Online Appendix:
House Price Beliefs And Mortgage Leverage Choice

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A Theoretical Derivations

A.1 Proofs

We first provide proofs for the simplified model considered in Sections 1 and 2 of the paper. We then study a more general environment with non-zero default costs. At times we rely on auxiliary mathematical results collected in Section A.3.

Proof of Proposition 1. (Mean and variance shifts with normally distributed beliefs).

Variable House Size. In the variable house size scenario, given Equation (8), it is sufficient to establish the behavior of $\Psi_{VHS}\left(\tilde{\delta}\right) \equiv \mathbb{E}_i\left[g|\, g \geq \tilde{\delta}\right]$ to characterize the effect of beliefs on leverage. Under standard regularity conditions, any change in parameters associated with an upwards point-wise shift in $\Psi_{VHS}(\cdot)$ for a range of $\tilde{\delta}$ implies a higher equilibrium level of δ_i. We respectively denote the pdf and cdf of the standard normal distribution by $\phi(\cdot)$ and $\Phi(\cdot)$. Under the assumption that $g \sim N\left(\mu_i, \sigma_i^2\right)$, we can express $\Psi_{VHS}\left(\tilde{\delta}\right)$ as follows

$$\Psi_{VHS}\left(\tilde{\delta}\right) = \mathbb{E}_i\left[g|g \geq \tilde{\delta}\right] = \mu_i + \sigma_i \lambda(\alpha_i)$$

where $\lambda(\alpha_i) = \frac{\phi(\alpha_i)}{1-\Phi(\alpha_i)}$ and $\alpha_i = \frac{\tilde{\delta}-\mu_i}{\sigma_i}$. The relevant comparative statics in μ_i and σ_i are given by

$$\frac{\partial \Psi_{VHS}\left(\tilde{\delta}\right)}{\partial \mu_i} = 1 - \lambda'(\alpha_i) > 0$$
$$\frac{\partial \Psi_{VHS}\left(\tilde{\delta}\right)}{\partial \sigma_i} = \lambda(\alpha_i) - \lambda'(\alpha_i)\alpha_i = \lambda(\alpha_i)\left(1 - \left(\lambda(\alpha_i) - \alpha_i\right)\alpha_i\right) > 0,$$

where we have used the properties of the Normal hazard rate from Fact 2 in Section A.3. These results establish the conclusions for the variable house size scenario.

Fixed House Size. In the fixed house size scenario, given Equation (9), it is sufficient to establish the behavior of

$$\Psi_{FHS}\left(\tilde{\delta}\right) \equiv 1 - F_i\left(\tilde{\delta}\right) = 1 - \Phi(\alpha_i),$$

where $\alpha_i = \frac{\tilde{\delta}-\mu_i}{\sigma_i}$, to characterize the effect of beliefs on leverage. Any change in parameters associated with an upwards point-wise shift in $\Psi_{FHS}(\cdot)$ for a range of $\tilde{\delta}$ implies a lower equilibrium level of δ_i. The relevant comparative statics in μ_i and σ_i are given by

$$\frac{\partial \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \mu_i} = \frac{1}{\sigma_i}\phi(\alpha_i) > 0$$
$$\frac{\partial \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \sigma_i} = \phi(\alpha_i)\frac{\alpha_i}{\sigma_i} = -\frac{\phi'(\alpha_i)}{\sigma_i},$$

which is negative when the probability of default is less than 50% (sufficiently low default probability), that is, when $\alpha_i < 0$ or, equivalently, when $\tilde{\delta} < \mu_i$.

Summary. Formally, we can summarize both sets of predictions as follows:

$$\frac{\partial \mathbb{E}_i\left[g|\, g \geq \tilde{\delta}\right]}{\partial \mu_i} > 0,\ \forall \tilde{\delta} \quad \Rightarrow \quad \frac{\partial \delta_i}{\partial \mu_i} > 0, \qquad \frac{\partial F_i\left(\tilde{\delta}\right)}{\partial \mu_i} < 0,\ \forall \tilde{\delta} \quad \Rightarrow \quad \frac{\partial \delta_i}{\partial \mu_i} < 0,$$
$$\frac{\partial \mathbb{E}_i\left[g|\, g \geq \tilde{\delta}\right]}{\partial \sigma_i} > 0,\ \forall \tilde{\delta} \quad \Rightarrow \quad \frac{\partial \delta_i}{\partial \sigma_i} > 0, \qquad \frac{\partial F_i\left(\tilde{\delta}\right)}{\partial \sigma_i} > 0,\ \forall \tilde{\delta} \quad \Rightarrow \quad \frac{\partial \delta_i}{\partial \sigma_i} > 0, \quad \text{if } \tilde{\delta} < \mu_i.$$

Proof of Proposition 2. (Additional predictions)

To be able to meaningfully study to impact of changes on the cost of default we must consider a model with non-zero default costs, so we provide the proof of part (a) of the proposition in the next section. We also refer to our derivations in the context of the more general model below regarding the part (b) results concerning the variable house size scenario.
In the fixed house size scenario, in which $\Psi_{FHS}\left(\tilde{\delta}\right) \equiv 1 - F_i\left(\tilde{\delta}\right) = 1 - \Phi\left(\alpha_i\right)$, we can find that

$$\frac{\partial^2 \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \mu_i^2} = -\frac{1}{\sigma_i^2}\phi'\left(\alpha_i\right),$$

which is negative when the probability of default is less than 50%, that is, when $\tilde{\delta} < \mu_i$. We also find that

$$\begin{aligned}
\frac{\partial^2 \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \sigma_i \partial \mu_i} &= \frac{1}{\sigma_i}\left[-\frac{1}{\sigma_i}\phi'\left(\alpha_i\right)\alpha_i - \phi\left(\alpha_i\right)\frac{1}{\sigma_i}\right] \\
&= \frac{\phi\left(\alpha_i\right)}{\sigma_i^2}\left[\alpha_i^2 - 1\right] \\
&= \frac{\phi\left(\alpha_i\right)}{\sigma_i^2}\left[\left(\frac{\tilde{\delta} - \mu_i}{\sigma_i}\right)^2 - 1\right],
\end{aligned}$$

which is positive as long as $\left|\tilde{\delta} - \mu_i\right| > \sigma_i$. Hence, it is needed that $\tilde{\delta} < \mu_i - \sigma_i$, which is valid whenever the probability of default is less than 16%. Note that the characterization of Proposition 2 exclusively focuses on the direct effects of changes in the default cost and average beliefs. As we describe in more detail below, when formally characterizing the more general environment, second-order comparative statics exercises are determined by both direct effects and curvature effects. Our focus is on the former set of effects.

A.2 General Environment: Non-zero default costs

We now proceed to systematically study a more general model in which default costs are non-zero. The goal of this section is to show that the results derived in Propositions 1 and 2 extend to the case of non-zero default costs, while providing a more detailed formal analysis of the environment that we consider in the body of the paper.[36]

Borrowers' Problem and Default Decision

When borrowers face non-zero default costs, they solve the following optimization problem

$$\max_{c_{0i}, b_{0i}, h_{0i}} u_i\left(c_{0i}\right) + \beta \mathbb{E}_i\left[\max\left\{w_{1i}^{\mathcal{N}}, w_{1i}^{\mathcal{D}}\right\}\right],$$

subject to Equation (1), where $w_{1i}^{\mathcal{N}}$ and $w_{1i}^{\mathcal{D}}$ denote borrowers' wealth in non-default and default states, given by

$$\begin{aligned}
w_{1i}^{\mathcal{N}} &= n_{1i} + p_1 h_{0i} - b_{0i} \\
w_{1i}^{\mathcal{D}} &= n_{1i} - \kappa_i h_{0i},
\end{aligned}$$

where $\kappa_i \geq 0$. Assuming that the default cost is proportional to the house size preserves the homogeneity of the borrowers' problem. This assumption can be relaxed without affecting our insights, though we believe that it is reasonable to assume that individual default costs are larger for homeowners with larger properties. At date 1, borrowers default according to the following threshold rule,

$$\text{if } \begin{cases} g \leq \delta_i - \chi_i, & \text{Default} \\ g > \delta_i - \chi_i, & \text{No Default} \end{cases} \qquad \text{where} \quad \chi_i = \frac{\kappa_i}{p_0} \quad \text{and} \quad \delta_i = \frac{b_{0i}}{p_0 h_{0i}},$$

and where g denotes the actual realization of house price growth. Note that $\chi_i \geq 0$ is simply a rescaled version of the default cost, so that $\chi_i = 0$ when $\kappa_i = 0$. Intuitively, borrowers decide to default only when date-1 house prices are sufficiently low.

[36] Throughout the paper, we present our results in a principal-agent formulation in which borrowers and lenders have predetermined roles. A common theme in existing work is that optimists endogenously become asset buyers and borrowers while pessimists become asset sellers and lenders. This type of sorting is natural if belief disagreement is the single source of heterogeneity. However, in housing markets, additional dimensions of heterogeneity (e.g., life-cycle motives, wealth, risk preferences, access to credit markets, etc.) also determine which agents become buyers and borrowers in equilibrium.

For any realization of house price changes, the probability of default by borrower i increases with the borrower's promised repayment δ_i and decreases with the magnitude of default costs κ_i.

Properties of $\Lambda_i(\delta_i)$

When allowing for non-zero default costs, we also allow for the possibility of loan-to-value ratios to vary with the identity of the borrower, so we denote them by $\Lambda_i(\delta_i)$. We also allow for potential deadweight losses associated with default, parameterized by $1-\eta$. Therefore, the loan-to-value ratio offered to borrower i for a given promised repayment δ_i corresponds to

$$\Lambda_i(\delta_i) = \frac{\eta \int_{\underline{g}}^{\delta_i-\chi_i} g dF_L(g) + \delta_i \int_{\delta_i-\chi_i}^{\overline{g}} dF_L(g)}{1+r},$$

which can be alternatively written in terms of default and non-default probabilities and truncated expectations as $\Lambda_i(\delta_i) = \frac{\eta F_L(\delta_i-\chi_i)\mathbb{E}[g|g\leq\delta_i-\chi_i]+(1-F_L(\delta_i-\chi_i))\delta_i}{1+r}$. Note that $\Lambda_i(\delta_i)$ is strictly positive and that $\Lambda_i'(\delta_i)$ can be expressed as

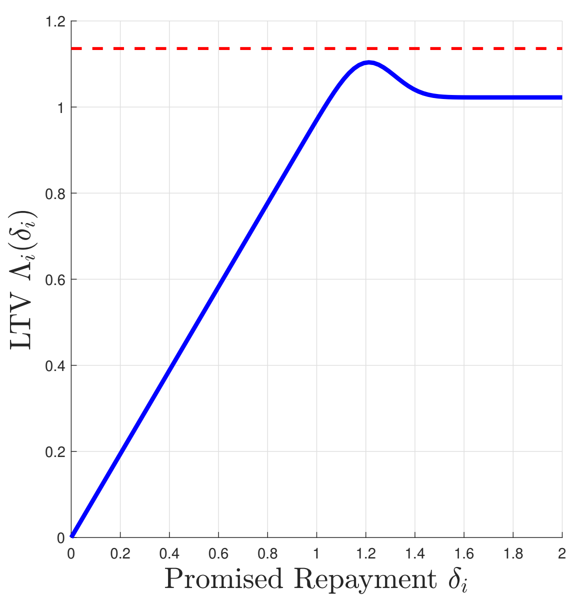
$$\Lambda_i'(\delta_i) = (1-(\eta\chi_i + (1-\eta)\delta_i)\lambda_L(\delta_i-\chi_i))\frac{1-F_L(\delta_i-\chi_i)}{1+r},$$

where $\lambda_L(\delta_i-\chi_i) = \frac{f_L(\delta_i-\chi_i)}{1-F_L(\delta_i-\chi_i)}$. In general, $\Lambda_i'(\delta_i)$ can take positive or negative values, depending on the sign of $1-(\eta\chi_i+(1-\eta)\delta_i)\lambda_L(\delta_i-\chi_i)$. When $\eta<1$ or $\chi_i>0$, the function $\Lambda_i(\delta_i)$ has a well-defined maximum. Note that, whenever $\delta_i>0$, it is the case that $\Lambda_i'(\delta_i)<1$. At any interior optimum for leverage there exists a positive relation between $\Lambda_i(\delta_i)$ and δ_i. Alternatively, we could have formulated homebuyers' choices in terms of the margin/haircut/downpayment, which is equal to $1-\Lambda_i(\delta_i)$, or the leverage ratio, which is equal to $\frac{1}{1-\Lambda_i(\delta_i)}$, since there exists a one-to-one relation among these variables. For instance, if a borrower pays \$100k dollars for a house, borrowing \$75k and paying \$25k in cash, the borrower's loan-to-value ratio is 0.75, his downpayment is 25%, and his leverage ratio is 4. Note that the limits of the LTV schedule offered by lenders correspond to

$$\lim_{\delta_i\to\infty}\Lambda_i(\delta_i) = \frac{\eta\int_{\underline{g}}^{\overline{g}} g dF_L(g)}{1+r} = \frac{\eta\mathbb{E}_L[g]}{1+r} \quad \text{and} \quad \lim_{\delta_i\to 0}\Lambda_i(\delta_i) = 0.$$

Figure A1 illustrates the behavior of $\Lambda_i(\delta_i)$ when lenders' beliefs are normally distributed.

