not grown as a proportion of real estate wealth. Other studies (e.g. Wheaton, 2006, and references contained therein) suggest that land prices largely just keep pace with inflation. Also, repeat-sales housing price indexes have an upward bias because they include price increases that may be attributable to improvements (additions, renovations, etc.).

2 A Growth Model with Housing

This section presents a general equilibrium growth model that is capable of capturing the important stylized facts about housing and the economy. The model has two sectors, a "manufacturing" sector that produces non-housing related goods and services, as well as the capital (structures and durable goods) that go into housing services. A second sector uses capital, labor, and land to produce a flow of housing services. The model exhibits balanced aggregate growth, but with unequal growth across sectors. We then consider the behavior of the model under a regime-switching specification for productivity growth in the manufacturing sector.

2.1 Firms

Competitive final goods firms produce two types of goods: A "manufactured" good Y_m, and housing services Y_h. Under perfect competition the final goods firms make zero profits and have perfectly elastic supplies of Y_m and Y_h at the above prices. The production functions for the two types of goods are

$$Y_j = A_j K_j^{\alpha} L_j^{\beta_j} \left(eN_j\right)^{1-\alpha-\beta_j}$$

for $j = m, h$, where K_j is capital allocated to j, L_j is land, and eN_j is labor input. The goods producers rent inputs in competitive markets. In particular, capital is rented from final goods producers of Y_m.

In the j sector, the representative firm's nominal profit in period t is given by

$$P_{jt}Y_{jt} - W_t e_t N_{mt} - R_{\ell t} L_{mt} - R_{kt} K_{mt} \tag{1}$$

where R_ℓ and R_k are nominal rental rates for land and capital respectively, and W_t is the nominal wage. Profit maximization implies

$$\alpha P_{jt} Y_{jt} / K_{jt} = R_{kt} \tag{2}$$

$$\beta_j P_{jt} Y_{jt} / L_{jt} = R_{\ell t} \tag{3}$$

$$\left(1-\alpha-\beta_m\right) P_{jt} Y_{jt} / \left(eN_{jt}\right) = W_t \tag{4}$$

$$\frac{K_{jt}}{L_{jt}} = \frac{\alpha}{\beta_j} \frac{R_{\ell t}}{R_{kt}} \tag{5}$$

$$\frac{K_{jt}}{e_t N_{jt}} = \frac{\alpha}{1-\alpha-\beta_j} \frac{W}{R_{kt}}. \tag{6}$$

This implies that marginal cost for all firms in sector $j = m, h$ is equal to

$$M_{jt} = \alpha^{-\alpha} \beta_j^{-\beta_j} \left(1-\alpha-\beta_j\right)^{-\left(1-\alpha-\beta_j\right)} W_t^{1-\alpha-\beta_j} R_{\ell t}^{\beta_j} R_{kt}^{\alpha}. \tag{7}$$

Because firms are assumed to be price takers in factor markets, marginal cost is not directly a function of output. But because of the short-run fixity of capital and land, and imperfectly elastic labor supply, factor prices and hence marginal cost will move procyclically.

2.2 Consumers

There are N_t representative agents at time t supplying $N_t e_t$ labor, where N is exogenous, growing exponentially at constant rate ν, and e endogenous, but, as usual in growth models, constant on a balanced growth path. Later we will consider the impact of shifting labor supply. Variation in hours of work per capita (see Figure) has been an important contributor to low frequency changes in output per capita in the postwar U.S., and may also affect housing prices, as we shall see. Let C denote the aggregate non-housing consumption good, and H aggregate housing services. We let $c \equiv C/N$ and $h \equiv H/N$ denote per capita quantities. The representative consumer then cares about c and h, and dislikes working. He solves the problem

$$\max\ U = E_t \left\{ \sum_{s=0}^{\infty} (1+\rho)^{-s} \left[\ln \left(\left[\omega_c c_{t+s}^{(\epsilon-1)/\epsilon} + \omega_h h_{t+s}^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)} \right) - \psi\left(e_{t+s}\right) \right] \right\} \tag{8}$$

subject to

$$P_{m,t+s}\left(c_{t+s} + \iota_{t+s}\right) + P_{h,t+s} h_{t+s} + V_{t+s}\left[(1+\nu)\,\ell_{t+s} - \ell_{t+s-1}\right] + b_{t+s}/\left(1+R_{t+s}\right)$$

$$\begin{aligned} &\leq b_{t+s-1} + W_{t+s} e_{t+s} + (1+\nu)\, R_{k,t+s} P_{m,t+s-1} k_{t+s-1} && (9)\\ &\quad + R_{\ell,t+s} V_{t+s-1} \ell_{t+s-1} + d_{m,t+s} + d_{h,t+s} && (10)\\ (1+\nu)\, k_{t+s} &= (1-\delta)\, k_{t+s-1} + \iota_{t+s} \qquad (j = m, h) && (11) \end{aligned}$$

where ι_t denotes total capital investment at date t, d_{jt} nominal dividends (for simplicity assumed to be distributed in a lump-sum fashion) from the profits of intermediate goods producers in sector j, b_t nominal one-period discount bonds, V_t the price of land at date t, and ℓ_t land holdings at date t. The constraints reflect the fact that population is growing, so that per capita stocks get deflated at rate ν. Both k_t and ℓ_t denote the sum of capital and land in both sectors.

The first-order conditions for the consumer's problem, letting

$$\phi(c,h) \equiv \left[\omega_c c^{(\epsilon-1)/\epsilon} + \omega_h h^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)}$$

are as follows:

$$\begin{aligned} \omega_c \phi^{-(\epsilon-1)/\epsilon} c_t^{-1/\epsilon} &= \tilde{\Lambda}_t P_{mt} && (12)\\ \omega_h \phi^{-(\epsilon-1)/\epsilon} h_t^{-1/\epsilon} &= \tilde{\Lambda}_t P_{ht} && (13)\\ \psi'\left(e_t\right) &= \tilde{\Lambda}_t W_t && (14)\\ \tilde{\Lambda}_t P_{mt} (1+\nu)(1+\rho) &= E_t \left\{ \tilde{\Lambda}_{t+1} \left[P_{mt} R_{kt+1} + P_{mt+1} (1-\delta) \right] \right\} && (15)\\ \tilde{\Lambda}_t V_t (1+\nu)(1+\rho) &= E_t \left\{ \tilde{\Lambda}_{t+1} \left[V_t R_{\ell t+1} + V_{t+1} \right] \right\} && (16) \end{aligned}$$

The last two equations function as equilibrium conditions under which the consumer is indifferent between investing more or less capital or land at date t. In real terms (expressed in units of m sector output) we have

$$\begin{aligned}
\omega_c \phi^{-(\epsilon-1)/\epsilon} c_t^{-1/\epsilon} &= \Lambda_t && (17)\\
\omega_h \phi^{-(\epsilon-1)/\epsilon} h_t^{-1/\epsilon} &= \Lambda_t p_{ht} && (18)\\
\psi'(e_t) &= \Lambda_t w_t && (19)\\
\Lambda_t (1+\nu)(1+\rho) &= E_t \{\Lambda_{t+1} [r_{kt+1} + 1 - \delta]\} && (20)\\
\Lambda_t v_t (1+\nu)(1+\rho) &= E_t \{\Lambda_{t+1} [v_t r_{\ell t+1} + v_{t+1}]\} && (21)
\end{aligned}$$

where the lower-case prices are relative to P_m (e.g. $w_t = W_t/P_{mt}$, etc.).

2.3 Equilibrium Growth

Aggregating over producers in each sector, we have

$$\begin{aligned}
C_t + K_t - (1-\delta) K_{t-1} &= A_{mt} K_{mt}^{\alpha} L_{mt}^{\beta_m} (e_t N_{mt})^{1-\alpha-\beta_m}\\
H_t &= A_{ht} K_{ht}^{\alpha} L_{ht}^{\beta_m} (e_t N_{ht})^{1-\alpha-\beta_h}
\end{aligned}$$

where

$$\begin{aligned}
L_{mt} + L_{ht} &= \bar{L}\\
K_{mt} + K_{ht} &= K_{t-1}\\
N_{mt} + N_{ht} &= N_t
\end{aligned}$$

The stocks of capital and land in the h sector would correspond to residential real estate. Labor in this sector would be partly non-market household labor, and partly service sector labor (particularly for apartment buildings). We assume (mainly for convenience) that capital's share is the same in both sectors, but labor's share is higher in manufacturing (implying of course that land's share is higher in the housing sector, i.e. $\beta_h > \beta_m$).

Let c and h denote per capita quantities of C and H, while k, ℓ, k_i, ℓ_h refer to per worker quantities in sector i (e.g. $k_{ht} \equiv K_{ht}/N_{ht}$, $k_t \equiv K_t/N_{t+1}$, i.e. no subscript refers to aggregates), while $n_{it} \equiv N_{it}/N_t$, $(i = m, h)$.[2] Given the assumption of perfect competition, we can assume the economy solves the following planner's problem:

$$\max\ U = E_0 \left\{ \sum_{t=0}^{\infty} (1+\rho)^{-t} \ln\left(\left[\omega_c c_t^{(\epsilon-1)/\epsilon} + \omega_h h_t^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)} \right) - \psi(e_t) \right\} \qquad (22)$$

[2] The derivations here draw on Ngai and Pissarides (2007), albeit in discrete time, and adding a fixed factor with heterogeneous technology.

subject to resource constraints

$$
\begin{aligned}
c_t + (1+\nu)\, k_t - k_{t-1}\,(1-\delta) &= A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} n_{mt} && (23)\\
h_t &= A_{ht} k_{ht}^{\alpha} \ell_{ht}^{\beta_h} e_t^{1-\alpha-\beta_h} n_{ht} && (24)\\
k_{mt} n_{mt} + k_{ht} n_{ht} &= k_{t-1} && (25)\\
\ell_{mt} n_{mt} + \ell_{ht} n_{ht} &= \ell_t && (26)\\
n_{mt} + n_{ht} &= 1. && (27)
\end{aligned}
$$

Total land $\bar{L}$ is assumed fixed, so $\ell_t/\ell_{t-1} = (1+\nu)^{-1}$. Average technological progress in sector i, i.e. the average growth rate of A_i, is denoted γ_i $(i = m, h)$. We assume e is the same in the two sectors, and that $\beta_h \geq \beta_m$. Note that the timing assumptions in (25) and (26) are such that while aggregate capital k is chosen one period ahead of time, and total land and labor are exogenous, for simplicity the sectoral allocations are determined contemporaneously.

