

Solutions to Exercises From “Introduction to Industrial Organization”

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■ **1.1*** Empirical evidence from a sample of more than 600 U.K. firms indicates that, controlling for the quantity of inputs (that is, taking into account the quantity of inputs), firm output is increasing in the number of competitors and decreasing in market share and industry concentration.² How do these results relate to the ideas presented in the chapter?

Solution: In Section 1.2, we argued that one of the implications of market power is the decline of productive efficiency. Controlling for input levels, the level of output is a measure of productive efficiency. The number of competitors and the degree of concentration are measures of the degree of competition (concentration is an inverse indicator). The empirical evidence from U.K. firms is therefore consistent with the view presented in the text.

As to the third explanatory variable (market share), see the discussion in Chapter 9.

■ **2.1** “A price-taking firm selling in a market with a price greater than the firm’s average cost should increase its output level.” Comment.

Solution: In a competitive market, firms are price takers; optimal output is such that price equals marginal cost (or marginal revenue equals marginal cost). It is perfectly possible that price be equal to marginal cost and greater than average cost. In fact, if price is greater than the minimum of average cost, then the optimal output is such that price is greater than average cost. In summary, the sentence is wrong.

¹I am grateful to Critian Dezso (New York University) for excellent assistance in preparing these solutions.

²Stephen J. Nickell, “Competition and Corporate Performance,” *Journal of Political Economy* **104** (1996), 724–746.

■ **2.2*** Consider the following values of the price elasticity of demand:

	Cigarettes	0.5
zinha	U.S. luxury cars in U.S.	1.9
	Foreign luxury cars in U.S.	2.8

Based on these values, provide an estimate of the impact on revenues of a 10% increase in the price of each of the above three products.

Solution: Revenue is given by

$$R = PQ$$

Taking the derivative with respect to P and rearranging, we get

$$\begin{aligned}\frac{dR}{dP} &= Q + P \frac{dQ}{dP} \\ &= Q + Q \frac{P}{Q} \frac{dQ}{dP} \\ &= Q(1 - \epsilon),\end{aligned}$$

where

$$\epsilon \equiv -\frac{dQ}{dP} \frac{P}{Q}$$

is the price elasticity of demand (see page 17). It follows that a 10% increase in price implies an increase in revenues from cigarette sales given by $10(1 - .5) = 5\%$. In the case of U.S. luxury cars and foreign luxury cars, a 10% price increase would lead to a decrease in revenues of -9% and -18%, respectively.

■ **2.3** You own and operate a facility located in Taiwan that manufactures 64-megabit dynamic random-access memory chips (DRAMs) for personal computers (PCs). One year ago you acquired the land for this facility for \$2 million, and used \$3million of your own money to finance the plant and equipment needed for DRAM manufacturing. Your facility has a maximum capacity of 10 million chips per year. Your cost of funds is 10% per year for either borrowing and investing. You could sell the land, plant and equipment today for \$8 million; you estimate that the land, plant, and equipment will gain 6% in value over the coming year. (Use a one-year planning horizon for this problem.)

In addition to the cost of land, plant, and equipment, you incur various operating expenses associated with DRAM production, such as energy, labor, raw materials, and packaging. Experience shows that these costs are \$4 per chip, regardless of the number of chips produced during the year. In addition, producing DRAMs will cause you to incur fixed costs of \$500,000 per year for items such as security, legal, and utilities.

(a) What is your cost function, $C(q)$, where q is the number of chips produced during the year?

Assume now that you can sell as many chips as you make at the going market price per chip of p .

(b) What is the minimum price, p , at which you would find it profitable to produce DRAMs during the coming year?

Solution:

- (a) The \$5 million you originally spent for the land, plant, and equipment is a sunk expenditure and thus not an economic cost. However, there is a “user cost of capital” associated with the land, plant and equipment, based on its current market value of \$8 million and your cost of funds and the rate of depreciation or appreciation of the asset over the planning horizon. Your (opportunity) cost of investing \$8 million for one year is \$800,000, but these assets will appreciate by \$480,000 over the year, giving a (net) user cost of capital of \$320,000. (The depreciation rate is 6%.) This is a fixed cost of making DRAM's, to which we must add the other fixed costs of \$500,000 to get a combined fixed cost of \$820,000 for the year. The variable costs are a constant \$4 per chip, so the cost function is $C(Q) = 820,000 + 4Q$, in the range of $0 < Q < 10,000,000$. (One could also report that $C(0) = 0$, by definition, and that $C(Q)$ is infinite for $Q > 10,000,000$, since your maximum capacity is ten million chips per year. Of course, in practice there would likely be a way to push production beyond “rated capacity,” at some cost penalty, but that is beyond the scope of this problem.)
- (b) The average cost function is $AC(Q) = 820,000/Q + 4$, again up to ten million chips per year. This declines with Q , so the minimum AC is achieved at full capacity utilization. At ten million chips per year, the fixed costs come to \$0.082 per chip, so average costs are \$4.082 per chip. This is your minimum average cost, and thus the minimum price at which it makes sense to stay open for the year.

■ **2.4** Consider the following 1988 data on the costs of a Sprinter (Class 150/2) train:³

Capital cost	525,000
Annual costs (per unit)	
Depreciation (20 years)	26,300
Overhaul and maintenance	32,600
Stabling and cleaning	9,400
Total annual cost of	
2 drivers	20,200
2 guards	15,600
Mileage costs of rolling stock (per unit mile)	
Maintenance	0.15
Fuel	0.126

³Source: Data provided by British Rail to the Mergers and Monopolies Commission.

(Notes: (a) Annual costs assume a 90,000 mile benchmark annual use. (b) There are 145 seats on the train.)

Based on these numbers, answer the following questions:

- (a) What is the average cost per train mile?
- (b) What is the average cost per passenger mile? (Note: the average number of passengers during this time period was 45.)
- (c) What is the marginal cost per train mile?
- (d) What is the marginal cost per passenger mile?

Solution:

- (a) Fixed costs are $26,300+32,600+9,400+20,200+15,600=104,100$. (Note: the capital cost should not be included in the yearly cost, only its depreciation.) Average variable cost per train mile is constant at $.15+.126=.276$ per train mile. It follows that average cost per train mile is $104,100/m+.276$, where m is the number of miles. Using the benchmark of 90,000 miles, this comes down to $1.157+.276=1.433$. As these number suggests, this is a capital-intensive, strong-scale-economies technology.
- (b) $1.433/45=.032$ (approximately).
- (c) Average variable cost per train mile is constant(see part a), thus equal to marginal cost: $.126$
- (d) $.276/45=.0061$ (approximately).

■ **2.5** You are considering opening your own restaurant. To do so, you will have to quit your current job, which pays \$46k per year, and cash in your life savings of \$200k, which have been in a certificate of deposit paying 6% per year. You will need this \$200k to purchase equipment for your restaurant operations. You estimate that you will have to spend \$4k during the year to maintain the equipment so as to preserve its market value at \$200k. Fortunately, you own a building suitable for the restaurant. You currently rent out this building on a month-by-month basis for \$2500 per month. You anticipate that you will spend \$50k for food, \$40k for extra help, and \$14k for utilities and supplies during the first year of operations. There are no other costs involved in this business.

What are the economic costs of operating the restaurant during the first year? In other words, what level of revenues will you need to achieve in the first year to make the first year profitable in an economic sense?

Solution: There are three opportunity costs:

1. The salary you could earn if you do not quit: \$46k.
2. The interest income your savings could earn if you do not cash in: $\$200k \times 0.06 = \$12k$.

3. The rent your building could earn if you do not use it for your restaurant: $\$2.5k \times 12months = \$30k$.

There are four direct costs:

1. Maintaining the equipment: \$4k.
2. Food: \$50k.
3. Hiring extra help: \$40k.
4. Utilities and supplies: \$14k.

Note that the \$200k cost of the equipment is not an economic cost because it is essentially reversible. That is, you can always sell the equipment for its current market value as long as you maintain it. Only the interest you would have earned on the money tied up in the equipment and the cost to maintain it are economic costs.

Adding up opportunity and direct costs yields \$196k. This is the break-even revenue for first year of operations.

■ **2.6** Eurotunnel, the company that owns the tunnel linking England and France, earned an operating profit of £46 million during the first semester of 1998. However, subtracting interest payments (mainly from the construction of the tunnel), its bottom line was a loss of £130 during the same period.⁴ Is it optimal to continue operating the tunnel, given all these losses?

Solution: The interest payments correspond to a cost (building the tunnel) that is sunk (literally!). It should therefore not be taken into consideration in the decision of whether or not to continue operations. However, if bankruptcy is a viable option for the owners of Eurotunnel, and if the situation is expected to remain the same (operating profit less than interest payments), then the optimal option is to declare bankruptcy.

■ **2.7*** 1998 was a turning point for Old McDonald's farm. Until then, the farm produced unprocessed tomato exclusively, selling its 100,000t for a profit margin of \$2.1/t. In January 1998, however, Old McDonald decided to start exporting processed tomato (tomato pulp) to Europe. At that time, the price of tomato pulp was \$6/t. In order to produce tomato pulp, Old McDonald bought a machine capable of processing 100,000t per year. The machine cost \$200,000 and was paid for with retained earnings that had been earning an 8% rate of return. This machine has a useful lifetime of 2 years. The market value of this machine drops to \$50,000 after one year of use (and zero after two years of use). In addition to the machine cost, there is a \$2.2/t harvesting and processing cost (mostly labor cost).

- (a) Determine Old McDonald's average cost, marginal cost, and profit margin.

⁴The Wall Street Journal Europe, September 22, 1998.

A few months later, things turned bad for Old McDonald. In December 1998, the European Union increased its tariffs on imported tomato pulp, implying that the net price received by American exporters is now only \$5/t. It is not expected that this price will change in the future. One accountant consulting for Old McDonald stated that as margins have declined drastically the farmer had better sell the machine right away and go back to producing unprocessed tomato. Old McDonald is trying to decide whether to take this consultant's advice.

(b) What would you advise Old McDonald to do?

(c) Would your advice change if the price of unprocessed tomato were expected to be \$0.50/t higher than described above? Explain why or why not.

Solution:

- (a) The user cost of capital corresponding to the machine is given by 8% times \$200,000 plus $(200,000 - 50,000)$, or simply \$166,000. Divided by 100,000t this gives \$1.66/t. Adding labor costs of \$2.2/t, this gives a total of \$3.86/t, the average cost. Marginal cost is \$2.2/t up to 100,000t/year, infinity thereafter. The profit margin is therefore $\$6 - \$2.2 = \$3.8/t$ (up to 100,000t).
- (b) We are considering the option of continuing to produce tomato pulp versus the option of producing unprocessed tomato. There are two opportunity costs that need to be accounted for. First, by selling tomato pulp the farmer is foregoing the chance of selling unprocessed tomato. This opportunity cost amounts to the the margin on unprocessed tomato, or \$2.1/t. The second opportunity cost is that of the machine — the user cost of capital. Since the machine is now worth only \$50,000 and will last for one more year, the user cost of capital is given by 50,000 plus 8% times 50,000 plus, or \$54,000, which corresponds to \$.54/t. The average economic profit, that is, including all imputed costs is, \$5 (price) - 2.2 (labor) - .54 (cost of capital) - 2.1 (margin on unprocessed tomato) = \$.16. Since this is positive, the firm should continue operating the machine and sell tomato pulp.
- (c) By a calculation analogous to the one above, we conclude that the farmer is better off by switching to unprocessed tomato.

■ **2.8*** Las-O-Vision is the sole producer of holographic TVs, 3DTVs. The daily demand for 3DTVs is $D(p) = 10200 - 100p$. The cost of producing q 3DTVs per day is $q^2/2$ (note this implies that $MC = q$).

(a) What is Las-O-Vision's total revenue schedule?

(b) What is Las-O-Vision's marginal revenue schedule?

(c) What is the profit-maximizing number of 3DTVs for Las-O-Vision to produce each day? What price does Las-O-Vision charge per 3DTV? What is its daily profit?

Solution:

- (a) Total Revenue is given by $p(x) \cdot x$, that is, the revenue that Las-O-Vision receives when it sells x units. To get $p(x)$, we invert the demand function $x = 10,200 - 100p$ by solving for p in terms of x , or $p(x) = 102 - x/100$. Substituting this into our total revenue equation, we obtain $TR(x) = (102 - x/100) \cdot x = 102x - x^2/100$.
- (b) Marginal revenue is the derivative of Total Revenue with respect to x , so $MR(x) = 102 - x/50$; or, since our demand equation is linear in x , we can obtain it by recalling that the marginal revenue curve is twice as steep as the inverse demand curve and starts at the same point on the vertical axis.
- (c) The profit maximizing quantity, x^* is that quantity at which marginal cost and marginal revenue are equal. Setting $MR(x) = MC$, we have $102 - x^*/50 = x^*$, or $x^* = 100$. The profit maximizing price is that which generates $x^* = 100$ in sales or, substituting into the inverse demand function calculated in (a), $p(100) = 102 - (100/100) = 101$. When selling 100 units, Las-O-Vision generates Total revenues equal to $TR(100) = 102 \cdot 100 - 100^2/100 = \$10,100$. Its total cost is $100^2/2 = 5000$. Therefore its total profit when it sells 100 units is $10,100 - 5000 = \$5,100$.

■ **2.9*** You own a private parking lot near U.C. Berkeley with a capacity of 600 cars. The demand for parking at this lot is estimated to be $Q = 1000 - 2p$, where Q is the number of customers with monthly parking passes and p is the monthly parking fee per car.

- (a) Derive your marginal revenue schedule.
- (b) What price generates the greatest revenues?

Your fixed costs of operating the parking lot, such as the monthly lease paid to the landlord and the cost of hiring an attendant, are \$25,000 per month. In addition, your insurance company charges you \$20 per car per month for liability coverage, and the City of Berkeley charges you \$30 per car per month as part of its policy to discourage the use of private automobiles.

- (c) What is your profit-maximizing price?

Solution:

- (a) Solving for p gives $p = 500 - Q/2$. Using the “twice-the-slope” formula for marginal revenue associated with a linear demand curve, we then have $MR = 500 - Q$.
Alternatively, one could directly write down the revenue function, $R(Q) = p(Q) \cdot Q$, and plug in for $p(Q) = 500 - Q/2$ to get $R(Q) = (500 - Q/2)Q = 500Q - Q^2/2$, then differentiate with respect to Q to get $MR(Q) = 500 - Q$.
- (b) Revenues are maximized when marginal revenues equal zero. Setting $MR = 0$ gives $500 - Q = 0$, or $Q = 500$. Then solving for price using the demand curve gives $p = 250$.
- (c) The (monthly) cost function here is $C(Q) = 25,000 + 50Q$. Marginal cost per car is simply \$50. Setting $MR = MC$ gives $500 - Q = 50$, or $Q^* = 450$. Using the demand curve to solve for the price that goes along with this quantity gives $p^* = \$275$.

To confirm that this is indeed the profit-maximizing price, you also should check that it is not optimal to shut down, i.e., that your economic profits are positive in comparison with shutting down. This can be done by directly calculating profits, which are given by $\pi^* = p^*Q^* - C(Q^*) = \$275(450) - \$50(450) - \$25,000 = \$76,250$. Another way to check profitability is to calculate the “contribution to fixed costs” generated by your customers. This contribution is \$225 per customer times 450 customers, or \$101,250, which easily exceeds the fixed costs of \$25,000 per month.

■ **2.10***** You are one of two companies bidding to try to win a large construction project. Call your bid B . You estimate that your costs of actually performing the work required will be \$800k. You are risk neutral.⁵ You will win if and only if your bid is lower than that of the other bidder. You are not sure what bid your rival will submit, but you estimate that the rival’s bid is uniformly distributed between \$1m and \$2m.⁶ What bid should you submit?

Solution: A risk-neutral bidder will use a bidding strategy that maximizes the expected value of its bid B . This entails picking a bid value B that balances two offsetting effects—changes in the value of winning due to changes in the bid (the larger your bid is the more valuable the contract is) and changes in the chances of winning due to changes in B (the larger your bid is the less likely you are to win). Formally, the expected value of a bid B can be expressed $E[B] = (B - 800,000) \cdot \text{Prob}(B < B_r)$, in which B_r is the rival bidder’s bid and $\text{Prob}(B < B_r)$ is the probability that its bid, B , is less than its rival’s bid. The first term in this equation is simply the payoff when a bidder wins. The second term is its chances of winning (which requires that $B < B_r$).

In this problem, the focal bidder believes that its rival’s bid can be anywhere between \$1m and \$2m so $\text{Prob}(B < B_r) = 1 - (B - 1,000,000)/1,000,000$ for all bids between bid \$1m and \$2m ($\text{Prob}(B < B_r) = 1$ for all bids less than \$1m since it believes that the rival never bids below \$1m and $\text{Prob}(B < B_r) = 0$ for all bids greater than \$2m since it believes that the rival never bids above \$2m). Substituting this expression into the expected value of the bid B we obtain: $E[B] = (B - 800,000)[1 - (B - 1,000,000)/1,000,000]$.

From this expression it is clear that the bidder’s payoff goes up with B but that its chance of winning declines with B . Picking the optimal B entails finding the maximum of $E[B]$, which we can easily obtain by taking the derivative of $E[B]$, setting it equal to zero and solving for B^* . This bid will be the point at which the two effects of changing B just offset each other. Dropping the zeros we have

$$\frac{\partial}{\partial B} E[B] = (1 - (B^* - 1)) - (B^* - .8) = 0$$

⁵We say that an agent is risk neutral if he or she is indifferent between receiving 100 for sure and receiving 0 or 200 with probability 50% each. More generally, a risk-neutral agent only cares about the expected value of each outcome.

⁶By “uniformly distributed between a and b ” we mean that all values between a and b are equally likely.

$$\begin{aligned} B^* - .8 &= 1 - B^* + 1 \\ B^* &= 1.4\text{m} \end{aligned}$$

An alternative approach to this problem is to construct a demand function from the information you have about the market. You can then solve the problem in the same way as you would with more straightforward problems in which you are given an explicit demand function (i.e., set $MR = MC$ and solve for Q^* , then solve for B^*). To see this approach, note that the bid the firm submits is just like a price. The higher its bid, the lower its expected demand will be. In this case, demand falls as the price goes up because the firm's chance of winning is falling. Formally, expected demand, Q , at any level B is equal to $Q = 1 - \text{Prob}(B < B_r) = 1 - (B - 1,000,000)/1,000,000 = 2 - B/1,000,000$. As this equation indicates, when the firm's bid is equal to 1,000,000 demand will be 1 unit. That is, the firm is sure to win the contract. As its bid (the price) increases, demand falls to some fraction of a unit until at 2,000,000 demand is zero. Since the contract is a winner take all item, the idea of fractional units is not really correct, but if there were say N consumers instead of a single consumer, and the firm was bidding against other firms for the business of each consumer, the aggregate demand function would then be $Q_N = N(2 - B/1,000,000) = 2N - B(N/1,000,000)$. This is like a simple linear demand function.

To continue with this approach, we need to invert the demand function and solve for B to get $B = 2,000,000 - 1,000,000 \cdot Q$. The bidder's total revenue is then $BQ = (2,000,000 - 1,000,000 \cdot Q)Q$. Taking the derivative of this total revenue function, we find that the marginal revenue of the firm is $2,000,000 - 2 \cdot 1,000,000 \cdot Q$. As we would expect, the marginal revenue curve has twice the slope of the inverse demand curve. We can then set this marginal revenue equal to the marginal cost of 800,000 to get Q^* :

$$\begin{aligned} 800,000 &= 2,000,000 - 2 \cdot 1,000,000 \cdot Q \\ 2 \cdot 1,000,000 \cdot Q &= 1,200,000 \\ Q^* &= 1.2/2 = .6 \end{aligned}$$

Substituting this value into our inverse demand function, we obtain the optimal bid of $B(.6) = 2,000,000 - (.6)1,000,000 = 1,400,000$.

■ **3.1** Explain why the assumption of profit maximization is or is not reasonable?

Solution: The answer to this question is given by Section 3.1 in the book. The main reason why we might think that the assumption of profit maximization is not reasonable is that the firm managers are frequently not the firm owners; and the goals of managers frequently

differs from those of the owners. However, it can be argued that the discipline imposed by the shareholders, the labor market, the product market and the capital market are sufficient to enforce profit maximization. In particular, the threat of a takeover has been found to have a significant effect on value maximization.

■ **3.2** Should firms have their own catering services or should they outsource it? What are the main trade-offs? Are there other alternatives in addition to “make or buy”?

Solution: The answer to this question is given by Section 3.2 in the book.

■ **3.3** Two parts in an automobile taillight are the plastic exterior cover and the light bulb. Which of these parts is a car company more likely to manufacture in-house? Why?

Solution: Light bulbs are a generally used homogeneous good. External suppliers enjoy economies of scale and specialization and supply the entire industry. In contrast, the plastic exterior cover must be custom-designed and manufactured for each make and model. Because it requires more Relationship Specific Investment (RSI), it is more likely to be made in-house.

■ **3.4** There are three main suppliers of commercial jet engines, Pratt & Whitney, General Electric, and Rolls-Royce. All three maintain extensive support staff at major (and many minor) airports throughout the world. Why doesn't one firm service each airport? Why do all three feel they need to provide service and support operations worldwide themselves? Why don't they subcontract this work? Why don't they leave it entirely to the airlines?

Solution: Jet engines are marvelously idiosyncratic. The knowledge, tools and parts needed to service one family (brand) of engines do not transfer fully across brands. One firm does not typically service each airport because the economies of scale (across brands) are small and the economies of specialization (within brand) are large. The only thing worse for an airline than an AOG (an aircraft sitting on the ground with a broken engine) is an aircraft flying with a broken engine or two. To ensure their reputation and revenues and to avoid ex post hold up, airlines demand before purchasing an aircraft that engine makers pre-commit capital to ensure that parts and service are available at major stations worldwide. Because the skills to do this are RSIs, and because the engine owner's reputation is

at stake, to sell engines and credibly commit to keeping them running, each manufacturer must provide service and support at major stations.