Figure A1: LTV schedule $\Lambda_i(\delta_i)$



Note: Figure shows the LTV schedule $\Lambda_i(\delta_i)$ offered by lenders with normally distributed beliefs with mean $\mu_L = 1.17$ and standard deviation $\sigma_L = 0.1$, for a recovery rate after default of $\eta = 0.9$ and a default cost that corresponds to $\chi_i = 0.1$. The red dashed line corresponds to the maximum LTV ratio with full recovery, which corresponds to $\frac{\mu_L}{1+r}$.

Note that our results remain valid under weaker assumptions on the determination of credit supply. For instance, our results are unchanged if lenders are restricted to offering a single loan-to-value schedule to all borrowers. A correct interpretation of our empirical findings requires that potentially unobserved characteristics used by lenders' to offer loan-to-value schedules to borrowers must be orthogonal to the recent house price experiences of a borrower's geographically distant friends.

Borrowers' Leverage Choice

The specification of the housing constraint in Equation (2) guarantees equilibrium existence provided that $n_{0i} > p_0 \underline{h_i}$, which guarantees that the feasible choice set is non-empty. Exploiting homogeneity, the problem solved by borrowers to determine their optimal leverage choice can be expressed in terms of the following Lagrangian:

$$\max_{c_{0i},\delta_i,h_{0i}} u_i(c_{0i}) + \beta p_0 h_{0i} \left[-\chi_i \int_{\underline{g}}^{\delta_i-\chi_i} dF_i(g) + \int_{\delta_i-\chi_i}^{\overline{g}} (g-\delta_i)\, dF_i(g) \right]$$
$$- \lambda_{0i}\left(c_{0i} + p_0 h_{0i}\left(1-\Lambda_i(\delta_i)\right) - n_{0i}\right) + \nu_{0i}\left(h_{0i} - \underline{h_i}\right).$$

The optimality conditions of this problem regarding consumption, promised repayment, and housing choice, are the counterparts to Equations (5) to (7) in the text. The derivation of Equation (7) uses the envelope theorem. Even though borrowers take into account that adjusting their leverage choice affects their probability of default ex-post — formally, varying δ_i modifies the limits of integration — no additional terms appear in Equation (7) to account for this effect, since borrowers make default decisions optimally. Intuitively, because borrowers are indifferent between defaulting and repaying at the default threshold, a small change in the default threshold has no first-order effects on borrowers' welfare. The same logic applies to the derivation of Equation (6).

Formally, when borrowers can adjust housing freely, the following relation must hold at the optimum:

$$\underbrace{u_i'(c_{0i})}_{\substack{\text{Marginal Benefit}\\ \text{of Consumption}}} = \underbrace{\frac{\beta \int_{\delta_i-\chi_i}^{\overline{g}} dF_i(g)}{\Lambda_i'(\delta_i)}}_{\substack{\text{Return on Larger}\\ \text{Downpayment}}} = \underbrace{\frac{\beta \left[-\chi_i \int_{\underline{g}}^{\delta_i-\chi_i} dF_i(g) + \int_{\delta_i-\chi_i}^{\overline{g}} (g-\delta_i)\, dF_i(g) \right]}{1-\Lambda_i(\delta_i)}}_{\text{Return on Leveraged House Purchase}}. \tag{A1}$$

Intuitively, at an optimum in which the housing constraint is slack, a borrower is indifferent between (i) consuming a dollar, (ii) using a dollar to increasing today's downpayment, and (iii) leveraging the dollar to make a larger housing investment. The second and third terms in Equation (A1) have an intuitive interpretation. A dollar of extra downpayment at date 0 allows borrowers to reduce their future per-dollar of housing repayment by $\frac{\partial \delta_i}{\partial \Lambda_i} = \frac{1}{\Lambda_i'(\delta_i)} > 1$ dollars. The per-dollar net present value of such a reduction corresponds to $\beta \int_{\delta_i-\chi_i}^{\overline{g}} dF_i(g)$, since borrowers only repay in states of the world in which they do not default. Said differently, pessimistic borrowers who think they are quite likely to default in the future perceive a very small benefit of reducing the promised payment tomorrow, and will therefore make small downpayments and take on larger leverage. Alternatively, a dollar invested in housing at date 0 can be levered $\frac{1}{1-\Lambda_i(\delta_i)}$ times, while the per-dollar net present value of a housing investment corresponds to $\beta \left[-\chi_i \int_{\underline{g}}^{\delta_i-\chi_i} dF_i(g) + \int_{\delta_i-\chi_i}^{\overline{g}} (g-\delta_i)\, dF_i(g) \right]$.

In the variable house size scenario, the borrowers' problem simplifies to

$$\max_{c_{0i}} J(c_{0i}), \quad \text{where} \quad J(c_{0i}) = u_i(c_{0i}) + (n_{0i} - c_{0i})\,\beta\rho_i,$$

and

$$\rho_i = \max_{\delta_i} \frac{-\chi_i \int_{\underline{g}}^{\delta_i-\chi_i} dF_i(g) + \int_{\delta_i-\chi_i}^{\overline{g}} (g-\delta_i)\, dF_i(g)}{1-\Lambda_i(\delta_i)}. \tag{A2}$$

With a solution is given by

$$\frac{\Lambda_i'(\delta_i)}{1-\Lambda_i(\delta_i)} = \frac{\int_{\delta_i-\chi_i}^{\overline{g}} dF_i(g)}{-\chi_i \int_{\underline{g}}^{\delta_i-\chi_i} dF_i(g) + \int_{\delta_i-\chi_i}^{\overline{g}} (g-\delta_i)\, dF_i(g)}. \tag{A3}$$

Equation (A3) determines borrowers' optimal leverage choice independently of the choices of consumption and housing. Once δ_i is determined, borrowers choose initial consumption by equalizing the marginal benefit of investing in housing with the marginal utility of consumption, that is, setting $u_i'(c_{0i}) = \beta\rho_i$, where ρ_i, defined in (A2), is the maximized levered return on a housing investment. Finally, given δ_i and c_{0i}, borrowers choose h_{0i} to satisfy their budget constraint, defined in Equation (1). Note that when $\kappa_i = \chi_i = 0$, Equation (A3) can be mapped to Equation (12) in Simsek (2013). The upshot of introducing a consumption margin in our formulation is that it highlights that borrowers' optimism only increases leverage indirectly – through Equation (6), not Equation (7) – through the fact that more optimistic borrowers need to lever up a given amount of resources to buy larger properties.

We derive all regularity conditions for the leading case $\kappa_i = \chi_i = 0$. Similar conditions apply to the general case. Given c_{0i} and δ_i, borrowers' housing choice satisfies $h_{0i} = \frac{1}{p_0}\frac{n_{0i}-c_{0i}}{1-\Lambda_i(\delta_i)}$. Note that the problem solved by borrowers can be decoupled into two problems. First, borrowers maximize the levered return on a housing investment. Second, borrowers solve a consumption savings problem. Therefore, introducing a consumption margin per se does not affect borrowers' leverage choice. If $\kappa_i = \chi_i = 0$ and $\eta = 1$, then the borrowers' leverage choice has a unique optimum if borrowers' beliefs dominate lenders' beliefs in a hazard rate sense, as in Simsek (2013). That condition is not necessary when $\eta < 1$ or $\chi_i > 0$, which we assume throughout. In that case, the problem that maximizes the levered return on a housing investment always has a well defined interior solution, since $\Lambda_i'(\delta_i)$ has to be strictly positive at the optimum, implying that an optimum is reached before the maximum feasible LTV level. The consumption-savings problem is equally well defined, since $\frac{\partial J}{\partial c_{0i}} = u_i'(c_{0i}) - \beta\rho_i = 0$ and $\frac{\partial^2 J}{\partial c_{0i}^2} = u_i''(c_{0i}) < 0$.

In the fixed house size scenario, the problem solved by borrowers can be expressed as

$$\max_{\delta_i} J(\delta_i), \quad \text{where} \quad J(\delta_i) = u_i\left(n_{0i} - p_0 h_{0i}\left(1-\Lambda_i(\delta_i)\right)\right) + \beta p_0 h_{0i}\left[-\chi_i\int_{\underline{g}}^{\delta_i-\chi_i} dF_i(g) + \int_{\delta_i-\chi_i}^{\overline{g}}(g-\delta_i)\,dF_i(g)\right],$$

where $h_{0i} = \underline{h_i}$. Borrowers' first order condition corresponds to

$$\frac{\partial J}{\partial \delta_i} = p_0 h_{0i}\left(u_i'\left(n_{0i} - p_0 h_{0i}\left(1-\Lambda_i(\delta_i)\right)\right)\Lambda_i'(\delta_i) - \beta\left(1 - F_i(\delta_i - \chi_i)\right)\right) = 0. \tag{A4}$$

Borrowers' second order condition satisfies

$$\begin{aligned}\frac{\partial^2 J}{\partial \delta_i^2} &= p_0 h_{0i}\left(u_i''(c_{0i})\left(\Lambda_i'(\delta_i)\right)^2 p_0 h_{0i} + u_i'(c_{0i})\Lambda_i''(\delta_i) + \beta f_i(\delta_i-\chi_i)\right)\\ &= p_0 h_{0i}\left(u_i''(c_{0i})\left(\Lambda_i'(\delta_i)\right)^2 p_0 h_{0i} + \beta\int_{\delta_i-\chi_i}^{\overline{g}} dF_i(g)\left(\frac{\Lambda_i''(\delta_i)}{\Lambda_i'(\delta_i)} + \frac{f_i(\delta_i-\chi_i)}{1-F_i(\delta_i-\chi_i)}\right)\right)\\ &= p_0 h_{0i}\left(u_i''(c_{0i})\left(\Lambda_i'(\delta_i)\right)^2 p_0 h_{0i} + \beta\int_{\delta_i-\chi_i}^{\overline{g}} dF_i(g)\left(M - \frac{f_L(\delta_i-\chi_i)}{1-F_L(\delta_i-\chi_i)} + \frac{f_i(\delta_i-\chi_i)}{1-F_i(\delta_i-\chi_i)}\right)\right),\end{aligned}$$

where the second line is valid at an optimum and $M \equiv \frac{-\left((1-\eta)\lambda_L(\delta_i-\chi_i)+(\eta\chi_i+(1-\eta)\delta_i)\lambda_L'(\delta_i-\chi_i)\right)}{(1-(\eta\chi_i+(1-\eta)\delta_i)\lambda_L(\delta_i-\chi_i))} < 0$. Hence, sufficient conditions that guarantee that borrowers' first order condition corresponds to an optimum are a) a non-decreasing lenders' hazard rate, and b) that borrowers' beliefs dominate lenders' beliefs in a hazard rate sense. Once δ_i is determined, borrowers choose initial consumption to satisfy their budget constraint, defined in Equation (1), where $h_{0i} = \underline{h_i}$.

