CHAPTER 10
Equity Derivatives

A. Derivatives: An Introduction

A *derivative security* is simply a financial instrument whose value is derived from that of another security, financial index or rate. A large number of different types of derivative securities have become very important for management of a variety of different types of equity-related - risks. Pricing of relevant derivative securities can provide useful information regarding characteristics of the equities from which they derive their values. Thus, derivative security pricing models, particularly the option pricing methodology, can very useful for the pricing of stocks and understanding their risks.

A few observers of financial markets have argued that derivative securities play no important productive role in our economy. It has been argued that derivative securities merely provide gambling opportunities to speculators, enabling them transfer wealth among themselves without contributing to the economy. However, it can also be argued that derivative securities play a very important role in improving the efficiency of capital markets. Clearly, active and efficient capital markets are essential to the output and growth of our economy. Business firms require capital resources to produce goods and services and to maintain jobs for our labor force. Firms obtain this capital from investors, who, either directly or through financial institutions, purchase the securities that firms sell. Poorly functioning capital markets inhibit production capabilities of business, and result in economic decline. However, risk and uncertainty are unavoidable characteristics of a capitalist economy. Businesses and the capital markets in which investors entrust their savings are profoundly affected by risk. Because investors and businesses tend to shy away from risk, the proper functioning of capital markets is impaired by uncertainty and volatility. Investors will be reluctant to invest and businesses will reduce output and employment when they are unable to control their exposure to risk. Whereas consumers control personal risks associated with illness or casualty losses by purchasing insurance contracts, financial risks are often controlled with positions in derivative securities.

There exist a variety of equity-related derivative securities. Some of the more frequently traded derivatives are the following:

Options, which confer the right but not obligation to buy or sell stock at a pre-specified price on or before an expiration date,
Hybrids that combine features of two or more securities (e.g., a convertible bond),
Futures contracts, which provide for the transfer of an index, portfolio or individual stock at a settlement price on the settlement date of the contract,
Swaps, which provide for the exchange of cash flows associated with a stock or portfolio for cash flows associated with some other asset, index or portfolio, and
Exotics, which are usually options with features or complications that set them apart from "plain vanilla contracts.

From a more general perspective, derivative securities in general are not an exotic invention of modern Wall Street security dealers; derivatives have played important roles in

world economies for centuries. Early examples include agricultural futures contracts dating back to *c.*2000 B.C. in India and option contracts on ship cargos created by ancient Phoenicians and Greeks. Options and futures contracts were used in 17th century markets in Amsterdam as well as in Osaka. Futures markets started to become more organized in the 19th century as exchanges developed to trade contracts. Such securities have helped manufacturers, farmers, mining companies, traders and transporters manage a multiplicity of risks ranging from stock market volatility to borrower default.

The markets for derivatives are crucial to business for the management of many types of risks. Risk factors frequently hedged with derivatives include, but are not limited to uncertainties associated with interest and exchange rates, client default, economy-wide and industry specific output. Markets for explicit insurance policies on such a wide array of risks do not exist largely due to contracting costs. Most insurance policies are fairly standardized (e.g., health, life and many casualty policies), while customized insurance contracts are expensive and time consuming to write. Businesses must be able to act quickly to manage their risks in this environment of rapid change. Flexibility and liquidity along with low contracting and transactions costs are key to the success of the risk management operations of a firm. An active and efficient market for derivative securities can meet these important requirements.

Business firms and individual investors desiring to hedge risks are not the only participants in markets for derivatives. A second type of market participant is the speculator who takes a position in a security based on his expectation regarding future price movement. Although the speculator is essentially concerned with his own trading profits, he plays an important role in maintaining liquidity in derivative markets, affording business and individual investors the opportunity to hedge risks quickly and efficiently. The speculator is often the counterparty to a hedger's trade, selling or purchasing derivatives as required by hedgers.

The arbitrager, who exploits situations where derivatives are mis-priced relative to one another not only provides additional liquidity to derivative markets, but plays an important role in their pricing. By constantly seeking price misalignments for a variety of types of securities, and by understanding the payoffs of securities relative to one another, arbitragers help ensure that derivative securities are fairly priced. This activity reduces price volatility and uncertainty faced by hedgers.

Whereas hedgers, speculators and arbitragers all play an important role in the pricing of and liquidity maintenance for derivatives, derivatives actually play a role in the evaluation of risks and prices for other securities. For example, it is well known that futures prices provide valuable information for predicting commodity prices and interest and exchange rates. This information is most useful for business planning. In fact, one very interesting study by Richard Roll of UCLA showed that market prices of orange juice futures anticipated severe winter weather conditions more accurately than did the National Weather Bureau forecasts. In addition, sophisticated stock analysts realize that the price of a stock option is most sensitive to the risk of the underlying stock. Thus, analysts frequently rely on the option price to provide information on the risk of the underlying stock. Market prices of derivative securities in general are quite useful for assessing the magnitude and pricing of a multitude of different risks.

Derivative securities are traded in the United States either on exchanges or in the so-called Over the Counter (OTC) markets. Substantial market interest is required for exchange listing, whereas securities with smaller followings or even customized contracts can be traded over the counter. The role of the derivatives dealer is essentially the same as that for other security dealers: to facilitate transactions for clients at competitive prices. Derivative dealers match counterparties for derivative contracts, act as a counterparty for many of their own custom contracts and provide an array of support services including expert advice and carefully engineered customized risk management products. It is necessary that the dealer providing full support services have a proper understanding of his client, his business and the client's needs. Since most clients do not have an understanding of the technical terms used in the industry, the dealer must be an effective communicator. It is equally important for the dealer to understand the nature of the securities with which he deals and how serving as a market maker for derivatives affects the risk structure of his employer. This understanding usually requires strong analytical skills.