Subcontracting would be difficult because of the RSI required (the subcontractor would fear hold-up) and because a poor subcontractor would impose a negative externality on the manufacturer. When the jet goes down, the manufacturer's reputation will suffer on a scale beyond any contractual penalty a subcontractor could likely be held to, so the work is not usually subcontracted. In addition, the manufacturers benefit directly from direct feedback within the firm on the performance of the engines they produce. This information may flow more readily within the firm than across firms.

Some airlines with sufficient scale do perform their own routine engine maintenance at their own maintenance bases. However, the airlines cannot efficiently do emergency engine repairs away from an airline's main bases. While there are enough GE engines going through Karachi International Airport to justify an on-site GE technical support staff, most airlines do not have enough flights through Karachi to justify the investment. The economies of scale in non-routine work are site and engine specific, not generally airline specific.

■ **3.5** The Smart car was created as a joint venture between Daimler-Benz AG and Swatch Group AG. Although Micro Compact Car AG (the name of the joint venture) was originally jointly owned, in November of 1998 Daimler-Benz AG took complete control by buying Swatch's share.⁷ The deal put an end to a very stressed relationship between Daimler and Swatch. What does Section 3.2 suggest as to what the sources of strain might have been?

Solution: Section 3.2 suggests that, when two parties invest in specific assets and contracts are incomplete, the equilibrium solution is inefficient in every situation short of vertical integration. (See also the mathematical supplement corresponding to this section.) It is likely that some of this happened in the "stressed relationship" between Daimler and Swatch. Since none of the parties was in complete control (and ownership) of the future developments in the joint venture, the incentives for each party to invest were less than efficient.

■ **3.6** Why do television networks have a few "owned and operated" stations but work through independent affiliates in most geographic locations?

Solution: See Exercise 3.7.

⁷ *The Wall Street Journal Europe*, November 5, 1998.

■ **3.7** Empirical evidence from franchise retailing suggests that, even when stores have similar characteristics, the mother company resorts to a mix between company-owned stores and franchised ones.⁸ How can this be justified?

Solution: Franchisers face a problem in judging the performance of their franchisees. Keeping some retail locations in-house provides the parent company with a baseline of more readily accessible and less biased information against which the performance of the franchises can be measured. This information then helps to set standards in negotiating and administering future franchise contracts. Franchising the majority of retail locations limits the parent's direct financial outlay and exposure. Franchisers might also have an interest in direct control of locations that could have a particularly strong impact on its brand or reputation.

■ **3.8** The U.K. Body Shop franchise network consists of three types of stores: franchised, company owned and partnership stores. All stores that are distant from headquarters by more than 300 miles are franchised. More than half of the company-owned stores are within 100 miles of headquarters.⁹ How can you explain these facts?

Solution: Owning a store has the advantages of vertical integration discussed in Section 3.2. However, it also has the problem that it requires increased monitoring by the store owner. We would expect the costs from monitoring to be lower the closer the store is to headquarters. Consequently, we would expect vertical integration to be more likely when the store is located closer to headquarters. The empirical evidence seems consistent with this hypothesis.

■ **3.9** Explain why Intel has maintained, if not increased, its competitive advantage with respect to rivals. Indicate the explanatory power of the different causes considered in the text (impediments to imitation, causal ambiguity, strategy, history).

Solution: This is a complex question. In fact, as argued in this chapter, this is *the* question in strategy. A good source for the particular case of Intel is the HBS case "Intel Corporation: 1968–1997," No. 9–797–137 (Rev. October 21, 1998).

⁸See, for example,

AFFUSO, LUISA (1998), "An Empirical Study on Contractual Heterogeneity Within the Firm: The "Vertical Integration-Franchise Contracts" Mix," University of Cambridge..

⁹Source:

WATTS, CHRISTOPHER F (1995), "The Determinants of Organisational Choice: Franchising and Vertical Integration," M.Sc. dissertation, University of Southampton..

■ **3.10***** Suppose that a firm's profits are given by $\pi = \alpha + \phi(e) + \epsilon$, where α denotes the intensity of product market competition, e effort by the manager, and ϵ a random shock. The function $\phi(e)$ is increasing and concave, that is, $\phi' > 0$ and $\phi'' < 0$.

In order for the firm to survive, it must be that profits are greater than $\underline{\pi}$. The manager's payoff is $\beta > 0$ if the firm survives and zero if it is liquidated, that is, if profits fall short of the minimum target. The idea is that if the firm is liquidated, then the manager loses his job and the rents associated with it.

Suppose that ϵ is normally distributed with mean μ and variance σ^2 , and that $\mu > \underline{\pi}$. Show that increased product market competition (lower α) induces greater effort by the manager, that is, $\frac{\partial e}{\partial \alpha} < 0$.

Solution: The manager's payoff is given by

$$P = \beta \mathcal{P}(\alpha + \phi(e) + \epsilon > \underline{\pi}) - e,$$

where $\mathcal{P}(x > y)$ is the probability that $x > y$. Since ϵ is normally distributed, we have

$$P = \beta (1 - F(\underline{\pi} - \alpha - \phi(e))) - e,$$

where $F(x)$ is the probability that ϵ is less than x (cumulative distribution function). Taking the derivative with respect to e , the manager's choice of effort level, we get

$$\frac{dP}{de} = \beta f(\underline{\pi} - \alpha - \phi(e)) - 1,$$

where $f(x)$ is the density function of ϵ . Since $\mu > \underline{\pi}$, $\mu > \underline{\pi} - \alpha - \phi(e)$. Therefore $f(\underline{\pi} - \alpha - \phi(e))$ is in the increasing portion of f . It follows that an increase in α leads to a decrease in $f(\underline{\pi} - \alpha - \phi(e))$; and this, in turn, implies a lower $\frac{dP}{de}$. Finally, a lower $\frac{dP}{de}$ implies a lower value of e . In words, a decrease in the degree of competition (higher α) decreases the marginal benefit from managerial effort ($\frac{dP}{de}$), and ultimately leads to a lower effort of managerial effort (e).

■ **4.1** What are the assumptions regarding player rationality implicit in solving a game by elimination of dominated strategies? Contrast this with the case of dominant strategies.

Solution: When applying the iterated elimination of dominated strategies one implicitly assumes that each player is rational and believes that the other player is rational. With dominant strategies the only assumption needed is that players are rational, utility-maximizing agents, *regardless* of their beliefs about other players.

■ **4.2** The UK Office of Fair Trading has recently unveiled a plan that will offer immunity from prosecution to firms who blow the whistle on their co-cartel conspirators. In the U.S., this tactic has proven extremely successful: since its introduction in 1993, the total amount of fines for anti-competitive behavior has increased twentyfold.

Show how the tactic initiated by the U.S. Department of Justice and soon to be followed by the Office of Fair Trading changes the rules of the game played between firms in a secret cartel.

Solution: Prior to the introduction of the plan, each cartel firm would have two options: (a) to stick by the agreement or (b) to deviate and set lower prices. With the introduction of the plan, the firm has a third option: (c) to blow the whistle. Let α be the probability that the DOJ discovers the price conspiracy. High values of α imply a low expected value from (a). The same is true of (b), though probably to a lesser extent. Finally, (c) is invariant to the value of α . We would thus expect that, for high values of α , (c) is the best strategy.

With the introduction of the plan, the firms now play a second prisoner's dilemma type of game. Before, it was whether to price high or price low. Now, it's whether to blow the whistle or not. Firms would be better off if neither of them blew the whistle. However, if α is high, the blowing the whistle is a dominant strategy.

■ **4.3** Figure 1 represents a series of two-player games which illustrate the rivalry between Time magazine and Newsweek. Each magazine's strategy consists of choosing a cover story: "Impeachment" or "Financial crisis" are the two choices.¹⁰

The first version of the game corresponds to the case when the game is symmetric (Time and Newsweek are equally well positioned). As the payoff matrix suggests, "Impeachment" is a better story but payoffs are lower when both magazines choose the same story. The second version of the game corresponds to the assumption that Time is a more popular magazine (Time's payoff is greater than Newsweek's when both magazines cover the same story). Finally, the third version of the game illustrates the case when the magazines are sufficiently different that some readers will buy both magazines even if they cover the same story.

For each of the three versions of the game,

- (a) Determine whether the game can be solved by dominant strategies.
- (b) Determine all Nash equilibria.
- (c) Indicate clearly which assumptions regarding rationality are required in order to reach the solutions in (a) and (b).

Solution:

- (i) Impeachment is a dominant strategy for both players. It follows that (Impeachment, Impeachment) is the unique Nash equilibrium. All we need to assume to reach this conclusion is that players are rational and know their own payoffs.

¹⁰In each cell, the first number is the payoff for the row player (Time).

		Newsweek	
Time		Impeachment	Financial Crisis
	Impeachment	35, 35	70, 30
	Financial Crisis	30, 70	15, 15

(i) Time and Newsweek are evenly matched

		Newsweek	
Time		Impeachment	Financial Crisis
	Impeachment	42, 28	70, 30
	Financial Crisis	30, 70	18, 12

(ii) Time is more popular than Newsweek

		Newsweek	
Time		Impeachment	Financial Crisis
	Impeachment	42, 28	70, 50
	Financial Crisis	50, 70	30, 20

(iii) Some customers will buy both magazines

Figure 1: The cover-story game.

- (ii) Impeachment is a dominant strategy for Time, but not for Newsweek. Given that Time chooses Impeachment, Financial Crisis is the optimal choice for Newsweek. It follows that (Impeachment, Financial Crisis) is the unique Nash equilibrium. This solution assumes that Time is rational and knows its payoffs; and Newsweek is rational, knows the payoffs for both players, and believes Time is a rational player.
- (iii) There are no dominant strategies in this game. There are two Nash equilibria (in pure strategies): (Impeachment, Financial Crisis) and (Financial Crisis, Impeachment). In this context, the concept of Nash equilibrium presupposes that players know the payoffs of both players; moreover, it is common knowledge (I expect that you expect that I expect...) that the particular equilibrium will be played.

■ **4.4*** In the movie “E.T.,” a trail of Reese’s Pieces, one of Hershey’s chocolate brands, is used to lure the little alien out of the woods. As a result of the publicity created by this scene, sales of Reese’s Pieces trebled, allowing Hershey to catch up with rival Mars.

Universal Studio’s original plan was to use a trail of Mars’ M&Ms. However, Mars turned down the offer, presumably because it thought \$1m was a very high price. The makers of “E.T.” then turned to Hershey, who accepted the deal.

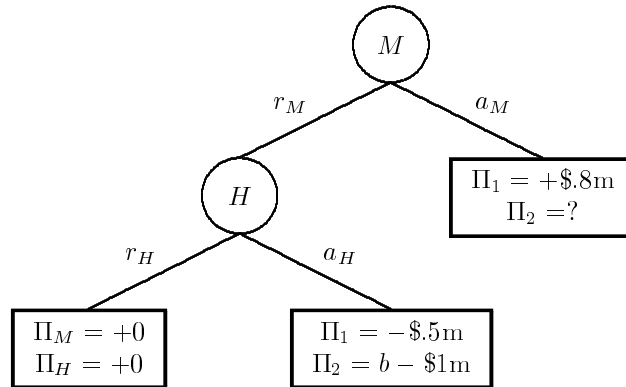


Figure 2: Mars vs Hershey. a_i and r_i signify acceptance and rejection by firm i , respectively.

Suppose that the publicity generated by having M&Ms included in the movie would increase Mars' profits by \$800,000. Suppose moreover that Hershey's increase in market share cost Mars a loss of \$500,000. Finally, let b be the benefit for Hershey's from having its brand be the chosen one.

Describe the above events as a game in extensive form. Determine the equilibrium as a function of b . If the equilibrium differs from the actual events, how do you think they can be reconciled?

Solution:

As can be seen from Figure 2, if $b > \$1,000,000$ then Hershey's equilibrium strategy is to accept the offer; likewise, Mars' equilibrium strategy is to accept the offer. If $b < \$1,000,000$, however, then the equilibrium strategies is for both firms to turn down the offer.

This differs from what actually happened (Mars rejected the offer, whereas Hershey accepted it). One possible explanation is that Mars underestimated either its own benefits from having M&Ms featured in the movie, or Hershey's benefits, or both.

■ **4.5** Hernan Cortéz, the Spanish navigator and explorer, is said to have burnt his ships upon arrival to Mexico. By so doing, he effectively eliminated the option of him and his soldiers returning to their homeland. Discuss the strategic value of this action knowing the Spanish colonists were faced with potential resistance from the Mexican natives.

		Japan	
		Low	High
U.S.	Low	4 3	2 4
	High	3 2	1 1

Figure 3: The HDTV game: each country chooses a high or a low level of R&D on HDTV.

Solution: By eliminating the option of turning back, Hernan Cortez established a credible commitment regarding his future actions, that is, to fight the Mexican natives should they attack. Had Cortez not made this move, natives could have found it better to attack, knowing that instead of bearing losses the Spaniards would prefer to withdraw.

■ **4.6** Consider the following game depicting the process of standard setting in high-definition television (HDTV).¹¹ The U.S. and Japan must simultaneously decide whether to invest a high or a low value into HDTV research. Each country's payoffs are summarized in Figure 3.

(a) Are there any dominant strategies in this game? What is the Nash equilibrium of the game? What are the rationality assumptions implicit in this equilibrium?

(b) Suppose now the U.S. has the option of committing to a strategy ahead of Japan's decision. How would you model this new situation? What are the Nash equilibria of this new game?

(c) Comparing the answers to (a) and (b), what can you say about the value of commitment for the U.S.?

(d) "When pre-commitment has a strategic value, the player that makes that commitment ends up 'regretting' its actions, in the sense that, given the rivals' choices, it could achieve a higher payoff by choosing a different action." In light of your answer to (b), how would you comment this statement?

Solution: (a) For the United States investing, a low value in HDTV research is a dominant strategy. The Nash equilibrium of the game is given by the U.S. choosing Low and Japan choosing High. The rationality assumptions implicit in this solution are that both players are rational and, moreover, Japan believes the U.S. acts rationally.

(b) See Figure 3. (See also Section 4.2.) By solving backwards, we get the following Nash equilibrium: U.S. chooses High, Japan chooses Low.

(c) Comparing the answers from a. and b. we can see that the value of commitment to the U.S. is 1 that is, 3 minus 2.

¹¹This exercise is adapted from

DIXIT, AVINASH K., AND BARRY J. NALEBUFF (1991), *Thinking Strategically*, New York: W W Norton..

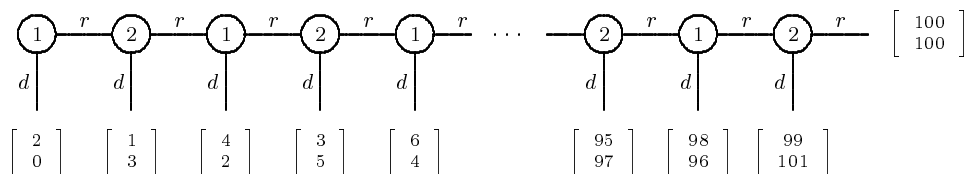


Figure 4: The centipede game. In the payoff vectors, the top number is Player 1's payoff, the bottom one Player 2's.

(d) *Given* that Japan chooses Low, the U.S. would be better off by choosing Low as well. However, it must be the case that the cost of switching from High to Low is so high that the U.S. won't do it (ex post). Otherwise, the commitment to stick to High would not be credible.

■ **4.7** Consider a one-shot game with two equilibria and suppose this game is repeated twice. Explain in words why there may be equilibria in the two-period game which are different from the equilibria of the one-shot game.

Solution: When the game is repeated twice the strategy space for each player becomes more complex. Each player's strategy specifies the action to be taken in period 1 as well as the action to be taken in period 2 *as a function of the outcome in period 1*. The possibility of linking period 2's actions to past actions allows for equilibrium outcomes that would not be attainable in the corresponding one-shot game (for example, the use of a 'punishment' action in period 2 if one of the players deviates from the designated period 1 payoff-maximizing action).

■ **4.8**** Consider the game in Figure 4.¹² Show, by backward induction, that rational players choose *d* at every node of the game, yielding a payoff of 2 for Player 1 and zero for Player 2. Is this equilibrium reasonable? What are the rationality assumptions implicit in it?

Solution: [IMPORTANT NOTE: there is a typo in the game tree: the payoffs in the second and third to last nodes should be increased by 2.]

¹²This game was first proposed by

ROSENTHAL, ROBERT (1981), "Games of Perfect Information, Predatory Pricing and the Chain-Store Paradox," *Journal of Economic Theory* **25**, 92–100.

Starting from the right-most node, we observe that Player 2's strategy, if that node is reached, is to play d , in which case its gets 101, whereas Player 1 gets 99. This implies that, in the second to last node, Player 1 is better off choosing d . In fact, by choosing r , Player 1 expects to get 99 (see sentence above) instead of 100 from d . And so forth. We conclude that the unique sub-game perfect Nash equilibrium is for each player to play d whenever it is called upon to make a move. The outcome of this equilibrium is Player 1 getting 2 and Player 2 getting 0.

Obviously, one might question whether this result is reasonable or not. Here, the implicit assumption is that each player is rational, believes that the other player is rational, believes that the other player believes that the first player is rational, and so forth.

To see how important this assumption is, suppose that Player 1 chooses r in the first period. Since this is not according to the equilibrium, Player 2 may not conjecture that Player 1 is not rational. But then choosing d may no longer be in Player 2's best interest. But then choosing r may be, after all, a rational strategy by Player 1 in the first place.

■ **5.1** “The degree of monopoly power is limited by the elasticity of demand.”

Comment.

Solution: Optimal monopoly pricing leads to the following relation between the price-cost margin and demand elasticity: $(p - MC)/p = 1/\epsilon$, where p is price, MC marginal cost, and ϵ demand elasticity. It follows that the greater the value of ϵ the lower the value of $(p - MC)$ and the lower monopoly profits. A monopolist facing a very elastic demand curve makes profits at the level of a competitive firm.

■ **5.2** A firm sells one million units at a price of \$100 each. The firm's marginal cost is constant at \$40, and its average cost (at the output level of one million units) is \$90. The firm estimates that its elasticity of demand is constant at 2.0. Should the firm raise price, lower price, or leave price unchanged? Explain.

Solution: Optimal monopoly pricing leads to the following relation between the price, marginal cost, and demand elasticity: $(p - MC)/p = 1/\epsilon$, where p is price, MC is marginal cost, and ϵ is the elasticity of demand. In this problem, we have $(p - MC)/p = (100 - 40)/100$ or 0.6, which is greater than $1/\epsilon = 1/2 = 0.5$. This tells us that the price/cost margin is too high, so a lower price (\$80) would be optimal. It would be a mistake to use AC rather than MC for the purposes of calculating the price/cost margin.

■ **5.3** A recent study estimates the long-run demand elasticity of AT&T in the period 1988–1991 to be around 10.¹³ Assuming the estimate is correct, what does this imply in terms of AT&T's market power?

Solution: A demand elasticity of 10 implies that AT&T's demand is very elastic. In fact, the author of the study that produced this estimate computes the welfare loss due to AT&T's market power to be less than 1% of sales volume.

■ **5.4** Sprint currently offers long-distance telephone service to residential customers at a price of 8c per minute. At this price, Sprint sells 200 million minutes of calling per day. Sprint believe that its marginal cost per minute of calling is 5c. So, Sprint's residential long-distance telephone service business is contributing \$6 million per day towards overhead/fixed costs.

Based on a statistical study of calling patterns, Sprint estimates that it faces a constant elasticity of demand for long-distance calling by residential customers of 2.0.

(a) Based on this information, should Sprint raise, lower, or leave unchanged its price?

(b) How much additional contribution to overhead, if any, can Sprint obtain by optimally adjusting its price?

Solution:

- (a) Given the elasticity of demand for long-distance, the optimal price is given by $p = MC\epsilon/(\epsilon - 1)$. The optimal price is thus $0.05 \cdot 2/1 = .10$. Sprint should raise its price from 0.08 to 0.10.
- (b) The demand curve in this case has constant elasticity. The general formula for demand with constant elasticity is $q = Qp^{-\epsilon}$, where A is a positive constant. We can find A in this problem by substituting p and Q ($p = 8$; $Q = 200$) into the formula. The result is that $A = 12,800$. Substituting the optimal value $p = .10$ into the above, $q = (12,800)(.10)^{-2}$, gives 128 million minutes a day.

The contribution to fixed cost is $128(.10 - .05) = \$6.4\text{m}$. Repricing yields higher profits of \$400,000 per day.

■ **5.5** After spending 10 years and \$1.5 billion, you have finally gotten Food and Drug Administration (FDA) approval to sell your new patented wonder drug, which reduces the aches and pains associated with aging joints. You will market this drug under the brand name of Ageless. Market research indicates that the elasticity of

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WARD, MICHAEL R. (1995), "Measurements of Market Power in Long Distance Telecommunications," Federal Trade Commission, Bureau of Economics Staff Report..

demand for Ageless is 1.25 (at all points on the demand curve). You estimate the marginal cost of manufacturing and selling one more dose of Ageless is \$1.

- (a) What is the profit-maximizing price per dose of Ageless?
- (b) Would you expect the elasticity of demand you face for Ageless to rise or fall when your patent expires?