Proof of Generalized Proposition 1. (Mean and variance shifts with normally distributed beliefs and non-zero default costs)

In the variable house size scenario, given Equation (A3) and the sustained regularity conditions, it is sufficient to establish the behavior of

$$\Psi_{VHS}\left(\tilde{\delta}\right) \equiv -\chi_i\frac{F_i\left(\tilde{\delta}-\chi_i\right)}{1-F_i\left(\tilde{\delta}-\chi_i\right)} + \mathbb{E}_i\left[g|\, g \geq \tilde{\delta}-\chi_i\right]$$

to characterize the effect of beliefs on leverage. Any change in parameters associated with an upwards point-wise shift in $\Psi_{VHS}(\cdot)$ for a range of $\tilde{\delta}$ implies a higher equilibrium level of δ_i. Under the assumption that $g \sim N\left(\mu_i, \sigma_i^2\right)$, we can express $\mathbb{E}_i\left[g|g \geq \tilde{\delta}-\chi_i\right]$ and $F_i\left(\tilde{\delta}-\chi_i\right)$ as follows

$$\begin{aligned}\mathbb{E}_i\left[g|g \geq \tilde{\delta}-\chi_i\right] &= \mu_i + \sigma_i\lambda(\alpha_i)\\ F_i\left(\tilde{\delta}-\chi_i\right) &= \Phi(\alpha_i),\end{aligned}$$

where $\lambda(\alpha_i) = \frac{\phi(\alpha_i)}{1-\Phi(\alpha_i)}$ and $\alpha_i = \frac{\tilde{\delta}-\chi_i-\mu_i}{\sigma_i}$, which allows us to express $\Psi_{VHS}\left(\tilde{\delta}\right)$ as

$$\Psi_{VHS}\left(\tilde{\delta}\right) = -\chi_i\frac{\Phi(\alpha_i)}{1-\Phi(\alpha_i)} + \mu_i + \sigma_i\lambda(\alpha_i).$$

The relevant comparative statics in μ_i and σ_i are given by

$$\frac{\partial \Psi_{VHS}\left(\tilde{\delta}\right)}{\partial \mu_i} = \chi_i \frac{\phi\left(\alpha_i\right)\frac{1}{\sigma_i}}{\left(1-\Phi\left(\alpha_i\right)\right)^2} + 1 - \lambda'\left(\alpha_i\right) > 0$$
$$\frac{\partial \Psi_{VHS}\left(\tilde{\delta}\right)}{\partial \sigma_i} = \chi_i \frac{\phi\left(\alpha_i\right)\frac{1}{\sigma_i}\alpha_i}{\left(1-\Phi\left(\alpha_i\right)\right)^2} + \lambda\left(\alpha_i\right) - \lambda'\left(\alpha_i\right)\alpha_i = \underbrace{\chi_i \frac{\lambda\left(\alpha_i\right)}{1-\Phi\left(\alpha_i\right)}\frac{\alpha_i}{\sigma_i}}_{\gtreqless 0} + \underbrace{\lambda\left(\alpha_i\right)\left(1-\left(\lambda\left(\alpha_i\right)-\alpha_i\right)\alpha_i\right)}_{>0},$$

where we have used the properties of the Normal hazard rate from Fact 2. Note that the sign of the comparative statics on σ_i is strictly positive for low or moderate values of χ_i, since $\lim_{\chi_i \to 0} \chi_i \frac{\lambda(\alpha_i)}{1-\Phi(\alpha_i)} \frac{\alpha_i}{\sigma_i} = 0$, so the variance result remains valid in this case for sufficiently low default costs. Importantly, sufficiently low default costs is only a sufficient condition, not a necessary one. These results establish the conclusions for the variable house size scenario.

In the fixed house size scenario, given Equation (A4) and the sustained regularity conditions, it is sufficient to establish the behavior of

$$\Psi_{FHS}\left(\tilde{\delta}\right) \equiv 1 - F_i\left(\tilde{\delta} - \chi_i\right) = 1 - \Phi\left(\alpha_i\right), \tag{A5}$$

where $\alpha_i = \frac{\tilde{\delta}-\chi_i-\mu_i}{\sigma_i}$, to characterize the effect of beliefs on leverage. Any change in parameters associated with an upwards point-wise shift in $\Psi_{FHS}(\cdot)$ for a range of $\tilde{\delta}$ implies a lower equilibrium level of δ_i.

The relevant comparative statics in μ_i and σ_i are given by

$$\frac{\partial \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \mu_i} = \frac{1}{\sigma_i}\phi\left(\alpha_i\right) > 0$$
$$\frac{\partial \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \sigma_i} = \phi\left(\alpha_i\right)\frac{\alpha_i}{\sigma_i} = -\frac{\phi'\left(\alpha_i\right)}{\sigma_i},$$

which is negative when the probability of default is less than 50%, that is, when $\alpha_i < 0$ or, equivalently, when $\tilde{\delta} < \mu_i + \chi_i$.

Proof of Generalized Proposition 2(a) (Interaction with cost of default)

Note that we exclusively focus on the direct effects of how changes in χ_i affect the sensitivity of leverage choices to beliefs. Abstractly, in an optimization problem in which agents choose x to solve $\max_x F(x;\theta,\alpha)$, where θ are parameters and where an interior optimum $x^\star$ is characterized by $F_x(x^\star;\theta,\alpha) = 0$ and $F_{xx} \leq 0$, we can characterize first-order comparative statics by $\frac{dx^\star}{d\theta} = \frac{F_{x\theta}}{-F_{xx}}$ and second-order comparative statics by $\frac{d\left(\frac{dx^\star}{d\theta}\right)}{d\alpha} = \frac{d^2x^\star}{d\theta d\alpha} = \frac{F_{xxx}\frac{dx^\star}{d\alpha}\frac{dx^\star}{d\theta} + F_{xx\alpha}\frac{dx^\star}{d\theta} + F_{x\theta x}\frac{dx^\star}{d\alpha} + F_{x\theta\alpha}}{-F_{xx}}$. When we state that we characterize the direct effect of recourse, we refer to the term of the form $F_{x\theta\alpha}$, which does not rely directly on assumptions about the curvature of the model, as F_{xxx} and $F_{xx\alpha}$ do.

In the fixed house size scenario, where $\Psi_{FHS}(\cdot)$ is defined in Equation (A5),

$$\frac{\partial^2 \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \mu_i \partial \chi_i} = -\frac{1}{\sigma_i^2}\phi'\left(\alpha_i\right),$$

which is negative when the probability of default is less than 50%. The marginal cost of borrowing goes up by less when the cost of default is higher.

$$\frac{\partial^2 \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \sigma_i \partial \chi_i} = \frac{1}{\sigma_i^2}\phi''\left(\alpha_i\right),$$

which is positive as long as $\left|\tilde{\delta} - \mu_i - \chi_i\right| > \sigma_i$. Hence, it is needed that $\tilde{\delta} < \mu_i + \chi_i - \sigma_i$, which is valid whenever the probability of default is less than 16%.

In the variable house size scenario, to be able to derive unambiguous predictions, we focus for simplicity on the case in which $\chi_i \to 0$. In that case,

$$\frac{\partial^2 \Psi_{VHS}\left(\tilde{\delta}\right)}{\partial \mu_i \partial \chi_i} = \frac{1}{\sigma_i}\lambda''\left(\alpha_i\right) + \frac{\phi\left(\alpha_i\right)\frac{1}{\sigma_i}}{\left(1-\Phi\left(\alpha_i\right)\right)^2} > 0$$
$$\frac{\partial^2 \Psi_{VHS}\left(\tilde{\delta}\right)}{\partial \sigma_i \partial \chi_i} = -\frac{1}{\sigma_i}\lambda'\left(\alpha_i\right) + \frac{\alpha_i}{\sigma_i}\lambda''\left(\alpha_i\right) + \frac{1}{\sigma_i}\lambda'\left(\alpha_i\right) + \frac{\phi\left(\alpha_i\right)\frac{\alpha_i}{\sigma_i}}{\left(1-\Phi\left(\alpha_i\right)\right)^2} = \frac{\alpha_i}{\sigma_i}\lambda''\left(\alpha_i\right) + \frac{\phi\left(\alpha_i\right)\frac{\alpha_i}{\sigma_i}}{\left(1-\Phi\left(\alpha_i\right)\right)^2},$$

which is negative when the probability of default is less than 50%, that is, when $\alpha_i < 0$ or, equivalently, when $\tilde{\delta} < \mu_i + \chi_i$.

Proof of Generalized Proposition 2(b) (Interaction with average beliefs)

In the fixed house size scenario, where $\Psi_{FHS}(\cdot)$ is defined in Equation (A5),

$$\frac{\partial^2 \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \mu_i^2} = -\frac{1}{\sigma_i^2}\phi'(\alpha_i),$$

which is negative when the probability of default is less than 50%, that is, when $\tilde{\delta} < \mu_i$. We also find that

$$\begin{aligned}\frac{\partial^2 \Psi_{FHS}\left(\tilde{\delta}\right)}{\partial \sigma_i \partial \mu_i} &= \frac{1}{\sigma_i}\left[-\frac{1}{\sigma_i}\phi'(\alpha_i)\alpha_i - \phi(\alpha_i)\frac{1}{\sigma_i}\right] \\ &= \frac{\phi(\alpha_i)}{\sigma_i^2}\left[\alpha_i^2 - 1\right] \\ &= \frac{\phi(\alpha_i)}{\sigma_i^2}\left[\left(\frac{\tilde{\delta} - \chi_i - \mu_i}{\sigma_i}\right)^2 - 1\right],\end{aligned}$$

which is positive as long as $\left|\tilde{\delta} - \chi_i - \mu_i\right| > \sigma_i$. Hence, it is needed that $\tilde{\delta} < \mu_i + \chi_i - \sigma_i$, which is valid whenever the probability of default is less than 16%.

In the variable house size scenario, to be able to derive unambiguous predictions, we focus for simplicity in the case in which $\chi_i \to 0$. In that case,

$$\begin{aligned}\frac{\partial^2 \mathbb{E}_i\left[g | g \geq \tilde{\delta}\right]}{\partial \mu_i^2} &= \lambda''(\alpha_i)\frac{1}{\sigma_i} > 0, \ \forall \tilde{\delta} \\ \frac{\partial^2 \mathbb{E}_i\left[g | g \geq \tilde{\delta}\right]}{\partial \sigma_i \partial \mu_i} &= -\lambda'(\alpha_i)\frac{1}{\sigma_i} + \frac{1}{\sigma_i}\lambda''(\alpha_i)\alpha_i + \frac{1}{\sigma_i}\lambda'(\alpha_i) = \frac{\alpha_i}{\sigma_i}\lambda''(\alpha_i),\end{aligned}$$

which is negative when the probability of default is less than 50%, that is, when $\tilde{\delta} < \mu_i$.

A.3 Auxiliary results

Facts 1 and 2 follow from Greene (2003). Fact 3 follows from Krishna (2010). When needed, we respectively denote the pdf and cdf of the standard normal distribution by $\phi(\cdot)$ and $\Phi(\cdot)$.

Fact 1. (Truncated expectation of a normal distribution) *If $X \sim N\left(\mu, \sigma^2\right)$, then*

$$\begin{aligned}\mathbb{E}\left[X | X > a\right] &= \mu + \sigma\lambda(\alpha), \quad \textit{where} \quad \lambda(\alpha) = \frac{\phi(\alpha)}{1 - \Phi(\alpha)} \quad \textit{and} \quad \alpha = \frac{a - \mu}{\sigma} \\ \mathbb{E}\left[X | X < b\right] &= \mu - \sigma\frac{\phi(\beta)}{\Phi(\beta)}, \quad \textit{where} \quad \beta = \frac{b - \mu}{\sigma}.\end{aligned}$$

More generally, $\mathbb{E}\left[X | a < X < b\right] = \mu + \sigma\frac{\phi(\alpha) - \phi(\beta)}{\Phi(\beta) - \Phi(a)}$*, where α and β are defined above.*

Fact 2. (Properties of normal hazard function) *The function $\lambda(\cdot)$, which corresponds to the hazard rate of the normal distribution, is also known as the Inverse Mills Ratio. It satisfies the following properties:*

1. $\lambda(0) = \sqrt{\frac{2}{\pi}}$, $\lambda(\alpha) \geq 0$, $\lambda(\alpha) > \alpha$, $\lambda'(\alpha) > 0$, and $\lambda''(\alpha) > 0$.
2. $\lim_{\alpha \to -\infty} \lambda(\alpha) = \lim_{\alpha \to -\infty} \lambda'(\alpha) = 0$, and $\lim_{\alpha \to \infty} \lambda'(\alpha) = 1$.
3. $\lambda(\alpha) < \frac{1}{\alpha} + \alpha$, $\lambda'(\alpha) = \frac{\phi'(\alpha_i)}{1 - \Phi(\alpha_i)} + (\lambda(\alpha_i))^2 = \lambda(\alpha)(\lambda(\alpha) - \alpha) > 0$, $\lambda'(\alpha) < 1$, and $\lambda''(\alpha) \geq 0$.

Fact 3. (Hazard rate dominance implies first-order stochastic dominance) *The hazard rate of a distribution with cdf $F(\cdot)$ is defined by $\lambda(x) = \frac{f(x)}{1 - F(x)} = -\frac{d\ln(1 - F(x))}{dx}$, so $F(x) = 1 - e^{-\int_0^x \lambda(t)dt}$. If $\lambda_i(g) > \lambda_j(g)$, $\forall g$, then it trivially follows that $F_i(\cdot) > F_j(\cdot)$.*

At times, we use the fact that $\phi'(\alpha) = -\alpha\phi(\alpha)$. The following results are also relevant for our derivations:

$$\frac{\partial\left(\frac{\Phi(\alpha)}{1 - \Phi(\alpha)}\right)}{\partial \mu} = \frac{-\phi(\alpha)\frac{1}{\sigma}}{(1 - \Phi(\alpha))^2} \quad \text{and} \quad \frac{\partial\left(\frac{\Phi(\alpha)}{1 - \Phi(\alpha)}\right)}{\partial \sigma_i} = \frac{-\phi(\alpha)\frac{\alpha}{\sigma}}{(1 - \Phi(\alpha))^2},$$

as well as

$$\frac{\partial \alpha}{\partial \mu} = \frac{\partial \alpha}{\partial \chi} = -\frac{1}{\sigma} \quad \text{and} \quad \frac{\partial \alpha}{\partial \sigma} = -\frac{\alpha}{\sigma}.$$

A.4 AR(1) Mathematics

Let us assume that investors perceive the following law of motion for house prices:

$$\Delta p_{t+1} = \mu + \rho \Delta p_t + \varepsilon_{t+1}, \tag{A6}$$

where $\epsilon_{t+1} \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$ and $\Delta p_{t+1} \equiv p_{t+1} - p_t$. Equivalently, this process can be expressed as a non-stationary AR(2) process: $p_{t+1} = \mu + (1+\rho) p_t - \rho p_{t-1} + \varepsilon_{t+1}$, where we define an agent's information set by $\mathcal{I}_t = \{p_t, p_{t-1}\}$.