It is worth mentioning that technical progress in the h sector is unrelated to technological progress in construction. (In fact, home construction occurs in the m sector in this model.) Rather, it refers to an increase in the housing services from given stocks of K_h, L_h, and labor inputs eN_h. What this means in practice depends on exactly what the term "housing services" encompasses, and on how one measures K_h. In the model it is assumed for simplicity to be indistinguishable from K_m other than by its allocation to the h sector. In particular, it is assumed to have the same price as K_m and C. In principle it would include both residential structures and housing service-related consumer durables (home appliances). L_h would include both non-market and market labor involved in household production—time devoted to housework, food preparation, home and yard maintenance, and the like.

The model obviously abstracts from a number of potentially important factors. First and foremost, the housing and construction sectors are heavily affected by government intervention, both via distortionary taxation and regulations. In particular, much land in the U.S. (and in most other countries as well) is neither residential nor commercial, and is either owned or heavily restricted in its use by the government. Second, there is tremendous heterogeneity in land and housing values. Land near navigable bodies of water, or ports, or along coastlines is much more valuable than land that does not have these features. +Obviously this model will have nothing directly to say about the cross-sectional distribution of land values or housing prices (though many of the factors that affect them over time undoubtedly come into play in the cross-section as well). Nonetheless if all of these factors remain relatively constant over time, then ignoring them in a model such as this should not be too great a sin.

Letting $\phi(c,h) \equiv \left[\omega_c c^{(\epsilon-1)/\epsilon} + \omega_h h^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)}$, the first-order conditions are:

$$
\begin{aligned}
\omega_c \phi^{-(\epsilon-1)/\epsilon} c_t^{-1/\epsilon} &= \mu_{mt} && (28)\\
\omega_h \phi^{-(\epsilon-1)/\epsilon} h_t^{-1/\epsilon} &= \mu_{ht} && (29)\\
\psi'(e_t) &= \mu_{mt}(1-\alpha-\beta_m) A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{-\alpha-\beta_m} n_{mt} && (30)\\
&\quad + \mu_{ht}(1-\alpha-\beta_h) A_{ht} k_{ht}^{\alpha} \ell_{ht}^{\beta_h} e_t^{-\alpha-\beta_h} n_{ht} &&\\
\mu_{mt}\beta_m A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m-1} e_t^{1-\alpha-\beta_m} &= \mu_{ht}\beta_h A_{ht} k_{ht}^{\alpha} \ell_{ht}^{\beta_h-1} e_t^{1-\alpha-\beta_h} && (31)\\
\mu_{mt} A_{mt} k_{mt}^{\alpha-1} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} &= \mu_{ht} A_{ht} k_{ht}^{\alpha-1} \ell_h^{\beta_h} e_t^{1-\alpha-\beta_h} && (32)\\
\mu_{mt} A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} &= \mu_{ht} A_{ht} e_t^{1-\alpha-\beta_h} \times && (33)\\
&\quad \left[\alpha k_{ht}^{\alpha-1} \ell_{ht}^{\beta_h} k_{mt} + \beta_h k_{ht}^{\alpha} \ell_{ht}^{\beta_h-1} \ell_{mt} + (1-\alpha-\beta_h) k_{ht}^{\alpha} \ell_{ht}^{\beta_h}\right] && (34)\\
\mu_{mt}(1+\nu)(1+\rho) &= E_t\left\{\mu_{mt+1}\left[A_{mt+1}\alpha k_{mt+1}^{\alpha-1} \ell_{mt+1}^{\beta_m} e_t^{1-\alpha-\beta_m} + 1 - \delta\right]\right\} && (35)
\end{aligned}
$$

μ_{mt} and μ_{ht} are shadow prices on the resource constraints (23) and (24). These can be shown to imply that

$$
\begin{aligned}
\frac{k_m}{k_h} &= \frac{1-\alpha-\beta_h}{1-\alpha-\beta_m} && (36)\\
\frac{\ell_m}{\ell_h} &= \frac{\beta_m}{\beta_h}\frac{1-\alpha-\beta_h}{1-\alpha-\beta_m}, && (37)
\end{aligned}
$$

Let p_t denote the relative price of housing services in terms of manufactured goods. We have

$$
\begin{aligned}
p_t &= \frac{\mu_{ht}}{\mu_{mt}} = \frac{A_{mt} k_{mt}^{\alpha-1} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m}}{A_{ht} k_{ht}^{\alpha-1} \ell_{ht}^{\beta_h} e_t^{1-\alpha-\beta_h}}\\
&= \frac{A_{mt}}{A_{ht}}\left(\frac{\beta_m}{\beta_h}\right)^{\beta_m}\left(\frac{1-\alpha-\beta_h}{1-\alpha-\beta_m}\right)^{\alpha+\beta_m-1}\left(\frac{\ell_{ht}}{e_t}\right)^{-(\beta_h-\beta_m)}
\end{aligned}
$$

Thus growth in the price of housing services reflects both relative productivity growth in manufacturing and the increasing scarcity of land.

Finally, from (30), (32) and (36) we have

$$\psi'(e_t) = \mu_{mt}(1-\alpha-\beta_h) A_{mt} k_{mt}^{\alpha-1} \ell_{mt}^{\beta_m} e_t^{-\alpha-\beta_m} k_{t-1} \tag{38}$$

which equates the marginal rate of substitution between consumption and leisure with the marginal product of labor expressed in terms of m sector output.

2.4 Balanced Aggregate Growth under Certainty

Let total expenditure $c+ph$ be denoted by x. It also turns out that $\mu_m = x^{-1}$, hence $\mu_{mt}/\mu_{mt-1} = x_{t-1}/x_t$. It turns out we can aggregate the two resource constraints as follows:

$$
\begin{aligned}
c_t + p_t h_t + (1+\nu) k_t - k_{t-1}(1-\delta) &= A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} n_{mt} + && (39)\\
&\quad A_{mt} k_{mt}^{\alpha-1} \ell_{mt}^{\beta_m} k_{ht} e_t^{1-\alpha-\beta_h} n_{ht} && (40)\\
x_t + (1+\nu) k_t - k_{t-1}(1-\delta) &= A_{mt} k_{mt}^{\alpha-1} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} k_{t-1} && (41)
\end{aligned}
$$

so that with nonstochastic technological progress (and constant labor-population ratio e) we have

$$\begin{aligned}(1+\nu)\,k_t/k_{t-1} &= A_{mt}k_{mt}^{\alpha-1}\ell_{mt}^{\beta_m}e_t^{1-\alpha-\beta_m} - x_t/k_{t-1}+1-\delta &(42)\\ (x_{t+1}/x_t)\,(1+\nu)\,(1+\rho) &= \alpha A_{mt}k_{mt}^{\alpha-1}\ell_{mt}^{\beta_m}e_t^{1-\alpha-\beta_m}+1-\delta &(43)\end{aligned}$$

We will define aggregate balanced growth under certainty as an equilibrium path in which x and k both grow at a constant rate, and in which the interest rate (i.e. the marginal product of capital) is also constant. Balanced growth clearly requires that $A_{mt}k_{mt}^{\alpha-1}\ell_{mt}^{\beta_m}e_t^{1-\alpha-\beta_m}$ be constant, which we can express as

$$(k_{mt}/e_t)\,/\,(k_{mt-1}/e_{t-1}) = \left[(1+\gamma_m)\left[(\ell_{mt}/e_t)\,/\,(\ell_{mt-1}/e_{t-1})^{\beta_m}\right]\right]^{1/(1-\alpha)},$$

i.e. a relationship between the growth rates in the m sector of the capital-labor ratio, technological progress, and the land-labor ratio. Therefore, let

$$Z_t \equiv \left[A_{mt}\,(\ell_{mt}/e_t)^{\beta_m}\right]^{1/(1-\alpha)} e_t = \left[A_{mt}\ell_{mt}^{\beta_m}e_t^{1-\alpha-\beta_m}\right]^{1/(1-\alpha)} \quad (44)$$

and define variables with "~" over them to be deflated by Z_t, e.g. $\tilde{k}_{mt} \equiv k_{mt}/Z_t$. We then have

$$\begin{aligned}(1+\nu)\,k_t/k_{t-1} &= \tilde{k}_{mt}^{\alpha-1} - x_t/k_{t-1}+1-\delta &(45)\\ (x_{t+1}/x_t)\,(1+\nu)\,(1+\rho) &= \alpha\tilde{k}_{mt}^{\alpha-1}+1-\delta &(46)\end{aligned}$$

With $\tilde{k}_m$ constant under balanced growth, k and x both grow at the same constant rate.

From (25)-(27) and (36)-(37) we have

$$k_{mt}\left[\frac{1-\alpha-\beta_m}{1-\alpha-\beta_h}n_{ht}+n_{mt}\right]=k_{t-1}. \quad (47)$$

Now let

$$Q_t \equiv \frac{1-\alpha-\beta_m}{1-\alpha-\beta_h}n_{ht}+n_{mt} \quad (48)$$

Then we have $k_{mt} = k_{t-1}/Q_t$, and we can define $\hat{k}_t \equiv k_t/(Z_tQ_t)$. This gives a normalization of k_t that is constant on the balanced growth path. Note that if $\beta_m = \beta_h$, then $Q = 1$ and we would have $k_{mt} = k_{ht} = k_{t-1}$. But with $\beta_m > \beta_h$, $Q > 1$ and n_h and n_m are changing over time (unless $\epsilon = 1$). In particular, if $\epsilon < 1$ and $\gamma_m \geq \gamma_h$, n_h (and hence Q) grows over time.

The model implies constant work effort along the balanced growth path. From (38) we have

$$\psi'(e_t) = \mu_{mt}\,(1-\alpha-\beta_h)\,A_{mt}k_{mt}^{\alpha-1}\ell_{mt}^{\beta_m}e_t^{-\alpha-\beta_m}k_{t-1}$$

which after normalization with Z yields

$$\begin{aligned}\psi'(e_t)\,e_t &= (1-\alpha-\beta_h)\,\tilde{k}_{mt}^{\alpha-1}k_{t-1}/x_t\\ &= (1-\alpha-\beta_h)\,\tilde{k}_{mt}^{\alpha}/\hat{x}_t\end{aligned}$$

Since $\tilde{k}_{mt}$ and $\hat{x}_t$ are constant along the balanced growth path, e_t is constant as well.