Solution:

- (a) Our general markup rule states that $(p - MC)/p = 1/\epsilon$, where ϵ is the elasticity of demand facing the firm at the point on the demand curve at which the firm operates. With a constant elasticity of demand and constant marginal cost, as in this problem, we can use this formula to solve directly for the profit-maximizing price, p^* . Here we get $(p^* - 1)/p^* = 1/1.25$. Solving for the optimal price gives $p^* = \$5$. Equivalently, one can directly use the other version of the markup formula, $p = MC\epsilon/(\epsilon - 1)$, to get $p = 1 \times 1.25/(1.25 - 1)$, which again gives $p^* = \$5$. Of course, the R&D expenditures are now sunk and thus do not enter into the pricing decision.
- (b) The level of demand for Ageless must fall now that there are many very close substitutes in the form of generic versions. Hopefully, your brand will still allow you to command a premium price, but surely at any given price you will sell less as a result of the presence of the generic competition.

The elasticity of demand for Ageless will very likely rise now that closer substitutes are available. Customers will presumably be more price sensitive, and thus will induce you to set a lower price.

■ **5.6** Is the Windows operating system an essential facility? What about the Intel Pentium microprocessor? To what extent does the discussion in Section ?? on essential facilities (vertical integration, access pricing) apply to the above examples?

Solution: [Note: this is a very controversial question and not all economists agree on a single answer.] Both Microsoft (the producer of the Windows operating system) and Intel (the producer of the Intel Pentium microprocessor) provide computer makers with essential components, without which the machines could not function. Nevertheless, strictly speaking, we cannot say that their output represents an essential facility. The discussion in section 5.3 applies to monopolists. The crucial difference from the examples presented in the section is the fact that Microsoft and Intel are not monopolists: computer makers always have the option of switching to another provider of components.

However, the widespread use of the Windows operating system, and the fact that Windows is only supplied by Microsoft, implies that the latter's position is much closer to the one of a monopolist than is Intel's. Even though Intel's chip design is very close to being an industry standard, Intel is not the only company supplying microprocessors with that

designing. Hence, the Windows operating system is closer to what is called an essential facility than Intel's Pentium processor.

■ **6.1** The technology of book publishing is characterized by a high fixed cost (typesetting the book) and a very low marginal cost (printing). Prices are set at much higher levels than marginal cost. However, book publishing yields a normal rate of return. Are these facts consistent with profit maximizing behavior by publishers? Which model do you think describes this industry best?

Solution: The model of monopolistic competition is probably the best approximation to describing this industry. The model of monopolistic competition shows that price-making, profit-maximizing behavior is consistent with a zero-profit long-run equilibrium. The strong scale economies in book publishing imply that the gap between price and marginal cost is particularly high.

■ **6.2** The market for laundry detergent is monopolistically competitive. Each firm owns one brand, and each brand has effectively differentiated itself so that it has some market power (i.e., faces a downward sloping demand curve). Still, no brand earns economic profits, because entry causes the demand for each brand to shift in until the seller can just break even. All firms have identical cost functions, which are U-shaped.

Suppose that the government does a study on detergents and finds out they are all alike. The public is notified of these findings and suddenly drops allegiance to any brand. What happens to price when this product that was brand-differentiated becomes a commodity? What happens to total sales? What happens to the number of firms in the market?

Solution: Based on the information provided, it seems that the initial situation in this market is like the long-run equilibrium of the monopolistic competition model; see Figure 6.3. The government's announcement has turned a differentiated product into a homogeneous one. In terms of the graph in Figure 6.3, this implies a flattening of the demand curve faced by each firm and a new long-run equilibrium where d (now horizontal) is tangent to the AC curve. At this new long-run equilibrium, price is given by p'_{LR} and each firm's output is given by q'_{LR} .

Clearly, the new equilibrium implies a lower price and a higher output per firm: $p'_{LR} < p_{LR}$ and $q'_{LR} > q_{LR}$.

Suppose that price were to drop from p_{LR} to p'_{LR} *without changing the degree of product differentiation or the number of firms*. This would imply an output per firm equal to q'_{SR} , where q'_{SR} is greater than q_{LR} but lower than q'_{LR} . If we take into account the disappearance

of product differentiation (and continue with the same number of firms), then the output per firm would be less than q'_{SR} . Whatever the exact value is, each firm would be losing money ($p'_{LR} < AC$). Therefore, in the post-announcement long-run equilibrium, some firms will need to exit the market.

Finally, it is not clear what will happen to total output. On the one hand, each firm's output goes up. On the other hand, the number of firms goes down. Which effect dominates depends on how consumers value product differentiation and how the demand curve shifts as a result of the government announcement.

■ **6.3**** Show that, in a long-run equilibrium with free entry and equal access to the best available technologies, the comparison of price to the minimum of average cost or the comparison of price to marginal cost are equivalent tests of allocative efficiency. In other words, price is greater than the minimum of average costs if and only if price is greater than marginal cost.

Show, by example, that the same is not true in general.

Solution: We first show the following **fact**: *marginal cost is greater than average cost if and only if average cost is increasing*. To see this, notice that Average Cost is given by the ratio Cost / Output. Taking the derivative with respect to Output q , we get

$$\frac{d AC}{d q} = \frac{d}{d q} \frac{C}{q} = \frac{\frac{d C}{d q} q - C}{q^2} = (MC - AC)/q,$$

which shows the fact.

In the long-run equilibrium of an industry with equal access, each firm will be producing at a point in the left-hand portion of its Average Cost curve. Given the above fact, it follows that marginal cost is lower than or equal to average cost. Since there is free entry, price is equal to average cost. Specifically, either price is equal to the minimum of average cost and equal to marginal cost; or price is greater than the minimum of average cost and greater than marginal cost.

The same is not true, for example, in a short-run equilibrium. Consider the case of perfect competition. and suppose that price is greater than the minimum of average cost. Since firms are price takers, price is equal to marginal cost. So, the comparison price minus marginal cost is zero whereas price minus the minimum of average cost is positive.

■ **7.1** According to Bertrand's theory, price competition drives firms' profits down to zero even if there are only two competitors in the market. Why don't we observe this in practice very often?

Solution: Section 7.2 suggests three possible explanations: (a) product differentiation, (b) dynamic competition, (c) capacity constraints.

■ **7.2** Three criticisms are frequently raised against the use of the Cournot oligopoly model: (i) firms normally choose prices, not quantities; (ii) firms don't normally take their decisions simultaneously; (iii) firms are frequently ignorant of their rivals' costs; in fact, they do not use the notion of Nash equilibrium when making their strategic decisions.

How would you respond to these criticisms? (Hint: in addition to this chapter, you may want to refer to Chapter ??.)

Solution:

- (i) If firms are capacity constraint, then price competition “looks like” like quantity competition. See Section 7.2.
- (ii) If there are significant information lags, then sequential decisions “look like” simultaneous decisions. See Chapter 4 (first section).
- (iii) The last section of Chapter 7 presents an argument for the relevance of Nash equilibrium which only requires each firm to know its own profit function.

■ **7.3** Which model (Cournot, Bertrand) would you think provides a better approximation to each of the following industries: oil refining, internet access, insurance. Why?

Solution: Capacity constraints seem relatively more important in oil refining and relatively less important in insurance. Given the discussion in Section 7.4, one would be inclined to select the Cournot model for oil refining and the Bertrand model for insurance. Internet access is an intermediate case between the previous two.

■ **7.4*** Two firms, CS and LC, make identical goods, GPX units, and sell them in the same market. The demand in the market is $Q = 1200 - P$. Once a firm has built capacity, it can produce up to its capacity each period with a marginal cost of $MC = 0$. Building a unit of capacity costs 2400 (for either CS or LC) and a unit of capacity lasts four years. The interest rate is zero. Once production occurs each period, the price in the market adjusts to the level at which all production is sold. (In other words, these firms engage in quantity competition, not price competition.)

(a) If CS knew that LC were going to build 100 units of capacity, how much would CS want to build? If CS knew that LC were going to build x units of capacity,

how much would CS want to build (that is, what is CS's best response function in capacity)?

(b) If CS and LC each had to decide how much capacity to build without knowing the other's capacity decision, what would the one-shot Nash equilibrium be in the amount of capacity built?

Solution:

- (a) If LC builds 100 units of capacity, then CS faces a residual demand of $Q_{CS} = Q - 100 = 1100 - p$. Its marginal revenue (contribution) is then $MR_{CS} = 1100 - 2Q_{CS}$. Equating this marginal revenue with CS's capacity costs of 600 yields the optimal capacity for CS as $Q_{CS}^* = 250$ units.

The generalization of this is to solve for CS's residual demand as a function of LC's capacity Q_{LC} . That is, $Q_{CS} = Q - Q_{LC} = 1200 - Q_{LC} - p$. CS's total revenue is then equal to $TR_{CS} = pQ_{CS} = (1200 - Q_{LC} + Q_{CS})Q_{CS}$ and its marginal revenue can be obtained by taking the derivative of TR_{CS} with respect to Q_{CS} (treating Q_{LC} as a constant). This yields $MR_{CS} = 1200 - Q_{LC} - 2Q_{CS}$. Equating this marginal revenue to marginal cost and solving for Q_{CS} yields $Q_{CS} = 300 - Q_{LC}/2$ as CS's optimal capacity in response to any capacity decision by LC.

- (b) Since the two firms are symmetric, LC's best response to CS is analogous to CS's best response to LC, or $Q_{LC} = 300 - Q_{CS}/2$. A Nash equilibrium requires that $Q_{LC}^* = 300 - Q_{CS}^*/2$ and $Q_{CS}^* = 300 - Q_{LC}^*/2$. Substituting Q_{LC}^* into Q_{CS}^* and solving for Q_{CS}^* yields $Q_{CS}^* = 200$. Substituting this amount into the LC's best response function yields $Q_{LC}^* = 200$. At these capacities the market price is $p = 1200 - 200 - 200 = 800$. Each firm's profits are then $(800 - 600)(200) = \$40,000$.

■ **7.5*** Consider a market for a homogeneous product with demand given by $Q = 37.5 - P/4$. There are two firms, each with constant marginal cost equal to 40.

- Determine output and price under a Cournot equilibrium.
- Compute the efficiency loss as a percentage of the efficiency loss under monopoly.

Solution: (a) Duopolist i 's profit is given by

$$\pi_i = q_i p(Q) - C(q_i) = q_i [150 - 4(q_i + q_j)] - 40q_i,$$

where the term in the square brackets comes from the demand function. The first order condition for profit maximization is given by:

$$150 - 4(q_i + q_j) - 40 = 0. \tag{1}$$

By symmetry, we have $q_i = q_j = 9.166$. Also, $p = 150 - 8q_i = 76.666$.

(b) The monopoly profit function is given by

$$\pi_m = Qp(Q) - C(Q_i) = Q(150 - 4Q) - 40Q.$$

The first order condition for profit maximization is given by:

$$150 - 8Q - 40 = 0. \quad (2)$$

Solving with respect to Q we get $Q = 13.75$, and then $p = 95$.

Under perfect competition the prevailing price would be given by marginal cost: $p = 40$; total quantity would be $Q = 27.5$ and welfare

$$W = CS = \frac{[p(0) - p]Q}{2} = 1512.5.$$

Under duopoly, total welfare is given by:

$$W_d = 2\pi + CS = 2q(p - c) + [p(0) - p]q = 1344.38.$$

Under monopoly, total welfare is given by

$$W_m = \pi + CS = (p - c)Q + \frac{[p(0) - p]Q}{2} = 1134.375.$$

Finally, the duopoly efficiency loss as a percentage of the monopoly efficiency loss is given by

$$EL = \frac{1512.5 - 1344.38}{1512.5 - 1134.375} = 44.5$$

■ **7.6**** Show analytically that equilibrium price under Cournot is greater than price under perfect competition but lower than monopoly price.

Solution: In a Cournot oligopoly, firm i 's profit is given by $\pi_i = q_i P(Q) - C(q_i)$, where Q is total output. The first-order condition for profit maximization is given by

$$P(Q) + q_i \frac{dP}{dq_i} - MC = 0. \quad (3)$$

The first-order condition for a monopolist is given by

$$P(Q) + Q \frac{dP}{dQ} - MC = 0. \quad (4)$$

Finally, under perfect competition we have

$$P(Q) - MC = 0.$$

Notice that $\frac{dP}{dq_i} = \frac{dP}{dQ} < 0$. Consider the case of oligopoly and suppose that price is equal to monopoly price. Monopoly price is such that the (4) holds exactly. The only difference between (3) and (4) is that the latter has Q instead of q_i . Since $Q > q_i$, it follows that, for p equal to monopoly price, the left-hand side of (3) is positive. Finally, if it is positive, each firm has an incentive to increase output, which results in a lower price.

By a similar argument we can also show that price under Cournot competition is greater than marginal cost.

■ **7.7**** Consider a duopoly for a homogenous product with demand $Q = 10 - P/2$. Each firm's cost function is given by $C = 10 + q(q + 1)$. Determine the values of the Cournot equilibrium.

Solution: Duopolist i 's profit is given by $\pi_i = q_i p(Q) - C(q_i) = q_i[20 - 2(q_i + q_j)] - 10 - q_i(q_i + 1)$. The first order condition for profit maximization is given by:

$$20 - 2(q_i + q_j) - 2q_i - 2q_i - 1 = 0. \quad (5)$$

The problem of duopolist j is symmetric, therefore we have $q_i = q_j = 2.375$ and $p = 10.5$.

■ **8.1** Explain why collusive pricing is difficult in one-period competition and easier when firms interact over a number of periods.

Solution: In one-period competition each firm has a strong incentive to deviate from the pre-agreed collusive price, since the gains from deviating are higher than the losses. In terms of the example in Section 8.1, had the duopolists interacted in only one period, the gain would be given by one half of monopoly profits, while the loss from deviating would be 0. We would then be led to the usual Nash-Bertrand equilibrium when both firms price at marginal cost.

If, however, firms interact over a number of periods, history, in the form of past pricing behavior, becomes important. Deviation from the collusive price in one period can be met by punishment (deviation) in future periods. Hence, the original defector must weigh short-term gains against long-term losses, made possible exactly by multi-period interaction.

■ **8.2** After several years of severe price competition that damaged Boeing's and Airbus' profits, the two companies have recently pledged that they will not sink into another price war. However, in June 1999, Boeing made an unusual offer to sell 100 small aircraft to a leasing corporation at special discount prices. (Although customers

never play list prices, it was felt that this deal was particularly attractive.) Boeing's move follows a similar one by Airbus.¹⁴

Based on the analysis of Section ??, why do you think it is so difficult for aircraft manufacturers to collude and avoid price wars?

Solution: Aircraft manufacturers receive orders infrequently. Moreover, the terms of each sale are seldom made public. For these reasons, it is very difficult for them to collude. The incentive to cheat on a tacit or explicit agreement would be very high because: (a) the short run is very important with respect to the long run (low discount factor); (b) the probability that cheating would be detected is low.

■ **8.3*** In a market with annual demand $Q = 100 - p$, there are two firms, A and B, that make identical products. Because their products are identical, if one charges a lower price than the other, all consumers will want to buy from the lower-priced firm. If they charge the same price, consumers are indifferent and end up splitting their purchases about evenly between the firms. Marginal cost is constant and there are no capacity constraints.

(a) What are the single-period Nash equilibrium prices, p_A and p_B ?

(b) What prices would maximize the two firms' joint profits?

Assume that one firm cannot observe the other's price until after it has set its own price for the year. Assume further that both firms know that if one undercuts the other, they will revert forever to the non-cooperative behavior you described in (a).

(c) If the interest rate is 10%, is one repeated-game Nash equilibrium for both firms to charge the price you found in part (b)? What if the interest rate is 110%? What is the highest interest rate at which the joint profit-maximizing price is sustainable?

(d) Describe qualitatively how your answer to (d) would change if neither firm was certain that it would be able to detect changes in its rival's price. In particular, what if a price change is detected with a probability of 0.7 each period after it occurs? Note: Do not try to calculate the new equilibria.

Return to the situation in part (c), with an interest rate of 10%. But now suppose that the market for this good is declining. The demand is $Q = A - p$ with $A = 100$ in the current period, but the value of A is expected to decline by 10% each year (i.e., to 90 next year, then 81 the following year, etc.).

(e) Now is it a repeated-game Nash Equilibrium for both firms to charge the monopoly price from part (b)?

Solution:

- (a) Given that there is plenty of capacity to serve the entire market, each firm will be willing to undercut the other to make all the sales in the market so long as $p > 10$. The one-shot Nash equilibrium is for both firms to charge $p = 10$, the "Bertrand trap."

¹⁴ *The Wall Street Journal Europe*, June 11–12, 1999.

- (b) The greatest profits possible are found at the monopoly price. The capacity expenditures are sunk. A monopoly would set Q so that $MR = MC$. In this case, MC is 10. So the collusive outcome would split the market and price at 55. $p = 100 - Q \Rightarrow MR = 100 - 2Q$. $MR = MC \Rightarrow 100 - 2Q = 10 \Rightarrow Q = 45 \Rightarrow p = 55$.

Assume that each firm can monitor the other's price very closely and can respond instantly (before any consumers make a purchase decision) to a price change.

- (c) Yes, one equilibrium is to stay at the monopoly price. If both firms are at the monopoly price, then each faces the following decision: "Assuming that the other firm will continue to charge the monopoly price, should I charge the monopoly price also, or should I charge slightly less today, knowing (believing) that we will then revert to $p = 10$ forever after?" Charging the monopoly price means getting half the monopoly profits forever, which is worth $PDV_{\text{cooperate}} = (1 + 1/r)(55 - 10)45/2 = 11137.5$ when the interest rate is 10%. Alternatively, $PDV_{\text{cheat}} = (54.99999 - 10)45 = 2025$. The logical conclusion is that it pays to cooperate indefinitely if you believe that the other firm will also. If, however, $PDV_{\text{cooperate}} < PDV_{\text{cheat}}$ then the monopoly price would not be sustainable. $PDV_{\text{cooperate}} < PDV_{\text{cheat}} \Rightarrow (1 + 1/r)(55 - 10)45/2 < 2025 \Rightarrow r > 100\%$. At any interest rate above 100%, the monopoly price would not be sustainable. Interest rates above 100% are rare, assuming that detection lags are on the order of weeks or months, so it looks like monopoly price could persist in this market.
- (d) If the probability of being detected is less than one, then a company that cheated would have a chance of getting the high profits of cheating for more than one period before it got caught. This would raise the incentive to cheat and lower the interest rate at which the monopoly price is sustainable. In fact, one can think of a detection probability of 70% as corresponding to an interest rate of 30% (added on top of whatever interest rate applies based on the time value of money).
- (d) Declining demand generally makes cooperative pricing more difficult to support. The rate of decline acts much like a discount rate on future earnings, since the cost to a firm of "cheating" in the current period, namely the loss of its share of future profits, is less in a declining market. However, a rate of decline of "only" 10% acts much like raising the interest rate by 10% (from 10% to 20% here), which is still safely below the interest rate at which cooperative pricing breaks down (assuming perfect detection and continuing to assume that these "grim trigger" strategies are credible punishments for cheating).

■ **8.4*** You compete against three major rivals in a market where the products are only slightly differentiated. The "Big Four" have historically controlled about 80% of the market, with a fringe of smaller firms accounting for the rest. Recently, prices have been rather stable, but your market share has been eroding slowly, from 25% just a few years ago to just over 15% now. You are considering adopting an aggressive discounting strategy to gain back market share.

Discuss how each of the following factors would enter into your decision.

- (a) You have strong brand identity and attribute your declining share to discounting by your rivals among the Big Four.
- (b) The Big Four have all been losing share gradually to the fringe, as the product category becomes more well known and customers become more and more willing to turn to smaller suppliers to meet their needs.
- (c) You believe your rivals are producing at close to their capacity, and capacity takes a year or two to expand.
- (d) You can offer discounts selectively, in which case it will take one or two quarters before your rivals are likely to figure out that you have become more aggressive on pricing.
- (e) Your industry involves high fixed costs and low marginal costs, as applies for most information goods.
- (f) The entire market is in rapid decline due to technological shifts unfavorable to this product.

Solution:

- (a) Discounting can cheapen your brand image and identity, but may be worthwhile if you still have relatively large margins and thus find it profitable to halt your slide in market share. Since the discounting is from other members of the Big Four, an aggressive response on your part, perhaps followed by an exploratory price increase, might signal that you will fight to avoid losing market share but are willing to accept today's shares if your rivals raise prices somewhat.
- (b) There is little you can do about this problem, since the fringe is hard to control in any way, and entry of new fringe players is not likely to be very difficult. This is the situation to emphasize your brand and to try to segment the market to retain your share of those customers willing to pay a premium for a well-known brand (yours!).
- (c) Generally, you can be more confident pushing prices up if rivals are at or near capacity. You will lose some sales, assuming that industry demand is not perfectly inelastic, but you will lose little or no customers to your rivals in the short run (a year or two) if they cannot expand production. Of course, if fringe firms are viewed as offering close substitutes, and do not face capacity constraints, then the capacity limitations faced by the other major players don't help you much at all.
- (d) Such "detection lags" always make discounting look more attractive, simply because any competitive responses will be delayed. Indeed, it seems that this is exactly how you lost market share, to rivals who were discounting before you realized what was going on.
- (e) Now discounting is more attractive because marginal cost is low, so setting marginal cost to the marginal revenue (associated with your residual demand curve) involves a lower price. Plus, even if you can engage in "cooperative pricing," the resulting price is lower, the lower are marginal costs.

- (f) In a declining market, the future is relatively less important commercially relative to the present. In terms of our theories of “cooperative pricing,” declining demand is much like a higher interest rate: the scale tips more towards maximizing current profits and away from a “patient” approach of sacrificing short-run profits to support or sustain long-run cooperation. So, discounting now to avoid a further loss of market share (or to gain market share back) looks more attractive in a declining market, even if this will trigger or inflame a price war.

■ **8.5** “Price wars imply losses for all of the firms involved. The empirical observation of price wars is therefore a proof that firms do not behave rationally.” True or false?

Solution: False. As Section 8.2 shows, price wars may be part of the equilibrium of a game played between rational firms.