While derivatives do play an important role in our economy by enabling investors to hedge risk, speculate and price other assets, a number of well-publicized difficulties have arisen from their use (and misuse). First, many derivative contracts have been created to be highly leveraged, so that very large profits or losses may result from relatively small price shifts in the underlying asset. This feature makes it crucial for financial managers to fully understand the features and implications of the contracts with which they deal. However, many of these contracts are somewhat complicated such that many of their users are not able to fully understand their implications. This has led to some very highly publicized losses and failures, including those at Procter & Gamble (over $100 million in interest rate contracts), Gibson Greetings (over $20 million), Orange County ($1.7 billion in interest rate derivatives) and Metallgesellschaft ($1.4 billion in oil futures). Second, tracking their values and reporting them on accounting statements are very difficult because many contracts do not conform to characteristics of assets typically reported by accountants. Even recent Financial Accounting Standards Board (FASB) opinions regarding "marking to the market" are not very helpful because most derivative contracts either have at best a very thin market and are subject to extreme pricing volatility. Accounting difficulties make it practically impossible for investors, regulators, managers and even auditors to understand the impact of derivatives investment on firms. These difficulties have led to a number of frauds that ultimately caused failures, including Barings Bank ($1.4 billion in Nikkei-index derivatives). It seems that financial innovators have been able to develop new derivative securities faster than accountants and regulators have been able to develop technique for tracking their values and implications.

B. Plain Vanilla Equity Options

A *stock option* is a legal contract that grants its owner the right (though, not obligation) to either buy or sell a given stock. There are two types of stock options: puts and calls. A *call* grants its owner to purchase stock (called underlying shares) for a specified exercise price (also known as a striking price or exercise price) on or before the expiration date of the contract. We introduced the fundamentals of options and option pricing in Appendix C to Chapter 4. Here, we discuss a few applications of equity options.

Calls for Leverage and Covered Calls

Because call options provide for their owners to benefit only from the excess of the share price over the option exercise price, they will cost less than the stock itself. Thus, the investor has the potential to claim share price gains above a given level (the option exercise price) with the purchase of calls than with the underlying stock itself. For this reason, call options are often said to be leveraged securities. When investors can gain a given level of price gains with a smaller investment, returns are said to be leveraged. Thus, per dollar invested, call options are more volatile than comparable investments in underlying stock.

On the other hand, writing *covered calls* will tend to reduce stock risk. A covered call is written or sold on shares of stock owned by the investor. If the underlying stock price increases above the exercise price, the investor will be obliged to sell the shares to the owner of the call. Regardless, the covered call writer keeps the premium received on the sale of the call. Thus, potential losses are reduced by the amount of the call premium and potential gains are limited by the exercise price of the call. Thus, stock risk is reduced on both up- and down-sides. In fact, the call's Black-Scholes hedge ration ($N(d_1)$) can be used to balance call and stock risk. An investor can sell $1/ N(d_1)$ calls for each share purchased to maintain a hedged portfolio for a short period. As $N(d_1)$ changes through time, the portfolio can be rebalanced.

Puts as Insurance

A *put* grants its owner the right to sell the underlying stock at a specified exercise price on or before its expiration date. An equity put is similar to an insurance contract. For example, an owner of stock may purchase a put contract ensuring that she can sell her stock for the exercise price given by the put contract. The exercise price of the put serves as the stock's insured value. The value of the put when exercised is equal to the amount by which the put exercise price exceeds the underlying stock price (or zero if the put is never exercised). One problem with using the put as an insurance contract is that its initial cost may exceed what the stockholder wishes to pay for it.

Collars

A *Collar* is the combination of a long position of a put and a short position in a call. The long put position is, at least in part, financed with the proceeds from the sale of the call. The collar provides the put owner the right to sell the underlying stock at the put exercise price (the put exercise price is called the floor). The short position in the call requires that the underlying stock be sold if the stock price at expiration exceeds the exercise price of the call (the call exercise price is called the cap). Thus, the stock will be sold at option expiry if the market price of the underlying stock is less than the exercise price of the put or exceeds the exercise price of

the call.

A *Zero Cost Collar* is a package of options, in this case, long call and short put positions (purchased call financed through the sale of a put) designed to require zero net investment. Typically, the exercise price of the put (floor) is set at a level relative to the exercise price of the call (the cap) so that their values exactly offset each other. As before, the short call position obliges the investor to sell her shares if the price is high enough at the expiry date while the long put offers her the opportunity to do so if the price is low enough. The investor has unwound her position of risk without actually selling his securities; her option positions provide for the sale of the stock at a later date regardless of share price behavior.

Collars and zero cost collars have been used by corporate executives to unwind their equity incentive packages. This means that executives can reduce or eliminate their positions of risk in their employers' stock without having to report stock sales to shareholders, the S.E.C. or to the I.R.S. Bettis, Bizjak and Lemmon [2001] found that managers using this strategy covered an average of 36% of their equity positions with collars. This study also found that managers strategically time their collars, with their stock price performance relative to the market being 19% higher during the 120 days prior to collar transactions than the six-month performance subsequent to the transactions. The I.R.S. and S.E.C. have since imposed limitations on the effectiveness of this strategy.

C. Hybrid Equities: Warrants and Convertibles

A *warrant* is a call option issued by a corporation on its own stock. Warrants provide their owners the right to purchase shares of stock directly from the corporation at a specified exercise price. Warrants are often traded among investors and are often issued by the corporation along with other securities such as bonds or preferred stock. The warrants may be intended by the issuing corporation as inducements to encourage investors to purchase other securities with which they are issued. Other warrants may be issued to managers, employees or investment banks as compensation for services rendered.