On the balanced aggregate growth path, ZQ grows at a constant rate. In fact it is straightforward to show that its growth rate g satisfies

$$\left[(1+\gamma_m)(1+\nu)^{-\beta_m}\right]^{1/(1-\alpha)} \equiv 1+g \tag{49}$$

We then have $\tilde{k}_{mt} = k_{mt}/Z_t = k_{t-1}/(Q_t Z_t) = \hat{k}_{t-1} Q_{t-1} Z_{t-1}/(Q_t Z_t) = \hat{k}_{t-1}/(1+g_t)$. Aggregate output per capita (in terms of manufactured goods), which we denote y_t, is $A_{mt} k_t^{\alpha} \ell_{mt}^{\beta_m} e_t n_{mt} + p_t A_{ht} k_{ht}^{\alpha} \ell_{ht}^{\beta_h} e_t n_{ht}$, or (after substituting for p_t and simplifying as before):

$$y_t = A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} Q_t = \tilde{k}_{mt}^{\alpha} Z_t Q_t, \tag{50}$$

so we can also define $\hat{y}_t = y_t/(Z_t Q_t) = \tilde{k}_{mt}^{\alpha}$ and $\hat{x}_t = x_t/(Z_t Q_t)$.

We can now characterize the dynamics in terms of stationary variables:

$$(1+\nu)(1+g)\hat{k}_t/\hat{k}_{t-1} = \left(\hat{k}_{t-1}/(1+g)\right)^{\alpha-1} - (1+g)\hat{x}_t/\hat{k}_{t-1} + 1 - \delta \tag{51}$$

$$(\hat{x}_{t+1}/\hat{x}_t)(1+\nu)(1+\rho)(1+g) = \alpha\left(\hat{k}_{t-1}/(1+g)\right)^{\alpha-1} + 1 - \delta \tag{52}$$

which yields (letting variables without subscripts denote steady state variables)

$$(1+\nu)(1+g) = \left(\hat{k}/(1+g)\right)^{\alpha-1} - (1+g)\hat{x}/\hat{k} + 1 - \delta \tag{53}$$

$$(1+\nu)(1+\rho)(1+g) = \alpha\left(\hat{k}/(1+g)\right)^{\alpha-1} + 1 - \delta \tag{54}$$

and

$$\hat{k} = (1+g)\left[\frac{\alpha}{(1+\nu)(1+\rho)(1+g) - (1-\delta)}\right]^{1/(1-\alpha)}. \tag{55}$$

This of course is just the standard neoclassical model when technological progress has a unit root. The innovation in this paper's model is to characterize the behavior of sectoral variables within the aggregate steady state, and also to specify a regime-switching model for the growth process.

2.5 Unbalanced Sectoral Growth

The sectoral variables (other than $\tilde{k}_m$ and $\tilde{k}_h$) will vary over time, but can be solved for directly as functions of the aggregates. Rewriting the relevant conditions in terms of p, n_m, ℓ_m, h, k_m, and c, we have

$$p_t h_t = \tilde{k}_{mt}^{\alpha} Z_t \frac{1-\alpha-\beta_m}{1-\alpha-\beta_h}(1-n_{mt}) \tag{56}$$

$$1 = \omega_c \phi(c_t, h_t)^{-(\epsilon-1)/\epsilon} c_t^{-1/\epsilon} \hat{x}_t Q_t Z_t \tag{57}$$

$$h_t/c_t = p_t^{-\epsilon}(\omega_h/\omega_c)^{\epsilon} \tag{58}$$

$$\ell_{mt} = \frac{\bar{L}}{N_t} \frac{\beta_m(1-\alpha-\beta_h)}{(1-n_{mt})\beta_h(1-\alpha-\beta_m) + n_{mt}\beta_m(1-\alpha-\beta_h)} \tag{59}$$

$$p_t = \frac{A_{mt}}{A_{ht}}\left(\frac{\beta_m}{\beta_h}\right)^{\beta_h}\left(\frac{1-\alpha-\beta_h}{1-\alpha-\beta_m}\right)^{\alpha+\beta_h-1}\left(\frac{\ell_{mt}}{e_t}\right)^{-(\beta_h-\beta_m)}. \tag{60}$$

It will be convenient to have normalized versions of h and c to eliminate Z from the above system. As with k and x, we denote by $\hat{h}$ and $\hat{c}$ the corresponding variables divided by ZQ. Then we have

$$
\begin{aligned}
p_t\hat{h}_t &= \tilde{k}_{mt}^{\alpha}\frac{1-\alpha-\beta_m}{1-\alpha-\beta_h}\left(1-n_{mt}\right)/Q_t && (61)\\
1 &= \omega_c\phi\left(\hat{c}_t,\hat{h}_t\right)^{-(\epsilon-1)/\epsilon}\hat{c}_t^{-1/\epsilon}\hat{x}_t && (62)\\
\hat{h}_t/\hat{c}_t &= p_t^{-\epsilon}\left(\omega_h/\omega_c\right)^{\epsilon}. && (63)
\end{aligned}
$$

We also have

$$
\begin{aligned}
\tilde{k}_{mt} &= \hat{k}_{t-1}/\left(1+g\right) && (64)\\
\tilde{k}_{ht} &= \frac{1-\alpha-\beta_m}{1-\alpha-\beta_h}\tilde{k}_{mt} && (65)\\
\ell_{ht} &= \frac{\beta_h}{\beta_m}\frac{1-\alpha-\beta_m}{1-\alpha-\beta_h}\ell_{mt} && (66)
\end{aligned}
$$

The six equations (48) and (59) – (63)provide solutions for the six sectoral variables $\hat{h}$, $\hat{c}$, n_m, ℓ_m, p, and Q as functions of the exogenous variables and the aggregate endogenous variables. Equations (64) – (66) along with $n_h = 1 - n_m$ then provide solutions for the remaining sectoral variables. Only in the knife-edge cases of $\epsilon = 1$ or $\gamma_m = \gamma_h - (\beta_h - \beta_m)\,\nu$ will these variables exhibit balanced growth in the sense of either being constant, or growing at the same rate as the aggregate economy.

To get a more intuitive feel for how sectoral labor allocation evolves over time, following Ngai-Pissarides (2007), let σ_h denote the share of expenditure on housing services h relative to total expenditure on goods x, and $\sigma_c = 1 - \sigma_h$ the share of x spent on c:.

$$
\sigma_{ht} \equiv \frac{p_t h_t}{x_t} = \left(\frac{\omega_h}{\omega_c}\right)^{\epsilon} p_t^{-(\epsilon-1)} \Big/ \left[1+\left(\frac{\omega_h}{\omega_c}\right)^{\epsilon} p_t^{-(\epsilon-1)}\right] \qquad (67)
$$

Then $p_t h_t = \tilde{k}_{mt}^{\alpha} Z_t n_{ht} = y_t n_{ht}/Q_t = \sigma_{ht} x_t$, and we have

$$
\begin{aligned}
n_{ht} &= \sigma_{ht}\frac{x_t Q_t}{y_t} && (68)\\
n_{mt} &= 1-\sigma_{ht}\frac{x_t Q_t}{y_t} && (69)
\end{aligned}
$$

On the balanced growth path x/y is constant, over time we have (for $\epsilon < 1$ and $\gamma_m > \gamma_h$), $\sigma_h \to 1,\ \sigma_m \to 0$. Consequently, in the long-run $n_h \to xQ/y,\ n_m \to 1-xQ/y$, and eventually all but an infinitesimal amount of labor is going toward producing housing services or structures. This is presumably not a realistic implication, but the model can still be a reasonable description of behavior over a very long time period.

Although land is not explicitly priced in the model, we can compute its shadow rental price q_t in terms of manufactured goods:

$$
\begin{aligned}
q_t &= \beta_m A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m-1} e_t^{1-\alpha-\beta_m} && (70)\\
&= \beta_m Z_t \tilde{k}_{mt}^{\alpha}/\ell_{mt}
\end{aligned}
$$

It will be convenient to define $\hat{q}_t \equiv q_t / (Q_t Z_t N_t)$, so that we have

$$\hat{q}_t = \beta_m \hat{k}_{mt}^{\alpha} / (\ell_{mt} N_t Q_t) \tag{71}$$

which is expressed as a function of the sectoral variables for which the solution is described above. To a first approximation we can say that the land rental price grows at rate $g + \nu$ on the balanced growth path—exactly $g + \nu$ if $\epsilon = 1$, a bit faster if $\epsilon < 1$.

2.6 Stochastic Growth

We suppose that the growth rate of A_h is fixed at γ_h, but that of A_m follows a Markov regime-switching process:

$$A_{mt}/A_{mt-1} = (1 + \tilde{\gamma}_{mt})\, \eta_t / \eta_{t-1} \tag{72}$$

where

$$\tilde{\gamma}_{mt} = \begin{cases} \gamma_m^1 & \text{if } \xi_t = 1 \\ \gamma_m^0 & \xi_t = 0 \end{cases} \tag{73}$$

η_t is a transitory disturbance, and ξ_t is a state variable with Markov transition matrix Θ, where $\Theta[i,j] = \Pr\left(\xi_t = j | \xi_{t-1} = i\right)$. Since the columns of Q must sum to one, we write Θ as

$$\Theta = \begin{bmatrix} \theta_1 & 1 - \theta_0 \\ 1 - \theta_1 & \theta_0 \end{bmatrix}.$$

If the diagonal elements of Θ are close to one, the growth states will be highly persistent, and a shift from one state to the other will carry with it a sizeable adjustment in the long-term level of A_m. Since the stationary distribtion of ξ is $\xi^* \equiv \begin{bmatrix} (1-\theta_0)/(2-\theta_1-\theta_0) & (1-\theta_1)/(2-\theta_1-\theta_0) \end{bmatrix}'$, the average growth rate of A_m is

$$\bar{\gamma}_m = \frac{1-\theta_0}{2-\theta_1-\theta_0}\gamma_m^1 + \frac{1-\theta_1}{2-\theta_1-\theta_0}\gamma_m^0. \tag{74}$$

For concreteness we will call $\xi = 1$ the “high-growth” regime, and $\xi = 0$ the "low-growth" regime, i.e. we assume $\gamma_m^1 > \gamma_m^0$.