■ **8.6** Empirical evidence from the U.S. airline industry suggests that fare wars are more likely when carriers have excess capacity, caused by GDP growth falling short of its predicted trend. Fare wars are also more likely during the Spring and Summer quarters, when more discretionary travel takes place.¹⁵ Explain how these two observations are consistent with the theories presented in Section ??.

Solution: The first model in Section 8.2 (secret price cuts) predicts that price wars start in periods of unexpected low demand. This is consistent with the first observation above. However, the effect of unexpected low demand is also consistent with a theory of price wars caused by financial distress (see the end of Section 8.2). The observation that prices fare wars take place during periods of higher demand is consistent with the second model in Section 8.2 (demand fluctuations).

■ **8.7** A 1998 news article reported that

Delta Air Lines and American Airlines tried to raise leisure air fares 4% in most domestic markets, but the move failed Monday when lone-holdout Northwest Airlines refused to match the higher prices.

The aborted price boost illustrates the impact Northwest’s woes already are having on the industry. Months of labor unrest ... are prompting passengers to book away from the fourth largest carrier.¹⁶

¹⁵

MORRISON, STEVEN A., AND CLIFFORD WINSTON (1996), “Causes and Consequences of Airline Fare Wars,” *Brookings Papers on Economic Activity* (Microeconomics), 205–276..

¹⁶ *The Wall Street Journal Europe*, August 12, 1998.

What does this say about the nature of price dynamics in the airline industry?

Solution: The event seems consistent with the view, presented at the end of Section 8.2, that price wars are asymmetric in nature. In this case, they are caused by firms, like Northwest Airlines, that are in financial distress.

■ **8.8** In the third quarter of 1999, most North American paper and forest-products companies experienced an improvement in their results. The industry, analysts said, was in a cyclical upswing: not only was demand increasing at a moderate pace; more importantly, the industry practiced restraint in keeping low production levels, thus providing support for higher prices.¹⁷

How do you interpret these events in light of the models presented in Section ???

Solution: The analysis of Section 8.1 predicts that collusion is easier in growing industries (the promise of future profits under collusion is worth more). This is consistent with the fact that “restraint in keeping low production levels” took place during the “cyclical upswing.”

■ **8.9** In 1918, the U.S. Congress passed a law allowing American firms to form export cartels. Empirical evidence suggests that cartels were more likely to be formed in industries where American exporters had a large market share, in capital-intensive industries, in industries selling standardized goods, and in industries that enjoyed strong export growth.¹⁸ Discuss.

Solution: The effect of export growth seems consistent with the analysis in Section 8.1. The effect of standardization may correspond to the fact that it is easier to monitor collusion with a standardized product (however, the effect of product differentiation on collusion is a controversial issue). The effect of market share is consistent with the analysis in Section 8.3 (concentration facilitates collusion).

■ **8.10** The endowments of the Ivy League universities have increased significantly in recent years. Princeton, the richest of all, boosted its endowment from \$400,000 per student in 1990 to more than \$750,000 in 1997. In the same period, both Harvard and Yale more than doubled their endowments. Notwithstanding these riches, the universities have restrained from using financial incentives as a means to compete for

¹⁷ *The Wall Street Journal*, October 11, 1999.

¹⁸

DICK, ANDREW (1997), “If Cartels Were Legal, Would Firms Fix Prices?,” Antitrust Division, U.S. Department of Justice..

students. For many years, the manual of the council of Ivy League Presidents stated that the schools should “neutralize the effect of financial aid so that a student may choose among Ivy Group institutions for non-financial reasons.” In 1991, the Justice Department argued that this amounted to price collusion and forced the agreement to end. However, no significant price competition took place until 1998, when Princeton University started offering full scholarships for students with incomes below \$40,000. Stanford, MIT, Dartmouth and Cornell followed suit. Allegedly, Harvard sent a letter to accepted 1998 applicants stating that “we expect that some of our students will have particularly attractive offers from the institutions with new aid programs, and those students should not assume that we will not respond.”¹⁹

How do you interpret these events in light of the theories discussed in this chapter?

Solution: If the Department of Justice was right in assuming the council manual’s clause was an explicit form of price collusion, then what happened after 1991 is that collusion ceased to be explicitly supported by the clause and turned into tacit collusion. In fact, the analysis in Chapter 8 suggests that explicit, contractual arrangements are not necessary to sustain a collusive agreement. The chapter also states that, under tacit collusion, each firm balances the short-run benefits from deviation against the long-term cost of entering into non-cooperative play. The fact that endowments have increased so much (especially Princeton’s) may be what has tipped the balance in the direction of giving away full scholarships.

■ **8.11** Based on data from the Spanish hotel industry, it was estimated that the rate set by hotel i in market k is positively influenced by a variable that measures the intensity of multimarket competition between hotel i and its competitors in market k : the more markets $m \neq k$ in which firm i and its competitors meet, the greater the measure of multimarket contact. It was also observed that the measure of multimarket contact is highly correlated with hotel chain size, that is, the larger hotel i ’s chain, the greater the measure of multimarket contact for firm i .²⁰

Provide two interpretations for the positive coefficient of multimarket contact on hotel rates, one based on collusion, one based on a different effect.

Solution: When interaction between oligopolists takes place over a number of periods, it is easier to sustain collusion: long-term losses weigh more compared to short-term gains from deviation. Multimarket contact adds another “dimension” to the balance between gains and losses. A firm’s gain from deviating in one market may be punished by its competitors in all the markets they meet, making the potential cost from deviation higher. However, the optimal behavior of the deviating firm would call for deviation in all markets. Thus, we have higher losses from deviating but also higher gains. As discussed in Section 8.3, if everything

¹⁹ *The Economist*, December 5, 1998.

²⁰

FERNÁNDEZ, NEREA, AND PEDRO MARÍN (1998), “Market Power and Multimarket Contact: Some Evidence from the Spanish Hotel Industry,” *Journal of Industrial Economics* **46**, 301–315..

is identical (firms, markets) then, multimarket contact does not increase the likelihood of collusion because the potential gains from deviation increase in the same proportion as the losses. However, asymmetries between firms or markets can make losses weigh more than gains, thus increasing the likelihood of collusion. This justifies the positive correlation between multimarket contact and average rates.

There is, however, an alternative interpretation. Maybe rates are higher in hotels of greater size. This could happen either because consumers attach a greater value to hotels that have larger chains or because bigger hotel chains command greater (unilateral) market power. Given the empirical correlation between hotel size and multimarket contact this would also imply a correlation between multimarket contact and rates, even if there is no implicit or explicit collusion between hotel chains.

■ **8.12** Consider the following excerpt from a 1998 news item.²¹

LONG-STALLED SHIPPING REFORM BILL TAKEN UP BY SENATE.
Washington — The Senate has formally begun consideration of a shipping reform bill that, if passed, would create changes for all countries shipping manufactured goods to and from the United States . . .

Until now U.S. shipping law has been founded on the principle of common carriage — “Everybody pays the same tariff (rate) to go from Oakland to Yokohama,” said the Department of Transportation (DOT) official, who asked not to be identified. Under this system, groups of liners called conferences — legal cartels with immunity from antitrust law — set the rates for their members and make those rates public through registration with the federal government. If the shipping bill passes, however, liners could make private, confidential deals with exporters-importers outside of conferences at market-set rates.

“This is going to bring marketplace economics into ocean shipping like we’ve never seen before,” the official said. “It’s going to really change the influence of ocean shipping conferences in the marketplace.” . . .

The Transportation Department official said the Clinton administration has generally supported legislation for shipping reform in line with its promotion of deregulation in airlines and trucking, but has stated concerns about specific provisions of the Senate bill. Probably the administration’s biggest concern is a provision of the bill allowing conferences also to engage in confidential contracting, he said. “In the administration view that conveys too much market power to the conferences,” the official said.

Do you agree with the Clinton administration’s view? Why or why not?

Solution: The example in Box 8.6 shows that making information public is not a panacea to the collusion problem. Although the market becomes more transparent, and collusive

²¹USIA *EPF513 04/03/98, written by USIA Staff Writer Bruce Odessey.

agreements are easier to monitor, this may come at a cost: It gives firms the opportunity to coordinate on a collusive equilibrium.

The approach taken by the U.S. Senate in its shipping reform bill is to switch from a public information exchange to the possibility of secretly priced individual/group contracts. The idea is that, although we may end up with a collusive equilibrium that is difficult to detect, this equilibrium is likely to feature price wars in order to be sustainable (cf Section 8.2). In the shared information approach, collusion is probably easier to detect but firms may (tacitly) coordinate on a higher price collusive equilibrium.

■ **8.13** In 1986, the U.S. Congress enacted a regulation (PL99-509) requiring railroads to disclose contractual terms with grain shippers. Following the passing of the legislation, rates increased on corridors with no direct competition from barge traffic, while rates decreased on corridors with substantial direct competition.²² How do you interpret these events?

Solution: One possible interpretation for these results is that, when there is no competition to railroad shipping, there is potential for collusion among railroad operators, whereas the opposite is true when there is direct competition from barge traffic. In this context, increased information about railroad contracts has the effect of

1. improving collusion among railroad operators when the latter have no competition. This is consistent with the idea that when price cuts are difficult to observe collusion is more difficult to sustain.
2. increasing competition in markets where railroad operators compete with barge operators. This is consistent with the idea that, in a competitive environment, better information about prices increases demand elasticity (consumer are more aware of price differences) and thus decreases margins.

■ **8.14*** Consider an n firm homogeneous-good oligopoly with constant marginal cost, the same for all firms. Let $\bar{\delta}$ be the minimum value of the discount factor such that it is possible to sustain monopoly prices in a collusive agreement. Show that $\bar{\delta}$ is decreasing in n . Interpret the result.

Solution: Let π^M be total industry profits. Under the collusive agreement, each firm receives π^M/n . If one of the firms undercuts its rivals, then it gets approximately π^M . Finally, if firms revert to a (perpetual) price war each firm gets zero. It follows that the

²²See

SCHMITZ, JOHN, AND STEPHEN W. FULLER (1995), "Effect of Contract Disclosure on Railroad Grain Rates: An Analysis of Corn Belt Corridors," *The Logistics and Transportation Review* **31**, 97–124..

condition such that it is an equilibrium for firms to price at the monopoly level is given by

$$\frac{1}{1-\delta} \frac{\pi^M}{n} \geq \pi^M.$$

Solving with respect to δ we get

$$\delta \geq \frac{n-1}{n}.$$

It follows that collusion is stable if and only if $\delta > \bar{\delta} \equiv \frac{n-1}{n}$. (Note that the condition is independent of the value of π^M , so the same condition would apply for any level of collusion.)

Taking the derivative of $\bar{\delta}$ with respect to n , we get

$$\frac{d\bar{\delta}}{dn} = \frac{n-(n-1)}{n^2} = 1/n^2 > 0.$$

It follows that $\bar{\delta}$ is increasing in n . In words, the more firms there are, the more difficult it is to sustain a collusive agreement. The idea is that the relative gain from cheating is greater the greater the number of firm (the profit from cheating is always the same, but the profit from collusion is lower the greater n is).

■ **8.15**** Consider the model of multimarket contact presented in Subsection ?? . Determine the minimum value of the discount factor such that the optimal collusive solution is stable.

Solution: The setting of the problem consists of Firms 1 and 2, and Markets A and B. Firm 1 has cost $\underline{c}$ in Market A, while Firm 2 has a cost of $\bar{c}$. The situation is reversed in Market B. Demand is the same both markets. It is assumed that $\underline{c} < \bar{c} < p^M$.

As discussed in section 8.3 the efficient collusive agreement is the following: In each market, the firm with a cost advantage sets the monopoly price, while the other sets a higher price and sells 0. Let us use the following notation: π^M represents the monopoly profit of the firm with cost advantage, $\pi^{M'} = \pi(p^M - \varepsilon, \bar{c})$ is the profit of the firm with high marginal cost when it charges (slightly less than) the monopoly price and $\pi^C = \pi(\bar{c}, \underline{c})$ is the profit of the firm with low cost when it charges a price equal to the other firm's costs.

In the efficient collusive agreement each firm gets:

$$\pi^M + \delta\pi^M + \delta^2\pi^M + \dots = \pi^M(1 + \delta + \delta^2 + \dots) = \frac{1}{1-\delta}\pi^M \quad (6)$$

If a firm decides to deviate, it will do so only in the market where it has a cost disadvantage, since in the other market it already earns monopoly profits. Suppose that the punishment for deviation is be for both firms to engage in a price war so that the prevailing price in each market is $\bar{c}$. If Firm 2 deviates in Market A, then it gets π^{MC} in that market

in the first period, plus 0 from then on; and $\pi^M + \delta\pi^C + \delta^2\pi^C + \dots$ in the other market. The situation is symmetric. Therefore, the deviating firm's total profits are given by:

$$\pi^{MC} + \pi^M + \delta\pi^C + \delta^2\pi^C + \dots = \pi^{MC} + \pi^M + \frac{\delta}{1-\delta}\pi^C.$$

The stability condition requires:

$$\frac{1}{1-\delta}\pi^M \geq \pi^{MC} + \pi^M + \frac{\delta}{1-\delta}\pi^C.$$

This gives the minimum value for the discount factor:

$$\underline{\delta} = \frac{\pi^{MC}}{\pi^{MC} + \pi^M - \pi^C}.$$

■ **9.2** Based on data from local cement markets in the U.S., a series of regressions were estimated for seven years in the period 1948–1980. Each regression has the form $\text{price} = \beta \cdot C_4 + (\text{other variables})$. The coefficient β was estimated to be positive in five of the seven years considered, negative in the remaining two. How can these results be explained?

Solution: It was shown in Section 9.1 that, under the assumption of Cournot competition, the higher the number of firms in the market, the lower price and the lower allocative inefficiency. Moreover, the more concentrated an industry (the smaller the number of firms), the easier is to sustain collusion. These arguments suggest that when market concentration (as measured by C_4 , for example) is greater equilibrium price is further apart from the competitive price (the structure-performance hypothesis).

However, as pointed out in Section 9.2, reverse causation is an important problem. If one assumes that market structure is endogenously determined (i.e., entry is possible) and market price is exogenous, then we obtain that a high price would induce entry from other firms and consequently decrease in concentration—a negative relation between the two variables.

■ **9.3**** Based on monthly data for Portuguese commercial banks, the following relation was estimated:

$$r_t = 0.098 + 0.814 m_t,$$

where r_t is the interest rate charged by commercial banks and m_t is the money market rate, that is, the interest rate that banks must pay to borrow in the short term. The standard deviation of the second coefficient estimate is .0878. Knowing that the money market interest rate is highly correlated with the marginal cost of giving out loans, and knowing that H is approximately .125, what can you say about market power in this sector?

Solution: Applying the equation on p. 161, we get

$$\theta = \frac{1 - 0.814}{0.814} \frac{1}{.125} = 1.828$$

This value is higher than Cournot (1) but lower than perfect collusion ($1/H = 8$). Another way of evaluating the result is to consider what the result would be under Cournot. From Table 9.2, we get that, under Cournot,

$$\xi = \frac{H}{1 + H} = \frac{1}{1.125} = .888$$

The statistical test that the estimated coefficient is greater than the Cournot value would correspond to the value $(.888 - .814)/.0878 = .84$. Although we don't have complete information about sample size, etc, this is a relatively low value. The conclusion is that behavior is between Cournot and collusive behavior but not statistically different from Cournot.

Finally, note that the above analysis is only valid under the assumption that demand and costs are linear.

■ **9.4**** Consider the following criteria for a good measure of market concentration:

1. Non-ambiguity. Given any two different industries, it must be possible to rank concentration between the two.
2. Invariance to scale. A concentration measure ought not to depend on measurement units.
3. Transfers. Concentration should increase when a large firm's market share increases at the expense of a small firm's market share.
4. Monotonicity. Given n identical firms, concentration should be decreasing in n .
5. Cardinality. If we divide each firm into k smaller firms of the same size, then concentration should decrease in the same proportion.

verify whether the indices C_n and H satisfy these requirements.

Solution: Recall that $C_m \equiv \sum_{i=1}^m s_i$ and $H \equiv \sum_{i=1}^n s_i^2$, where n is the total number of firms and $m \leq n$. [Note: there is a typo on p. 164. It should be C_m , not C_n .]

(a) We can compute both C_m and H for any two industries with the result being a rational number. Since rational numbers form an ordered set we can rank any two industries based on the two measures.

(b) This condition is satisfied since when computing the share of each firm the measure becomes units-free. For example, if we consider the share of firm i as the proportion of the firm's sales in the total industry sales, it is irrelevant whether we measure sales in billions or millions or thousands of dollars.

(c) Let firm k 's market share increase at the expense of firm j 's, so that $s_k^t = s_k + \alpha$ and $s_j^t = s_j - \alpha$, where the superscript indicates post-transfer values. Moreover, as required by the condition, let $s_k > s_j$.

$$H^t = \sum_{i=1, i \neq j, k}^n s_i^2 + (s_j^t)^2 + (s_k^t)^2 = \sum_{i=1, i \neq j, k}^n s_i^2 + (s_j - \alpha)^2 + (s_k + \alpha)^2 = H + 2\alpha(\alpha + s_k - s_j) > H.$$

We conclude that H satisfies the transfer condition. Now suppose that $j, k < m$. Then we have

$$C_m^t = \sum_{i=1, i \neq j, k}^m s_i + s_j^t + s_k^t = \sum_{i=1, i \neq j, k}^m s_i + s_j - \alpha + s_k + \alpha = C_m.$$

We conclude that C_m does not satisfy the transfer condition.

(d) If all firms are identical, then $H = \frac{1}{n}$ and $C_m = m/n$. Clearly, both indices satisfy the condition.

(e) It is easy to find examples where C_m violates cardinality. For example, suppose that $s_1 = .4, s_2 = .2$. In this case, $C_2 = .6$. Suppose all firms are divided by $k = 2$. The new value of C_2 is $.2 + .2 = .4$, which is different from $.6/2$.

We now show that H does satisfy cardinality. Let H' be the new value of H when each of the initial n firms is divided by k .

$$H' = \sum_{i=1}^n \left(\sum_{j=1}^k \left(\frac{s_i}{k} \right)^2 \right) = \sum_{i=1}^n k \left(\frac{s_i}{k} \right)^2 = \sum_{i=1}^n \frac{1}{k} s_i^2 = \frac{H}{k}.$$

■ **9.5**** Suppose you only know the value of the market shares for the largest m firms in a given industry. While you do not possess sufficient information to compute the Herfindahl index, you can find a lower and an upper bound for its values. How?

Solution: A lower bound would result from an industry where, in addition to the top m firms, there is a very large number of firms with a very small market share. In the limit of infinitesimal shares, the value of H would be $H = \sum_{i=1}^m s_i^2$. An upper bound would result from an industry where all the remaining firms have the same market share as the m -th firm. The value of H would then be $H = \sum_{i=1}^m s_i^2 + (1 - \sum_{i=1}^m s_i) s_m$. (Notice that the remaining firms would be $(1 - \sum_{i=1}^m s_i) / s_m$ in number.)

The above lower and upper bounds are frequently very close, so a fairly good approximation is often possible.

■ **10.1** First-time subscribers to the *Economist* pay a lower rate than repeat subscribers. Is this price discrimination? Of what type?

Solution: This is an example of third-degree price discrimination. The market is segmented into new subscribers and repeat subscribers. New subscribers, know the product less well and are thus likely to be more price sensitive. Moreover, the fact that they have not subscribed in the past indicates that they are likely to be willing to pay less than current subscribers. It is therefore optimal to set a lower price for new subscribers.

■ **10.2** Many firms set a price for the export market which is lower than the price for the domestic market. How can you explain this policy?

Solution: A possible explanation is that there is a “domestic product bias” that makes consumers less price sensitive to domestic products (see Box 10.1). It is then rational to set higher prices in the domestic market.

■ **10.3** Cement in Belgium is sold at a uniform delivered price throughout the country, that is, the same price is set for each customer, including transportation costs, regardless of where the customer is located. The same is practice is also found in the sale of plasterboard in the United Kingdom.²³ Are these cases of price discrimination?

Solution: Yes, these are cases of price discrimination. Consider the total price being paid by each customer, P , as being composed of the price actually charged and the transportation cost; $P = p_i + t_i$. Since locations are different, transportation costs are different, thus, each consumer is charged a price p_i that depends on his or her location. This is a clear example of geographic price discrimination.

■ **10.4** A restaurant in London has recently removed prices from its menu: each consumer is asked to pay what he or she thinks the meal was worth. Is this a case of price discrimination?

Solution: It is likely that each consumer will pay a price that reflects his or her willingness to pay. In that sense, this is a situation of close to perfect price discrimination.

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PHILIPS, LOUIS (1983), *The Economics of Price Discrimination*, Cambridge: Cambridge University Press., pp. 23–30.

■ **10.5** In the New York Fulton fish market, the average price paid for whiting by Asian buyers is significantly lower than the price paid by White buyers.²⁴ What type of price discrimination does this correspond to, if any? What additional information would you need in order to answer the question?

Solution: This appears to be a case of third-degree price discrimination, whereby a group of buyers (a market segment) pays a different price than another group. Theory predicts that in a non-competitive market (monopoly, oligopoly) buyers with higher price elasticity should be charged a lower price; as a result, we can conclude that Asian buyers have higher price elasticity than white buyers.

In order to have a more accurate picture, however, more information is needed. Different prices could simply result from quantity discounts and the possible fact that different quantities are bought by the different groups. It could also be the case that different groups use different types of payment type (cash or credit), so that different prices reflect different costs. Also, the time of purchase (e.g., before 5am or after 5am) could be correlated with race, so that it is not race that determines the price difference. The same reasoning applies to the type of establishment the buyer represents (store, fry shop, etc.).