Warrants are usually of long duration, in some cases, perpetual. They are often issued with privately placed bonds (in this case, known as an equity kicker). In most instances, they are detachable from the securities with which they are issued. The warrants may be intended by the issuing corporation as inducements to encourage investors to purchase other securities with which they are issued. For example, warrants may be issued with an IPO for a very speculative issue or as financing for a merger (example: old share of common is exchanged for a bond and a warrant).

Dilution of Shares

An important difference between call options and warrants on stock is that warrants are issued in treasury stock rather than outstanding shares. Warrant exercise causes more of the company's shares to become outstanding. Thus, exercise of warrants implies a dilution of share prices, though the corporation does receive exercise money from the exercise of options. However, stock prices may normally be expected to decline as a result of warrant exercise because exercise takes place only if the exercise money paid by investors is less than the value of shares that warrant-holders receive. Accounting for both the m additional shares outstanding and exercise money received from warrant exercise can be rather difficult because the warrant value is a function of the stock value and the stock value is a function of the warrant value. Our first step is to extend the Black-Scholes formula to price warrants is to adjust for dilution where the firm currently has outstanding n shares of stock and warrants are issued on m additional shares of stock:

$$w_0 = \frac{c_0}{1+\frac{m}{n}} = \frac{c_0}{1+q}$$

After adjusting for dilution, we revise the Black-Scholes model to reflect the increase in share value associated with the warrants that might be exercised. Thus, we replace S_0 in the Black-Scholes formula with $S_0 + (m/n)\times w'_0$ as follows:

$$d_1 = \frac{\ln\left(\frac{S_0+\frac{m}{n}w_0'}{X}\right)+(r_f+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$w_0 = \left(S_0 + \frac{m}{n} w_0^{'} \right) \times N(d_1) - X / e^{r_f T} N(d_2)$$

Because there is a circular relationship between the stock value and the warrant value, we replace our old warrant value in the Black-Scholes equation with the revised warrant value and repeat. Normally, only one or two repetitions will be required for adequate accuracy.

Now, let's consider the example in the chapter involving managerial stock options. The problem with our computations in the chapter is that they apply to call options on outstanding shares. Managerial stock options, technically warrants, are usually issued on treasury stock, meaning that more shares will become outstanding if the options are exercised. We need to recognize that exercising managerial stock options will have two effects on the value of the firm's stock, thereby affecting the option value. First, the firm's asset value will increase if options are exercised and the firm collects exercise money. In this example, if the manager were to exercise his option to purchase 1,000,000 shares of stock at a price of \$50 per share, the firm would increase its equity value by 1,000,000 × \$50 = \$50,000,000, though we don't know if the warrants will be exercised. Second, share value will be diluted as more shares of stock become outstanding. With 10,000,000 shares of stock currently outstanding, exercise of 1,000,000 managerial stock options would increase the number of outstanding shares by q = 1,000,000/10,000,000 = 10%. Adjusting the Black-Scholes formula for both the asset value increase and the dilution is rather complicated. In any case, the value of the firm's equity in effect rises by the value of warrants w_0 issued, so the per-share value of equity rises to $S_0 + (m/n) \times w_0 = \$30 + (1.000,000/10,000,000) \times w_0$.

Thus, the value of the firm's equity is, in effect, a function of the value of the warrant and the value of the warrant is a function of the value of the stock. The first step in estimating the warrant value w_0 is to determine its value after adjusting for dilution where q = m/n = .1, or the proportional increase in the number of outstanding shares when the options are exercised:

$$w_0^{'} = \frac{c_0}{1+q} = \frac{2.783}{1+.1} = 2.53$$

Our first estimate for the fully diluted option value is \$2.53. Next, we adjust the firm's stock value to reflect the value of the outstanding warrants. We replace S_0 in the Black-Scholes formula with $S_0 + (m/n) \times w'_0$ as follows:

$$d_1 = \frac{\ln\left(\frac{S_0 + \frac{m}{n} w_0^{'}}{X} \right) + (r_f + \sigma^2/2)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{30 + \left(\frac{1{,}000{,}000}{10{,}000{,}000} \right) \times 2.53}{50} \right) + (.05 + .4^2/2) \cdot 2}{.4 \cdot \sqrt{2}} = -.4285$$

$$d_2 = d_1 - \sigma\sqrt{T} = -.4285 - .4 \cdot \sqrt{2} = -0.9942$$

$$N(d_1) = .3341 \qquad N(d_2) = .1600$$

$$w_0 = \left(S_0 + \frac{m}{n} w_0^{'} \right) \times N(d_1) - X \,/\, e^{r_f T} N(d_2)$$

$$= \left(30 + \frac{1{,}000{,}000}{10{,}000{,}000} \cdot 2.53 \right) \times .3341 - 50 / 2.71828^{.05 \cdot 2} \cdot .1600 = 2.606$$

Next, we substitute this revised warrant value 2.606 back into our equations as follows:

$$d_1 = \frac{\ln\left(\frac{S_0 + \frac{m}{n} w_0^{'}}{X} \right) + (r_f + \sigma^2 / 2)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{30 + \left(\frac{1{,}000{,}000}{10{,}000{,}000} \right) \times 2.606}{50} \right) + (.05 + .4^2 / 2) \cdot 2}{.4 \cdot \sqrt{2}} = -.4281$$

$$d_2 = d_1 - \sigma\sqrt{T} = -.4281 - .4 \cdot \sqrt{2} = -0.9938$$

We find $N(d_1)$ and $N(d_2)$ and determine that the revised warrant value is:

$$\left(30 + \frac{1{,}000{,}000}{10{,}000{,}000} \cdot 2.606 \right) \times .3342 - 50 / 2.71828^{.05 \cdot 2} \cdot .1601 = 2.608$$

Our estimate has changed very little. If we repeat this process indefinitely, we will find that the revised warrant value changes very little and converges to approximately 2.608. This means that the value of the 1,000,000 managerial stock options is $2,608,000.