Following Hamilton (1994), we can describe the above Markov chain as an AR(1) process. First, define

$$z_t = \begin{bmatrix} z_{1t} \\ z_{0t} \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \xi^* & \text{if } \xi_t = 1 \\ \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \xi^* & \text{if } \xi_t = 0 \end{cases}, \tag{75}$$

which has an unconditional mean of zero. Then $z_t = \Theta z_{t-1} + v_t$, where $v_t = z_t - E_{t-1} z_t$, so that $E\left\{v_t | \xi_{t-1}\right\} = 0$. Thus, for example, if $\xi_{t-1} = 1$, we have

$$v_t = \begin{bmatrix} v_{1t} \\ v_{0t} \end{bmatrix} \begin{cases} \begin{bmatrix} 1-\theta_1 \\ -(1-\theta_1) \end{bmatrix} & \text{if } \xi_t = 1 \quad (\theta_1) \\ \begin{bmatrix} -\theta_1 \\ \theta_1 \end{bmatrix} & \text{if } \xi_t = 0 \quad (1-\theta_1) \end{cases} \tag{76}$$

and if $\xi_{t-1} = 0$.

$$v_t = \begin{bmatrix} v_{1t} \\ v_{0t} \end{bmatrix} \begin{cases} \begin{bmatrix} \theta_0 \\ -\theta_0 \end{bmatrix} & \text{if } \xi_t = 1 \quad (1-\theta_0) \\ \begin{bmatrix} -(1-\theta_0) \\ 1-\theta_0 \end{bmatrix} & \text{if } \xi_t = 0 \quad (\theta_0) \end{cases} \tag{77}$$

where the terms in parenthesis are conditional probabilities. Note that while $E\left(v_t|\xi_{t-1}\right) = 0$, v_t is not identically distributed over time, as the conditional distribution depends on ξ_{t-1}.

The log deviation version of g_t satisfies

$$\begin{aligned} gg_t/(1+g) &= \left(\frac{1}{1+\bar{\gamma}_m}(\tilde{\gamma}_{mt} - \bar{\gamma}_m) + \eta_t - \eta_{t-1}\right)/(1-\alpha) \\ &= \left[\vec{\gamma}_m' z_t/(1+\bar{\gamma}_m) + \eta_t - \eta_{t-1}\right]/(1-\alpha) \end{aligned}$$

where $\vec{\gamma}_m \equiv \begin{pmatrix}\gamma_m^1 & \gamma_m^0\end{pmatrix}$. We suppose that $\eta_t = \chi_1\eta_{t-1} + \chi_2\eta_{t-1} + \upsilon_t$, where υ_t is i.i.d. with a zero mean. In what follows, we will first assume that economic agents observe both z_t and η_t before making their period t decisions. Later we will consider the possibility that they only observe g_t and must estimate z_t and η_t given the history of g_t.

2.7 Asset Prices

Thus far we have only described the behavior of the price of housing services and rental prices for land. The term "housing prices" generally refers to asset prices of homes and the land they are packaged with. In this model we can calculate the value of what might be called "real estate wealth," which would be the total value of capital and land allocated to the housing services sector. The value of the capital is just $K_h = k_h n_h$. The asset value of the land $L_h = \ell_h n_h$ requires some computation. Letting V_t denote the asset price of land in terms of m sector output, we must have

$$V_t = q_t + E_t\left\{\Phi_{t,1}V_{t+1}\right\} = E_t\left\{\sum_{\tau=0}^{\infty}\Phi_{t,\tau}q_{t+\tau}\right\} \tag{78}$$

where

$$\Phi_{t,\tau} = \frac{\mu_{m,t+\tau}}{\mu_{mt}(1+\nu)^\tau(1+\rho)^\tau} = \frac{x_t}{x_{t+\tau}(1+\nu)^\tau(1+\rho)^\tau} \tag{79}$$

is the stochastic discount factor. If we normalize the variables by dividing by $Q_t Z_t N_t$, we get

$$\hat{V}_t = \hat{q}_t + E_t\left\{\hat{\Phi}_{t,1}\hat{V}_{t+1}\right\} = E_t\left\{\sum_{\tau=0}^{\infty}\hat{\Phi}_{t,\tau}\hat{q}_{t+\tau}\right\}$$

where

$$\hat{\Phi}_{t,\tau} \equiv \frac{\hat{x}_t}{\hat{x}_{t+\tau}(1+\rho)^\tau}. \tag{80}$$

On the balanced growth path we have $\Phi^{-1} = (1+g)(1+\rho)(1+\nu)$, and $\hat{q}_t$, as mentioned previously, is (for plausible parameters) almost constant but technically a function of A_{mt}/A_{ht} and N_t (for $\epsilon < 1$ it is increasing in both arguments). While the capital stock and the economy grow at $g+\nu$, the price of land, and hence the price of "houses" (capital plus land in the h sector) grows at a rate (slightly) faster than $g+\nu$. We will examine the behavior of land prices off the steady state later after describing the model under stochastic growth.

2.8 Calibration

Most of the parameters take on standard values: $\alpha = 0.33$, $\nu = 0.01$, $\delta = 0.05$ (a compromise for structures and equipment). The parameters β_h and β_m should reflect the shares of land in the cost of housing services and non-housing output respectively. We set $\beta_h = 0.5$ and $\beta_m = 0.05$. Since housing services represent about 20 percent of overall consumer expenditures, we set $\omega_h = 0.2$, $\omega_c = 0.8$. We set the time preference rate ρ equal to 0.01. Finally, we choose the parameters of the regime-switching process for productivity to correspond roughly to the results in Kahn and Rich (2007): $\left(\gamma_m^1 - \beta_m \nu\right) / (1-\alpha) = 0.029$, $\gamma_m^0 = 0.013$, $\theta_1 = 0.99$, $\theta_0 = 0.983$. Thus high growth regimes are slightly more persistent than low-growth, and implied the overall mean growth rate of A_m, $\bar{\gamma}_m$, is 2.31 percent.

2.9 The Elasticity of Substitution between Housing and other Consumption

The first-order conditions of the model imply a relationship between the expenditure ratio for housing and non-housing consumption and the relative price.

$$\frac{\omega_c^\epsilon}{\omega_h^\epsilon} \frac{p_t h_t}{c_t} = p_t^{1-\epsilon} \tag{81}$$

The long-term behavior of aggregate expenditures on housing services suggests a unit income elasticity for such expenditures, but price inelastic (i.e. $\epsilon < 1$). This is because the ratio expenditures on housing services to non-housing consumption expenditures has no long-run trend, but is positively correlated with the relative price of housing services, at least as measured by NIPA. Figure 4 presents annual data going back to 1929 of the two series, which show a positive relationship for most of the sample, though recently (since roughly 1990) they have diverged. The magnitude of the elasticity, however, is difficult to infer from time series data, given that both the ratio and the price are endogenous variables. In addition, whereas the nominal expenditures on h and c may be measured accurately, there may be substantial error in measuring the true relative price. Whereas the Boskin Commission had estimated an upward bias in CPI rents, Gordon and and vanGoethem (2005) argue for a downward bias averaging roughly 0.5 percent annually, but varying over time.

As a consequence, we instead examine evidence from micro data. Specifically, we examine data from the Consumer Expenditure Survey (CEX) to gauge the extent of housing service expenditure share variability as a function the relative price of housing services. To do so we construct rent (or owner's equivalent rent) relative to other expenditures, and match this up with data on housing prices by region, total expenditures, and demographic controls. As the above condition suggests, we can obtain an estimate of ϵ by a suitable regression of the nominal expenditure ratio on relative price.

Looking at micro data solves several problems. First, it is reasonable to take the price, which is based on regional CPI measures of owner's equivalent rent relative to the CPI excluding shelter, as exogenous to individual households. Second, the price measurement issue alluded to above is arguably mitigated by reliance on relative prices across regions. Third, we can include specific demographic controls to account for variation in, for example, ω_c/ω_h.

Consider the following model for individual i at date t in region j:

$$\ln\left[p_{jt}h_{it}/\left(x_{it}-p_{jt}h_{it}\right)\right]=a_j+b\ln x_{it}+(1-\epsilon)\ln p_{jt}+z_{it}'\theta+u_{itj}.$$

Here a_j reflects some constant region-specific factor that might affect expenditure shares. For example, if living in a region provides some amenities such as inexpensive public transportation or moderate weather that substitute for other expenditures (automobiles, heating oil), a_j might be positive. Note that we have to assume that a_j is constant over time or else we would not be able to identify the price effect. The coefficient b would reflect a wealth effect to the extent it differs from zero, and the coefficient on p_{jt} has the interpretation indicated: if $\epsilon < 1$, relative expenditures on housing services increase with their cost. The model also includes a set of demographic variables z_{it}, which would include things like family size and number of wage-earners. Note that we would expect a negative coefficient on an indicator of two adults working, as this would typically result in less household production and more expenditures on non-housing goods and services. Finally, the error term u_{ijt} represents idiosyncratic variation in preferences for housing services (in the model represented by ω_h/ω_c), measurement error of the dependent variable, and other omitted variables.

If the CEX had a true panel structure we could difference out the a_j. We could also allow for individual fixed effects. Unfortunately each individual household observation is present for at most four quarterly observations, so the panel aspect is probably useless (the other explanatory variables are not likely to change in a meaningful way over the course of a household's participation). Consequently we choose to pool the sample in levels and use regional dummies to capture the region effects. We also just use one (the first) observation per household, to avoid giving more weight to households simply because they remain in the sample.

The total expenditure variable x_{it} is likely to be measured with error. Such error would tend to produce a potentially large downward bias in the estimate of b because x_{it} enters the denominator of the dependent variable. Fortunately we have candidates for instruments: demographic variables such as race and education that are plausibly unrelated to the u_{itj}.

For nominal expenditures on housing services we use rent for renters and owner's equivalent rent (OER) on the primary residence for owners.[3] Consequently we subtract mortgage and home equity interest from total expenditures, as well as property taxes and expenditures on various categories of home repairs and maintenance that would normally be included in rent. We assume that OER is not intended to include utilities, so for owners utility expenditures are included in total but not housing expenditures. For renters, reported utility expenditures are also included only in total expenditures, but a dummy variable is included for renters that report zero utilities expenditures (on the assumption that they are included in rent).

The results of this estimation exercise are shown in Table 1. Because of an unexplained break in the level of the dependent variable that occurs in 1993 (presumably because of some change in variable definitions), all of our estimates included a dummy variable for post-1993 data. We present results with and without constraining $b = 0$, and with and without region dummies. Without instrumenting for x, the estimate of b is an implausibly large negative number, and the estimate of ϵ is only 0.134. Instrumenting brings $\hat{b}$ down to a more reasonable -0.254, and increases $\hat{\epsilon}$ to around 0.2. Imposing $b = 0$ results in $\hat{\epsilon} = 0.284$. The simulations in the next section will conservatively set $\epsilon = 0.3$.