For a more complete discussion, see the cited reference.

■ **10.6** Supermarkets frequently issue coupons that entitle consumers to a discount in selected products. Is this a promotional strategy, or simply a form of price discrimination? Empirical evidence suggests that paper towels are significantly more expensive in markets offering coupons than in markets without coupons.²⁵ Is this consistent with your interpretation?

Solution: This may be interpreted as a case of price discrimination. By offering coupons (hence a lower price), supermarkets can serve the buyers with a higher price elasticity at a different price. In order for this strategy to improve revenues with respect to single price, supermarkets should then set a higher regular price. Hence, empirical evidence is consistent with the explanation that this is a form of price discrimination.

■ **10.7**** A market consists of two population segments, A and B. An individual in segment A has demand for your product $q = 50 - p$. An individual in segment B has

²⁴

GRADDY, KATHRYN (1995), "Testing for Imperfect Competition at the Fulton Fish Market," *Rand Journal of Economics* **26**, 75–92..

²⁵

LEVEDAHL, J W (1984), "Marketing, Price Discrimination, and Welfare: Comment," *Southern Economic Journal* **3**, 886–891..

demand for your product $q = 120 - 2p$. Segment A has 1000 people in it. Segment B has 1200 people in it. Total cost of producing q units is $C = 5000 + 20q$.

- (a) What is total market demand for your product?
- (b) Assume that you must charge the same price to both segments. What is the profit-maximizing price? What are your profits?
- (c) Imagine now that members of segment A all wear a scarlet "A" on their shirts or blouses and that you can legally charge different prices to these people. What price do you charge to the scarlet "A" people? What price do you charge to those without the scarlet "A"? What are your profits now?

Solution:

- (a) Segment A people buy zero at or above $p = 50$. At $p < 50$, the total demand from segment A types is $Q_A = 1000(50 - p) = 50000 - 1000p$. Segment B people buy zero at or above $p = 60$. At $p < 60$, the total demand from segment B types is $Q_B = 1200(120 - 2p) = 144000 - 2400p$. At $p \geq 60$, quantity demanded is zero. At $50 < p < 60$, total demand is just the demand from B, $Q = 144000 - 2400p$. At $p < 50$ total demand is from both types $Q = (144000 - 2400p) + (50000 - 1000p) = 194000 - 3400p$.
- (b) First note that $MC = 20$ at all output levels. For $p > 50$, the only consumers in the market are segment B consumers so $TR = Q(60 - Q/2400) = 60Q - Q^2/2400$. Using calculus, one can then take the derivative and find $MR = 60 - Q/1200$ in this range. But note that at the break point $p = 50$, where segment A customer begin to enter the market, $Q = 24000$, and $MR = 40$, which is still greater than MC . Therefore, the firm would keep lowering its price to sell more units. This would induce segment A consumers to buy so the demand function should we consider at $p < 50$, is now a combination of A and B segment customers so $TR = 57.06Q - Q^2/3400$, $MR = 57.06 - Q/1700$. Taking the derivative of this total revenue function and setting it equal to MC we have $57.06 - Q/1700 = 20$ which yields an optimal output of $Q = 63002$, which yields $p = 38.53$. To avoid doing the calculus, one could set up a spreadsheet with every possible quantity and find the profit maximizing Q .
- (c) The problem can now be solve as two separate markets. In each, you pick the profit maximizing quantity to sell to the segment by setting marginal cost equal to the marginal revenue for that segment..

$$Q_A = 50000 - 1000p \Rightarrow p = 50 - Q_A/1000 \Rightarrow TR = 50Q_A - Q_A^2/1000 \Rightarrow MR = 50 - Q_A/500. MR = MC \Rightarrow 50 - Q_A/500 = 20 \Rightarrow Q_A = 15000 \Rightarrow p_A = 35.$$

$$Q_B = 144000 - 2400p \Rightarrow p = 60 - Q_B/2400 \Rightarrow TR = 60Q_B - Q_B^2/2400 \Rightarrow MR = 60 - Q_B/1200. MR = MC \Rightarrow 60 - Q_B/1200 = 20 \Rightarrow Q_B = 48000 \Rightarrow p_B = 40.$$

■ **10.8*** Coca-Cola recently announced that it is developing a "smart" vending machine. Such machines are able to change prices according to the outside temperature.²⁶

²⁶ *Financial Times*, October 28, 1999.

Suppose for the purposes of this problem that the temperature can be either “High” or “Low.” On days of “High” temperature, demand is given by $Q = 280 - 2p$, where Q is number of cans of Coke sold during the day and p is the price per can measured in cents. On days of “Low” temperature, demand is only $Q = 160 - 2p$. There is an equal number days with “High” and “Low” temperature. The marginal cost of a can of Coke is 20 cents.

(a) Suppose that Coca-Cola indeed installs a “smart” vending machine, and thus is able to charge different prices for Coke on “Hot” and “Cold” days. What price should Coca-Cola charge on a “Hot” day? What price should Coca-Cola charge on a “Cold” day?

(b) Alternatively, suppose that Coca-Cola continues to use its normal vending machines, which must be programmed with a fixed price, independent of the weather. Assuming that Coca-Cola is risk neutral, what is the optimal price for a can of Coke?

(c) What are Coca-Cola’s profits under constant and weather-variable prices? How much would Coca-Cola be willing to pay to enable its vending machine to vary prices with the weather, i.e., to have a “smart” vending machine?

Solution:

- (a) On a Hot day, $Q = 280 - 2p$, or $p = 140 - Q/2$. Marginal revenue is $MR = 140 - Q$. Equating to marginal cost (20) and solving, we get $Q^* = 120$ and $p^* = 80$. On a Cold day, $Q = 160 - 2p$, or $p = 80 - Q/2$. Marginal revenue is $MR = 80 - Q$. Equating to marginal cost (20) and solving, we get $Q^* = 60$ and $p^* = 50$.

[(b)] Observe from part (a) that even on a Hot day the optimal price is no greater than 80 cents. So, we can restrict our attention to prices of 80 cents or less. In this price range, the expected demand is given by $Q = .5(280 - 2p) + .5(160 - 2p) = 220 - 2p$. Solving for p gives $p = 110 - Q/2$. The marginal revenue associated with this expected demand curve is given by $MR = 110 - Q$. Equating this marginal revenue to marginal cost, we get $Q^* = 90$ and $p^* = 65$.

[(c)] Under price discrimination, from part (a), profits on a Hot day are $(80 - 20)120 = \$72$, and profits on a Cold day are $(50 - 20)60 = \$18$. Expected profits per day are therefore $(\$72 + \$18)/2 = \$45$. Under uniform pricing, expected profits per day are $(65 - 20)90 = \$40.50$. It follows that Coca-Cola should be willing to pay up to an extra \$4.50 per day for a “smart” vending machine.

■ **10.9*** Suppose the California Memorial Stadium has a capacity of 50,000 and is used for exactly seven football games a year. Three of these are OK games, with a demand for tickets given by $D = 150,000 - 3p$ per game, where p is ticket price. (For simplicity, assume there is only one type of ticket.) Three of the season games are not so important, the demand being $D = 90,000 - 3p$ per game. Finally, one of the games is really big, the demand being $D = 240,000 - 3p$. The costs of operating the Stadium are essentially independent of the number of tickets sold.

(a) Determine the optimal ticket price for each game, assuming the objective of profit maximization.

Given that the Stadium is frequently full, the idea of expanding the Stadium has arisen. A preliminary study suggests that the cost of capacity expansion would be \$100 per seat per year.

(b) Would you recommend that the University of California go ahead with the project of capacity expansion?

Solution:

- (a) Demand for OK games is given by $D = 150 - 3p$, where number of tickets is measured in thousands. Inverse demand is $p = 50 - Q/3$. Marginal revenue is $MR = 50 - 2/3 \cdot Q$. Marginal cost is zero, since costs do not depend on the number of tickets sold. Equating marginal cost to marginal revenue, we get $Q = 75$. This is greater than capacity. Therefore, the optimal solution is simply to set price such that demand equals capacity: $150 - 3p = 50$, which implies $p = \$33.3$

Demand for not-so-important games is given by $D = 90 - 3p$. Inverse demand is $p = 30 - Q/3$. Marginal revenue is $MR = 30 - 2/3 \cdot Q$. Equating marginal revenue to marginal cost, we get $Q = 45$. Substituting back in the inverse demand curve we get $p = \$15$.

Since demand for the Big Game is greater than for the OK games, it will surely be the case that $MR = MC$ implies a demand level greater than capacity. The optimal price is therefore determined by equating demand to capacity: $240 - 3p = 50$, or simply $p = \$63.3$

- (b) The marginal revenue of an additional seat is the sum of the difference between marginal revenue and marginal cost for all games where capacity was a constraint. For OK games, marginal revenue is given by $MR = 50 - 2/3 \cdot 50 = 16.7$. For the Big Game, $MR = 80 - 2/3 \cdot 50 = 46.7$. Adding these up (three times the first plus the second) we get \$96.7. Since this is less than the marginal cost of capacity expansion, it is not worth it to pursue the project.

■ **10.10**** Your software company has just completed the first version of SpokenWord, a voice-activated word processor. As marketing manager, you have to decide on the pricing of the new software. You commissioned a study to determine the potential demand for SpokenWord. From this study, you know that there are essentially two market segments of equal size, professionals and students (one million each). Professionals would be willing to pay up to \$400 and students up to \$100 for the full version of the software. A substantially scaled-down version of the software would be worth \$50 to consumers and worthless to professionals. It is equally costly to sell any version. In fact, other than the initial development costs, production costs are zero.

(a) What are the optimal prices for each version of the software?

Suppose that, instead of the scaled-down version, the firm sells an intermediate version that is valued at \$200 by professionals and \$75 by students.

(b) What are the optimal prices for each version of the software? Is the firm better off by selling the intermediate version instead of the scaled-down version?

Suppose that professionals are willing to pay up to $\$800(a - .5)$, and students up to $\$100a$, for a given version of the software, where a is the software's degree of functionality: $a = 1$ denotes a fully functional version, whereas a value $a < 1$ means that only $100a\%$ features of the software are functional. It is equally costly to produce any level of a . In fact, other than the initial development costs, production costs are zero.

(c) How many versions of the software should the firm sell? Which versions? What are the optimal prices of each version?

Solution:

- (a) It is optimal to price the full version at 400 and the scaled-down version at 50. Total profits are 450.
- (b) One first possibility would be to price the intermediate version at 75 and the full version at 400. However, this would lead professionals to choose the intermediate version since the difference between willingness to pay and price is greater for the intermediate version. In order to induce professionals to buy the full version, the full version's price would need to be $75 + (400 - 200) = 275$, where the value in parentheses is the professionals' difference in willingness to pay between the two versions. This would lead to a total profit of $275 + 75 = 350$, which is lower than initially. Still another possibility would be to price the full version at 400 and the intermediate version at $400 - (400 - 200) = 200$. In this case, professionals would buy the full version but students would not buy the intermediate version. Profits would then be 400: better than 350 but still less than the 450 the firm would get with the truly scaled-down version.
- (c) There are two candidates for optimal price: \$400 and \$100. Profits are given by \$400m in the first case and \$200m in the second case (recall that there are one million professionals and one million students). It follows that $a = 1, p = 400$ is the optimal solution.

Since there are only two types of consumers, it will not be necessary to offer more than two different versions. Since it is equally costly to produce any version and willingness to pay is increasing in a , it follows that one of the versions should have full functionality ($a = 1$), the other one $a \leq 1$. Since professionals value at zero any version with $a \leq .5$, we conclude that the “damaged” version has $.5 \leq a \leq 1$.

At the margin, professionals are willing to pay more for greater functionality than students. Therefore, if there is to be self-selection between two different versions, it will be the case that professionals choose the fully functional version and students the other one. If professionals prefer the fully functional version, it must be that $p_1 - p_a \leq 800(1 - .5) - 800(a - .5)$, that is, the price difference must be smaller than the difference in willingness to pay (p_1 and p_a are the prices of the fully functional and damaged versions, respectively). Moreover, it must be $p_1 \leq 400$. By the same token, if students prefer to purchase the “damaged” version, it must be that $p_1 - p_a \geq 100 - 100a$ and $p_a \leq 100a$.

Suppose the first and fourth inequalities are binding. Profits as a function of a are then given by $p_1 + p_a$, which is equal to $200a + 800(1 - .5) - 800(a - .5)$. This is decreasing in a , implying that the optimal value would be $a = .5$. But this would lead to $p_1 = 450$, which violates the second inequality. It follows that the optimal solution is to choose the minimum value of a such that the second constraint is just satisfied, that is, $100a + 800(1 - .5) - 800(a - .5) = 400$, or simply $a = 4/7$. (Notice that the third constraint is satisfied for these values.) Optimal prices are therefore given by $p_1 = 400$ and $p_a = 100(4/7) \approx 57.14$.

Profits under one version are \$400m. Under two versions, the firm gets \$457.14m, an increase of \$57.14m. Basically, the increase corresponds to student sales.

■ **10.11*** One of the arguments used in Microsoft's defense against allegations of monopoly behavior is that it "cannot charge a monopoly price because it faces competition from ... its own installed base." Based on the above discussion on durable goods, how would you qualify/extend Microsoft's defense?

Solution: In Section 10.4, we discussed the problem faced by a monopolist selling a durable good. If the monopolist can set different prices over time (inter-temporal price discrimination), then its profits may be lower than they would be if the monopolist could not set different prices over time. Rational buyers know that, once high-valuation buyers have purchased the good the seller has an incentive to lower price and capture lower-valuation consumers who would otherwise not purchase the product.

In order for this to take place, it is important that potential buyers have some flexibility regarding the time of purchase (as is usual with durable goods). Operating systems seem a good candidate for this: typically, consumers are already using a given operating system when they buy a new one, and thus delay is a reasonable option. However, many computer purchases are bundled with the latest operating system, in which case buyers don't really make a decision of when to purchase the operating system. In summary, it's unclear how important the durable-good constraint is in this case.

■ **10.12** In 1998, the European Commission fined Volkswagen more than \$100m for preventing its dealers in Italy from selling to foreign buyers. Is this consistent with the European Commission's policy regarding price discrimination? Is this the right decision from a social welfare point of view?

Solution: Section 10.5 presents several cases concerning the European Union's policy towards price discrimination. The E.U. appears concerned with price discrimination within the union but less so between the E.U. and the rest of the world. Since both Italy and Germany (home to Volkswagen) are part of the E.U., the decision is consistent with the E.U. policy goal of creating a single market.

From a social welfare point of view, as Section 10.5 suggests, things are not straightforward. Price discrimination may be more efficient if total welfare is increased. However, price discrimination may be considered unfair by consumers: German buyers may not like the idea of paying more for the same car as Italian buyers.

■ **10.13*** Can coupons be used to price discriminate? How? Empirical evidence suggests that, in U.S. cities where coupons are used more often, breakfast cereals are sold at a lower price.²⁷ Is this consistent with the interpretation that coupons are used for price discrimination? If not, how can the empirical observation be explained?

Solution: Paralleling the explanation in Exercise 10.6, one could argue that coupons can be used for price discrimination. However, the empirical evidence from the breakfast cereal market is not consistent with this explanation (as was the example with the market for paper towels in Exercise 10.6). The interpretation of coupons as a promotion strategy is probably a better explanation.

For more information, see the cited reference.

■ **10.14**** In September 1997, the New York state's attorney general pressed charges against Procter & Gamble over the fact that P&G eliminated the use of coupons. The argument was that P&G was colluding with rivals to eliminate coupons, for doing so "only works if everybody goes along with it."²⁸ What does this suggest about the practice of price discrimination in the context of oligopoly? (In the end, P&G, while not admitting any wrongdoing, agreed on a \$4.2m settlement of the charges.)

Solution: Price discrimination may be viewed as a "prisoners' Dilemma." If oligopolists can commit not to use coupons (and price discriminate) then everybody is better off (as are both prisoners in the case they do not defect). However, using coupons may be a dominant strategy, implying that every player would use it. The equilibrium where no player uses coupons can then only be achieved through collusion / cooperation.

■ **10.15*** Suppose that perfect price discrimination implies a transaction cost T , incurred by the seller. Show that perfect price discrimination may be optimal for the seller but welfare decreasing for society as a whole.

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NEVO, AVIV, AND CATHERINE WOLFRAM (1999), "Prices and Coupons for Breakfast Cereals," University of California, Berkeley, and Harvard University..

²⁸ *The Economist*, August 1st, 1997.

Solution: Refer to Figure 10.1. By going from no price discrimination to (perfect) price discrimination, the seller's gross profits increase by $B + C$, whereas consumer surplus decreases by B . The net social gain is C . Suppose however that the seller must incur a cost T in order to implement perfect price discrimination. If $C < T < B + C$, then perfect price discrimination is profitable but not socially desirable.

■ **10.16*** Consider the model of a monopolist with two markets presented in Section ???. Suppose that the seller has a limited capacity and zero marginal cost up to capacity (or very low marginal cost). An example of this would be an airline with two types of passengers or a football stadium with two types of attendees.

Derive the conditions for optimal pricing. How do they relate to the case when there are no capacity constraints?

Solution: Let K denote capacity and $p_1(q_1)$, $p_2(q_2)$ denote the inverse demand functions.

The monopolist's problem becomes:

$$\max_{q_1, q_2} q_1 p_1(q_1) + q_2 p_2(q_2) - c(q_1 + q_2)$$

subject to

$$q_1 + q_2 \leq K.$$

The Lagrangean for this problem is

$$\mathcal{L} = q_1 p_1(q_1) + q_2 p_2(q_2) - c(q_1 + q_2) + \lambda (K - q_1 - q_2).$$

The first-order conditions are:

$$\begin{aligned} MR_1 &= MC + \lambda \\ MR_2 &= MC + \lambda, \end{aligned}$$

or simply

$$\begin{aligned} MR_1 &= \lambda \\ MR_2 &= \lambda, \end{aligned}$$

since marginal cost is zero up to capacity. Depending on whether capacity constraints are binding or not, we will have λ positive or zero. Whichever is the case, the above equations show that optimality implies that *marginal revenue* be equated across markets. Notice that, if demand elasticity differs across markets, then this implies different prices for the different markets.

The same result can be obtained intuitively. Suppose that the seller is capacity constrained. Is the current set of prices optimal? One alternative is to take one unit from one

market and sell it the other market, changing prices accordingly. Would the seller want to do this? By taking one unit away from Market 1, the seller would lose MR_1 . By selling it in Market 2, the seller would get MR_2 . Optimality then requires that $MR_1 = MR_2$.

■ **10.17***** Consider the model of non-linear pricing introduced in Section ?? . Suppose there are two types of consumers, in equal number. Type 1 have demand $D_1(p) = 1 - p$, and type 2 $D_2(p) = 2(1 - p)$. Marginal cost is zero.

(a) Show that if the seller is precluded from using non-linear pricing, then the optimal price is $p = \frac{1}{2}$ and profit (per consumer) $\frac{3}{8}$.

(b) Show that if the seller must set a single two-part tariff, then the optimal values are $f = \frac{9}{32}$ and $p = \frac{1}{4}$, for a profit of $\frac{9}{16}$.

(c) Show that if the seller can set multiple two-part tariffs, then the optimal values are $f_1 = \frac{1}{8}$, $p_1 = \frac{1}{2}$, $f_2 = \frac{7}{8}$, $p_2 = 0$, for a profit of $\frac{5}{8}$.

d) Show that, like profits, total surplus increases from (a) to (b) and from (b) to (c).

Solution: (a) Total demand from a consumer of Type 1 and a consumer of Type 2 is given by $D(p) = D_1(p) + D_2(p) = 1 - p + 2(1 - p) = 3(1 - p)$. The monopolist's problem is:

$$\max_p 3p(1 - p) \quad (7)$$

The solution to this problem is given by the first order condition, $1 - 2p = 0$, so that we get $p = \frac{1}{2}$ and the profit is $\frac{3}{4}$. Social welfare is given by the sum of the firm's profit and the consumer surplus and is equal to: $W_a = 3p(1 - p) + (1 - p)^2 = 1$.

(b) In this case the monopolist's demand is the same. However, the monopolist now can also charge a fixed fee, f , from both consumers. The problem becomes:

$$\begin{aligned} \max_p & 3p(1 - p) + 2f \\ \text{s.t.} & \frac{(1 - p)^2}{2} \geq f, \end{aligned}$$

where the constraint comes from the fact that the consumer of Type 1 must have a positive surplus, otherwise it will not buy. Once the constraint for the Type 1 consumer is satisfied, the constraint for Type 2 is also satisfied; we can therefore ignore it. The monopolist is better off when it extracts as much surplus as possible from consumers. Thus, its optimal policy requires that the fixed fee be equal to the Type 1 consumer surplus, that is, the constraint should be binding. The monopolist's problem becomes:

$$\max_p 3p(1 - p) + (1 - p)^2,$$

and the solution is given by the first order condition, $3 - 6p - 2 + 2p = 0$, so that we get $p = \frac{1}{4}$, $f = \frac{9}{32}$ and the profit is $\frac{9}{8}$. Welfare is given by $W_b = 3p(1 - p) + (1 - p)^2 + 0 + \frac{(1 - p)^2}{2} = \frac{45}{32} > W_a$.

(c) In this case the monopolist's problem is more complex:

$$\begin{aligned}
& \max_{p_1, p_2} \quad p_1(1 - p_1) + f_1 + 2p_2(1 - p_2) + f_2 \\
& \text{s.t.} \quad \quad \quad CS_1(p_1) \geq f_1 \quad (PC1) \\
& \quad \quad \quad CS_2(p_2) \geq f_2 \quad (PC2) \\
& \quad \quad \quad CS_1(p_2) - f_2 \leq CS_1(p_1) - f_1 \quad (IC1) \\
& \quad \quad \quad CS_2(p_1) - f_1 \leq CS_2(p_2) - f_2, \quad (IC2)
\end{aligned}$$

where the participation constraints assure that the consumer will prefer to consume and the incentive compatibility constraints assure that each plan is chosen by the targeted type of consumers, that is, Type 1 consumers will prefer plan 1 to plan 2 while Type 2 consumers will prefer plan 2 to plan 1.