Perception of Future Prices: Analytical Results

First, we can analytically characterize the distribution of individual perceptions over house prices k-period forward, given by p_{t+k}. We analytically characterize up to to $k = 5$, but illustrate our results for higher k in Figure A2. Note that, from the perspective of date t, any statistical moment of the perceived distribution of p_{t+k} translates one-for-one into moments of the distribution of $p_{t+k} - p_t$. Note also that $p_{t+k} - p_t = \sum_{m=1}^{k} \Delta p_{t+m}$, so we can refer in the text to the distribution of cumulative price changes.

One and two periods forward, we can write

$$\begin{aligned} p_{t+1}|\mathcal{I}_t &\sim N\left(\mu + (1+\rho) p_t - \rho p_{t-1}, \sigma_\varepsilon^2\right). \\ p_{t+2}|\mathcal{I}_t &\sim \mathcal{N}\left((1 + (1+\rho))\mu + \left((1+\rho)^2 - \rho\right) p_t - (1+\rho)\rho p_{t-1}, \left(1 + (1+\rho)^2\right)\sigma_\epsilon^2\right). \end{aligned}$$

Three periods forward, we can show that $p_{t+3}|\mathcal{I}_t$ is normally distributed with the following moments:

$$\begin{aligned} \mathbb{E}\left[p_{t+3}|\mathcal{I}_t\right] &= \left(2 + (1+\rho)^2\right)\mu + \left((1+\rho)^3 - 2\rho(1+\rho)\right) p_t + \left(\rho^2 - (1+\rho)^2 \rho\right) p_{t-1} \\ \mathbb{V}ar\left[p_{t+3}|\mathcal{I}_t\right] &= \left(1 + (1+\rho)^2 + \left((1+\rho)^2 - \rho\right)^2\right)\sigma_\varepsilon^2. \end{aligned}$$

Four periods forward, we can show that $p_{t+4}|\mathcal{I}_t$ is normally distributed with the following moments:

$$\begin{aligned} \mathbb{E}\left[p_{t+4}|\mathcal{I}_t\right] &= \mu\left[1 + (1+\rho)\left(2 + (1+\rho)^2\right) - \rho(2+\rho)\right] + p_t\left[(1+\rho)^4 - 3\rho(1+\rho)^2 + \rho^2\right] + p_{t-1}\left[2\rho^2(1+\rho) - \rho(1+\rho)^3\right] \\ \mathbb{V}ar\left[p_{t+4}|\mathcal{I}_t\right] &= \left[1 + (1+\rho)^2 + \left((1+\rho)^2 - \rho\right)^2 + \left((1+\rho)^3 - 2\rho(1+\rho)\right)^2\right]\sigma_\varepsilon^2. \end{aligned}$$

Finally, we can show that $p_{t+5}|\mathcal{I}_t$ is normally distributed with the following moments:

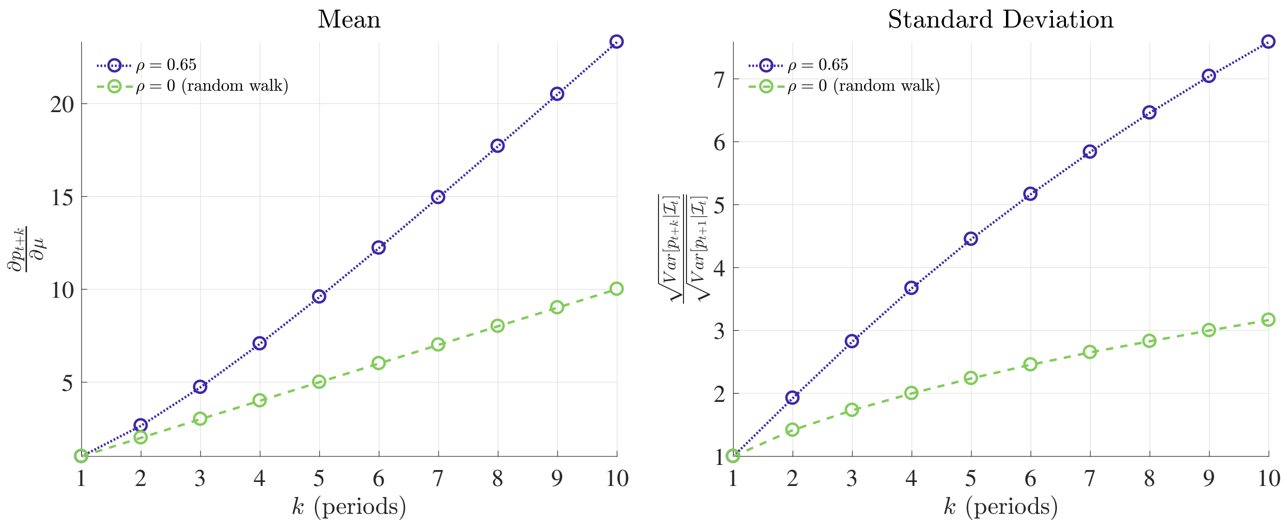
$$\begin{aligned} \mathbb{E}\left[p_{t+5}|\mathcal{I}_t\right] &= \mu\left[1 - 2\rho + (1+\rho) + (2-\rho)(1+\rho)^2 + (1+\rho)^4 - \rho(1+\rho)(2+\rho)\right] \\ &\quad + p_t\left[(1+\rho)\left[(1+\rho)^4 - 3\rho(1+\rho)^2 + \rho^2\right] - \rho\left((1+\rho)^3 - 2\rho(1+\rho)\right)\right] \\ &\quad + p_{t-1}\left[(1+\rho)\left[2\rho^2(1+\rho) - \rho(1+\rho)^3\right] - \rho\left(\rho^2 - (1+\rho)^2\rho\right)\right] \\ \mathbb{V}ar\left[p_{t+5}|\mathcal{I}_t\right] &= \left[1 + (1+\rho)^2 + \left((1+\rho)^2 - \rho\right)^2 + \left((1+\rho)^3 - 2\rho(1+\rho)\right)^2 + \left((1+\rho)^4 - 3\rho(1+\rho)^2 + \rho^2\right)^2\right]\sigma_\varepsilon^2. \end{aligned}$$

Perception of Future Prices: Simulation

Given our assumption about the behavior of beliefs in Equation (A6), when we obtain information about house price growth one-period forward, the average belief reported by an individual must be mapped to $\mu + \rho \Delta p_t$ and the standard deviation reported by an individual must be mapped to σ_ε.

Figure A2a shows the sensitivity of the expected house price k periods forward to a change in the parameter μ as of date t. This exercise corresponds to attributing the reported expected house price growth by an individual over the next twelve months to a change in μ: hence, the values in Figure A2a provide an upper bound for the impact that an increase of $\mathbb{E}\left[p_{t+1}|\mathcal{I}_t\right]$ may have on $\mathbb{E}\left[p_{t+k}|\mathcal{I}_t\right]$. Figure A2b shows the sensitivity of the standard deviation of the house price k periods forward to a change in the parameter σ_ε as of date t. Because there is a one-to-one mapping between $\mathbb{V}ar\left[p_{t+k}|\mathcal{I}_t\right]$ and σ_ε^2, these values are exact within the context of the process, and not only a bound.

Our results map a unit change in μ into roughly a twenty-fold increase for the expected price ten periods forward. Our results map a unit change in σ_ε into a roughly seven-fold increase for the standard deviation of house prices ten periods



(a) Sensitivity of $\mathbb{E}\left[p_{t+k}|\mathcal{I}_t\right]$ to a change in μ

(b) Sensitivity of $\sqrt{\mathbb{V}ar\left[p_{t+k}|\mathcal{I}_t\right]}$ to a change in σ_ε.

Note: Figure A2a illustrates the path of expected values for the house price k-periods forward induced by a change in μ. In other words, it shows how a unit change in μ propagates over individual expectations of house prices. Figure A2b illustrates the path of standard deviations induced by a unit change in σ_ε, that is, it shows how a change in σ_ε affects future perceived house price volatility. Both figures correspond to the stochastic process $\Delta p_{t+1} = \mu + \rho \Delta p_t + \varepsilon_{t+1}$. The blue dotted line is drawn for $\rho = 0.65$, while the green dashed line is drawn for a random walk ($\rho = 0$), for comparison.

Figure A2: Expectation and Standard Deviation of p_{t+k}

forward.

A.5 Further Extensions

Two assumptions crucially allow us to derive tractable analytical results: the risk neutrality of borrowers, common in existing work on leverage cycles, and our formulation for housing preferences. Here, we show how our framework can be extended to incorporate curvature in borrowers' date-1 utility and smooth preferences for housing. Our theoretical results can be interpreted as a first-order approximation to the more general case.

It is conceptually easy to make borrowers in our model risk averse. We assume that, at date 1, borrowers derive a continuation utility of wealth $v_i(\cdot)$. We also assume that their date-0 flow utility corresponds to a sufficiently regular $u_i(c_{0i}, h_{0i})$, so

$$\max_{\delta_i, h_{0i}} u_i\left(n_{0i} - p_0 h_{0i}\left(1 - \Lambda_i\left(\delta_i\right)\right), h_{0i}\right) + \beta\left[\int_{\underline{g}}^{\delta_i - \chi_i} v_i\left(w_{1i}^{\mathcal{D}}\right) dF_i(g) + \int_{\delta_i - \chi_i}^{\overline{g}} v_i\left(w_{1i}^{\mathcal{N}}\right) dF_i(g)\right],$$

where $w_{1i}^{\mathcal{N}} = n_{1i} + p_1 h_{0i} - b_{0i} = n_{1i} + p_0 h_{0i}(g - \delta_i)$ and $w_{1i}^{\mathcal{D}} = n_{1i} - \kappa_i h_{0i} = n_{1i} - \chi_i p_0 h_{0i}$.

Borrowers' optimality conditions in this case correspond to

$$\frac{\partial u_i}{\partial c_{0i}}\left(1 - \Lambda_i\left(\delta_i\right)\right) = \frac{\frac{\partial u_i}{\partial h_{0i}}}{p_0} \underbrace{-\chi_i \beta \int_{\underline{g}}^{\delta_i - \chi_i} v_i'\left(w_{1i}^{\mathcal{D}}\right) dF_i(g) + \beta \int_{\delta_i - \chi_i}^{\overline{g}} \left(g - \delta_i\right) v_i'\left(w_{1i}^{\mathcal{N}}\right) dF_i(g)}_{\text{Expected Return}},$$

$$\frac{\partial u_i}{\partial c_{0i}} \Lambda_i'\left(\delta_i\right) = \underbrace{\beta \int_{\delta_i - \chi_i}^{\overline{g}} v_i'\left(w_{1i}^{\mathcal{N}}\right) dF_i(g)}_{\text{Mg. Cost of Borrowing}},$$

which are the counterparts of Equations (6) and (7) in the main text. We highlight the terms through which the expected return and downpayment protection channels materialize. Equation (A1) corresponds in this case to

$$\frac{\partial u_i}{\partial c_{0i}} = \beta \frac{\int_{\delta_i-\chi_i}^{\overline{g}} v_i'\left(w_{1i}^{\mathcal{N}}\right) dF_i\left(g\right)}{\Lambda_i'\left(\delta_i\right)} = \frac{\frac{\partial u_i}{\partial h_{0i}}}{p_0\left(1-\Lambda_i\left(\delta_i\right)\right)} + \beta \frac{\left(-\chi_i \int_{\underline{g}}^{\delta_i-\chi_i} v_i'\left(w_{1i}^{\mathcal{D}}\right) dF_i\left(g\right) + \int_{\delta_i-\chi_i}^{\overline{g}} \left(g-\delta_i\right) v_i'\left(w_{1i}^{\mathcal{N}}\right) dF_i\left(g\right)\right)}{1-\Lambda_i\left(\delta_i\right)}.$$

In this more general scenario, changes in borrowers' beliefs only affect borrowers' decisions insofar as they affect the expected return and downpayment protection terms, which now include marginal utilities. We now sequentially study the impact of several alternative modeling assumptions.