[3] We do not include rent or OER on vacation or second homes. Though it might be desirable to do so, especially for second homes, it is difficult to distinguish, for example, someone who owns a vacation home but rents it out for 48 weeks of the year and uses it for four weeks, on the one hand, from someone who uses it every weekend. The former should be treated as equivalent to someone who rents a vacation home for four weeks, which we regard as something qualitatively different from housing services.

2.10 Model Simulations

The model separates conveniently into its dynamic aggregate component, which is essentially the neoclassical growth model, and the sectoral variables, which do not have a simple steady state representation, but are static functions of the aggregate state variables. Thus we can use standard methods (e.g. Uhlig, 1997) to obtain a solution for the linearized aggregate model of the form

$$\begin{aligned} \hat{k}_t &= \mathbf{P}\hat{k}_{t-1} + \mathbf{Q}\Lambda_t \\ \hat{x}_t &= \mathbf{R}\hat{k}_{t-1} + \mathbf{S}\Lambda_t. \\ \Lambda_{t+1} &= \mathbf{N}\Lambda_t + \Xi_{t+1} \end{aligned}$$

where

$$\Lambda_t = \begin{bmatrix} z_{1t} & z_{0t} & \hat{\eta}_t & \hat{\eta}_{t-1} \end{bmatrix}'$$

$$\mathbf{N} = \begin{bmatrix} \theta_1 & 1-\theta_0 & 0 & 0 \\ 1-\theta_1 & \theta_0 & 0 & 0 \\ 0 & 0 & \chi_1 & \chi_2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and where $\hat{k}_t$, $\hat{x}_t$, etc. are now logarithmic deviations from their steady state values. **P**, **Q**, **R**, and **S** are scalars that can be found using standard methods. It should be noted that parameters related to the sectoral dimension of the model do not enter into this part of the problem. These include β_h, ϵ, γ_h, ω_c, and ω_h.

For given realizations of the exogenous disturbances, time paths for the aggregates can be computed, they can be converted back to levels and become inputs to solving the for the sectoral variables (e.g. p, n_m, n_h, etc.) period-by-period using (59) – (66) and the definition of Q (48).

The key to doing interesting simulations is to take the peculiar error structure of the disturbance process into account. Even though the conditional expectation of the errors in the z_t process (the regime states) is zero, actual realizations of zero are not possible, and in fact given the values of q_1 and q_0, a small error (of absolute value $1 - q_0$ or $1 - q_1$) that leaves z unchanged is highly likely in any given time period. So rather than consider a one-time shock to v, it makes sense to consider a single large shock (a regime-switch) followed by a sequence of identical small shocks that leave the regime unchanged for an extended period of time. Such a path is more like a modal outcome rather than the improbable mean.

Figure 7 gives an example of this type of simulation. The economy is in the low growth regime in periods 1 to 12, and then switches to the high growth regime, where it remains. The figure plots the behavior of the asset price of a house (a fixed-weight combination of capital and land—see the Appendix) against per capita income, for $\epsilon = 0.3$ and 0.9. In the $\epsilon = 0.3$ case, the acceleration in housing prices is, depending on the horizon, approximately 2.25 percent, which compares with the 2.8 percent in the data pre- and post-1996. Figure 8 shows similar behavior for housing wealth (plotted against per capita consumption), and shows that the $\epsilon = 0.3$ case yields the prediction that housing wealth grows relative to per capita consumption during high-growth regimes. Finally, Figure 9 shows the behavior of housing investment, which is somewhat different. The prediction is for a relatively short-lived burst of investment at the time of the regime shift, not a sustained boom in housing investment. Clearly adding adjustment costs or imperfect information about the regime would smooth out the predicted behavior of the prices and quantities, but would not fundamentally change their low-frequency behavior.

Figure 10 depicts the actual behavior of residential investment relative to non-residential investment, which while obviously very cyclical, does bear out the low-frequency prediction of the model that this ratio will be higher in the high-growth regime than in the low-growth.

2.11 Regime Uncertainty

The model solution and simulations above assume perfect knowledge about the growth regime. This is unlikely to be a realistic assumption. Fortunately, earlier work (Kahn and Rich, 2007) provides a natural mechanism for extracting what economic agents know about the growth regime from the behavior of other economic variables. Figure 11 provides an updated estimate of the so-called smoothed (incorporating all available data through 2007:Q3) and zero-lag (incorporating data for each observation only up to that date) estimates of regime probabilities, incorporating information about the growth regime from data on productivity, labor compensation, aggregate consumption, and aggregate hours of work. The zero-lag estimates provide a reasonable basis for what economic agents might have thought at the time. Note in particular the recent signs of a shift back to the low-growth regime, roughly coincident with the sudden end of the housing boom.

Using these zero-lag probabilities, we can simulate the model's predictions about housing prices taking regime uncertainty into account in a realistic way. This is depicted in Figure 12. While it captures some of the flavor of Figure 1, but one noteable difference is that the downturn in housing prices in the following the 1973 productivity slowdown occurs very soon after 1973 in the simulation, whereas housing prices in the data continued to grow through the 1970s. One drawback of this approach is that the model parameters are based on the full sample. Because the productivity slowdown in 1973 was unprecedented, the probabilities in Figure 11 are probably unrealistic in suggesting that people figured out pretty soon after 1973 that there had been a regime shift.

3 Conclusions

This paper has developed a growth model with land, housing services, and other goods and shown that it is capable both qualitatively and quantitatively of explaining a substantial portion of the movements in housing prices over the past 40 years, including potentially the recent downturn. The paper also uses micro data to calibrate a key cross-elasticity parameter that governs the relationship between productivity growth and home price appreciation. The matching of the model to the data relies not on fitting the overall trend (which depends on an unobservable), but on the changes in trend relative to changes in trend productivity growth (for goods excluding housing services). The paper has more limited success in matching the behavior of housing investment, though it does qualitatively fit its low-frequency behavior.

4 Appendix

4.1 Solving the Dynamic Model

First we linearize the system

$$\begin{aligned}(1+\nu)(1+g_t)\hat{k}_t/\hat{k}_{t-1} &= \left(\hat{k}_{t-1}/(1+g_t)\right)^{\alpha-1} - (1+g_t)\hat{x}_t/\hat{k}_{t-1} + 1 - \delta \\ (1+\rho)(1+\nu) &= E_t\left\{(\hat{x}_t/\hat{x}_{t+1})(1+g_{t+1})^{-1}\left[\alpha\left(\hat{k}_t/(1+g_{t+1})\right)^{\alpha-1} + 1 - \delta\right]\right\}\end{aligned}$$

around the steady state values $\hat{k}$ and $\hat{x}$. The linearized versions of the two equations are

$$(1+\nu)(1+g)\left(\hat{k}_t - \hat{k}_{t-1} + \frac{gg_t}{1+g}\right)$$

$$= -(1-\alpha)\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1}\left(\hat{k}_{t-1} - \frac{gg_t}{1+g}\right) - \left(\hat{x}/\hat{k}\right)(1+g)\left(\frac{gg_t}{1+g} + \hat{x}_t - \hat{k}_{t-1}\right) \tag{82}$$

$$0 = E_t\left\{\left[\hat{x}_t - \hat{x}_{t+1} - \frac{gg_{t+1}}{1+g}\right]\left(\alpha\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} + 1 - \delta\right)\right. \tag{83}$$

$$\left. -\alpha(1-\alpha)\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1}\left(\hat{k}_t - \frac{gg_{t+1}}{1+g}\right)\right\} \tag{84}$$

Note that

$$\begin{aligned}\alpha\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} + 1 - \delta &= (1+\rho)(1+\nu)(1+g) \\ (1+g)\hat{x}/\hat{k} &= \left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} + 1 - \delta - (1+\nu)(1+g)\end{aligned}$$

So

$$\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} = \frac{(1+\rho)(1+\nu)(1+g) - (1-\delta)}{\alpha}$$

This system is of the form

$$0 = \mathbf{A}\hat{k}_t + \mathbf{B}\hat{k}_{t-1} + \mathbf{C}\hat{x}_t + \mathbf{D}\Lambda_t \tag{85}$$

$$0 = E_t\left\{\mathbf{F}\hat{k}_{t+1} + \mathbf{G}\hat{k}_t + \mathbf{H}\hat{k}_{t-1} + \mathbf{J}\hat{x}_{t+1} + \mathbf{K}\hat{x}_t + \mathbf{L}\Lambda_{t+1} + \mathbf{M}\Lambda_t\right\} \tag{86}$$

$$\Lambda_{t+1} = \mathbf{N}\Lambda_t + \Xi_{t+1} \tag{87}$$

where

$$\Lambda_t = \left[\begin{array}{cccc} z_{1t} & z_{0t} & \hat{\eta}_t & \hat{\eta}_{t-1}\end{array}\right]'$$

and

$$
\begin{aligned}
\mathbf{A} &= (1+\nu)(1+g) \\
\mathbf{B} &= -(1+\nu)(1+g) + (1-\alpha)\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} - \left(\hat{x}/\hat{k}\right)(1+g) && (88) \\
&= -\left(\alpha\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} + 1 - \delta\right) && (89) \\
\mathbf{C} &= \left(\hat{x}/\hat{k}\right)(1+g) && (90) \\
\mathbf{D} &= \left[(1+\nu)g - (1-\alpha)\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1}\frac{g}{1+g} + \left(\hat{x}/\hat{k}\right)g\right]\frac{1}{1-\alpha}\frac{1}{1+\bar{\gamma}_m}\left[\begin{array}{cccc} \gamma_m^1 & \gamma_m^0 & 1 & -1 \end{array}\right] && (91) \\
&= \left[\alpha\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} + (1-\delta)\right]\frac{g}{1+g}\frac{1}{1-\alpha}\frac{1}{1+\bar{\gamma}_m}\left[\begin{array}{cccc} \gamma_m^1 & \gamma_m^0 & 1 & -1 \end{array}\right] && (92)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{F} &= 0 \\
\mathbf{G} &= -\alpha(1-\alpha)\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} \\
\mathbf{H} &= 0 \\
\mathbf{J} &= -\left(\alpha\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} + 1 - \delta\right) \\
\mathbf{K} &= \left(\alpha\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} + 1 - \delta\right) \\
\mathbf{L} &= -\left[\alpha^2\left(\frac{\hat{k}}{1+g}\right)^{\alpha-1} + 1 - \delta\right]\frac{g}{1+g}\frac{1}{1-\alpha}\frac{1}{1+\bar{\gamma}_m}\left[\begin{array}{cccc} \gamma_m^1 & \gamma_m^0 & 1 & -1 \end{array}\right] \\
\mathbf{M} &= 0
\end{aligned}
$$

$$
\mathbf{N} = \begin{bmatrix} \theta_1 & 1-\theta_0 & 0 & 0 \\ 1-\theta_1 & \theta_0 & 0 & 0 \\ 0 & 0 & \chi_1 & \chi_2 \\ 0 & 0 & 1 & 0 \end{bmatrix}
$$

and

$$
\Xi_t = \begin{bmatrix} v_{1t} \\ v_{2t} \\ u_t \\ 0 \end{bmatrix}
$$

where v_{1t} and v_{2t} are as defined earlier. We can then use the method of undetermined coefficients outlined by Uhlig (1997) to find the solution of the model in the form

$$\begin{aligned}\hat{k}_t &= \mathbf{P}\hat{k}_{t-1} + \mathbf{Q}\Lambda_t \\ \hat{x}_t &= \mathbf{R}\hat{k}_{t-1} + \mathbf{S}\Lambda_t \\ \Lambda_t &= N\Lambda_{t-1} + \Xi_t.\end{aligned}$$

where in this case, of course, $\mathbf{P}$ and $\mathbf{R}$ are scalars.