One can show that PC1 and IC2 are binding, while PC2 and IC1 are not. Suppose that PC1 and IC2 are satisfied. We have: $CS_2(p_2) - f_2 \geq CS_2(p_1) - f_1 \geq CS_2(p_1) - CS_1(p_1) \geq 0$, where the last inequality comes from the fact that, at any price, the surplus of the Type 2 consumers is higher, since they consume more. Therefore, PC2 is automatically satisfied. PC2 will not be binding unless consumers of Type 1 are not served. To see this, suppose PC2 is binding. From IC2 and PC1 we get $CS_2(p_1) \leq f_1 \leq CS_1(p_1)$ which is obviously impossible. In contrast, PC1 must be binding: if PC1 and PC2 would not bind the monopolist could increase its profits by increasing both f_1 and f_2 with the same small amount without violating the ICs. If IC2 is not binding the monopolist could increase f_2 with a small amount and keep all other constraints satisfied, while increasing her profits.

Therefore, we have $f_1 = CS_1(p_1) = \frac{(1-p_1)^2}{2}$ and $f_2 = CS_2(p_2) - CS_2(p_1) + f_1 = (1 - p_2)^2 - \frac{(1-p_1)^2}{2}$. The monopolist's problem becomes:

$$\max_{p_1, p_2} \quad p_1(1 - p_1) + 2p_2(1 - p_2) + (1 - p_2)^2$$

The first order conditions are: $1 - 2p_1 = 0$ and $2 - 4p_2 - 2 + 2p_2 = 0$, and the solutions are: $p_1 = \frac{1}{2}$, $f_1 = \frac{1}{8}$, $p_2 = 0$, $f_2 = \frac{7}{8}$ and the profit is $\frac{5}{4}$. The welfare is given by $W_c = p_1(1 - p_1) + \frac{(1-p_1)^2}{2} + 2p_2(1 - p_2) + (1 - p_2)^2 = \frac{11}{8} < W_b$.

(d) The proof is already contained in the previous points.

■ **10.18***** Many retail stores set lower-than-usual prices during a fraction of the time (sale). One interpretation of this practice is that it allows for price discrimination between patient and impatient buyers.

Suppose that each buyer wants to purchase one unit per period. Each period is divided into two subperiods, the first and the second part of the period. Suppose there are two types of buyers, $i = 1, 2$. Each type of buyer is subdivided according to

the part of the period they would ideally like to make their purchase. One half the buyers would prefer to purchase during the first part of the period, one half during the second part. A buyer of type i is willing to pay $\bar{v}_i$ for a purchase during his or her preferred part of the period; and $\underline{v}_i$ for a purchase at another time.

Buyers of type 1, which constitute a fraction α of the population, are high-valuation, impatient buyers; that is, $\bar{v}_h$ is very high and $\underline{v}_h$ very low. High valuation implies that $\bar{v}_h$ is very high; impatience implies that $\underline{v}_h$ is very low; buyers of type 1 are not willing to buy at any time other than their preferred time. Buyers of type 2, by contrast, are very patient: $\bar{v}_l \approx \underline{v}_l$. Assume that α is relatively low; specifically, $\alpha < \bar{v}_l/\bar{v}_h$. To summarize: $\bar{v}_h > \bar{v}_l \approx \underline{v}_l > \alpha\bar{v}_h > \underline{v}_h \approx 0$.

a) Show that, under a constant-price strategy, the seller optimally sets $p = \bar{v}_l$.

b) Determine firm profits when it sets prices $p = \bar{v}_h$ and $p = \underline{v}_l$ in the first and second parts of the period, respectively.

c) Show that profits are greater under the “sales” strategy.

Solution: a) If $p > \bar{v}_l$, then there is no sale. If $\bar{v}_2 < p < \bar{v}_1$, then the only purchasers are the impatient, high-valuation buyers, and the seller's profit is $\pi = \alpha p$, with maximum value $\alpha\bar{v}_1$. If $p < \bar{v}_2$, then all buyers make a purchase and the seller's profit is $\pi = p$, with maximum value $\bar{v}_2$. Since $\alpha\bar{v}_1 < \bar{v}_2$, it is clear that the best constant-price strategy is to set $p = \bar{v}_2$.

b) Under this strategy the seller's profit is $\pi = \frac{1}{2}\alpha\bar{v}_1 + (1 - \alpha)\underline{v}_2 + \frac{1}{2}\alpha\underline{v}_2 = \underline{v}_2 + \frac{1}{2}\alpha(\bar{v}_1 - \underline{v}_2) > \bar{v}_2$, where the last inequality is based on the fact that $\underline{v}_2 \approx \bar{v}_2$.

c) The proof is contained in part b.

■ **11.1*** Assume for the purposes of this problem that, contrary to their protestations, Microsoft has a monopoly in providing operating systems, called “Windows,” for personal computers. Assume also that the marginal cost to Microsoft of supplying its operating system for one more computer is zero. Denote by w the price charged by Microsoft for its operating system. (Assume that Microsoft sets a single price per computer, i.e., does not employ two-part tariffs, quantity discounts, or other forms of price discrimination.)

Computer Original Equipment Manufacturers (OEMs) assemble computers. Suppose that the “bill of materials” for a computer, i.e., the cost to the OEM of all the parts necessary to build a computer, adds up to \$900 per machine, and that assembly costs another \$100 per machine. Finally, assume (contrary to the efforts of Dell and Compaq) that computers are a homogeneous good and the annual demand for computers is given by $Q = 50,000,000 - 10,000p$, where Q is quantity and p is price as usual.

Suppose that the OEM business is perfectly competitive.

(a) For any given price, w , of operating systems, what will be the price and sales of computers?

(b) What price w should Microsoft set for its operating system? How much money will Microsoft make? How much money will OEMs make? What will be the price of a computer?

Amusing if irrelevant note: Microsoft in fact charges in the \$50 to \$60 range per PC for Windows98. Microsoft argued in their antitrust trial that they must not really have a monopoly or else they would be charging a lot more.

(c) How much money would a vertically integrated firm controlling both the supply of Windows and the assembly of computers make? What price would such a firm charge for computers?

(d) Could Microsoft make more money by integrating downstream into computer assembly? Why or why not?

Suppose now (definitely contrary to reality) that a single firm, Compaq, has a monopoly over the assembly of computers.

(e) For a given price, w , for Windows, what price, p , would Compaq set for computers and how many computers would be sold?

(f) What price, w , should Microsoft set for its operating system? How much money will Microsoft make? How much money will Compaq make? What will be the price of a computer?

(g) Could Microsoft and Compaq make more money by merging? If so, how much? Would such a merger benefit or harm computer users? By how much?

Solution:

- (a) Competition in the downstream computer market will drive prices in that market to the OEM's marginal cost. For the downstream computer makers, marginal cost is equal to w the price they pay Microsoft for its operating system plus the $900 + 100 = 1000$ per machine that they incur in materials and assembly. Therefore, the price in the downstream market will be $p = w + 1000$ and the total number of computers sold will be $Q = 50,000,000 - 10,000(w + 1,000) = 40,000,000 - 10,000w$.
- (b) The demand curve calculated in part (a) is the derived demand for Microsoft's operating system since each computer sold has one copy of the Microsoft's operating system. Therefore, the correct price for Microsoft to charge is the price that maximizes its profits in this market, which is the monopoly price. Inverting this demand curve you get $w = 4000 - Q/10000$, which means that Microsoft's marginal revenue will be $MR_M = 4000 - Q/5000$. Since its marginal cost for the operating system is zero, the optimal quantity to sell will be $4000 - Q/5000 = 0$ or $Q = 20,000,000$ machines, or 20 million copies of its operating software. In order to sell this many copies, Microsoft needs to set its price, w such that $w = 4000 - 20,000,000/10000 = 2,000$. At this price, Microsoft earns $\$2000 \cdot 20,000,000 = \$40,000,000,000$ in profits. The OEMs earn zero profits since they price at marginal cost, which is equal to $\$2,000 + \$100 + 900 = \$3,000$.
- (c) A vertically integrated firm has marginal costs of is \$0 for the operating system, \$100 for assembly, and \$900 for computer parts. Assuming that the vertically integrated firm only sells operating systems to its downstream computer subsidiary, this subsidiary would be a monopolist in the computer market. Therefore, its marginal revenue would be $MR = 5000 - Q/5000$. Setting this marginal revenue equal to marginal cost of \$1000 and solving for Q^* yields $Q^* = 20,000,000$ units. In order to sell this many units the firm would charge

$p = 5000 - 20,000,000/10000 = 5000 - 2000 = \$3,000$ per computer. Its profits would be $20,000,000(\$3000 - \$1000) = \$40,000,000,000$.

- (d) Since all the market power in this industry is in the software business, Microsoft can make just as much money by staying in the upstream market as it could by entering the downstream market. You just demonstrated that fact in part (c).
- (e) As a monopolist in the downstream market, Compaq faces marginal revenue equal to $MR = 5000 - Q/5000$. Equating this with its marginal cost of $w + 1000$ and solving for Q^* yields, $Q^* = 20,000,000 - 5000w$. This is the optimal amount for Compaq to sell, which requires a price of $p = 5000 - (20,000,000 - 5000w)/10000 = 3000 + w/2$.
- (f) When setting its price, w , Microsoft knows that its demand will be $Q^* = 20,000,000 - 5000w$ since this is the optimal amount for Compaq to sell when it pays Microsoft a price of w per unit of the operating system. For Microsoft, the optimal number of units of the operating system to sell are those that maximize its profits given this demand. Inverting Q^* and solving for w yields $w = 4000 - Q^*/5000$, which means that Microsoft's marginal revenue is $MR = 5000 - Q^*/2500$. Equating this with its marginal cost of zero, you find that the optimal amount for Microsoft to sell is $Q^* = 12,500,000$ units. In order to sell this many units, Microsoft's price to Compaq needs to be $w = 4000 - 12,500,000/5000 = \1500 per unit. Its profits will be $12,500,000\$1500 = \$18,750,000,000$.
- (g) If Microsoft and Compaq merged, their profits would be those calculated for the integrated computer maker calculated above (Part (c)), or $\$40,000,000,000$, which is an improvement of $\$21,250,000,000$. Consumers actually benefit as well since the total number of computers they buy will increase from 12,500,000 to 20,000,000 and the price they pay will fall. The total improvement for consumer is equal to the change in consumer surplus associated with the expansion of the number of computers sold and this price decline. In the vertically separated setting, the total cost of a computer is $p = 1500/2 + 3000 = 3750$ (from Part (e) above). Therefore, the consumer surplus is equal to $(12,500,000)(5000 - 3750)/2 = \$7,812,500,000$. In the vertically integrated setting, the price of a computer is $\$3,000$ (from Part (b) above) so the consumer surplus will be $(20,000,000)(5000 - 3000)/2 = \$20,000,000,000$. Therefore, the net improvement in consumer welfare will be $\$12,187,500,000$.

■ **11.2** Empirical evidence suggests that franchiser-owned McDonald's restaurants charge lower prices than independent ones. How can this difference be explained?

Solution: This is an example of double marginalization. If firms are vertically integrated (as is the case with franchise-owned McDonalds restaurants), then the retailer price is the monopoly price for the vertical structure. On the other hand, if the firms are not vertically integrated, then retailer's profit maximization leads to a second monopoly margin which takes as marginal cost the wholesale price. If the wholesale price is equal to the marginal cost of the upstream firm, then the two retail prices are the same. However, in such a

case the upstream firm makes zero profits. We would thus expect the wholesale price to be greater than marginal cost. It follows that the retail price for independent retailers is higher than for franchise-owned retailers.

■ **11.3** Suppose that a manufacturer sells to n retailers by means of a two-part tariff (f, w) including a fix fee f and a wholesale price w . Explain the intuition of the result that the greater the degree of retailer competition, the greater the optimal wholesale price.

Solution: See Section 11.2.

■ **11.4** The following industries are known to practice or have practiced resale price maintenance: fashion clothing, consumer electronics, fine fragrances. In each case, indicate the probable motivation for RPM and the likely welfare consequences.

Solution: In the case of consumer electronics, as pointed out in Section 11.3, there is an important positive externality from investing in sales effort. Retailers can free-ride on the investment efforts made by competing retailers, since one consumer can benefit from the point-of-sale services provided by a retailer (who invested in sales effort) and shop at a lower-price retailer (who did not invest). The result of this externality is that no retailer invests and the demand for the good is lower. An RPM policy induces investment in sales effort (which increases demand); instead of competing in price (which is now the minimum price required by the manufacturer), the retailers compete in investments in sales effort to attract customers. The final beneficiaries of this policy are, obviously, the retailers and the manufacturer. Nevertheless, consumers also benefit from better services at the point of sale.

in the fashion clothing and fine fragrance industries the degree of externality is likely to be much lower. Still, the incentives to invest may not high enough, since the retailer's benefit from investing in effort sales depends on the margin it receives. Specifically, if the margin is low, then the retailer will invest a small amount. Using an RPM policy, the manufacturer can create a larger margin for the retailer, thus inducing the optimal level of investment (see Section 11.4).

■ **11.5** Vermont Castings is a manufacturer of wood-burning stoves, a somewhat complex product. One of Vermont Castings's dealers once complained about the terms of the relations between the manufacturer and dealers, stating that "the worst disappointment is spending a great deal of time with a customer only to lose him to Applewood [a competing retailer] because of price." Specifically, the dealer lamented

“the loss of 3 sales of V.C. stoves . . . to people whom we educated and spent long hours with.”²⁹

How do you think this problem can be resolved? How would you defend your solution in an antitrust/competition policy court?

Solution: Obviously, this is a case when one retailer makes an investment in sales effort while the other free-rides and gets the customer by charging a lower price. As we have seen in the discussions in sections 11.2 and 11.3, one possible solution to this problem is to use an RPM policy. In this way, the price would be “fixed” at the minimum level, while the retailers would compete in sales efforts to attract customers.

One would expect that the minimum price set in the RPM policy would be high, hence, an antitrust court would not agree with this policy. However, Vermont Casting may argue that, absent the RPM policy, the retailers will have no incentives to further invest in sales effort and to provide services to customers, making them (the customers) worse off. The price may be lower but the services may be poor. On the contrary, by using an RPM policy, the price may indeed increase, but the customer will now benefit from proper services provided at the point of sale. Obviously there is a trade-off between using and not using an RPM policy, with the crucial issue being the level at which the minimum price should be set.

■ **11.6** Should the European Union outlaw the practice of exclusive territories in car dealerships? Why or why not?

Solution: As we saw in Section 11.3 exclusive territories represent an instance of vertical restraints that helps in resolving the inter-retailer externality represented by underinvestment in sales effort. Thus, if awarded an exclusive territory, a car dealer has all the incentives to invest in advertising, educating customers, etc., while absent this policy it would, most likely, underprovide these services. The issue is to quantify the positive and negative effects of such a policy, that is, to observe how prices and service levels are set in areas where this policy is in use compared to areas where it is not.

■ **11.7** Beer producers are wont to impose an exclusive dealing clause on retailers. Discuss the efficiency and market power effects of this practice.

Solution: Exclusive dealing has, obviously, the effect of foreclosing upstream competition, that is, competition between manufacturers, which, as discussed in Sections 11.5 and 11.6, is likely to reduce welfare and increase market power. One possible defense of exclusive dealing

²⁹Cf Judge R Posner's opinion, cited

MATHEWSON, FRANK, AND RALPH WINTER (1998), “The Law and Economics of Resale Price Maintenance,” *Review of Industrial Organization* **13**, 57–84..

is that there may be important investments to be made by the manufacturer at the retail store, so that, if there is competition between manufacturers, an externality may appear, leading to sub-optimal investments. In the case of car dealerships, such externality arise in the context of dealer training to be done by the manufacturer. In the case of beer, however, it is unlikely there are significant manufacturer externalities.

■ **11.8*** Two major music companies—Sony and Warner Music—have recently been subject to an antitrust inquiry by the FTC over allegations that they illegally discouraged retail discounting of compact disks. The investigation is centered on the practice of announcing suggested prices. Suggested prices are not illegal—only agreements among firms on such prices are illegal. But in practice retailers that advertise or promote CDs at a price below the suggested price are denied cash payments by the manufacturers, in effect “forcing” such suggested prices.³⁰

How would you decide on this case?

Solution: De facto, this situation corresponds to one of RPM, even though it is not explicitly presented as such. The analysis of the costs and benefits from RPM should therefore be applied.

■ **11.9***** Consider the model presented in the beginning of Section ??, but assume that retailers compete à la Cournot. Show that the optimal wholesale price is strictly between marginal cost and monopoly price.

Solution: As in the text, suppose that the upstream firm offers retailers a contract stipulating a fixed fee, f , as well as a wholesale price, w . From Chapter ??, we know that the equilibrium price under Cournot competition is given by $p^N = \frac{1}{3}a + \frac{2}{3}w$, where w is the effective marginal cost paid by retailers. Output per firm is given by $q^N = \frac{1}{2}(a - p^N) = (a - w)/3$. Finally, equilibrium profit per firm is $\pi^R = (a - w)^2/3$. This implies that the upstream firm can ask for as much as $f = (a - w)^2/3$ as a fixed fee.

The upstream firm’s total profit is therefore given by

$$\pi^M = 2((w - c)q^N + \pi^R) = 2\left(w\frac{a - w}{3} + \left(\frac{a - w}{3}\right)^2\right).$$

Maximizing with respect to w , we get the optimal value $w = \frac{1}{4}a + \frac{3}{4}c$. Notice that the optimal w is a convex combination of a and c , that is, the coefficients of a and c add up to 1. Moreover, from Chapter ?? we know that monopoly profit is given by $p^M = \frac{1}{2}a + \frac{1}{2}c$. Since the relative weight of w on c is greater than the weight of p^M on c (and $c < a$), it follows that w is less than monopoly price. By the same argument, it is also clear that w is greater than marginal cost.

³⁰ *The Wall Street Journal*, December 16, 1999.

■ **11.10***** Consider the following highly simplified picture of the personal computer industry.

There are many, price-taking firms that assemble computer systems. Call these firms “computer OEMs.” (“OEMs” stands for “original equipment manufacturers.”) Each of these firms must buy three inputs for each computer system that it sells: (1) a variety of components that are themselves supplied competitively and collectively cost the computer OEM \$500 per computer; (2) the Windows operating system, available only from Microsoft, at a price p_M , to be discussed below; and (3) a Pentium microprocessor, available only from Intel, at a price p_I , also to be discussed below. Since each computer system requires precisely one operating system and one microprocessor, the marginal cost of a computer to an OEM is $500 + p_M + p_I$. Assume that competition among OEMs drives the price of a computer system down to marginal cost, so we have $p = 500 + p_M + p_I$, where p is the price of a computer system.

The demand for computer systems is given by $Q = 100,000,000 - 50,000p$.

Microsoft is the sole supplier of the Windows operating system for personal computers. The marginal cost to Microsoft of providing Windows for one more computer is zero.

Intel is the sole supplier of the Pentium microprocessors for personal computers. The marginal cost to Intel of a Pentium microprocessor for one more computer system is \$300.

(a) Suppose that Microsoft and Intel simultaneously and independently set the prices for Windows and Pentium chips, p_M and p_I . What are the Nash equilibrium prices, $\hat{p}_M$ and $\hat{p}_I$?

Now suppose that Microsoft and Intel sit down to negotiate an agreement to sell Windows and Pentium chips as a package to computer OEMs for a package price of p_{MI} .

(b) What package price would maximize Microsoft’s and Intel’s combined profits? By how much would an agreement between Microsoft and Intel boost their combined profits?

(c) Would final consumers benefit from such an agreement between Microsoft and Intel, or would they be harmed? What about computer OEMs? Relate your answer to your calculations in parts (a) and (b), and explain the economic principles underlying your answer.

Solution: [(a)] First consider Microsoft’s best response to any given price p_I by Intel. Using the underlying demand for computers, the demand for Windows is given by $Q = 100,000,000 - 50,000(500 + p_M + p_I)$. For a given value of p_I , the demand for Windows is $Q = 75,000,000 - 50,000p_I - 50,000p_M$. The corresponding marginal revenue for Microsoft is $MR_M = 1500 - p_I - Q/25,000$. Setting this equal to Microsoft’s marginal cost of zero gives $q_M^* = 37,500,000 - 25,000p_I$, and the corresponding optimal price of $p_M^* = 750 - p_I/2$. Next, repeat these steps to consider Intel’s best response to any given price p_M by Microsoft. The only difference is that Intel has a marginal cost of \$300. These calculations imply that $MR_I = 1500 - p_M - Q/25,000$. Setting this equal to Intel’s marginal cost of \$300 gives with the corresponding optimal price of $p_I^* = 900 - p_M/2$. Finally, solve these two equations together to get the Nash Equilibrium prices, which are $p_M^* = \$400$ and $p_I^* = \$700$. Note

that the resulting price of a computer is \$1600, so total computer sales are 20 million.

[(b)] This is a basic monopoly pricing problem for Microsoft and Intel collectively. If they set a package price of p_{MI} , the price of a computer system will be $500 + p_{MI}$. The number of computers sold will be $Q = 100,000,000 - 50,000(500 + p_{MI})$. The marginal revenue corresponding to this demand curve is $MR_{MI} = 1500 - Q/25,000$. Setting this equal to the (combined) marginal cost of \$300 gives a quantity of $Q_{MI}^* = 30,000,000$ and a corresponding package price of $p_{MI}^* = \$900$. At this price, the contribution to Microsoft's and Intel's combined profits is \$600 per computer times 30 million machines, or \$18 billion. In comparison, the Nash Equilibrium in part (a) involved a contribution of \$800 per computer times 20 million machines, or \$16 billion. Cutting a deal is worth \$2 billion to Microsoft and Intel together.