Convertible Bonds

Corporations issue bonds with option like features. For example, a convertible bond permits its owner the option to convert the debt security into a specified number of shares of common stock. A convertible bond is, in many respects, similar to a package consisting of a straight bond and a series of warrants on the firm's stock. One should note that exercise of the convertibility feature does not result in a cash flow to the firm; it results in a reduction in debt. Convertible bondholders may benefit from increases in stock price. Convertible bonds are often referred to as hybrid securities because their values, in effect, represent the sum of two different security values.

The example in Table 1 defines a number of terms frequently used in the description of convertible bonds:

Conversion Ratio (CR): the number of shares received per bond. IBM: 6.51, Bally: 30.60
Conversion Value: The value of the shares if converted
Conversion Value = Share Price @ Conversion Ratio. IBM: $726.68 = $111.63 @ 6.51
Conversion Parity Price (CPP): Break-even Price
Conversion Parity Price = Bond Price / Conversion Ratio. IBM: $156.49 = $1018.75 / 6.51
Conversion Premium = [CPP - Stock Price]/Stock Price; IBM: 40.2%=[$156.49-$111.63]/$111.63
Conversion Premium: Difference between the conversion parity price and stock price as a percentage of the stock price

Convertible Bond Data
Source: *Standard & Poor's Bond Guide*

Issuer	IBM	Bally
Coupon Rate	7.875%	10.00%
Maturity Date	2004	2006
Debt Rating	AAA	D
Conversion Ratio	6.51	30.60
Bond Price	$1018.75	$292.50
Stock Price	$111.63	$4.75
Parity Price	$156.49	$9.56
Conversion Premium	40.2%	101.2%
Conversion Value	$726.68	$145.35

Table 1: Convertible Bond Example

Now, consider an example where the Bios Co. has issued a 6 percent convertible bond (CV) that matures in 20 years. If Bios were to sell a straight 20-year issue in the current market, the bond would have to yield eight percent to be competitive. What would be the price of a 20-year straight bond with a six- percent coupon rate if the investor required an eight- percent YTM? This is determined as follows:

$$V_{SD} = \frac{60}{.08}[1 - \frac{1}{(1 + .08)^{20}}] = \$803.63$$

The straight bond would sell for $803.63. Thus, if a CV had a face value of $1000 that pays the same interest, the warrant part of the CV would be worth 1000 - 803.63=196.37. Similarly, the value of the option feature of the convertible bond could be determined with an appropriate option pricing model.

We will verify this option feature valuation of $196.37 by considering some equity and bond features. Suppose that the stock is currently selling for $40 per share and the conversion ratio is 10 (the bond can be converted for 10 shares of stock). Further suppose that there are 1,000,000 shares of stock currently outstanding and 10,000 convertible bonds. The riskless return rate equals the bond's yield to maturity of .08 and the standard deviation of stock returns equal .1606.

First, we will compute the conversion option as though it were calls on 10 shares of stock:

$$d_1 = \{\ln(400/803.63) + (.08 + .5 \times .0258) \times 20\} \div \{.1606 \times \sqrt{20}\} = 1.311$$

$d_2 = d_1 - .1606 * \sqrt{20} = .59283$

Next, by either using a z-table (See Appendix C to Chapter 4) or by using an appropriate polynomial estimating function from a statistics manual, we find normal density functions for d_1 and d_2:

$$N(d_1) = .90508 ; \qquad N(d_2) = .723353$$

Finally, we use $N(d_1)$ and $N(d_1)$ in Equation (8) to value the call on 10 shares of stock:

$$c_0 = 400(.90508) - [803.63 \times .201897] \times (.723353) = 215.9898$$

Next, we value a warrant to account for q, the dilution caused by the conversion of bonds. Since there are 10,000 convertible bonds and 1,000,000 shares of stock, conversion will increase the number of outstanding shares by q = .1 or 10%. Hence, we divide the call value or $216 by (1+q) = (1 + .1) to obtain the warrant value of $196.37 as determined earlier. Thus, the overall value of the bond equals the sum of its straight debt value and its option value:

$$V_{CB} = V_{SD} + V_O = \$803.63 + \$196.37$$

D. Equity and Index Futures

A *forward contract* represents an agreement specifying delivery of given quantity of an asset at a later date at a given price. Essentially, a forward contract is an agreement providing for a seller (an entity said to be taking the short position on the asset) to deliver the specified asset at a future date to a purchaser (an entity said to be taking the long position on the asset). The actual exchange of the asset for cash occurs at a date subsequent to the date the forward contract is initiated. A large number of forward transactions involve direct negotiations between banks, brokerage firms and other financial institutions.

A *futures contract* also represents an agreement specifying delivery of given quantity of an asset at a later date at a given price. However, the futures contract differs from a forward contract in several important ways. First, a forward contract is created by its long and short participants according to whatever terms they specify. However, a futures contract is created by a *clearing house* which acts as a middleman between the contract participants. The clearing serves as counterparty on all transactions that effectively eliminates default risk. A futures contract is traded on an exchange. To facilitate trading and to ensure that a reasonably large number of investors will be interested in the futures contract, the futures contract is standardized with respect to the exact quantity and nature of the asset to be delivered along with the date that the actual sale of the asset will take place (settlement date). To ensure that both parties to the futures contract will honor their commitments, participants are expected to post *margin*, which is, in effect, collateral required by the brokerage firm. Furthermore, futures contracts provide for *marking to the market*, which involves daily re-computations of the margin requirement based on updated asset value. Several exchanges exist for trading of futures contracts, including the Chicago Board of Trade, the Chicago Mercantile Exchange and the New York Mercantile Exchange. Futures markets are tightly regulated and overseen by the Commodities Futures Trading Commission (CFTC) and to a lesser extent, the Securities Exchange Commission (SEC).