Final date risk aversion

We now assume that housing does not enter borrowers' utility directly. In that case, we can combine borrowers' optimality conditions for housing and leverage to find a condition that generalizes (8). In particular, assuming $\kappa_i = 0$ for simplicity,

$$\frac{\Lambda_i'\left(\delta_i\right)}{1-\Lambda_i\left(\delta_i\right)} = \frac{1}{\frac{\mathbb{E}_i\left[\left(g-\delta_i\right) v_i'\left(w_{1i}^{\mathcal{N}}\right) \middle| g \geq \delta_i\right]}{\mathbb{E}_i\left[v_i'\left(w_{1i}^{\mathcal{N}}\right) \middle| g \geq \delta_i\right]}},$$

where

$$\frac{\mathbb{E}_i\left[\left(g-\delta_i\right) v_i'\left(w_{1i}^{\mathcal{N}}\right) \middle| g \geq \delta_i\right]}{\mathbb{E}_i\left[v_i'\left(w_{1i}^{\mathcal{N}}\right) \middle| g \geq \delta_i\right]} = \mathbb{E}_i\left[g-\delta_i \middle| g \geq \delta_i\right] + \frac{\mathbb{C}ov_i\left[g-\delta_i, v_i'\left(w_{1i}^{\mathcal{N}}\right) \middle| g \geq \delta_i\right]}{\mathbb{E}_i\left[v_i'\left(w_{1i}^{\mathcal{N}}\right) \middle| g \geq \delta_i\right]}.$$

This more general expression includes a new term that separately affects borrowers' leverage choices. Unfortunately, it is not possible to analytically characterize how a change on borrowers' beliefs affects the new term.

In the fixed house size scenario, a simple result can be found when borrowers are prevented from defaulting. In that case, the first-order condition for borrowing corresponds to

$$\frac{\partial u_i}{\partial c_{0i}} \Lambda_i'\left(\delta_i\right) = \beta \int_{\underline{g}}^{\overline{g}} v_i'\left(n_{1i} + p_0 h_{0i}\left(g-\delta_i\right)\right) dF_i\left(g\right).$$

In this case, it is easy to show that more optimistic borrowers, in a first-order stochastic dominance sense, decide to borrow more. Intuitively, optimism is associated with a desire to transfer resources from the future and lower precautionary savings, both of which are associated with higher borrowing.

Smooth preferences for housing

In this case, we preserve borrowers' risk neutrality and focus on the case in which preferences for housing are separable, so $u_i\left(c_{0i}, h_{0i}\right) = v_i\left(c_0\right) + \frac{\alpha}{2}\left(h_{0i} - \bar{h}\right)^2$ and $\frac{\frac{\partial u_i}{\partial h_{0i}}}{p_0} = \alpha \frac{h_{0i}-\bar{h}}{p_0}$. Setting again $\kappa_i = 0$ for simplicity, we can express borrowers' optimality condition for housing as

$$h_{0i} = \bar{h} + \frac{p_0}{\alpha}\left(\frac{\partial u_i}{\partial c_{0i}}\left(1-\Lambda_i\left(\delta_i\right)\right) - \beta \int_{\delta_i}^{\overline{g}} \left(g-\delta_i\right) dF_i\left(g\right)\right).$$

In the limit in which $\alpha \to \infty$, it must be that $h_{0i} \to \bar{h}$, so changes in borrowers' beliefs do not affect housing choices. In that case, borrowers' make leverage choices according to

$$\frac{\partial u_i}{\partial c_{0i}} \Lambda_i'\left(\delta_i\right) = \beta \int_{\delta_i}^{\overline{g}} dF_i\left(g\right),$$

as in the fixed house size scenario.

Alternative savings opportunities

Finally, although we have introduced consumption at date 0 as homebuyers' alternative use of funds, they could be indifferent at the margin between their investment return on housing and alternative investment opportunities. If we assume that households have access to a savings opportunity with a constant gross returns R_s, their date 0 budget constraint becomes

$$n_{0i} = p_0 h_{0i}\left(1-\Lambda_i\left(\delta_i\right)\right) + s, \tag{A7}$$

where s denotes savings. Therefore, at the optimum, the counterpart of Equation (A1) corresponds to

$$R_s = \frac{\beta \int_{\delta_i - \chi_i}^{\overline{g}} dF_i(g)}{\Lambda_i'(\delta_i)} = \frac{\beta \left[-\chi_i \int_{\underline{g}}^{\delta_i - \chi_i} dF_i(g) + \int_{\delta_i - \chi_i}^{\overline{g}} (g - \delta_i) \, dF_i(g) \right]}{1 - \Lambda_i(\delta_i)}. \tag{A8}$$

Homebuyers' housing and borrowing decisions must at the margin yield a return R_s, so every argument in the paper remains valid. We adopt the formulation with consumption in the paper because for this alternative formulation to be well-behaved in the variable house size case, we would need to impose some curvature in the return R_s. By slightly changing the default formulation, one can allow for borrowers to save at date 0 to use some of those funds to avoid default at date 1. In that case, Equations (A7) and (A8) still determine the equilibrium after accounting for the different default behavior.

Role of Lenders' Beliefs

Although our empirical findings build on cross-sectional variation on homebuyers' beliefs, it may be valuable to separately characterize the effect of changes in lenders' beliefs on the equilibrium leverage choices of borrowers. We show that an increase in lenders' optimism can increase equilibrium leverage in both the fixed house size and the variable house size scenarios. We assume that lenders have normally distributed beliefs with mean μ_L.

Proposition. (Parametric predictions of lenders' beliefs.) *In the variable house size scenario and the fixed house size scenario, higher optimism by lenders, in the form of a higher average belief μ_L, holding all else constant, including borrowers' beliefs, can be associated with higher leverage.*

We now formally show that a change in the average belief of lenders μ_L is associated with point-wise shifts in $\Lambda_i(\cdot)$ and $\Lambda_i'(\cdot)$. Intuitively, when lenders become more optimistic, borrowers have access to cheaper funding. In the variable house size scenario, we show that access to cheaper credit increases the maximum levered return that a borrower can achieve, generating a substitution effect towards higher leverage in equilibrium. In the fixed house size scenario, in addition to the substitution effect, there is an income effect that works in the opposite direction: lower interest rates make borrowers feel richer, which generates a force towards higher consumption and lower leverage. Therefore, our results show that shifts in the beliefs of borrowers and lenders, when considered in isolation, can have opposite predictions for equilibrium leverage.

To provide comparative statics in the case of lenders' beliefs, we must characterize the behavior of $\frac{\partial \Lambda_i(\delta_i)}{\partial \mu_L}$ and $\frac{\partial \Lambda_i'(\delta_i)}{\partial \mu_L}$.[37] Formally, we find that both the LTV schedule offered by lenders and its derivative with respect to δ_i shift pointwise with changes in μ_L:

$$\begin{aligned}
\frac{\partial \Lambda_i(\delta_i)}{\partial \mu_L} &= \frac{\eta \Phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right) - \frac{(\eta \mu_L - \delta_i)}{\sigma_L} \phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right) + \eta \phi'\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right)}{1 + r} > 0, \\
\frac{\partial \Lambda_i'(\delta_i)}{\partial \mu_L} &= (\eta \chi_i + (1 - \eta)\delta_i) \left(\lambda\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right) - \frac{\delta_i - \chi_i - \mu_L}{\sigma_L} \right) \frac{1}{\sigma_L} \frac{\phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right)}{1 + r} \\
&\quad + \left[1 - (\eta \chi_i + (1 - \eta)\delta_i) \lambda\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right) \right] \frac{1}{\sigma_L} \frac{\phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right)}{1 + r} > 0,
\end{aligned}$$

where we use the fact that the pdf of the Normal distribution satisfies $\phi'(x) = -x\phi(x)$ and that $1 > (\eta \chi_i + (1 - \eta)\delta_i) \lambda\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right)$ at any interior solution. Both results, when combined, imply that

$$\frac{d\left(\frac{\Lambda_i'(\delta_i)}{1 - \Lambda_i(\delta_i)}\right)}{d\mu_L} > 0.$$

In the variable house size scenario, Equation (A3) directly implies that upward point-wise shifts on $\frac{\Lambda_i'(\delta_i)}{1 - \Lambda_i(\delta_i)}$ are associated with a higher equilibrium leverage choice δ_i. Hence, a higher μ_L is associated with a higher equilibrium

[37] Note that we can express LTV schedules as

$$\begin{aligned}
\Lambda_i(\delta_i) &= \frac{\eta \Phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right) \left(\mu_L - \sigma_L \frac{\phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right)}{\Phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right)} \right) + \left(1 - \Phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right)\right) \delta_i}{1 + r} \\
&= \frac{\delta_i + (\eta \mu_L - \delta_i) \Phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right) - \eta \sigma_L \phi\left(\frac{\delta_i - \chi_i - \mu_L}{\sigma_L}\right)}{1 + r}
\end{aligned}$$

leverage choice δ_i. In the fixed house size scenario, Equation (A4) directly implies that upward point-wise shifts on $u_i'(n_{0i} - p_0 h_{0i}(1 - \Lambda_i(\delta_i)))\Lambda_i'(\delta_i)$ are associated with a higher equilibrium leverage choice δ_i. In this case,

$$\frac{\partial\left(u_i'(\cdot)\Lambda_i'(\delta_i)\right)}{\partial\mu_L} = u_i'(n_{0i} - p_0 h_{0i}(1 - \Lambda_i(\delta_i)))\frac{\partial\Lambda_i'(\delta_i)}{\partial\mu_L} + p_0 h_{0i} u_i''(n_{0i} - p_0 h_{0i}(1 - \Lambda_i(\delta_i)))\frac{\partial\Lambda_i(\delta_i)}{\partial\mu_L}\Lambda_i'(\delta_i),$$

whose sign is ambiguous. An increase in μ_L generates both a substitution and an income effect. More optimistic lenders offer more attractive LTV schedules at the margin, so borrowers substitute towards higher leverage. More optimistic lenders offer more attractive schedule, which makes borrowers effectively richer at date 0, reducing their need to borrow, which reduces their equilibrium leverage.

B Numerical Simulation

Although we have derived our analytical results in the context of two polar scenarios, we now simulate an extension of the model in which homebuyers have conventional CES preferences over consumption and housing (see, for example, Piazzesi, Schneider and Tuzel, 2007). In that case, the return sensitivity of housing investments is not determined by a potentially-binding housing size constraint, but instead arises endogenously from household preferences.

Our goal with this simulation is twofold. First, we show that we can recover the predictions of both polar scenarios for the relationship between beliefs and leverage choices when employing more conventional preferences for housing. The fact that we can generate opposite predictions reinforces our motivation for the empirical analysis, which seeks to understand which effects dominate in the data. At the same time, we can provide a sense of the parameters that are consistent with the predictions of the fixed house size scenario that we find in the data. Second, the extended model allows us to formally reconcile the findings of this paper with the findings of Bailey et al. (2017), who document that more optimistic individuals purchase larger homes. Specifically, we show that in a world with partial collateral return sensitivity, it is possible to have more optimistic individuals purchasing larger houses with lower LTV ratios.

Environment. We consider the same environment as in Section 3, except that we drop the housing constraint, given by Equation (2), and instead introduce standard CES preferences between consumption and housing at date 0. Formally, homebuyers solve the following problem:

$$\max_{c_{0i},\delta_i,h_{0i}} J\left(c_{0i},\delta_i,h_{0i}\right)$$

where

$$J\left(c_{0i},\delta_i,h_{0i}\right) = U\left(c_{0i},h_{0i}\right) + \beta p_0 h_{0i}\int_{\delta_i}^{\overline{g}}\left(g-\delta_i\right)dF_i\left(g\right) - \lambda_{0i}\left(c_{0i}+p_0 h_{0i}\left(1-\Lambda\left(\delta_i\right)\right)-n_{0i}\right),$$

and where

$$U\left(c_{0i},h_{0i}\right) = \frac{\left(u_i\left(c_{0i},h_{0i}\right)\right)^{1-\gamma}}{1-\gamma} \quad \text{and} \quad u_i\left(c_{0i},h_{0i}\right) = \left[\alpha c_{0i}^{\frac{\varepsilon-1}{\varepsilon}} + \left(1-\alpha\right)h_{0i}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where σ corresponds to the elasticity of substitution between consumption and housing.[38] The lenders' LTV schedule is given by

$$\Lambda_i\left(\delta_i\right) = \frac{\eta\int_{\underline{g}}^{\delta_i} g dF_L\left(g\right) + \delta_i\int_{\delta_i}^{\overline{g}} dF_L\left(g\right)}{1+r},$$

with $F_L\left(\cdot\right)$ normally distributed. Using the fact that $c_{0i} = n_{0i} - p_0 h_{0i}\left(1-\Lambda\left(\delta_i\right)\right)$, the solution of the model can be found by solving the following system of two equations and two unknowns, δ_i and h_{0i}:

$$\begin{aligned}
\frac{\partial u_i\left(c_{0i},h_{0i}\right)}{\partial h_{0i}} &= p_0\beta\left(1-F_i\left(\delta_i\right)\right)\left(\frac{1-\Lambda\left(\delta_i\right)}{\Lambda'\left(\delta_i\right)} - \frac{\int_{\delta_i}^{\overline{g}}\left(g-\delta_i\right)dF_i\left(g\right)}{\int_{\delta_i}^{\overline{g}} dF_i\left(g\right)}\right)\\
\frac{\partial u_i\left(c_{0i},h_{0i}\right)}{\partial c_{0i}} &= \beta\frac{1-F_i\left(\delta_i\right)}{\Lambda'\left(\delta_i\right)}
\end{aligned}$$