[(c)] Since Windows and Pentium are complements, Microsoft's profits are decreasing in the price of Pentium chips, and Intel's prices are decreasing in the price set by Microsoft. This implies that the two companies together would benefit from lower prices than they would set separately. Indeed, comparing parts (a) and (b) we see a lower price in part (b) than in part (a). Final consumers thus benefit from the cooperation between Microsoft and Intel. OEMs are indifferent, because their profits are driven to zero by competition, whatever the prices of components. (In practice, OEMs would benefit in the short run from the lower input prices, and OEMs able to differentiate themselves with their own brand names would benefit for a longer period of time.) The underlying principle is that cooperation among suppliers of complements tends to benefit consumers, just as cooperation among suppliers of substitutes (i.e., collusion) harms consumers. This is closely related to the theory of "double marginalization" that we discussed in this chapter; the only difference is that Microsoft and Intel stand in a "complements" relationship rather than a buyer/seller relationship.

■ **12.2** Empirical evidence suggests that, during the 1970s, a firm with an IBM 1400 was as likely as any other firm to purchase an IBM when making a new purchase, while a firm with an IBM 360 was more likely to purchase an IBM than a firm that did not own an IBM 360. Software for the IBM 1400 could not run on the succeeding generations of IBM models (360, 370, 3000, and 4300), while software for the IBM 360 could run on the 370, 3000 and 4300.³¹

How do you interpret these results?

Solution: These results suggest how backwards compatibility influences the degree of switching cost. Switching away from an IBM 1400 was less costly because there was no backwards compatibility between later models and the software developed for the IBM 1400. The same was not true for the 360, 370, 3000 and 4300 models. Consumers who bought one of these models had a higher opportunity cost of switching to a non-IBM computer. As

³¹See

GREENSTEIN, SHANE M. (1993), "Did Installed Base Give an Incumbent Any (Measurable) Advantages in Federal Computer Procurement?," *Rand Journal of Economics* **24**, 19–39..

we would expect, these consumers were more likely to buy IBM in the future than other consumers.

■ **12.3** Says a market analyst in Brussels:

I think the euro [the new European single currency] will bring lower prices over all but that the price differences will be more or less the ones we have right now.

Do you agree? Why or why not?

Solution: As discussed in Section 12.4, there is significant price dispersion across European countries. Cross-country differences are partly due to price discrimination, partly to taxation and regulation, and partly to search costs (and possibly other factors). Search costs may be reduced because consumers are no longer confused by transforming prices from one currency to another. Therefore, the reduction in search costs should reduce the market power of firms (perhaps not to a great extent, though), resulting in lower prices. Overall differences will, however, persist, due to the above mentioned price discrimination, taxes, regulations, etc.

■ **12.4** A study on retail price for books and CDs finds that price dispersion (weighted by market shares) is lower for internet retailers than for conventional retailers.³² Discuss.

Solution: Lower price dispersion may result from two factors. First, it is easier to obtain information about online store prices than it is about conventional retailers. Second, online stores have one less dimension of differentiation with respect to traditional stores: geographical location.

As shown in Section 12.4, imperfect information leads to higher prices and possible price dispersion. In Section 12.2, we argued that product differentiation leads to higher prices. And, although this was not formally shown, product differentiation may also lead to price dispersion.

In summary, lower price dispersion by online sellers may result both from imperfect information and product differentiation. In fact, one of the points of this chapter is that the effects of imperfect information and product differentiation are often similar.

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BRYNJOLFSSON, E., AND M. SMITH (1999), "Frictionless Commerce? A Comparison of Internet and Conventional Retailers," Working Paper, MIT..

■ **12.5** “Price dispersion is a manifestation — and indeed it is a measure — of ignorance in the market.”³³ Do you agree? Compare with possible alternative explanations for price dispersion.

Solution: If we consider search costs as being a measure of market ignorance, then indeed the above claim holds. As in note f) in section 12.4, quotation of prices in different currencies makes comparison shopping more difficult by increasing “search costs”. The fact that buyers do not know or bother to learn how to transform prices from one currency into another is a sign of ignorance, which supports price dispersion.

Other alternative explanations for price dispersion may be: price discrimination, different regulatory or taxation regimes (geographical price dispersion) or different shopping experience (see the example for CDs bought in a small music shop or in a supermarket).

■ **12.6** Consider the model of price dispersion sketched in Section ?? . Show that there can be at most two different prices in equilibrium.

Solution: See S. Salop and J. Stiglitz, “Bargains and Ripoffs,” *Review of Economic Studies* 44 (1976), 493–510; or the summary discussion in H. Varian, *Microeconomic Analysis*, New York: Norton (1978), Chapter 8.

■ **12.7*** Two firms are engaged in Bertrand competition. There are 10,000 people in the population, each of whom is willing to pay at most 10 for at most one unit of the good. Both firms have a constant marginal cost of 5. Currently, each firm is allocated half the market. It costs a customer s to switch from one firm to the other. Customers know what prices are being charged. Law or custom restricts the firms to charging whole-dollar amounts (e.g., they can charge 6, but not 6.50).

(a) Suppose that $s = 0$. What are the Nash equilibria of this model? Why does discrete (whole-dollar) pricing result in more equilibria than continuous pricing?

(b) Suppose that $s = 2$. What is (are) the Nash equilibrium (equilibria) of this model?

(c) Suppose that $s = 4$. What is (are) the Nash equilibrium (equilibria) of this model?

(d) Comparing the expected profits in (b) to those in (c), what is the value of raising customers’ switching costs from 2 to 4?

Solution:

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STIGLER, GEORGE (1961), “The Economics of Information,” *Journal of Political Economy* **69**, 213–225..

- (a) There are three Nash equilibria: (1) both firms charge $p = 5$, (2) both firms charge $p = 6$, and (3) both firms charge $p = 7$. The reason whole-dollar pricing results in multiple Nash equilibria is that one has to undercut by a discrete amount, not by just a fraction of a cent.
- (b) Now to undercut your rival, you must drop price by at least 3 to get the whole market. (If you undercut by 2, you get half the other's customers). There is only one Nash equilibrium: both firms charge $p = 10$.
- (c) Same as in part (b). One Nash equilibrium: both firms charge $p = 10$.
- (d) There is no advantage to further increasing switching costs once $s = 2$.

■ **12.8*** Twenty five different stores sell the same product in a given area to a population of two thousand consumers. Consumers are equally likely to first visit any of the twenty five stores. Half of the consumers have no search costs and purchase at the lowest price. The other half is willing to buy one unit of the product up to a maximum of \$70 and must incur a cost of \$44 in order to find out about the prices charged by other stores. Each store can sell up to 50 units and has a unit cost of \$25.

- (a) Show that, in equilibrium, there exist at most two different prices.
- (b) Show that, if there exist two different equilibrium prices, then the higher price must be 70.
- (c) Show that the following is an equilibrium: 5 firms set a price of 70 and the remaining 20 firms set a price of 45.

Solution: a. As in Exercise 12.6;

b. If the high price is lower than 70, a firm that deviates by slightly increasing price does not lose market share since consumers are not willing to pay the search cost. Therefore, the firm is strictly better off. Hence, all firms would want to deviate upwards, so that the high price must be 70.

c. [There are two typos in this problem: each store's capacity is given by 90 units, not 50. Moreover, consumers with zero search cost have willingness to pay of 45.]

First notice that, given the search costs for first type of consumers, we can safely assume that these consumers will not search, rather will compare price to their willingness to pay (70).

At the proposed prices, profits are as follows: for a firm setting $p = 70$, demand is given by $1,000/25=40$ and total profit is $40(70 - 25) = 1,800$. For a firm setting $p = 45$, total demand is $1,000/25 + 1,000/20 = 90$ and total profit is $90(45 - 25) = 1,800$.

A $p = 45$ firm could deviate by setting a lower price. It would get more demand but, since it is selling at capacity, profit would be lower. It could set a higher price but would then only keep the high valuation consumers. It could at most make a profit equal to the profit currently earned by the $p = 70$ firms, which in turn is equal to its current profit. We thus conclude that such firm would not want to charge a different price.

A $p = 70$ firm could deviate by setting a lower price. Any price below 70 and above 45 leads to the same demand but a lower margin. By setting a price equal or lower than 45, the firm would get less than what $p = 45$ currently get, which in turn is the same as a $p = 70$ firm currently gets.

■ **13.1** Explain how advertising expenditures with no direct informational content can increase market efficiency.

Solution: As discussed in Section 13.1, advertising expenditures may signal product quality. In the presence of repeat purchases, a firm that produces a high-quality good and sells the good not only in the present but also in the future, will have more to gain from getting customers to try its product than a firm that produces a low-quality good. This is because once a good is purchased, consumers become aware of its quality; in the future they will buy the high-quality good. If, however, a consumer does not get to try the good in the present, in the future he or she will still be uncertain about the good's quality. Therefore, high-quality goods producers will try to lure customers in the present since their gain is higher. They thus have an incentive to differentiate themselves from low-quality goods producers.

Although advertising has no direct informational content, the equilibrium with advertising may be more efficient than the equilibrium without advertising. Absent advertising, high-quality firms have no incentive to produce, since they cannot differentiate themselves; their products are ex-ante identical to the ones produced by low-quality firms. Therefore, if consumers value high-quality goods, even if there are savings in advertising expenditures, the overall efficiency effect may be negative, due to the loss in the availability of high-quality goods.

■ **13.2*** Empirical evidence suggests that the probability of a household switching to a different brand of breakfast cereal is increasing in the advertising intensity of that brand. However, the effect of advertising is significantly lower for households who have previously tried that brand.³⁴ What does this suggest about the nature of advertising expenditures (persuasion vs information)?

Solution: To answer this question one can simply parallel the explanation provided in Box 13.1. The effect on the probability of switching is high when the consumer did not try the product before and low if the consumer has already tried the product. This is consistent with the hypothesis that advertising has an informative effect.

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SHUM, MATTHEW (1999), "Advertising and Switching Behavior in the Breakfast Cereals Market," University of Toronto..

■ **13.3** Consider the following industries: pharmaceuticals, cement, perfumes, fast food, compact cars. How would you expect them to be ordered by advertising intensity? Why?

Solution: According to the Dorfman-Steiner formula, advertising intensity is proportional to the demand elasticity of advertising expenditures and inversely proportional to the price elasticity of demand. Price elasticity of demand is lowest for pharmaceuticals and perfumes, highest for cement. Advertising elasticity is lowest for cement, highest for perfumes (and some pharmaceuticals). We would expect advertising intensity to be highest for pharmaceuticals and perfumes, lowest for cement, intermediate for fast food and compact cars.

■ **13.4** In Section ??, it was argued that advertising intensity under duopoly should be greater than under monopoly. DeBeers, the dominant firm in the diamond industry (a cartel that in many respects is like a monopoly), spends vast resources on advertising. More recently, DeBeers has also started to advertise diamonds *and* the name DeBeers. Is this consistent with the analysis of Section ??? What aspects of the diamond industry are not reflected in the analysis of Section ???

Solution: The value of diamonds is, to a great extent, a consequence of the perception of scarcity. Advertising has played a very important role in the diamond industry, both by increasing demand and by inducing a perception of scarcity. In this sense, there is a strong “public good” element in the advertising of diamonds. By controlling the distribution of diamonds, DeBeers is able to internalize this extrenality. Recent events in the industry (the cartel defection of the Australian mines and the emergence of non-cartel mines in Canada) is likely to lead to a more fragmented market structure. The “public good” effect would then imply lower levels of advertising. However, with DeBeers controlling a smaller market share, the market-share-shifting effect of advertising is now more important, leading possibly to higher levels of advertising. Finally, in addition to changes in the *level* of advertising we are also likely to observe a shift in the nature of the advertising expenditures, with a greater emphasis on branding and less on generic characteristics of diamonds.

■ **13.5** Which of the two cars, BMW series 5 and Nissan Sentra, would you expect to have a greater price elasticity? Based on this, which car would you expect to have a greater advertising to sales ratio? Is the empirical evidence consistent with this?

Solution: One would expect the price elasticity of demand to be higher for compact cars, both because branding effects are likely to be smaller in this price range and because the number of competing models is greater. See Box 12.1 for data from the US car market.

Table 1: Advertising, income and price elasticities in specific industries.

Industry	Income	Price	Advertising	
			Short-run	Long-run
Bakery products	.757	-.263	.223	.265
Books	2.205	-.774	.250	.348
Canning	.359	-.820	.614	.963
Cereals and grain mill products	.177	-1.469	.224	.320
Cigars and cigarettes	.001	-1.809	.408	.575
Costume jewelry	-1.407	-3.007	.282	.307
Distilled liquor	.179	-.253	.641	.745
Drugs	.719	-1.079	.663	1.042
Jewelry (precious metal)	1.792	.661	.147	.201
Malt liquor	-.184	-.562	.004	.010
Soaps	1.684	-.758	.284	.294
Soft drinks	2.008	-1.478	.567	.591
Wines	.407	-.680	.972	1.202

■ **13.6** Consider the values in Table 1. In which industries do you expect advertising intensity to be higher?

Solution: We know from Equation 13.1 that $\frac{a}{R} = \frac{\eta}{\epsilon}$, that is, advertising intensity is proportional to advertising elasticity and inversely proportional to price elasticity. Therefore, for the table in the exercise we have:

- Bakery products: ;
- Books: $\frac{a}{R} = \frac{.223}{.263} = 0.85$;
- Canning: $\frac{a}{R} = \frac{.250}{.774} = 0.32$;
- Cereals and grain mill products: $\frac{a}{R} = \frac{.614}{.820} = 0.75$;
- Cigars and cigarettes: $\frac{a}{R} = \frac{.224}{1.469} = 0.15$;
- Costume jewelry: $\frac{a}{R} = \frac{.408}{1.809} = 0.225$;
- Distilled liquor: $\frac{a}{R} = \frac{.282}{3.007} = 0.09$;
- Drugs: $\frac{a}{R} = \frac{.663}{1.079} = 0.61$;
- Jewelry (precious metal): $\frac{a}{R} = \frac{.147}{.661} = 0.22$;
- Malt liquor: $\frac{a}{R} = \frac{.004}{.562} \approx 0$;
- Soaps: $\frac{a}{R} = \frac{.284}{.758} = 0.37$;
- Soft drinks: $\frac{a}{R} = \frac{.567}{1.478} = 0.38$;

- Wines: $\frac{a}{R} = \frac{.972}{.680} = 1.43$.

All of the above are computed for the short-run advertising elasticity of demand.

■ **13.7**** Your company sells expensive, branded fountain pens. Currently, there are 100,000 people aware of your pens. Each of these 100,000 people has his or her own willingness to pay for your pens. These willingness-to-pay numbers are uniformly distributed between \$0 and \$500. So, your demand curve is given by $Q = 100000(1 - p/500)$. Your marginal cost per pen is \$100. Well-versed in economics, you are pricing your pens at \$300 each, and selling 40,000 pens, generating a contribution of \$8 million.

You have just become brand manager for these fountain pens. The previous brand manager engaged in very little advertising, but you are considering running a major promotional campaign to build your brand image and visibility. You are considering two possible advertising campaigns, call them “Build Value,” “Expand Reach.” (You will either run one of these campaigns or none at all; you cannot run both.)

The “Build Value” campaign will not reach any new potential customers, but will increase the willingness to pay of each of your current 100,000 existing customers by 25%. This campaign costs \$2.5 million to run.

The “Expand Reach” campaign will expand the set of potential customers by 25%, from 100,000 to 125,000. The 25,000 new customers reached will have the same distribution of willingness-to-pay as the pre-existing 100,000 potential customers (namely, uniformly distributed between \$0 and \$500). This campaign costs \$1.8 million to run.

(a) If your choice were between running the “Build Value” campaign and running no campaign at all, would you choose to run the “Build Value” campaign?

(b) If your choice were between running the “Expand Reach” campaign and running no campaign at all, would you choose to run the “Expand Reach” campaign? Show your calculations.

(c) What choice would you make in this situation: run the “Build Value” campaign, run the “Expand Reach” campaign, or run neither?

Solution:

- (a) If you run the “Build Value” campaign, the willingness-to-pay of your 100,000 potential customers will be uniformly distributed between \$0 and \$625, since \$625 is 25% higher than \$500. Thus, your demand will shift from $Q = 100,000(1 - p/500)$ to $Q = 100,000(1 - p/625)$. Put differently, demand will shift from $p = 500(1 - Q/100,000)$ to $p = 625(1 - Q/100,000)$. With this new demand curve, the corresponding marginal revenue curve is $MR = 625(1 - Q/50,000)$. Setting MR equal to the marginal cost of \$100 and solving for Q gives $Q^* = 42,000$. The corresponding price is $p^* = \$362.50$. This generates a contribution of \$11,025,000, or \$3,025,000 higher than without the campaign. Since this exceeds the \$2.5 million cost of the campaign, the “Build Value” campaign is worth running, rather than no campaign at all.

- (b) If you run the “Expand Reach” campaign, you will now face 125,000 customers with willingness-to-pay uniformly distributed between \$0 and \$500. Thus, your demand will shift from $Q = 100,000(1 - p/500)$ to $Q = 125,000(1 - p/500)$. Solving for p gives $p = 500(1 - Q/125,000)$, with corresponding marginal revenue of $MR = 500(1 - Q/62,500)$. Setting this equal to the marginal cost of \$100 and solving for Q gives $Q^* = 50,000$. The corresponding price is $p^* = \$300$. This generates a contribution of \$10,000,000, or \$2,000,000 higher than without the campaign. Since this exceeds the \$1.8 million cost of the campaign, the “Expand Reach” campaign is worth running, rather than no campaign at all.
- (c) In comparison with running no campaign, the “Build Value” campaign adds \$525,000 to profits. In comparison with running no campaign, the “Expand Reach” campaign adds \$200,000 to profits. Since you can only pick one, you should pick the “Build Value” campaign.

■ **13.8**** The effect of advertising expenditures can be decomposed into (a) effect on total market demand and (b) effect on market shares. Accordingly, the following cases can be distinguished, where q_i is firm i 's demand and a_i its advertising expenditure:³⁵

Cooperative advertising:	$\partial q_j / \partial a_i > 0$
Predatory advertising:	$\partial q_j / \partial a_i < 0$
Perfectly cooperative advertising:	$\partial q_i / \partial a_i = \partial q_j / \partial a_i$
Completely predatory advertising:	$\partial q_i / \partial a_i + \partial q_j / \partial a_i = 0$

Empirical studies suggest the following values of demand elasticity with respect to advertising levels.³⁶

Product	Advertising Elasticity	
	Own	Cross**
Coca Cola	.25	-.06
Pepsi Cola	.32	-.62
Saltine crackers*	.16	-.05
High-tar cigarettes	.005***	-.001***

* Long-run elasticity for major brands.

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FRIEDMAN, JAMES (1983), “Advertising and Oligopolistic Equilibrium,” *Bell Journal of Economics* **14**, 464–373..

³⁶Source:

ROBERTS, MARK, AND LARRY SAMUELSON (1988), “An Empirical Analysis of Dynamic, Nonprice Competition in an Oligopolistic Industry,” *Rand Journal of Economics* **19**, 200–220..

GASMI, F, JEAN-JACQUES LAFFONT, AND QUANG VUONG (1992), “Econometric Analysis of Collusive Behavior in a Soft-Drink Market,” *Journal of Economics, Management and Strategy* **1**, 277–312..

SLADE, MARGARET E (1995), “Product Rivalry with Multiple Strategic Weapons: An Analysis of Price and Advertising Competition,” *Journal of Economics, Management and Strategy* **4**, 445–476..

** Cross elasticity is the elasticity of q_i with respect to a_j .

*** NB: these are derivatives of market share with respect to advertising level.

Based on the above classification, how do you characterize advertising expenditures on cola drinks, saltine crackers and cigarettes?

Solution: These are all instances of predatory advertising since the cross-elasticities are negative, that is, increasing advertising decreases the market share of competitors.

■ **14.1** Explain in words why the number of firms in a free-entry equilibrium may be less than proportional to market size.

Solution: The explanation lies in the fact that as the number of firms increases, so does competition. As a result, prices will fall, reducing the margin, $p - c$. Therefore, variable profit per unit of market size decreases, making the number of firms the market can sustain increase less than proportionally to market size.

■ **14.2*** Suppose that two countries, initially in autarchy, decide to create a single market. For simplicity, assume that, in both economies, there is only one product. Demand for this product is given by $D_i = S_i(a - p_i)$, ($i = 1, 2$), where S_i is a measure of country i 's size. Upon the creation of a single market, total demand is given by the horizontal sum of the two initial demands.

Assuming there is free entry and that firms compete à la Cournot, determine the equilibrium number of firms in autarchy and after the completion of the single market. Interpret the results.

Solution: In autarchy we have $p_i = a - \frac{D_i}{S_i}$. Assuming that the cost function takes the form $c(q_{ik}) = F + cq_{ik}$ (where i indexes the country and k indexes the firm), each firm solves the problem $\max[(p - c)q_{ik} - F]$ which is equivalent to $\max[(a - c - \frac{\sum_k q_{ik}}{S_i})q_{ik} - F]$. The first order condition is given by $a - c - \frac{(n+1)q_{ik}}{S_i} = 0$ (due to the symmetry assumption), therefore, we have the solution for each firm's quantity $q_{ik} = \frac{(a-c)S_i}{n+1}$. The profits for each firm will be $\pi_i(n_i) = \left(\frac{a-c}{n_i+1}\right)^2 S_i - F$. In a free entry equilibrium these profits should be 0.