Futures contracts are traded on a variety of different types of assets. Futures contracts have been traded for many centuries on a multiplicity of commodities, including agricultural products such as corn, wheat and pork bellies, metals such as gold and copper, minerals such as diamonds, petroleum and phosphates, as well as paper, timber. Futures contracts are traded on an assortment of financial securities such as treasury bonds, market indices and interest rates. Futures contracts also trade on a large number of different currencies.

How might a futures contract help a business control risk? Consider the following simple example involving a wheat farmer and a cereal manufacturer. Suppose that the manufacturer needs to purchase wheat in three months for cereal production and the farmer, who is just now planting seed wishes to sell wheat at harvest in three months. Since the market price of wheat is likely to be affected by general economic and trade conditions, weather, and many other factors beyond the control of the two trading partners, neither knows what the price of wheat will be in three months. Hence, the manufacturer faces uncertainty with respect to cereal production costs and the farmer cannot know what revenues he will derive from the sale of his produce. Such uncertainty clearly affects the abilities of the trading partners to make appropriate business decisions. In fact, in the face of such uncertainties, businesses often scale back their levels of operations to avoid losses. It would seem simple for the farmer and the cereal manufacturer to

agree on a transaction price of wheat in advance. In fact, this is exactly what the futures market allows trading partners to do; the party wishing to lock in a purchase price for wheat takes what is called a long position in the futures contract and the party wishing to lock in a selling price for wheat takes a short position in the futures contract. The futures contract, in effect, obliges each of its participants to transact for wheat at the agreed upon price (settlement price) at the pre-specified date. However, since the manufacturer never knows exactly whose wheat it will purchase, and the farmer never knows exactly who will be the end user of his wheat, they simply take positions in futures contracts with anonymous counterparties. Although the farmer actually sells wheat at the prevailing market price at harvest, his short position in the futures contract enables to him offset decreases (increases) in the market price of wheat with gains (losses) in his short position in the futures contract. Similarly, the cereal manufacturer purchases wheat at the prevailing market price at harvest, though its long position in the futures contract enables to him offset increases (decreases) in the market price of wheat with gains (losses) in its long position in the futures contract. Similarly, forward and futures contracts on individual equity positions can help shareholders control and manage risk and index contracts can help investors manage portfolio risk.

Index Contracts

Stock indices exist to permit simple evaluations of market performance. Broad market indices such as the S&P 500 and the Wilshire 5000 index exist to indicate overall market performance. An index future is a contract on an index. For example, the S&P 100 index has associated with it a futures contract (the OEX) which is traded on the Chicago Mercantile Exchange (CME). The Chicago Board of Trade markets the Major Market Index contract on the 20 largest NYSE stocks. The contract value equals the index value times a constant known as the futures value multiplier. In the case of each of these two contracts, the futures value multiplier equals $500.

Specific industry or segment indices exist to indicate performance in a narrow market. Stock index futures contracts exist to permit investors to take positions in a particular market without having to take positions in each of the component securities. Among the well-known index futures contracts are the VLCI on Kansas City Board of Trade, S&P 500 on IMM, MMI on Chicago Board of Trade, and the NYSE on New York Stock Exchange.

Options on these futures contracts trade on exchanges as well. Among the well-known options contracts on stock indices are the Value Line Composite Index (VLCI) on the Philadelphia Exchange (PHLX), OEX (S&P 100) on the Chicago Board Options Exchange (CBOE), the Major Market Index (MMI) on AMEX, NYSE on NYSE, Computer Technology Index on the AMEX and the Oil Index on the AMEX. The two primary purposes of Index Options are:

1. for speculating on 1 option rather than series of contracts
2. for hedging the risk of a portfolio whose construction resembles that of the index.

Traders wishing to take positions in the stock market of a particular country without committing to a single company's stock may take a position in an index futures contract or in options on that contract. For example, the Osaka Stock Exchange trades options contracts on the Nikkei 225,

which is an unweighted index of the shares of the 225 largest companies traded on the Tokyo Stock Exchange. Futures index contracts exist for most major country markets. Among these indices for markets outside of the U.S. include the FTSE 100 (the *"Footsie 100" or Financial Times - Stock Exchange 100*) in the U.K., the DAX (*Deutscher Aktienindex*, based on 30 equities), the CAC 40 (*Cotation Assistée en Continu*), the TSE 300 on Toronto and the SPI (Swiss Performance Index on approximately 400 firms). A "composite" international index computed by Morgan Stanley is the *EAFE* (Morgan Stanley Capital International Europe, Australia, Far East Index) based on approximately 2000 companies in 21 countries. Among the important index option contract features are:

1. Contract multiple: number of options in contract
2. Cash settlement
3. End of day exercise only
4. Regular expiration cycles
5. Taxes: all realized and unrealized gains are taxable

Securities whose prices are to be included in the index have to be added or averaged in some manner. There are three primary methods for weighting the various securities comprising a particular index:

1. Price Weighted: $I_{pt} = [\Sigma_{=1} P_{it}/n]$ (e.g., simple average); Examples:
   $DJIA_t = [\Sigma_{1=1} P_{it}/AD_t]$ where AD_t is adjusted divisor for time t
   $MMI_t = [\Sigma_{1=1} P_{it}/AD_t]$
2. Value Weighted: $I_{vt} = [\Sigma_{i=1} Q_{it}P_{it}] \div [\Sigma_{=1} Q_{i0}P_{i0}]$; Example:
   $SP500_t = [\Sigma Q_{it}P_{it}] \div [\Sigma Q_{i0}P_{i0}] * 10$
3. Equal Weighted: $I_{et} = [\Sigma_{i=1} P_{it}/P_{i0}]$

The following formula may be used for valuing index options where δ represents dividend leakage (yield):

$$c_0 = S_0 e^{-\delta t} N(d_3) - X e^{-rft} N(d_4)$$
$$d_3 = \{\ln(S_0 e^{-\delta t}/X) + (r_f + .5\sigma^2)t\} \div \sigma\sqrt{t}$$
$$d_4 = d_3 - \sigma\sqrt{t}$$

Thus, this index option formula makes use of the Merton continuous dividend leakage formula to compute option values.