To solve for the work effort e_t and sectoral variables, which we can denote by the vector

$$\hat{\boldsymbol{s}}_t = \left(e_t, \hat{h}_t, \hat{c}_t, n_{mt}, n_{ht}, \ell_{mt}, \ell_{ht}, p_t, \hat{q}_t, Q_t, \tilde{k}_{mt}, \tilde{k}_{ht}\right),$$

we pick initial values for A_m, A_h, and N, which then evolve exogenously. We start with the steady state levels $\hat{k}$ and $\hat{x}$, which do not depend on the levels of the exogenous variables. We then construct an entire path of $\left\{\hat{k}_t, \hat{x}_t, \Lambda_t\right\}$ from the above solution, exponentiated and multipled by the steady state levels. These can be used first to construct at each t

$$\hat{s}_t = \Psi\left(\hat{k}_{t-1}, \Lambda_t; A_{m0}/A_{h0}, N_0\right).$$

Finally, we compute the path of $Q_t Z_t$, which depends on A_{mt}, ℓ_{mt}, and n_{mt}. From that we can compute the path of the non-normalized variables, multiplying the "^" variables by $Q_t Z_t$ and the "~" variables by Z_t.

The nonlinearity of Ψ implies that in general the sectoral variables (except for $\tilde{k}_m$ and $\tilde{k}_h$, which are linear in $\hat{k}$) do not grow at a constant rate along the balanced growth path. This does not present problems for computing the time paths of these variables, but it does make computing something like V, the price of land, considerably more difficult—mainly because the levels of A_m/A_h and N have nonlinear effects on V. We have

$$\hat{V}_t = \hat{q}_t + E_t\left\{\hat{\Phi}_{t,1}\hat{V}_{t+1}\right\} = E_t\left\{\sum_{\tau=0}^{\infty}\hat{\Phi}_{t,\tau}\hat{q}_{t+\tau}\right\}$$

where

$$\hat{\Phi}_{t,\tau} \equiv \frac{\hat{x}_t}{\hat{x}_{t+\tau}(1+\rho)^{\tau}} \quad (93)$$

$$\hat{q}_t = \beta_m \hat{k}_{mt}^{\alpha} / (\ell_{mt} Q_t). \quad (94)$$

What about the price of a "house"? The total value of land and capital in housing services is $V_t L_{ht} + K_{ht}$. Given a path $\{K_{ht}, L_{ht}\}$, we can define a "constant-quality" house price index P_{ht} by choosing a base year, say $t = 0$, and defining $P_{ht} = 100\,(V_t L_{h0} + K_{h0}) / (V_0 L_{h0} + K_{h0})$.

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Table 1: Parameter Estimates from CEX Household Data

Parameter			
$\hat{\epsilon}$	0.134	0.195	0.284
	(0.042)	(0.046)	(0.052)
$\hat{b}$	−0.743	−0.254	—
	(0.003)	(0.009)	
Instruments for x	N	Y	—
R^2	0.575	0.464	0.317

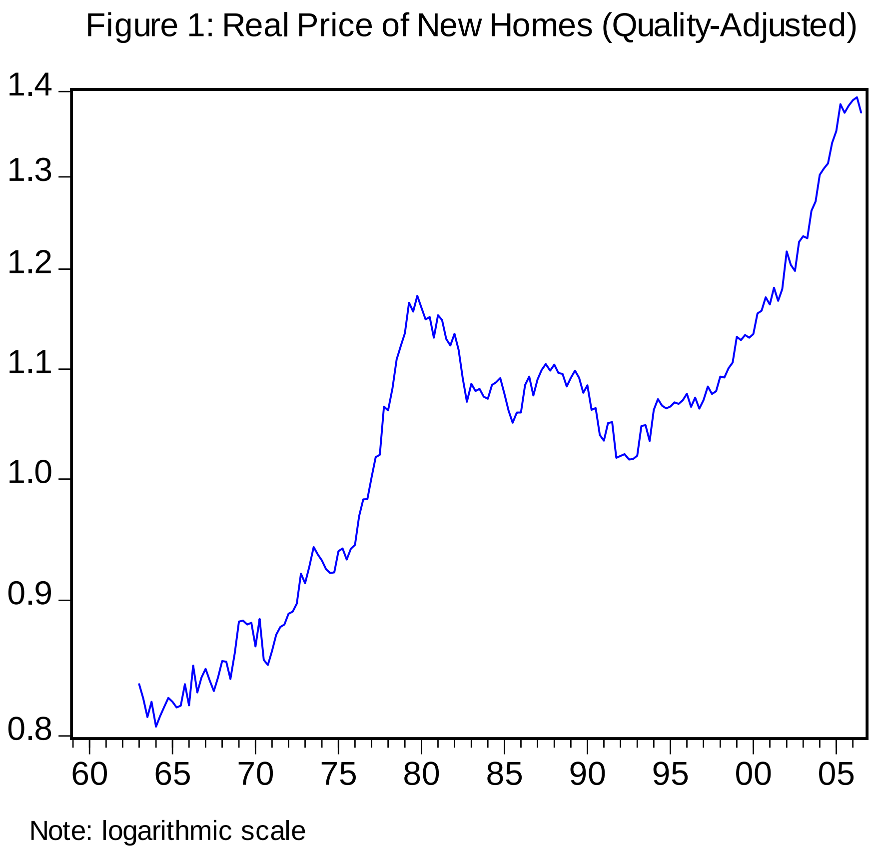


Figure 1: Real Price of New Homes (Quality-Adjusted)
1.4
1.3
1.2
1.1
1.0
0.9
0.8
60
65
70
75
80
85
90
95
00
05
Note: logarithmic scale

Figure 10:

Figure 2a: Ratio of Housing Wealth to Consumption

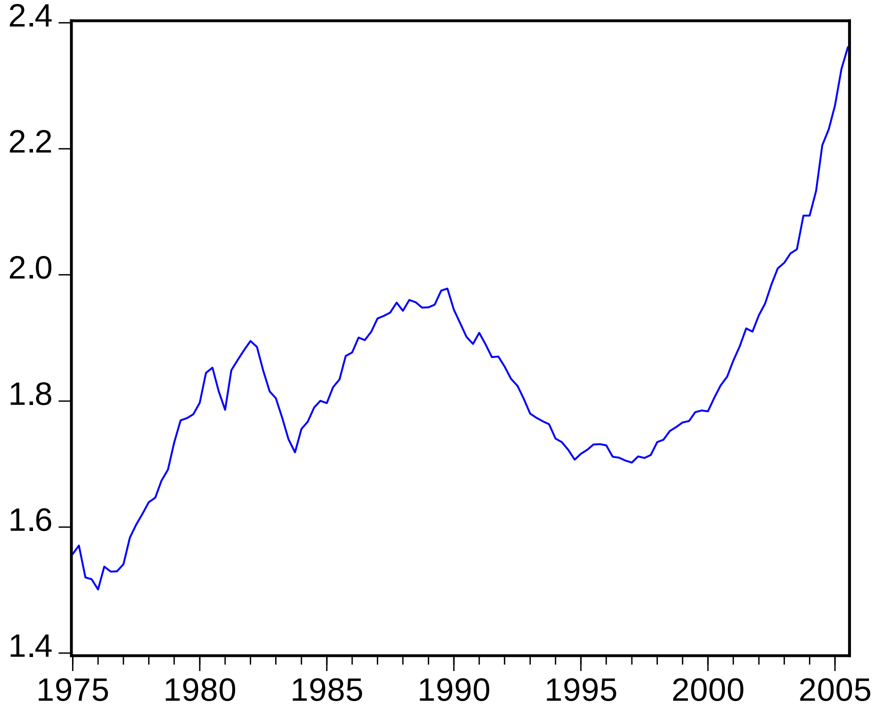


Figure 2b: Ratio of Housing Wealth to Total Net Worth

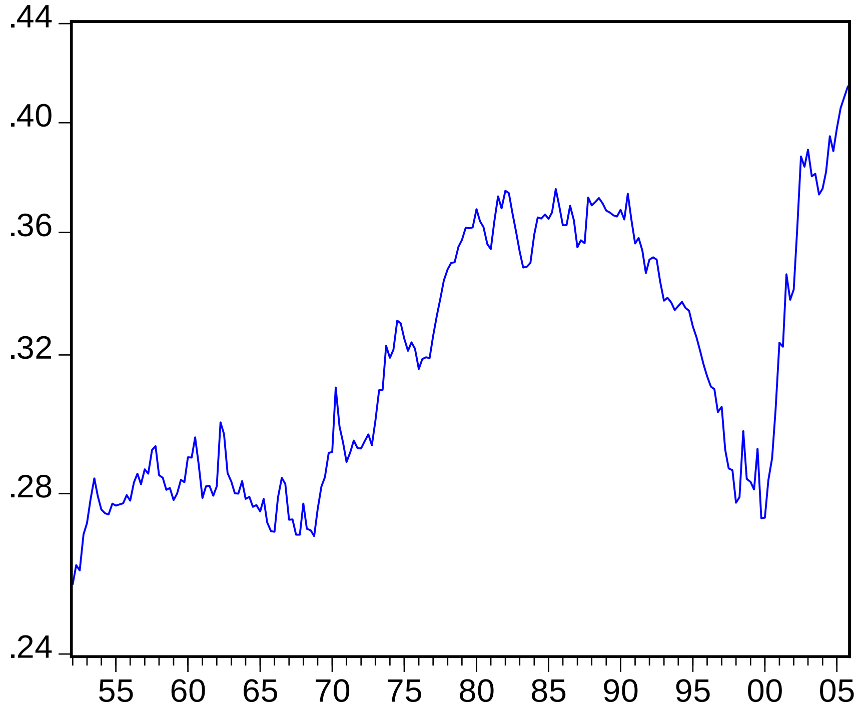
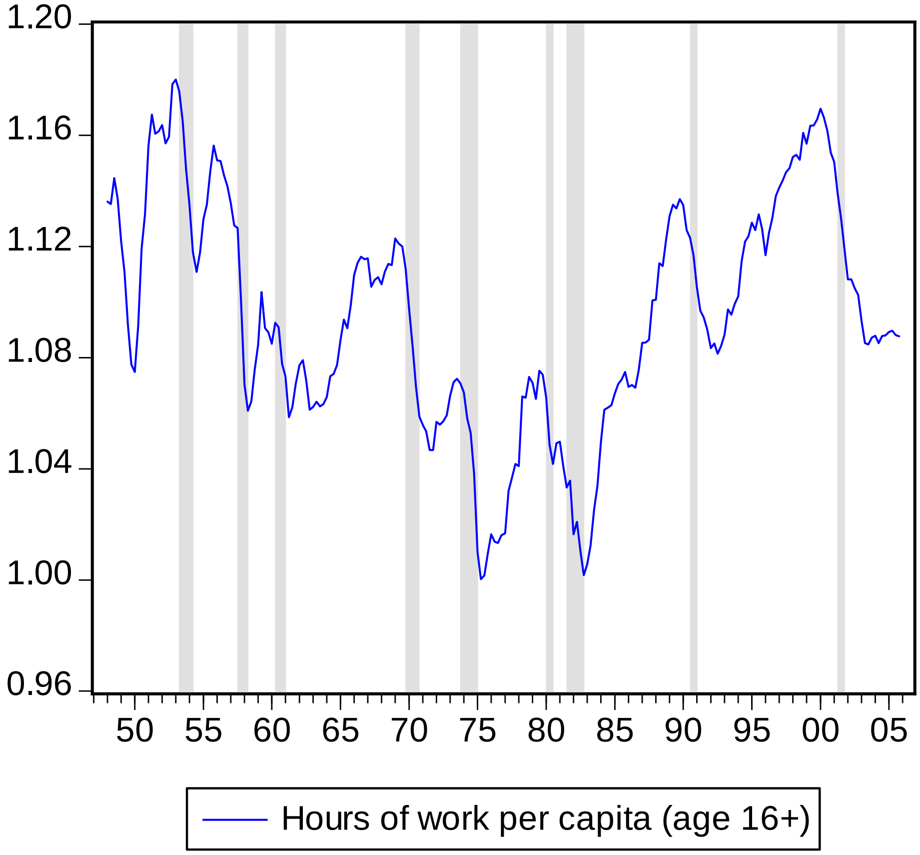


Figure 3: Trends in Per Capita Hours



Note: Shaded areas are NBER-defined recessions

Figure 4: Housing Services: Expenditures and Prices

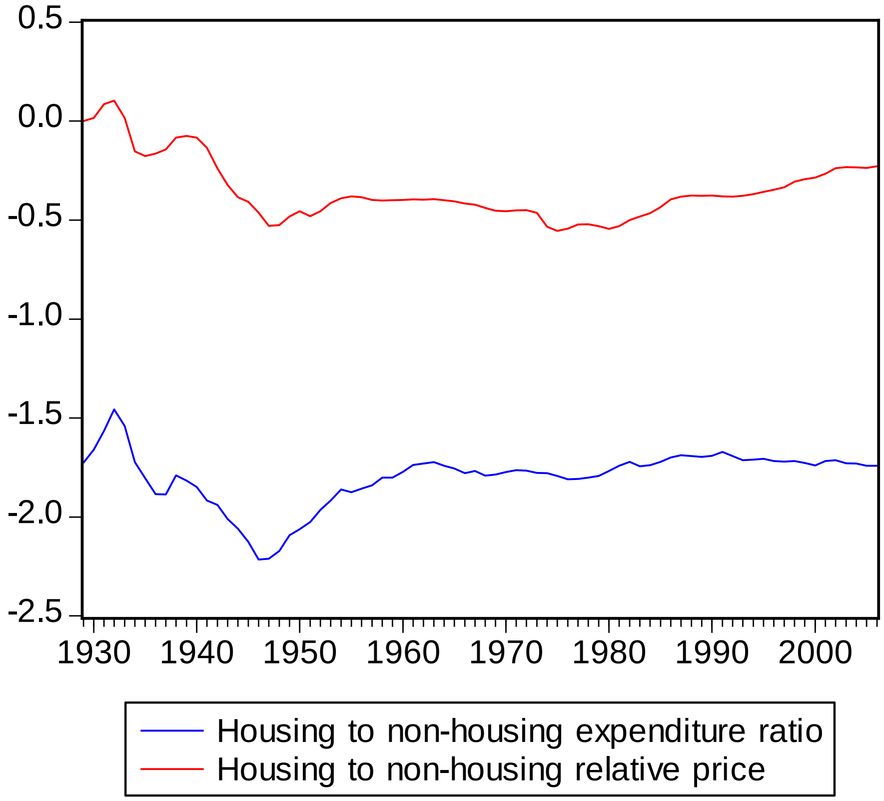
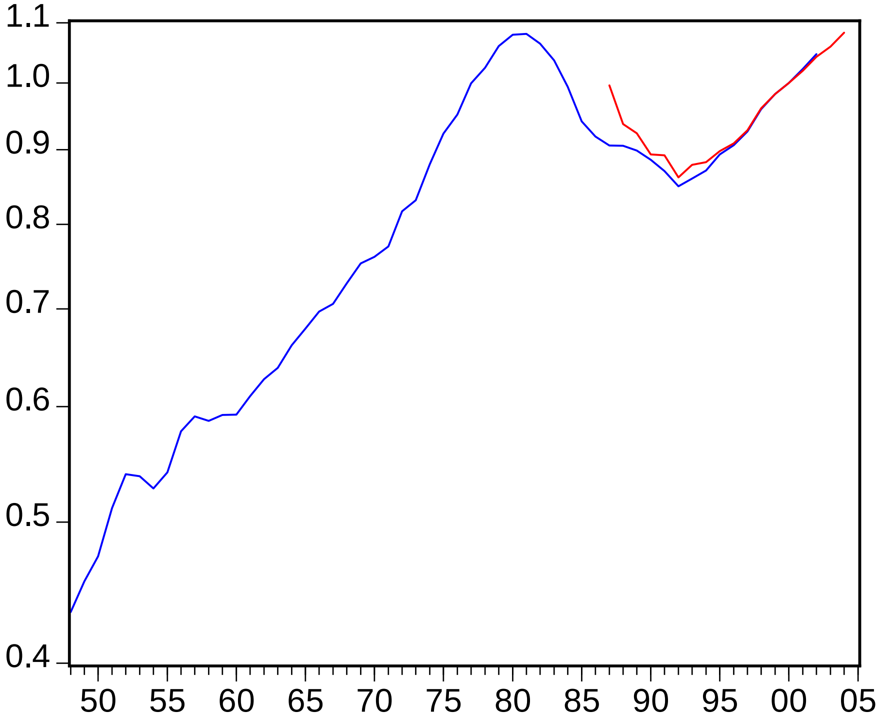


Figure 5: Real Price of Land



Source: BLS

Note: Logarithmic scale. The two series are from different vintages of BLS data.