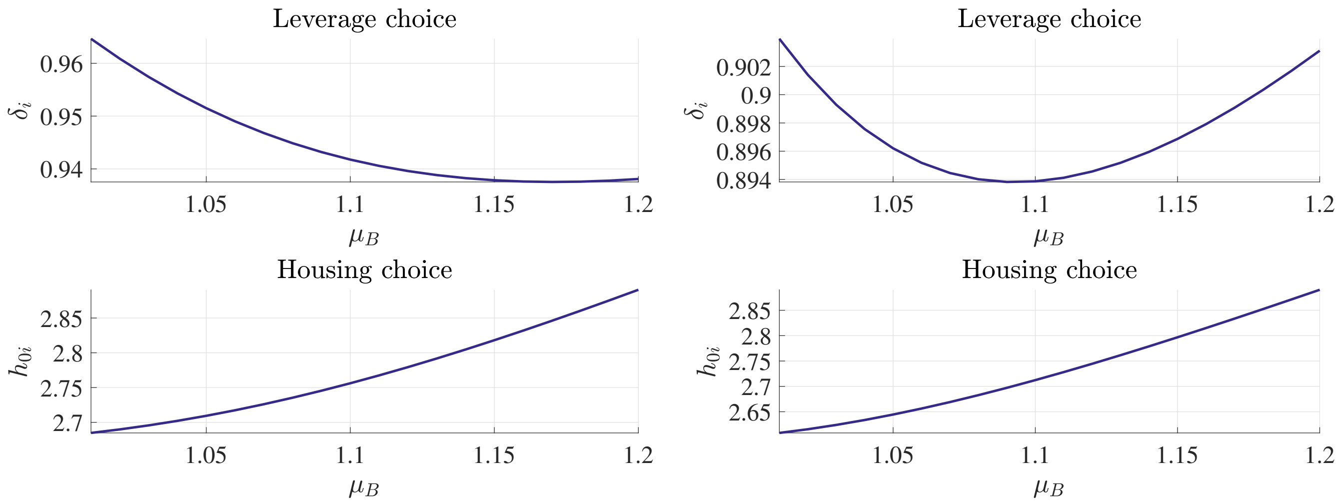
Parameters. Although our model is highly stylized, we stay close to existing quantitative models with housing when selecting parameters for our simulation. We interpret the single period in the model as five years. Our choices for preference parameters build on Piazzesi, Schneider and Tuzel (2007). We select a static elasticity of substitution between consumption and housing of $\varepsilon = 1.25$, an EIS/Risk Aversion parameter of $\gamma = 1$ (corresponding to log-utility), and $\alpha = 0.5$, which would imply equal shares of consumption and housing services in a static setup. We use a recovery rate for lenders of $\eta = 0.75$ after default, consistent with Campbell, Giglio and Pathak (2011). We normalize $p_0 = n_{0i} = 1$, so that lenders' date-0 resources are equal to unity. We assume that lenders' average belief is $\mu_L = 1.1$ and that both homebuyers and lenders have the same standard deviation of beliefs, $\sigma_B = \sigma_L = 0.25$. We assume that $r = 1.15 \approx 1.03^5$ and consider two scenarios: one with patient homebuyers/borrowers ($\beta = 0.95 \approx 0.99^5$), and one with impatient homebuyers/borrowers ($\beta = 0.7 \approx 0.94^5$). We set (effectively non-binding) upper and lower thresholds for all distributions $\underline{g} = 0.1$ and $\overline{g} = 2$.

Results. We illustrate the results of our simulation in Figure A3. The left panels show the relationship between impatient borrowers' beliefs and choices for LTV ratio and house size. Over almost the entire range of beliefs considered, $\mu_B \in [1, 1.2]$, we find a negative relationship between borrowers' beliefs and leverage choices, consistent with the empirical results in Section 3. Only when borrowers are very optimistic, we find that LTV ratios are increasing in borrowers' beliefs. In

[38] This preference specification implies that $U_c = \alpha u_i^{\frac{1}{\varepsilon}-\gamma} c_{0i}^{-\frac{1}{\varepsilon}}$ and $U_h = \left(1-\alpha\right)u_i^{\frac{1}{\varepsilon}-\gamma} h_{0i}^{-\frac{1}{\varepsilon}}$.

Figure A3: Simulation

(a) Relatively Impatient Homebuyers (b) Relatively Patient Homebuyers



Note: Both panels show homebuyers' optimal leverage and housing decisions for different values of homebuyers' average belief. Both panels share the following parameters: $\varepsilon = 1.25$, $\gamma = 1$, $\alpha = 0.5$, $\eta = 0.75$, $r = 0.12$, $n_{0i} = 1$, $p_0 = 1$, $\mu_L = 1.1$, $\sigma_L = 0.25$, and $\sigma_B = 0.22$. The left panel corresponds to impatient homebuyers, with $\beta = 0.7$. The right panel corresponds to patient homebuyers, with $\beta = 0.95$.

addition, over the entire range of beliefs considered, more optimistic borrowers purchase bigger houses. This suggests that for reasonable parameters, the predictions from our model are consistent with both our empirical results and the findings in Bailey et al. (2017). In terms of magnitudes, a 5 percentage points increase in borrowers' beliefs about house price changes over the following 5 years (starting from $\mu_B = 1.05$) reduces patient homebuyers' LTV choice by roughly 100 basis points and impatient homebuyers' LTV choice by roughly 2 basis points. These values are within the ballpark of our empirical estimates, although more sophisticated structural modeling could provide more precise predictions.

Which parameters are important for the model to generate both observed relationships? To explore this question, the right panels show simulations of these relationships for impatient homebuyers. For relatively pessimistic borrowers we continue to observe house size to increase and leverage to decrease with optimism. However, the relationship between changes in optimism and changes in leverage turns positive at lower baseline levels of optimism. This suggests that homebuyer impatience is helpful in generating the empirical findings within our model. The intuition is that, all else equal, more impatient homebuyers choose to borrow more, and are therefore closer to the default threshold. More generally, assumptions on primitives that generate high perceived probabilities of default are likely to generate the negative link between optimism and leverage that we identify in this paper.

C Non-Parametric Predictions

C.1 Theoretical results

While the parametric assumption underlying Propositions 1 and 2 are natural, given that the distribution of g is unimodal and symmetric in the data, these results impose a priori unnecessary distributional restrictions. An alternative approach is to consider cross-sectional comparisons among borrowers that do not rely on parametric assumptions. This approach uses the ability to potentially observe shifts of the whole distribution of beliefs, which is a unique feature of our empirical setup.

In general, there are numerous ways in which one borrower may be "more optimistic" than another in a non-parametric sense. For example, two borrowers could disagree primarily about the probability of very large declines or very large increases in house prices. In this section, we identify the appropriate definition of stochastic dominance that allows us to provide unambiguous directional predictions for borrowers' leverage choices across both the fixed house size and variable house size scenarios. Because of the inherent difficulties with establishing general comparisons between infinite-dimensional distributions, our non-parametric results only provide a partial order when comparing borrowers in the data — there are many pairwise comparisons of borrowers' distributions that cannot be ranked according to the appropriate dominance notion.

We use three different stochastic orders to define optimism. Given two distributions with cumulative distribution functions $F_j(\cdot)$ and $F_i(\cdot)$ with support $\left[\underline{g}, \overline{g}\right]$, we define truncated expectation stochastic dominance, first-order stochastic dominance and hazard rate dominance as follows:

1. **Truncated expectation stochastic dominance:** F_j stochastically dominates F_i (borrower j is more optimistic than i) in a truncated expectation sense if

$$\mathbb{E}_j\left[g|\, g \geq \delta\right] \geq \mathbb{E}_i\left[g|\, g \geq \delta\right], \quad \forall \delta \in \left[\underline{g}, \overline{g}\right].$$

2. **First-order stochastic dominance:** F_j stochastically dominates F_i (borrower j is more optimistic than i) in a first-order sense if

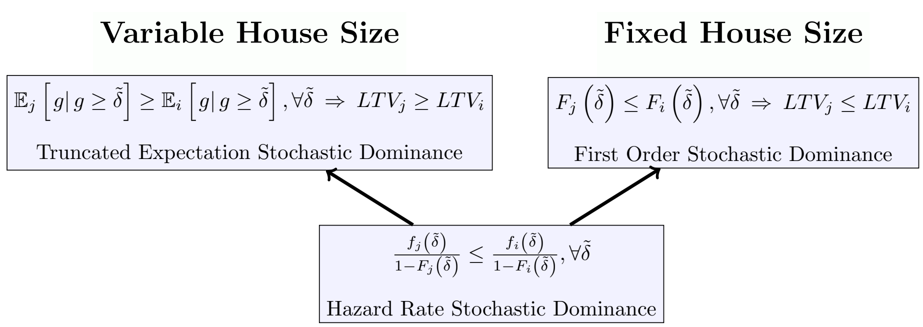
$$F_j(\delta) \leq F_i(\delta), \quad \forall \delta \in \left[\underline{g}, \overline{g}\right].$$

3. **Hazard rate stochastic dominance**: F_j stochastically dominates F_i (borrower j is more optimistic than i) in a hazard rate sense if

$$\frac{f_j(\delta)}{1-F_j(\delta)} \leq \frac{f_i(\delta)}{1-F_i(\delta)}, \quad \forall \delta \in \left[\underline{g}, \overline{g}\right].$$

All three definitions capture different notions of optimism. We show in Appendix A.3 that if $F_j(\cdot)$ dominates $F_i(\cdot)$ in a hazard rate sense, then $F_j(\cdot)$ also dominates $F_i(\cdot)$ in a first-order and a truncated expectation sense. The converse is not true: first-order stochastic dominance and truncated expectation dominance do not imply hazard rate dominance. Figure A4 illustrates the relation between the different orders. Hazard rate dominance is equivalent to saying that $\frac{1-F_j(g)}{1-F_i(g)}$ is increasing on g. It captures the idea that optimists are increasingly optimistic about higher house price growth realizations.

Figure A4: Relation Between Stochastic Orders



We have shown that in the variable house size scenario, when $\kappa_i = 0$, there are clear predictions for the leverage choices across two borrowers whose belief distributions could be ranked according to truncated-expectation stochastic dominance. We also showed that in the fixed house size scenario, clear predictions were obtained for belief distributions that could be ranked according to first-order stochastic dominance. However, neither of these dominance concepts implies the other, so it is unclear how to compare belief distributions if we are ex-ante agnostic about whether the fixed house size or the variable house size scenario applies. Therefore, we adopt hazard rate dominance as the appropriate definition of optimism that allows to test non-parametrically for the effect of beliefs on leverage regardless of the underlying scenario, as formalized in Proposition 3.

Proposition 3. (Non-parametric predictions.) *Compare two borrowers i and j with different distributions $F_i(\cdot)$ and $F_j(\cdot)$ about the growth rate of house prices changes.*
a) In the variable house size scenario, if $F_j(\cdot)$ dominates $F_i(\cdot)$ in a hazard rate sense (or in a truncated expectation sense when $\kappa_i = 0$), all else equal, borrower j chooses a higher LTV ratio and a larger house than borrower i.
b) In the fixed house size scenario, if $F_j(\cdot)$ dominates $F_i(\cdot)$ in a hazard rate sense (or in a first-order sense), all else equal, borrower j chooses a lower LTV ratio and lower consumption than borrower i.

Proposition 3 shows that borrowers' optimism, measured as hazard rate dominance, has opposite predictions in the polar scenarios we consider for any value of κ_i. More optimistic borrowers take on more leverage in the variable house size scenario, but they take on less leverage in the fixed house size scenario. These results generalize the insights from the case with normally distributed beliefs, but they are not exactly identical. We show here that, when beliefs are normally distributed, a distribution with a higher mean dominates in a hazard rate sense a distribution with a lower mean (holding its variance constant). Hence, when comparing means in the normal case, the results of Proposition 1 are implied by those of Proposition 3. However, we also show that, in the normal context, a distribution with a higher variance does not dominate in a hazard rate sense a distribution with a lower variance (holding its mean constant). This illustrates that hazard rate dominance is only a sufficient (not a necessary) condition to derive unambiguous predictions for the relationships between beliefs and leverage choice.

Proof of Proposition 3. (Non-parametric predictions)

In the variable house size scenario, it follows that an upward pointwise shift in $1 - F_i\left(\tilde{\delta}\right)$, $\forall\tilde{\delta}$, is associated with a lower equilibrium value of δ_i. If the beliefs of borrower j first-order stochastically dominate the beliefs of borrower i (borrower j is more optimistic), then $F_j\left(\tilde{\delta}\right) < F_i\left(\tilde{\delta}\right)$ and $1 - F_j\left(\tilde{\delta}\right) > 1 - F_i\left(\tilde{\delta}\right)$, which implies that borrower j takes on more leverage on equilibrium.