Consider the following example of index option use for hedging purposes. Suppose that an investor owns a portfolio that resembles the MMI. The total value of this portfolio is $1,000,000. Assume that the investor wishes to neutralize his position without selling shares. The current MMI is 500; MMI calls and puts with this exercise price are selling for 4 and 2, respectively. Their terms to expiration correspond to the date on which the investor wishes to wish to be neutral. The multiplier for the MMI is 250.

One way to neutralize the portfolio ids to make use of put-call parity, where the position is neutralized by buying puts and writing calls on the index as follows:

$1{,}000{,}000 \div (500 \times 250) = 8 = \#$ call contracts
$1{,}000{,}000 \div (500 \times 250) = 8 = \#$ put contracts

$8 \times 4 \times 250 = \$8000 =$ premium received for calls
$8 \times 2 \times 250 = \$4000 =$ premium paid for puts

Thus, $4000 equals the net premium received for locking in the value of the portfolio at $1,000,000 by buying 8 puts and writing 8 calls.

Another type of index option, the Market Volatility Index (MVI) traded on the CBOE is used by investors to take positions on the volatility of the market. This index is based on the implied volatilities of options on the S&P 100 (OEX). These options are particularly useful to investors who wish to hold vega neutral portfolios (portfolios whose values are invariant with respect to the risk of component securities).

There has been a fairly extensive empirical literature concerning index option pricing. For example, Evnine and Rudd [JoF July 1985] found a significant number of arbitrage opportunities based on the binomial option pricing model for 1798 SP100 and MMI option transactions between June 26, 1984 and August 30, 1984. However, many of these may not really have existed because of the inability to trade underlying assets or because of the high level of activity at that time. Also, the binomial model requires restrictive assumptions and option trades were not simultaneous. In another paper, Chance also found numerous basic model violations. However, he concluded that abnormal returns were zero after transactions costs.

In addition to index and futures contracts, investors can trade shares of a trust created to replicate an index. For example, shares of and contracts on the Diamonds Trust, traded on the American Stock Exchange, replicate the Dow Jones Industrials Average.

Some observers have noted reservations concerning the various index funds and ETFs. One cited reservation is that certain demand shifts for fund shares are unrelated to the future cash flows of firms comprising the portfolios, particularly when the composition of an index changes. For example, when Yahoo was added to the S&P 500 in December 1999, index fund managers purchased shares of the stock even though it had a limited public float. This extra demand drove the price of Yahoo up by over 50% in a week and over 100% in a month. Eighteen months later, the stock price was down by over 90% from its price shortly after having been added to the S&P 500.

Portfolio Insurance and Program Trading

Index futures and options are often used for portfolio insurance, an asset allocation strategy enabling investors to alter their portfolio risk structure. Usually, it is intended to limit downside portfolio risk. An insured portfolio is one whose loss is limited to a known finite level while its return is a predictable function of the return of the underlying uninsured portfolio.

The following are definitions of terms often used in conjunction with portfolio insurance:

1. Floor: minimum portfolio value
2. Cushion: difference between portfolio value and floor
3. Exposure: potential loss
4. Multiple: exposure ÷ cushion
5. Hedge Ratio: total face value of contracts ÷ portfolio face value

Managers may wish to rebalance a portfolio to maintain a constant multiple. The objective of each of the following portfolio management strategies is to maintain upward stock price potential while eliminating downside risk below a floor:

1. Stop-loss order: its disadvantages are that it does not allow for upward profit potential after the sell order is executed; this strategy is *path dependent* in that its success is a function of market movement. Also, it does not guarantee execution at the floor value.
2. Purchase put options: Buy appropriate equity or index puts. This presents a *tracking problem* in that the correlation between the portfolio and the index may not be perfect. Imperfect correlation leads to finding appropriate hedge ratios. Also, most traded options are of the American variety, thus, are more expensive to purchase. Also, short-term standard expirations and the small number of exercise prices limit the applicability of these contracts. However, if the correlation between the index and portfolio is perfect, the advantage of this strategy is that it does not require continuous rebalancing.
3. Create synthetic puts: One can create synthetic puts by continuously updating hedge ratios in the Black-Scholes framework. This process is analogous to the process of creating a synthetic call by buying stock and borrowing money in the binomial or Black-Scholes frameworks (Note: the hedge ratio must be continuously updated). To create a synthetic put, one sells stock and lends. Again, the hedge ratio is continuously updated in the Black-Scholes framework. The hedge ratio for the number of shares to sell is given:

$$1\text{-}dp/dS = 1\text{-}e^{-rt}N(d_1) = 1 - \{\ln(S/X)+(r_f+.5\sigma^2)t\}/\sigma\sqrt{t}$$

   where X is the terminal floor value of the portfolio and S is the current market value of the stocks. The selected put is on the index that most resembles the portfolio. This strategy is likely to present several difficulties:
   a. It is likely to involve frequent trading
   b. There may be restrictions on selling and borrowing.
   c. Stock prices do not follow an Ito process.
   d. There may be uncertainty regarding future interest rates, volatilities or dividends.
4. Dynamic hedging using futures contracts involves the shorting of index futures, updating is required. The dynamic hedge ratio is the number of contracts to short:

$$n_f = \{ [V/(S+p)][1+e^{-rt}(N(d_1)-1)] - V/S \}e^{-rt}$$

   where p is the value of a put with an exercise price of S. Dynamic hedging has the advantage of relatively liquid futures contracts and allows for frequent updating with fairly low transactions costs. However, it still requires perfect correlation between the portfolio and the index. The major problem associated with dynamic hedging is the inability to trade instantaneously, causing the hedge to work less than perfectly and limiting upward portfolio value potential.