Figure 6: Detrended HP-Filtered Output per Hour

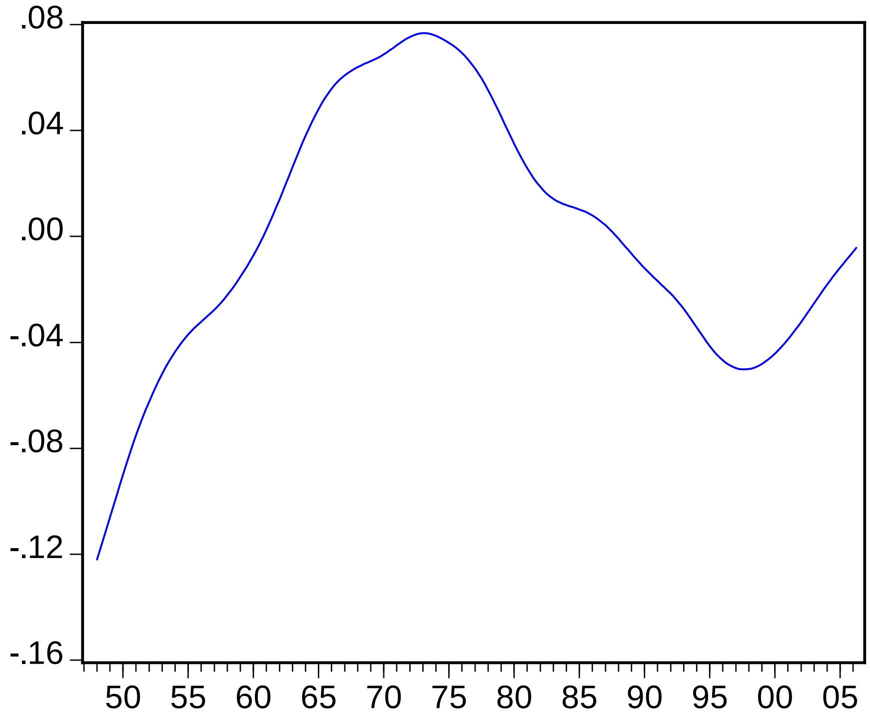


Figure 7: Home Price Appreciation in Reponse to a Regime Switch

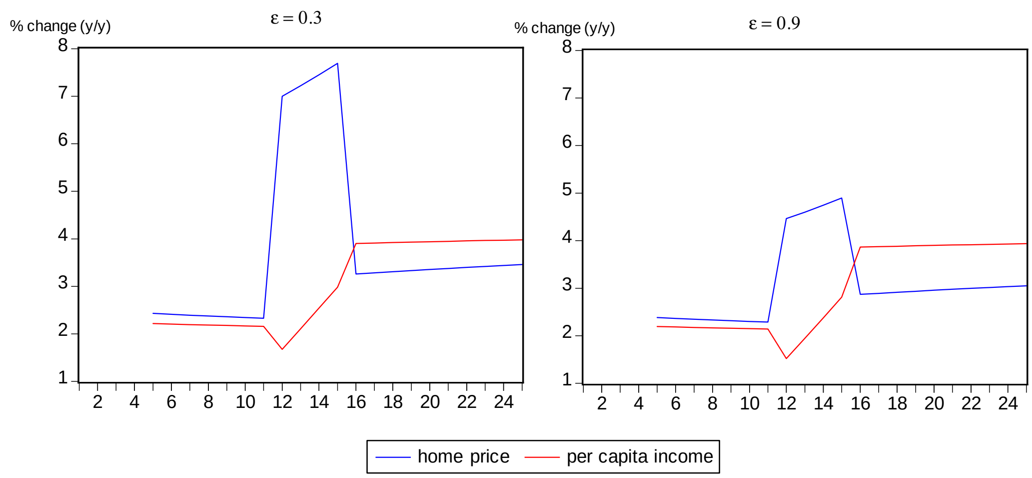


Figure 8: Housing Wealth and Productivity Regime Change

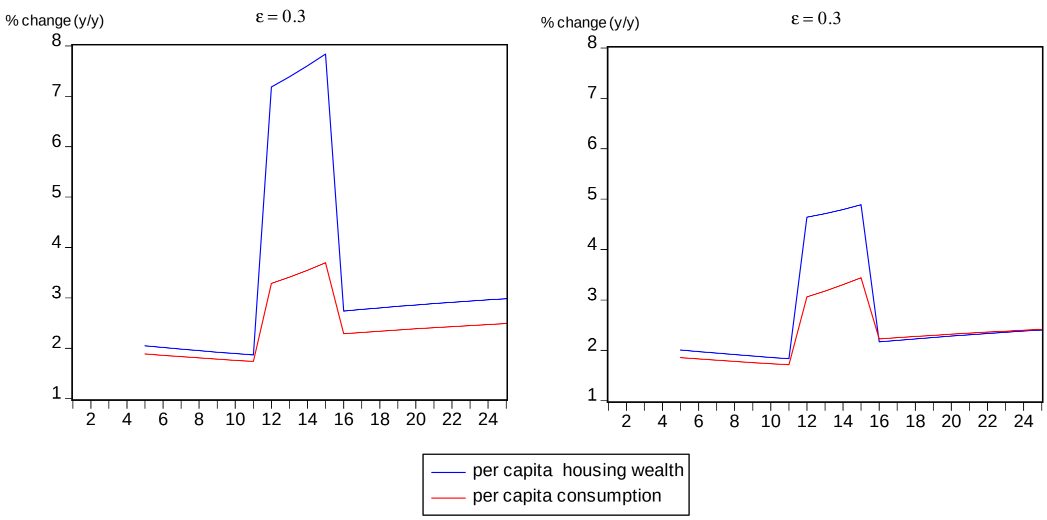
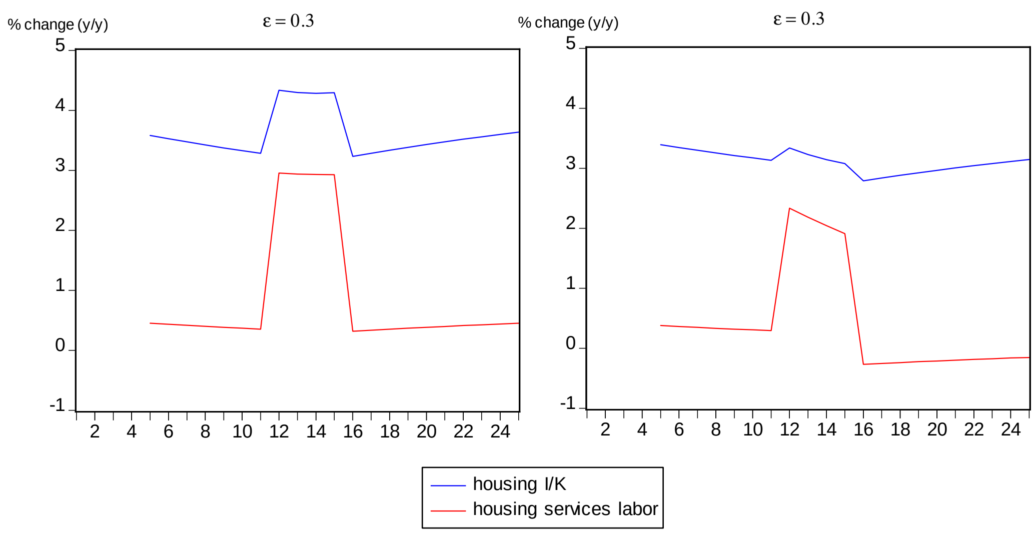
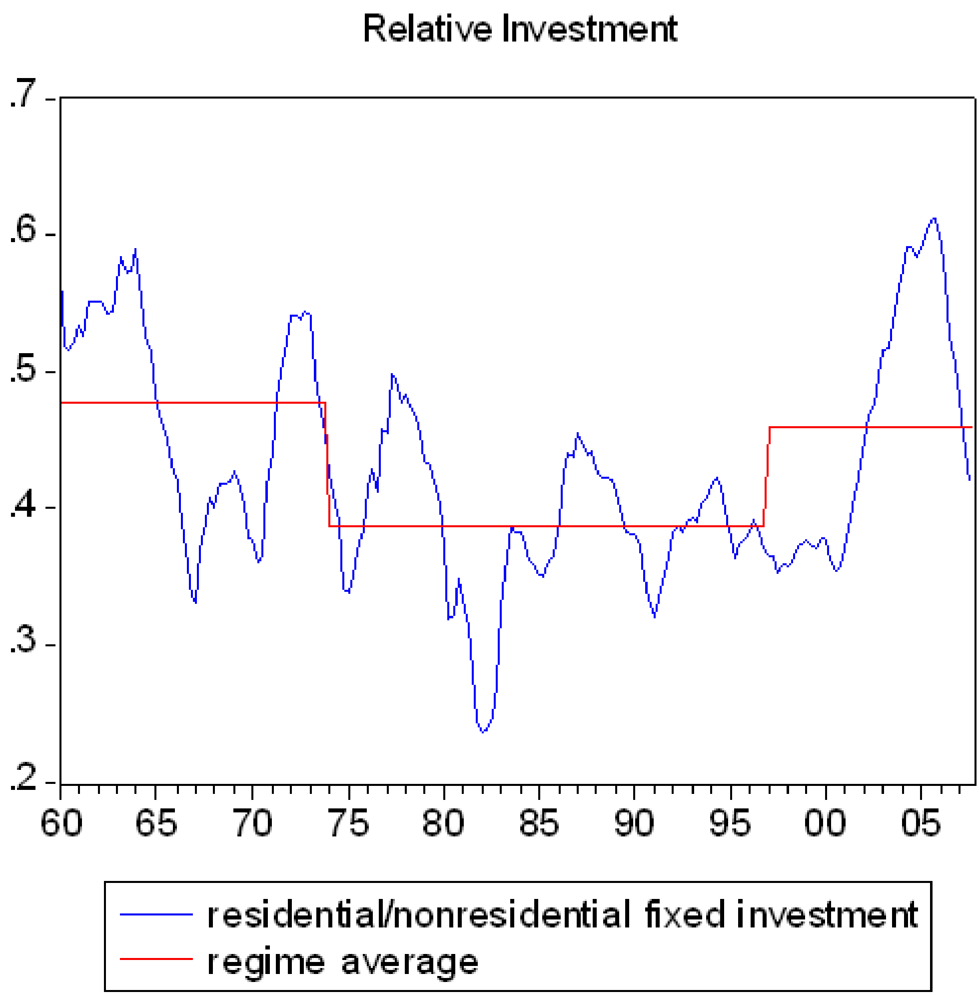


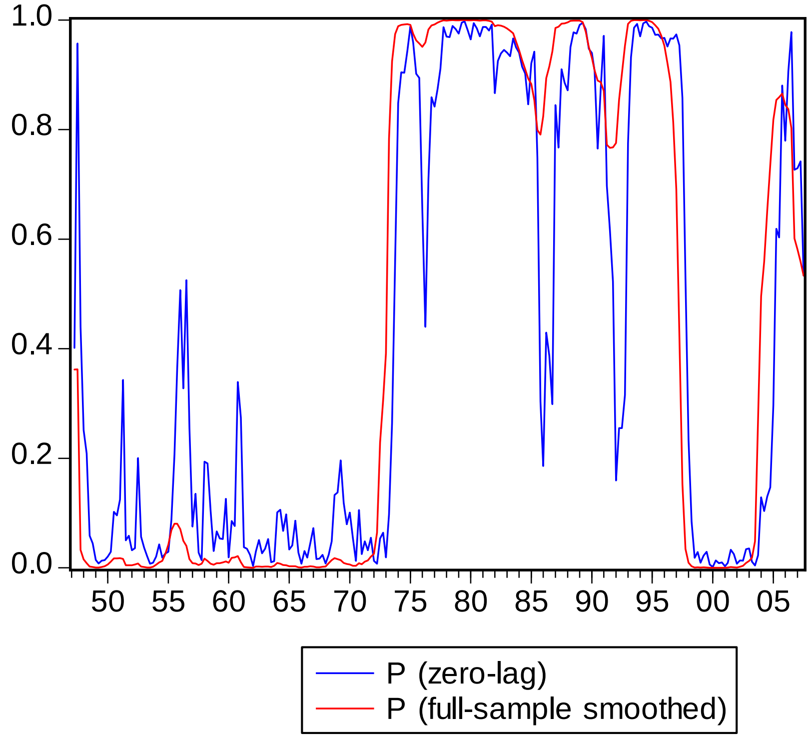
Figure 9: Housing Investment and Sectoral Labor





Relative Investment
.7
.6
.5
.4
.3
.2
60
65
70
75
80
85
90
95
00
05
residential/nonresidential fixed investment
regime average

Figure 11: Low-Growth Regime Probabilities



Calculations based on Kahn and Rich (2007)

Figure 12: Simulated Housing Price with Uncertain Regimes