In the variable house size scenario, from Equation (8), it follows that an upward pointwise shift in $-\chi_i \frac{F_i\left(\tilde{\delta}-\chi_i\right)}{1-F_i\left(\tilde{\delta}-\chi_i\right)} + \mathbb{E}_i\left[g|\, g \geq \tilde{\delta} - \chi_i\right]$, $\forall\tilde{\delta}$ is associated with a higher equilibrium value of δ_i. To compare the leverage choices of two different borrowers i and j, it is thus sufficient to show that $F_j \succ_{HRD} F_i$ implies that

$$\mathbb{E}_j\left[g|\, g \geq \tilde{\delta}\right] - \mathbb{E}_i\left[g|\, g \geq \tilde{\delta}\right] > 0, \quad \forall\tilde{\delta},$$

to conclude that the more optimistic borrower j takes on more leverage, since hazard-rate dominance implies first order stochastic dominance, which trivially implies that $\frac{F_j\left(\tilde{\delta}-\chi_i\right)}{1-F_j\left(\tilde{\delta}-\chi_i\right)} < \frac{F_i\left(\tilde{\delta}-\chi_i\right)}{1-F_i\left(\tilde{\delta}-\chi_i\right)}$. To this purpose, we can define $h(\delta)$ as

$$h\left(\tilde{\delta}\right) = \mathbb{E}_j\left[g|\, g \geq \tilde{\delta}\right] - \mathbb{E}_i\left[g|\, g \geq \tilde{\delta}\right].$$

We can express the derivative of $h\left(\tilde{\delta}\right)$ as

$$\begin{aligned} h'\left(\tilde{\delta}\right) &= \frac{\partial \mathbb{E}_j\left[g|\, g \geq \tilde{\delta}\right]}{\partial \delta} - \frac{\partial \mathbb{E}_i\left[g|\, g \geq \tilde{\delta}\right]}{\partial \delta} \\ &= \frac{f_j\left(\tilde{\delta}\right)}{1 - F_j\left(\tilde{\delta}\right)}\left[\mathbb{E}_j\left[g|\, g \geq \tilde{\delta}\right] - \tilde{\delta}\right] - \frac{f_i\left(\tilde{\delta}\right)}{1 - F_i\left(\tilde{\delta}\right)}\left[\mathbb{E}_i\left[g|\, g \geq \tilde{\delta}\right] - \tilde{\delta}\right] \\ &= \underbrace{\left(\frac{f_j\left(\tilde{\delta}\right)}{1 - F_j\left(\tilde{\delta}\right)} - \frac{f\left(\tilde{\delta}\right)}{1 - F\left(\tilde{\delta}\right)}\right)}_{>0}\mathbb{E}_j\left[g|\, g \geq \tilde{\delta}\right] + \frac{f_i\left(\tilde{\delta}\right)}{1 - F_i\left(\tilde{\delta}\right)} h\left(\tilde{\delta}\right). \end{aligned} \tag{A9}$$

When $\tilde{\delta} \to 0$, it follows from hazard-rate dominance that $\mathbb{E}_j[g] - \mathbb{E}_i[g] > 0$. Note that all elements in (A9) are strictly positive under hazard rate dominance, which implies that the solution to the ordinary differential equation for $h\left(\tilde{\delta}\right)$ must be weakly positive everywhere, implying that $h\left(\tilde{\delta}\right)$ is positive, which shows our claim.

C.2 Non-Parametric Test

The empirical results presented in Sections 3.3 and 3.4 test the predictions from our model under the assumption that individuals' beliefs about future house price changes are normally distributed. We argued that this is a realistic approximation, given the shape of the distribution of actual house price changes. In Section C.1, we also derived non-parametric predictions on the relative mortgage leverage choices of individuals with arbitrary belief distributions. In particular, we showed that an individual whose belief distribution dominated that of an otherwise identical individual in the sense of hazard rate dominance would choose higher leverage in the variable house size scenario, but lower leverage in the fixed house size scenario.

While we are not able to measure the full belief distribution of each individual, we can compare how the leverage choices of individuals vary with the full distribution of the house price experiences of their friends.[39] Specifically, we conduct pairwise comparisons of the distributions of the house price experiences of all friends (and all out-of-commuting zone friends) of individuals purchasing houses in the same county in the same month, and who borrow from the same lender. We then test whether one distribution dominates the other in a hazard rate dominance sense. We only focus on county-month-lender combinations with at least three mortgage originations: at the average (median) such combination, we have 9.8 (5) mortgage originations. Of all the pairwise comparisons across these originations, we can rank the distribution of all friends' house price experiences in a hazard rate sense for 3.5% of transaction pairs, and the distributions of out-of-commuting zone friends' house price experiences for 8.1% of transaction pairs. When comparing the mortgage leverage choices across the individuals with a clear ranking using the experience distribution of all friends, we find that individuals with an experience distribution that is hazard rate dominant will choose 90 basis points lower leverage; this difference is highly statistically significant. When comparing the experience distribution of individuals' out-of-commuting zone friends, those with a hazard rate dominant distribution choose a 68 basis points lower leverage. Both of these results provide additional evidence that the fixed house size scenario, in which collateral choices are return insensitive, is dominant in the data.

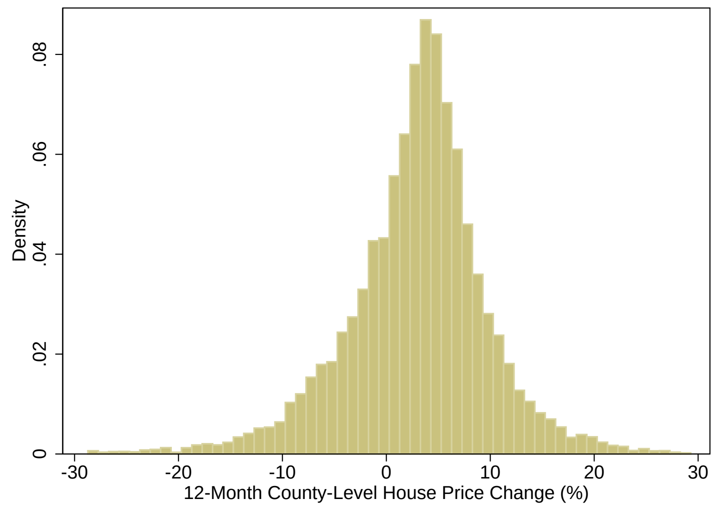
[39] This test builds on the survey evidence in Section 3.2, which showed that, on average, the first and second moments of the house price experiences of an individual's friends affect the corresponding moments of her belief distribution about future house price changes. The assumption underlying the test in this section is that additional moments of the distribution of individuals' friends' house price experiences will also shift the corresponding moments of those individuals' belief distribution.

D Additional Evidence

D.1 Distribution of County-Level House Price Changes

Figure A5 shows the distribution of county-year level annual house price changes in the United States between 1993 and 2017, as measured by Zillow. The distribution is unimodal and approximately symmetric, motivating our choice of modeling beliefs about house price changes to follow a normal distribution.

Figure A5: Distribution of House Price Changes (1993-2017)

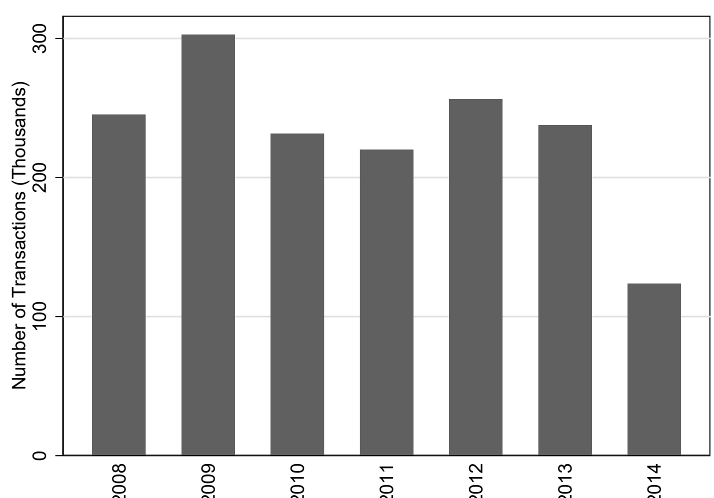


Note: Figure shows the distribution of annual county-level house price changes in the United States since 1993, as measured by Zillow.

D.2 Transactions Over Time

Figure A6 shows the number of transactions per calendar year in the baseline sample studied in Section 2.1.

Figure A6: Number of Transactions



Note: Figure shows the number of transactions by purchase year in our matched transaction-Facebook sample.

D.3 Hedonic Valuation Model

In Section 3.4, we address the concern that the relationship between optimism and leverage is the result of more optimistic individuals not being able to increase their mortgage amount. The concern is that these optimistic individuals overpay for their properties, yet banks determine maximal mortgage amounts based on their own assessed value of the property. To rule out such concerns, we control for the extent of overpayment by individual homebuyers in column 4 of Table 5. In column 5 of that Table, we restrict the sample to transactions where the homebuyer paid less for the property than its predicted value. In this Appendix, we discuss our procedure of arriving at the automated valuation of the property, and describe how we construct our measure of overpayment. Specifically, we begin by running a hedonic regression of the log of the transaction price on observable characteristics of the property (see Giglio, Maggiori and Stroebel, 2015; Stroebel, 2015, for similar hedonic pricing approaches):

$$\log\left(Price_{i,t}\right) = \alpha + \beta X_{i,t} + \psi_{zip_i,t} + \varepsilon_{i,t}.$$

We control flexibly for the property characteristics that are observable in our data set: property size, lot size, property age, property type, and whether the property has a pool. For property size and lot size, we include year-specific dummy variables for 50 quantiles of the distribution. For the other variables, we include year-specific dummy variables for each of the possible values. In addition, we include zip code by month fixed effects, to control for localized time-trends in house prices. This regression has an R^2 of 83.6%, which is high, particularly given the fact that we observe prices in buckets. We then calculate, for each property, the predicted price based on this hedonic pricing model. We then construct a measure of whether a particular buyer "overpaid" relative to the predicted value as:

$$Overpay = 100 \times \frac{TransactionPrice - PredictedPrice}{PredictedPrice}.$$

D.4 County Wealth vs. County House Price Changes

One concern addressed in Section 3.4 was that wealthier people would have friends in wealthier counties. If these wealthier counties also experienced house price appreciation, then the estimates of β_1 in Equation R1 might be picking up that wealthier people make larger downpayments. We argued that this is unlikely to be the explanation of our results, since no persons' friend experience consistently higher or lower house price experiences. Specifically, we argued that it was not the case that wealthier regions always have higher house price growth — in some years, wealthier counties see larger house price increases, in other years they see smaller house price increases. In this Appendix, we provide the evidence for these claims.

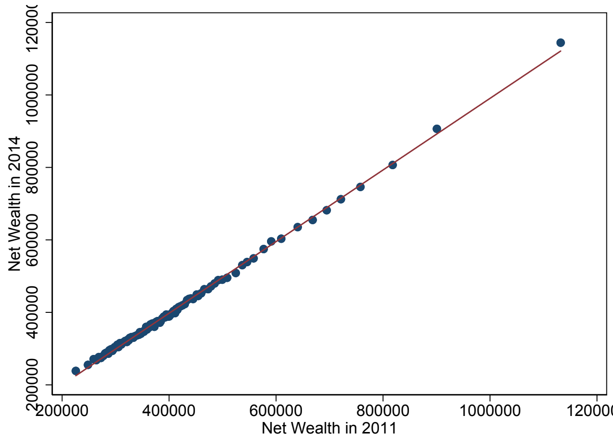
Specifically, to measure household wealth at the county level, we use county-level measures of average household net wealth constructed by Applied Geographic Solutions for the years 2011 to 2016.[40] The ranking of counties in their net wealth is stable over time: counties that were relatively wealthy in 2011 were generally relatively wealthy in 2014 — see Panel (a) of Figure A7. However, house price changes over the prior 24 months were quite different between 2011 and 2014: counties that had a relatively positive house price experience in the 24 months leading up to January 2011 had a relatively negative house price experience in the 24 months leading up to January 2014 – see Panel (b) of Figure A7. As a result, the relationship between county-level wealth and house price growth depends on time horizon considered. For example, Panel (c) shows that in January 2011, wealthier counties had experienced relatively larger house price declines over the previous 24 months while Panel (d) shows that in January 2014, wealthier counties had seen larger relative house price increases.

The instability of the relationship between county wealth and house price experiences is a persistent pattern in our data. To document this, we calculate, for every month between January 2008 and December 2016, the correlation between net wealth and house price changes over the previous 24 months. Panel (e) of Figure A7 plots that correlation over time. One can see that the correlation between wealth and past house price experiences is zero or negative until January 2013, after which it becomes positive. Due to the strong persistence in county net wealth over time, this pattern is unaffected by when we measure of county level net wealth.