E. Swaps

A huge variety of other types of derivative securities exist for a number of purposes, including hedging, speculating and arbitrage. One important type of derivative, a swap contract, provides for the exchange of one set of cash flows for another set of cash flows. The amounts of these cash flows are usually tied to cash flows associated with other assets or portfolios. Swap contracts are specified for commodities, currencies, debt and equity securities, interest rates and a large number of other types of assets as well. These swap contracts have a number of uses. For instance, swap contracts enable financial market participants to synthesize other securities which are either unavailable or inappropriately priced. For example, Japanese regulations have restricted investment in many types of securities; in particular, Japanese institutions have been restricted with respect to non-yen bond purchases. Suppose that a firm wished to borrow dollars to purchase American products. Japanese tax code often makes borrowing less expensive in Japan. The borrower could sell to a Japanese institution a yen denominated bond (resulting in an attractive interest rate due to preferential tax treatment of Japanese zero coupon notes) then execute a dollar/yen currency swap such that its initial loan receipts and loan repayments are denominated in dollars. Thus, all of the borrower's net cash flows are denominated in dollars (it has synthesized a dollar loan) and the Japanese institution fulfills regulatory requirements by issuing a yen denominated note.

An equity swap is a contract providing for the delivery of cash flows associated with shares of equity in exchange for the cash flows associated with another asset (such as a debt or index instrument). Equity swaps permit investors to reduce their risk in an equity investment without actually selling shares. Most frequent participants in this market have been corporate managers. In a well-publicized case involving Autotote Company, the CEO arranged to deliver dividends and any capital gains (which would be negative in the event of a capital loss) associated with Autotote stock in exchange for certain cash flows associated with treasury securities. Thus, technically, the CEO did not sell his shares, though he divested himself of any of the return risk associated with share ownership. By engaging this equity swap, the CEO reduces his risk in the employing company without having to report a sale of shares. This means that the CEO is not subject to capital gains taxes at the time of the transaction.[1] The CEO is not required to report to the SEC a transaction as an insider, or bear the selling price consequences associated with an insider sell transaction. Furthermore, the CEO maintains his voting control in the company's shares. Thus, in a sense, the equity swap permits the CEO the opportunity to, in effect, execute a sale of shares without bearing the undesirable consequences associated with the sale.

An alternative to the collar unwinding strategy discussed earlier is the equity swap, a contract providing for the delivery of cash flows associated with shares of equity in exchange for the cash flows associated with another asset (usually a treasury or stock index instrument). Equity swaps also permit managers to reduce their share risk without actually selling shares. In a well-publicized case involving Autotote Company, the CEO arranged to deliver dividends and any capital gains (which would be negative in the event of a capital loss) associated with Autotote stock in exchange for certain cash flows associated with treasury securities. Thus,

[1]This tax benefit may be eliminated by the IRS in the future.

technically, the CEO did not sell his shares, though he divested himself of any of the return risk associated with share ownership. By engaging this equity swap, the CEO reduces his risk in the employing company without having to report a sale of shares. This means that the CEO is not subject to capital gains taxes at the time of the transaction.[2] The CEO was not required to report to the SEC a transaction as an insider, or bear the selling price consequences associated with an insider sell transaction. Furthermore, the CEO maintained his voting control in the company's shares. Thus, in a sense, the equity swap permits the CEO the opportunity to, in effect, execute a sale of shares without bearing the undesirable consequences associated with the sale.

[2]This tax benefit may be eliminated by the IRS in the future.

E. Other Equity Derivatives

Primes and Scores

Primes and scores are securities which, in effect, represent a division of stock cash flows into their primary components: dividends and capital gains. The securities are created by a trust which purchases the stock and issues units, each consisting on one prime and one score. The owner of the prime receives all dividend income and share value up to a specified level X (typically set somewhat above the stock price at the creation of the trust) at the termination of the trust (typically five years after its creation). The owner of the score receives any price appreciation above X at trust termination. A study by Jarrow and O'Hara [1989] indicated that the sum value of primes and scores exceeded the underlying stock prices. This excess seemed to exceed transactions costs. The market for these securities was substantially reduced when a 1986 tax regulation amendment effected subjected these trusts to triple taxation (the corporate level, the trust level and the personal level).

Preferred Equity Redemption Cumulative Stock

Preferred equity redemption cumulative stock (PERCS) is essentially preferred stock that automatically converts into shares of common stock (unless called by its issuer) at the end of a specified period. The issuer will usually set a cap such that if the stock trades above that price, the preferred shareholder will receive less than one share of common for each share of preferred. These issues may seem more attractive when common stock dividend yields are low relative to prevailing interest rates.

ADRs

As we discussed earlier, American Depository Receipts (ADRs) are shares issued by banks evidencing ownership of shares of foreign company stock which are held by the bank. ADRs are the American version of a more general security known as an IDR, International Depository Receipt. Many ADRs are traded over-the-counter, but some meeting various regulatory and reporting requirements are exchange listed. Among the ADRs listed on the NYSE are Telefonos de Mexico, Glaxo, British Telecom and Royal Dutch Petroleum, all of which are among the most active securities on the exchange. These are level three ADRs, meaning that they conform to U.S. accounting standards and file 20-F statements (similar to 10-K reports) in English with the SEC. Other ADRs, such as Nestlé and Volkswagen, are level one, which trade over the counter and do not comply with U.S. accounting standards.