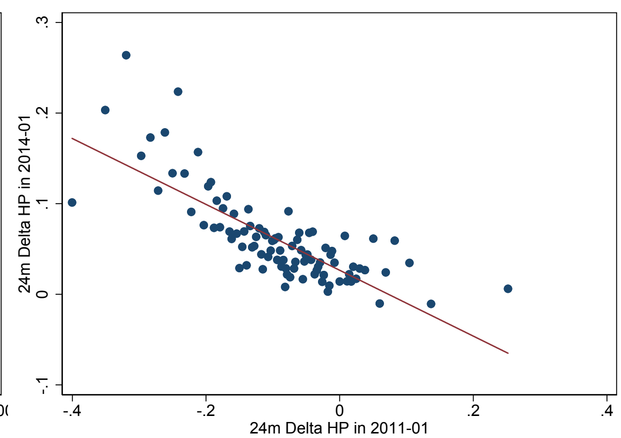
[40] Applied Geographic Solutions defines net worth as the difference between assets and debts of households at the county level, where assets include transactions accounts, life insurance with cash value, primary residence value, and debts include mortgage and credit card debt. This measure is constructed using a number of data sets, including from the American Community Survey, the U.S. Census, IRS statistics, and credit bureau data.

Figure A7: County Level Wealth and House Price Changes

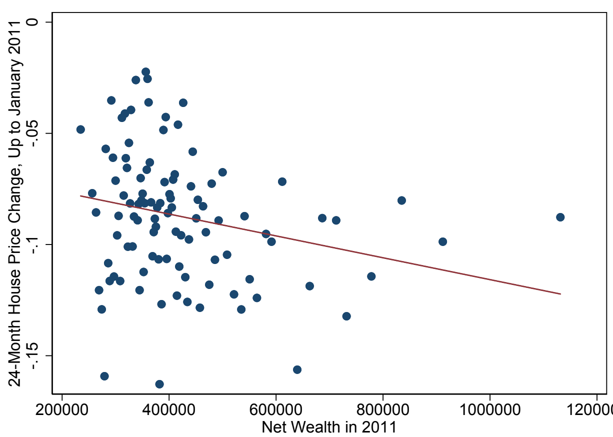
(a) Correlation in County-Level Net Wealth



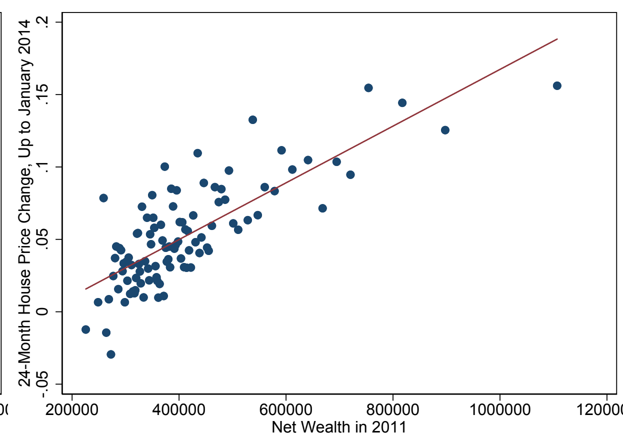
(b) County Wealth vs. House Price Changes



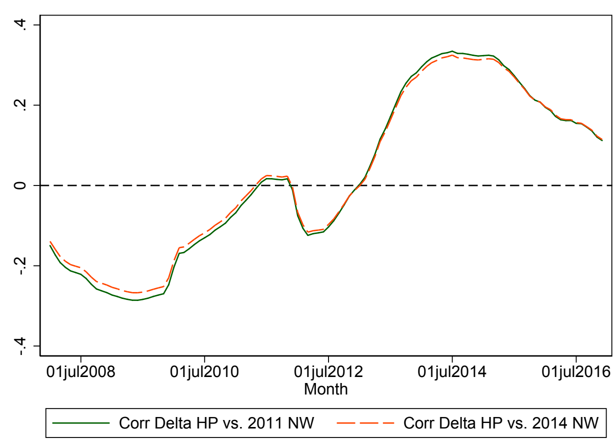
(c) County Wealth vs. House Price Changes: January 2011



(d) County Wealth vs. House Price Changes: January 2014



(e) Correlation between House Price Changes and Net Wealth



Note: Panel (a) shows a binned scatter plot of counties' average net wealth in 2011 and 2014. Panel (b) shows a binned scatter plot of counties' house price changes in the 24 months up to January 2011 and the 24 months up to January 2014. Panels (c) and (d) show binned scatter plots of county-level wealth in 2011 and county-level house price changes in the 24 months prior to January 2011 and prior to January 2014, respectively. Panel (e) shows a monthly time-series in the correlation between county-level net wealth (either measured in 2011 or in 2014) and the counties' house price changes over the previous 24 months.

D.5 Downpayments and Expected Default

As discussed in Section 4, many personal financial advice websites and blogs discuss the tradeoffs between making larger and smaller downpayments. Many of these highlight explicitly the tradeoffs that we formally model in Section 1. For example, the real estate brokerage website Home Point Real Estate describes one of the benefits of high leverage as follows:

> This last reason for putting down a small down payment is kind of twisted, but sadly practical. I doubt we are going to have any down turn in the market soon, and I really doubt it will be like the one we just came through if it does come; but the less you put down the less you lose. Yet, if you are upside down on your home and have to walk away (or lose to foreclosure) the less down payment you put into it the less you lose.

Reuters (2011) outlines similar reasons for making a smaller downpayment:

> Even if you have the money for a bigger down payment, there can be good reasons to save your cash. Mortgage rates continue to skirt all-time lows: Why not put your money to work for yourself and borrow as much as you can reasonably afford, on a monthly basis, at today's rates? You can put the money you're not paying into a down payment to work elsewhere. If home values rise, you will have done your best to leverage a small down payment into bigger equity. If they fall, you'll have less skin in the game, and that could put more pressure on your banker to improve your loan terms lest you walk away.

US News (2017) described the benefits of a smaller downpayment as follows:

> The last major housing crash scared some people away from low down payments after many homeowners found themselves owing more than their homes were worth when values plummeted. But even a 20 percent down payment won't protect you against a 50 percent drop in home values. In fact, if you lost your home to foreclosure or did a short sale, you may have lost less money if you made a small down payment.

MK Real Estate (2016) describes why making smaller downpayments is the more conservative choice for home buyers worried about potential house price drops.

> The link between the economy and home prices is the third reason to consider a small down payment. In general, as the U.S. economy improves, home values rise. When the U.S. economy sags, home values sink. Buyers with large down payments find themselves over-exposed to economic downturns compared to buyers whose down payments are small.
>
> Consider the purchase of a $400,000 home and two home buyers, each with different ideas about how to buy a home. One buyer makes a 20% down payment to avoid Private Mortgage Insurance (PMI). The other buyer wants to stay as liquid as possible, and puts down just 3.5%. The first buyer takes $80,000 from the bank and converts it to illiquid home equity. The second buyer puts $14,000 into the home.
>
> Over the next few years, the economy falters. Our two buyers lose 20% of the value of each of their homes, bringing their values down to $320,000. Neither buyer has any equity left. Our first buyer lost all $80,000 of their money. That money is lost and cannot be recouped except through the housing market's recovery. The second buyer lost only $14,000. While the second buyer's home is "underwater," with more money owed on the home than the home is worth, the bank has the risk of loss and not the borrower.
>
> Let's say that both borrowers default and no longer make their payments. Which homeowner will the bank be more likely to foreclose upon? It's counter-intuitive, but the buyer who made a large down payment is less likely to get relief during a time of crisis and is more likely to face foreclosure. A bank's losses are limited to when the home is sold at foreclosure. The homeowner's twenty percent home equity is already gone, so the remaining losses (legal and home prep costs) are easily absorbed by the bank. Foreclosing on an underwater home will lead to great losses. All of the borrower's money is already lost. The bank will not only have legal and home prep costs, but will also have to write down $66,000 in lost value.
>
> Conservative borrowers recognize that risk increases with the size of the down payment. The smaller your down payment, the smaller your risk.

D.6 Downpayment Motivation Survey

In Section 4.2, we described results from our Downpayment Motivation Survey. In this Appendix, we provide additional information on the respondent demographics, and more details on how the responses vary with these demographics. For complete disclosure, the full data set is available from the authors upon request.

We ran two waves of the survey between September 19, 2017, and September 21, 2017. The survey was designed using the SurveyMonkey survey platform. Each wave was targeted at approximately 800 homeowners, and no additional filters for the target audience were selected. In each wave, respondents were presented with two different house price growth scenarios, and asked to select one of three different mortgage choices for each house price scenario. In Table A1, we show the changes in the downpayment recommendation between the optimistic and the pessimistic scenario ("Decreased Downpayment" represents individuals who suggested a smaller downpayment in the more pessimistic house price scenario).

Table A1: Downpayment Motivation Survey - Results by Demographics

	Wave 1				**Wave 2**			
	Total	Decreased DP	Same DP	Increased DP	Total	Decreased DP	Same DP	Increased DP
All	826	20.5%	69.4%	10.2%	794	14.0%	77.6%	8.4%
Income								
$0 to $9,999	17	11.8%	82.3%	5.9%	12	8.3%	91.7%	0.0%
$10,000 to $24,999	38	13.2%	63.2%	23.7%	37	10.8%	62.2%	27.0%
$25,000 to $49,999	92	14.1%	76.1%	9.8%	84	8.3%	88.1%	3.6%
$50,000 to $74,999	56	23.2%	73.2%	3.6%	46	15.2%	60.9%	23.9%
$75,000 to $99,999	35	20.0%	71.4%	8.6%	40	17.5%	80.0%	2.5%
$100,000 to $124,999	15	6.7%	86.7%	6.7%	18	16.7%	83.3%	0.0%
$125,000 to $149,999	50	26.0%	66.0%	8.0%	51	7.8%	84.3%	7.8%
$150,000 to $174,999	103	28.2%	59.2%	12.6%	88	13.6%	77.3%	9.1%
$175,000 to $199,999	122	22.1%	68.0%	9.8%	122	13.1%	80.3%	6.6%
$200,000 and up	117	18.0%	75.2%	6.8%	108	16.7%	74.1%	9.3%
Prefer not to answer	125	16.0%	76.8%	7.2%	137	16.8%	77.4%	5.8%
Age								
18-29	51	23.5%	62.8%	13.7%	42	14.3%	78.6%	7.1%
30-44	168	18.5%	68.4%	13.1%	154	11.7%	76.6%	11.7%
45-59	215	16.7%	76.3%	7.0%	216	16.2%	76.4%	7.4%
60+	336	21.4%	70.5%	8.0%	331	13.0%	79.2%	7.8%
Gender								
Male	324	19.4%	67.9%	12.6%	294	14.3%	77.2%	8.5%
Female	446	21.5%	70.8%	7.7%	449	9.2%	83.1%	7.7%
Region								
East North Central	130	21.5%	70.8%	7.7%	130	9.2%	83.1%	7.7%
East South Central	46	15.2%	73.9%	10.9%	41	9.8%	82.9%	7.3%
Middle Atlantic	94	18.1%	73.4%	8.5%	90	16.7%	75.6%	7.8%
Mountain	58	13.8%	82.8%	3.5%	63	14.3%	74.6%	11.1%
New England	44	13.6%	77.3%	9.1%	47	12.8%	78.7%	8.5%
Pacific	133	23.3%	65.4%	11.3%	111	10.8%	79.3%	9.9%
South Atlantic	131	22.1%	64.1%	13.7%	134	18.7%	72.4%	9.0%
West North Central	59	17.0%	78.0%	5.1%	55	18.2%	76.4%	5.4%
West South Central	74	20.3%	71.6%	8.1%	68	11.8%	79.4%	8.8%

Note: Table shows responses in the Downpayment Motivation Survey separately by the reported demographic of the respondents. The results show how the downpayment recommendation changes in the pessimistic house price scenario relative to the optimistic house price scenario.

Online Appendix References

- Bailey, Michael, Ruiqing Cao, Theresa Kuchler, and Johannes Stroebel. 2017. "The Economic Effects of Social Networks: Evidence from the Housing Market." Journal of Political Economy (forthcoming).
- Campbell, John Y, Stefano Giglio, and Parag Pathak. 2011. "Forced Sales and House Prices." The American Economic Review, 2108–2131.
- Giglio, Stefano, Matteo Maggiori, and Johannes Stroebel. 2015. "Very long-run discount rates." The Quarterly Journal of Economics, 130(1): 1–53.
- Greene, W.H. 2003. Econometric Analysis. Pearson Education.
- Krishna, Vijay. 2010. Auction theory. Burlington, MA: Academic Press/Elsevier
- Piazzesi, M., M. Schneider, and S. Tuzel. 2007. "Housing, consumption and asset pricing." Journal of Financial Economics, 83(3): 531–569.
- Simsek, Alp. 2013. "Belief disagreements and collateral constraints." Econometrica, 81(1): 1–53.
- Stroebel, Johannes. 2015. "Asymmetric information about collateral values." Journal of Finance.