ADRs were originally developed by J.P. Morgan in 1927 to facilitate U.S. investment in Selfridge's , a British retailer. ADRs are priced in dollars, though the underlying shares are still subject to currency risk. Over 1700 trade in U.S. markets as of 1997.

Sponsored ADRs are those creation has been facilitated by the underlying company. Sometimes, these ADRs are referred to as ADS, or American Depository Shares. Foreign companies may wish to cross-list their securities in the U.S. to create a broader secondary market and improve liquidity for existing shares, particularly for American investors. ADRs may also increase visibility among the company's customers, suppliers and creditors. The empirical evidence on the impact of cross listing of is mixed. For example, Alexander, Eun and Janakiramanan [JFQA 1988] found that foreign firms cross-listing in the U.S. experienced

positive returns. Howe and Kelm [FM 1987] found negative returns associated with U.S. firms cross-listing outside of the U.S.

Exotic Options

As described earlier, options which confer the right but not obligation to buy or sell an asset at a pre-specified price on or before a given date. We discussed earlier the "plain vanilla" options with the most simple terms. A variety of other options exist to meet a variety of investor needs:

An *Asian Option* (average rate) has a payoff function that is based on the average price (or average exchange rate) of the underlying asset (or currency). For example, an Asian call on currency permits its owner to receive the difference between the average currency exchange rate over the life of the option (A_T) and the exercise price (E) associated with the option: $C_{A,T}$ = MAX(0, A_T-E) and $p_{A,T}$ = MAX(0, E-A_T) where A_T - $\Box S_t/n$. A potential user of the Asian option might be an importer who purchases from a particular country the same number of units of its resource each day. For example, an electric utility company purchasing oil from Mexico each day using pesos may wish to use Asian options to reduce its currency risk. Since the exchange rate will vary daily, the oil expenses incurred by the oil importer will vary. The Asian option helps enable the exporter to stabilize its cash flows without entering the derivatives market on a daily basis. The cash flow structures of these options vary from contract to contract. For example, some contracts call for the payoff to be related to the difference between the time T spot rate and the average exchange rate realized during the life of the option. The payoff functions for the Average Strike Price options are: $C_{A,T}$ = MAX(0, S_T -A_T) and $p_{A,T}$ = MAX(0, A_T-S_T) where S_T is the spot rate prevailing at the time the option expires.

A *Lookback Option* enables its owner to purchase (or sell in the case of a put) the underlying currency at the lowest rate (or highest rate in the case of a put) realized over the life of the option. The payoff function for lookback options might be $C_{A,T}$ = MAX(0, S_T -S_{MIN}) and $p_{A,T}$ = MAX(0, S_{MAX}-S_T) where S_{MIN} and S_{MAX} are the minimum and maximum exchange rates realized over the lives of the contracts.

As discussed earlier, the *Zero Cost Collar* is a package of options designed to require zero net investment. Typically, the collar consists of a package with a long position in a put enabling its owner to sell the underlying security if its price drops to a specified price along with a short position in a call whose exercise price is set so that it exactly offsets what is paid for the put. Hence, such a collar requires no net investment. Similarly, the *Range Forward Contract* enables (and obliges) its owner to purchase the underlying security with a time T value for the following price: (X_1 if S_T>X_1; S_T if X_1>S_T>X_2; or X_2 if S_T<X_2).

A *Barrier Option* is similar to a "plain vanilla" option except that it expires or is activated (in the case of a down-and-out option, or, in the case of down-and-in options can only be activated) once the underlying asset value reaches a pre-specified price. These are often referred to as either *knock out* or *knock in* options.

Other exotics include the *Compound Option,* which is simply an option on an option. The *Rainbow Options* are written on two or more assets. A rainbow call may give its owner the right

to choose between any of two or more assets. An *Interest Rate Cap* pays its owner a value based on the difference between the market rate and the cap strike rate if the market rate rises above the strike rate. A *Swaption* gives its owner the right (but not the obligation) to enter into a swap arrangement at a later date.

Exercises

1. The Franklin Company issued 5000 convertible bonds with $1000 par value, in December 2007. Each bond is convertible into 200 shares of Franklin common stock on the maturity date, December 2010. Budge currently has 1 million shares of common stock outstanding. If at maturity the bondholders will receive $1000 in principal plus $80 in interest if they do not convert. How much must the total assets of the firm be worth in December 2010 to make conversion worthwhile for the bondholders?

Solutions

1. The value of receiving principal and interest on the bond equals:

#of conv bonds * (principal + interest) = 5000 * (1000 + 80) = 5,400,000.

The value of converting into stock equals the proportion of assets owned if bonds are converted times the Value of Assets:

$$\text{conv. Value} = \frac{(\text{shs./bond})*(\#\text{bonds})*\text{Asset Value}}{(\text{shs./bond})*(\#\text{bonds}) + \text{Current shs.}}$$

Now, solve for the indifference Value of Assets when the bonds' conversion value is 5,400,000:

$$5{,}400{,}000 = \frac{(200)*(5000)*\text{Asset Value}}{(200)*(5000) + 1{,}000{,}000}$$

Thus, Asset Value is **10,800,000**. So if Franklin's Value of Assets is more than 10,800,000, the bonds should be converted.

REFERENCES

Bettis, Carr, John Bizjak and Michael Lemmon, 2001. "Managerial Ownership, Incentive Contracting, and the Use of Zero-Cost Collars and Equity Swaps by Corporate Insiders," *Journal of Financial and Quantitative Analysis*, no. 3.