Therefore we have the solution: $n_i = \left[(a - c)\sqrt{\frac{S_i}{F}} - 1\right]$.

After the completion of the single market the size of the market increases, and, as it is assumed, demand becomes $D_{1+2} = D_1 + D_2 = (S_1 + S_2)(a - p)$. Using the same general formula that we derived for the autarchy case we obtain that $n_{1+2} = \left[(a - c)\sqrt{\frac{S_1+S_2}{F}} - 1\right]$. This tells us that some firms will exit, the explanation for this being the same as in Exercise 14.1.

■ **14.3*** The number of imported automobiles in California is four times higher than in Montana, in per capita terms. The population of California is mainly urban, whereas the population of Montana is mainly rural. How do demographic differences and the model presented in Section ?? explain the differences in consumption patterns?³⁷

Solution: The model predicts that smaller markets will have fewer firms and higher margins. The fact that the population of Montana is mainly rural implies that the typical market for a car dealer is smaller than in California.

■ **14.4*** Retail in Switzerland is mostly dominated by highly profitable cartels. The Swiss authorities anticipate the gradual collapse of these cartels as the country becomes better integrated with the rest of Europe. OECD, by contrast, hold a more sceptical view, claiming that the collapse of cartels does not necessarily lead to more competitive markets; rather, they add, cartel breakdowns are frequently associated with an increase in concentration. Which prediction seems more reasonable? Are the two views inconsistent?

Solution: Integration is likely to imply greater competition from foreign suppliers. Lower margins will then imply that the Swiss market cannot hold the same number of firms as currently. It is therefore possible that the two predictions hold true: that prices go down and that the industry becomes more concentrated.

■ **14.5** “Barriers to entry may be welfare improving.” What particular industry characteristics might make this statement valid?

Solution: Following the discussion in Section 14.3, free entry may decrease welfare when the business stealing effect dominates. For this to happen, as in the example of retail banking, the product or service should be relatively homogenous (so that product differentiation is unimportant) and price competition should be soft. In this case, paying a fee for setting up a branch represents a barrier to entry and may act as an efficient means of blocking excessive entry.

■ **14.6**** Show that the coefficient of scale economies, AC/MC , is greater than one if and only if average cost is decreasing.

³⁷Adapted from an exercise written by T. Bresnahan.

Solution: Average Cost is given by the ratio Cost / Output. Taking the derivative with respect to Output q , we get

$$\frac{dAC}{dq} = \frac{d}{dq} \frac{C}{q} = \frac{\frac{dC}{dq}q - C}{q^2} = (MC - AC)/q,$$

It follows that AC is greater than MC if and only if $\frac{dAC}{dq} < 0$, that is, average cost is decreasing.

■ **14.7***** Consider the model presented in Section 14.7. Suppose that firms can choose one of two possible technologies, with cost functions $C_i = F_i + c_i q$.

a) Derive the conditions for a free-entry equilibrium.

b) Show, by means of numerical example, that there can be more than one equilibrium, with different numbers of large and small firms.

Solution: a) Suppose that demand is given by $Q = a - p$. There are two types of firms, Firm i 's profit is given by $(a - Q)q_i - C_i$. The first-order condition for profit maximization is $q_i = a - c_i - Q$. Suppose that in equilibrium each of the n_i firms with technology i produce output q_i . Then $Q = n_1 q_1 + n_2 q_2$. Solving the system of first-order conditions, we get

$$q_i^*(n_1, n_2) = \frac{a - c_j - n_i(c_j - c_i)}{1 + n_1 + n_2},$$

for $i, j = 1, 2$ and $i \neq j$. From these equations, we can get $Q^*(n_1, n_2)$, the equilibrium total output when there are n_i firms of each type:

$$Q^*(n_1, n_2) = n_1 \frac{a - c_2 - n_1(c_2 - c_1)}{1 + n_1 + n_2} + n_2 \frac{a - c_1 - n_2(c_1 - c_2)}{1 + n_1 + n_2}.$$

The equilibrium conditions are then given by

$$\begin{aligned} (a - Q^*(n_1, n_2)) q_i^*(n_1, n_2) &\geq F_i + c_i q_i^*(n_1, n_2) \\ (a - Q^*(n_i + 1, n_j)) q_i^*(n_i + 1, n_j) &\leq F_i + c_i q_i^*(n_i + 1, n_j) \\ (a - Q^*(n_i + 1, n_j - 1)) q_i^*(n_i + 1, n_j - 1) - \\ (F_i + c_i q_i^*(n_i + 1, n_j - 1)) &\leq (a - Q^*(n_1, n_2)) q_j^*(n_1, n_2) \\ &\quad - (F_j + c_j q_j^*(n_1, n_2)) \end{aligned}$$

The first of these conditions implies that incumbent firms make positive profits. The second condition implies that a potential entrant would make negative profits. The third condition implies that an incumbent would not gain from switching technologies. Note that all conditions apply for $i = 1, 2$. We thus have a total of six equilibrium conditions.

b) The following values satisfy the equations above: $a = 1000, F_1 = 173, c_1 = 0, F_2 = 10.3, c_2 = 10, n_1 = 60, n_2 = 60$.

■ **14.8*** Consider the monopolistic competition model, presented in Chapter ??.
What is, according to this model, the relation between the degree of product differentiation and market structure?

Solution: Refer to Figure 6.3 the solution to Exercise 6.2. The greater the degree of product differentiation, the steeper the demand curve d faced by each firm. In the long run, price equal average cost. Therefore, the steeper d is the lower each firm's output is in the long run equilibrium. We would therefore expect a more fragmented market structure when the degree of product differentiation is higher.

■ **14.9**** T. Bresnahan and P. Reiss collected data for small, geographically isolated U.S. towns, on population as well as on the number of doctors, dentists, plumbers, etc., in each town. Based on these data, they estimated that the minimum town size that justifies the entry of a second doctor is approximately 3.96 times the required size for the first doctor to enter. For plumbers, the number is 2.12. How can these numbers be interpreted?

Solution: The higher number for doctors has two interpretations. The first one is that competition between two doctors is very intense, so that it would take a much larger market before the second doctor could recoup entry costs. The second interpretation is that there are specific barriers to entry by a second doctor which are not present in the case of a plumber.

■ **14.10**** Derive Equation (??).

Solution: $\pi_i = (p - c)q_i - F$ and $p = a - \frac{Q}{S}$, therefore, $\pi_i = (a - c - \frac{\sum q_j}{S})q_i - F$. The first-order condition for profit maximization is $a - c - \frac{\sum q_j}{S} - \frac{q_i}{S} = 0$. Using the symmetry assumption, we get $q_i = \frac{(a-c)S}{n+1}$. Plugging this into the profit function we obtain $\pi_i = (\frac{a-c}{n+1})^2 S - F$.

■ **14.11***** Consider the following model of entry into an advertising-intensive industry. To simplify the analysis, and to concentrate on the effects of advertising, suppose that there is no price competition. Specifically, the value of the market, in total sales, is given by S . (One can think of a demand curve $D(p)$ and an exogenously given price, whereby $S = pD(p)$.) S is therefore a measure of market size.

Each firm must decide whether or not to enter the industry. Entry cost is given by F . If a firm decides to enter, then it must also choose how much to invest in advertising; let a_i be the amount chosen by firm i . Finally, firm i 's market share, s_i , is assumed to be equal to its share of the industry total advertising effort:

$$s_i = \frac{a_i}{\sum_{i=1}^n a_i} = \frac{a_i}{A},$$

where n is the number of firms in the industry and $A \equiv \sum_{i=1}^n a_i$ is total industry advertising.

(a) Show that each firm i 's optimal level of advertising solves

$$\frac{A - a_i}{A^2} S - 1 = 0.$$

(b) Show that, in a symmetric equilibrium,

$$a = \frac{n-1}{n^2} S.$$

where a is each firm's level of advertising.

(c) Show that equilibrium profit is given by

$$\pi = \frac{S}{n^2}.$$

(d) Show that the equilibrium number of entrants is given by

$$\hat{n} = \left\lfloor \sqrt{\frac{S}{F}} \right\rfloor,$$

where $\lfloor x \rfloor$ means the highest integer lower than x .

(e) Interpret this result in light of the previous discussion on the effects of endogenous entry costs.

Solution: a) The profit of each firm is given by $\pi_i = pq_i - a_i - F = pQ \frac{q_i}{Q} - a_i - F = S s_i - a_i - F$. Therefore, each firm is solving $\max_i [S \frac{a_i}{\sum_{j=1}^n a_j} - a_i - F]$. The first-order condition is given by $\frac{S}{A} - \frac{S a_i}{A^2} - 1 = 0$, which is equivalent to $\frac{S(A - a_i)}{A^2} - 1 = 0$.

b) In a symmetric equilibrium we have $a_i = \frac{A}{n}$ and using the result from a) we obtain $a = \frac{S(n-1)}{n^2}$.

c) $\pi = \frac{S}{n} - \frac{S(n-1)}{n^2} - F = \frac{S}{n^2} - F$.

d) The equilibrium requires profits to be 0, hence we have $\pi = \frac{S}{n^2} - F = 0$ so that $n = \left\lfloor \sqrt{\frac{S}{F}} \right\rfloor$.

e) With this specification of the model we have, from b), that advertising expenditures increase with market size. This is an instance of endogenous entry costs, where because of this costly investment in advertising, the net industry profit grows by less than the market size (as can also be observed from c)). As a result, even if price is exogenously given, as it is in our model, the number of firms increases by less than the market size, as the result in d) shows.

■ **15.1*** LC Burgers is currently the sole fast-food chain in Linear city, a city that is one mile long and consists of one street, with one thousand consumers distributed uniformly along the street. The price for the BigLC, the only product sold by the LC Burger chain, is set nationally at \$4, so that the local Linear city manager's decision is limited to choosing the number and location of its stores.

Each store costs \$600,000 to open and lasts indefinitely. Each consumer buys one burger per week at the current price of \$4. However, no consumer will walk for more than a quarter of a mile to buy a burger. Operating costs are \$1 per burger. The interest rate is 0.1% per week. The market conditions are unchanging, so present discounted profits can be regarded as level perpetuities.

(a) Suppose that LC Burgers faces no competition and no threat of entry. How many stores should LC Burgers open, and at what locations?

CS Burgers is contemplating entering Linear city. CS Burgers' costs and price are the same as those of LC Burgers. Moreover, consumers regard the products at both chains as equally good, so, if both brands are in town, each consumer buys from the closest store.

(b) At what locations should CS Burgers open stores, given that LC Burgers has opened the locations found to be optimal in part (a)?

(c) Recognizing the threat of entry by CS Burgers, at what locations should LC Burgers open stores?

(d) Would your analysis of these product-location decisions be affected if you also considered the possibility of pricing competition, i.e., if prices were then set independently given the locations of the stores (rather than taking prices as fixed, as was done above)?

(e) Moving beyond this particular model, does product positioning involve a first-mover advantage, a second-move advantage, or does this depend upon particular aspects of the market in question?

Solution: With two stores, one at .25 and the other at .75 (miles from the left end of the street), LC Burgers is able to cover the entire market. Any additional store would not increase demand and would thus be sub-optimal. By opening two stores, LC Burgers makes a discounted profit of $1000(\$4 - \$1)/.1\% - 2\$600,000 = \1.8m . If LC Burgers were to open one store only, the maximum it could possibly get is $500 * (\$4 - \$3)/.1\% - \$600,000 = \$900,000$.

CS Burgers is contemplating entering Linear city. CS Burgers' costs and price are the same as those of LC Burgers. Moreover, consumers regard the products at both chains as equally good, so, if both brands are in town, each consumer buys from the closest store.

- (b) CS Burger should open four stores, to the immediate left and right of LC Burger's stores, thus stealing all of the market demand. Given these locations, CS Burger would receive a demand of 1,000 and a discounted profit (net of entry costs) of $1,000(\$4 - \$3)/.1\% - 4\$600,000 = \$600,000$. Notice that, under this outcome, LC Burger's profit is $-\$1.2\text{m}$ (two stores, no revenues).
- (c) LC Burger should open three stores and locate them at .1666, .5 and .8333. Given these locations, the maximum an entrant can get is one sixth of the market (check). Given this demand, discounted profits are $166(\$4 - \$3)/.1\% - \$600,000 = -\$100,000$. Under this location strategy, LC Burgers gets a total profit of $1,000(\$4 - \$3)/.1\% - 3\$600,000 = \1.2m . This is substantially less than (unchallenged) monopoly profits (as in (a)). However, it is more than LC Burgers would get by choosing the same locations as in (a) ($-\$1.2\text{m}$).
- Notice that, while these locations are optimal, they are not the only optimal solution. The important thing is that an entrant cannot achieve a market share of 20% or more, where 20% is the minimum market share necessary to recover entry costs (check). Therefore, any solution with a store between .1 and .2, one at .5, and a third one between .8 and .9 would also be optimal. The solution proposed above, however, is the only three-store solution that deters entry when entry costs are as low as \$500,000.
- (d) If there were price competition, then we would expect firms not to locate their stores very close to each other. In particular, CS Burger's entry strategy in (b). would unlikely take place as firms would then compete as in the Bertrand model, yielding zero profits for incumbent and entrant.
- (e) In the case considered above, there is clearly a first-mover advantage: the first-mover makes positive profits, whereas the second mover stays out of the market and makes zero profits. Suppose however that each firm has limited resources and can open on store only. Then it can be seen that, whichever location the first firm chooses, the second firm can choose a location that gives it profits at least as large.

■ **15.2** In less than one year after the deregulation of the German telecommunications market at the start of 1998, domestic long-distance rates have fallen by more than 70%. Deutsche Telekom, the former monopolist, accompanied some of these rate drops by increases in monthly fees and local calls. MobilCom, one of the main competitors, fears it may be unable to match the price reductions. Following the announcement of a price reduction by Deutsche Telekom at the end of 1998, shares of MobilCom fell by 7%. Two other competitors, O.tel.o and Mannesmann Arcor, said they would match the price cuts. VIAG Interkom, however, accused Telekom of "competition-distorting behavior," claiming the company is exploiting its (still remaining) monopoly power in the local market to subsidize its long-distance business.³⁸

³⁸ *International Herald Tribune*, December 29, 1998.

Is this a case of predatory pricing? Present arguments in favor and against such assertion.

Solution: One could indeed argue that this is a case of predatory pricing. If Deutsche Telekom has monopoly in local markets, it likely has financial resources strong enough to afford losing money in the long distance market by pricing below marginal cost. However, since there are two other competitors that matched Deutsche Telekom's prices, one can argue that there exists technology with marginal cost less than the low-price charged. Evidently, other explanations can also be invoked, namely low-cost signaling and reputation for toughness. (See the discussion in the chapter.)

■ **15.3** “The combined output of two merging firms decreases as a result of the merger.” True or false?

Solution: If the merger implies little or no cost efficiencies (namely at the level of marginal cost), we would expect the combined output of the merging firms to decline. If however the merger reduces the marginal cost of the combined firm significantly, then it is possible that the combined output increases as a result of the merger.

■ **15.4*** One of the efficiencies created by mergers in the paper industry results from reorganization of production. A machine is more efficient the narrower the range of products it produces, among other reasons because the length of each production run can be made longer.

The paper industry underwent a wave of mergers in the 1980s. Of the firms that merged, about two thirds increased their market share as a result of the merger. Assuming that (i) firms compete by setting production capacity and (ii) paper products are relatively homogeneous across firms, explain how the previous paragraph explains the pattern of changes in market shares. Which firms would you expect to increase their market share?³⁹

Solution: According to the paragraph, there are increased cost efficiencies from mergers. Applying the analysis from Section 15.3, it seems that for two thirds of the merging firms the cost efficiencies were so big that the merging firms increased their output and market share, while for the rest the efficiencies were not big enough, resulting in a decreased market share.

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PESENDORFER, MARTIN (1998), “Horizontal Mergers in the Paper Industry,” Department of Economics, Yale University, September..

■ **15.5** “The renewed prospect of a link-up between British Aerospace PLC and the Marconi defense arm of General Electric Co. PLC of the U.K. as led to revived talks between the top defense companies in Germany and France.”⁴⁰ Discuss.

Solution: Refer to the discussion on merger waves in this chapter.

■ **15.6** Consider a homogeneous product industry with inverse demand given by $p = 100 - 2Q$. Variable cost is given by $C = 10q$. There is currently one incumbent firm and one potential competitor. Entry into the industry implies a sunk cost of F .

- (a) Determine the incumbent's optimal output in the absence of potential competition.
- (b) Suppose the entrant takes the incumbent's output choice as given. Show that the entrant's equilibrium profit is decreasing in the incumbent's output.
- (c) What output should the incumbent firm set in order to deter entry?
- (d) Assuming that the incumbent firm decides to deter entry, determine the Lerner index as a function of F . Discuss the result.
- (e) Determine the lowest value of E such that the incumbent firm prefers to deter entry.

Solution: a) The incumbent solves $\max[pq - c(q)] = \max[(100 - 2q)q - 10q]$. The first order condition is $90 - 4q = 0$ and the solution is $q = 22.5$.

b) Taking the incumbent's output choice as given, the potential entrant solves the following problem: $\max[100 - 2(q_i + q_e)]q_e - 10q_e - F$. The first-order condition is given by $90 - 2q_i - 4q_e = 0$, and the solution is $q_e = 22.5 - \frac{q_i}{2}$. Plugging this result into the entrant's profit function we obtain $\pi_e = 2(22.5 - \frac{q_i}{2})^2 - F$. As one can see, the bigger is q_i , the lower the entrant's profits are.

c) Knowing that $\pi_e = 2(22.5 - \frac{q_i}{2})^2 - F$, in order to deter entry the incumbent has to set q_i such that $\pi_e = 0$. Therefore we have $q_i = 2(22.5 - \sqrt{\frac{F}{2}})$.

d) Since there is only one firm in the market (entry is deterred) the market share is equal to 1, therefore, the lerner index is $L = \frac{p-MC}{p}$. In our case, $p = 100 - 2q_i = 100 - 4(22.4 - \sqrt{\frac{F}{2}}) = 10 + \sqrt{8F}$, $MC = 10$, hence $L = 1 - \frac{10}{10 + \sqrt{8F}}$. This basically says that the higher the sunk costs, the higher the concentration index. In order to deter entry, the incumbent deviates from its optimal monopoly output choice. However, sunk cost act as a barrier to entry. Therefore, the higher the sunk costs, the smaller the incumbent's deviation from the monopoly output choice and the higher the concentration.

⁴⁰ *The Wall Street Journal Europe*, January 15–16, 1999.

■ **15.7***** A large fraction of industry entry corresponds to acquisition of incumbent firms. For example, from a sample of 3,788 entry events, about 70% were acquisitions.⁴¹ Econometric analysis suggests that entry by acquisition is more common in more concentrated industries.⁴² Can you explain this observation?

Suggestion: Consider a Cournot oligopoly with n symmetric firms. Determine the maximum that an entrant would be willing to pay for one of the incumbent firms. Determine also the minimum that an incumbent would require from a buyer, *knowing that the alternative to selling the firm is for the entrant to create a new firm*. Show that the difference between the two values above is greater when the industry is more concentrated.⁴³

What other factors would you expect to influence the “build or buy” decision when entering an industry?

Solution: Suppose that $p = a - bQ$ and $c(q) = cq + F$. Every firm solves $\max[(a - c - bQ)q - F]$, with the solution being $q_i = \frac{a-c}{b(n+1)}$. In a symmetric equilibrium all firms produce the same quantity, and the profits would be $\pi_i^{(n)} = \left(\frac{a-c}{b(n+1)}\right)^2 \frac{n}{b} - F$. Therefore, a buyer is willing to pay up to $\left(\frac{a-c}{b(n+1)}\right)^2 \frac{n}{b} - F$ to acquire an incumbent firm. If the buyer decides to enter, the number of firms in the industry becomes $n + 1$, hence each firm's profit is given by $\pi_i^{(n+1)} = \left(\frac{a-c}{b(n+2)}\right)^2 \frac{n+1}{b} - F$. This is also the minimum that a target would require.

The difference is given by $\Delta = \pi_i^{(n+1)} - \pi_i^{(n)} = \left(\frac{a-c}{b(n+1)}\right)^2 \frac{n}{b} - \left(\frac{a-c}{b(n+2)}\right)^2 \frac{n+1}{b} = \frac{(a-c)^2}{b} \left(\frac{n(n+2)^2 - (n+1)^3}{(n+1)^2(n+2)^2} \right) = \frac{(a-c)^2}{b} \left(\frac{n^2 + n - 1}{(n+1)^2(n+2)^2} \right)$. $\text{sign}(\frac{\partial \Delta}{\partial n}) = \text{sign}(-2n^3 - 3n^2 + 5n + 8)$, which is less than 0 for any n greater or equal to 2.

In industries with a large number of firms, the value of a potential target does not go down to much if the potential acquirer enters by building a new plant. Adding one more firm in an industry with a big number of players results in a marginal decrease in profits. On the other hand, if the industry is concentrated, adding one more firm leads to a large drop in profits, hence, the potential target prefers to be acquired.

One other reason why acquisition may be thought of as a good strategy is the time necessary to build a new plant. Acquisition gives the right to the firms profits in a “short” time, while a new plant needs time to become established as a player in the industry.

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PORTER, MICHAEL (1987), “From Competitive Advantage to Corporate Strategy,” *Harvard Business Review*, May-June, 43–59..

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CAVES, RICHARD E., AND SANJEEV MEHRA (1986), “Entry of Foreign Multinationals into U.S. Manufacturing Industries,” in Porter (Ed), *Competition in Global Industries*, Cambridge, MA: Harvard Business School Press..

⁴³This exercise is adapted from

GILBERT, RICHARD, AND DAVID NEWBERY (1992), “Alternative Entry Paths: The Build or Buy Decision,” *Journal of Economics and Management Strategy* 1, 127–150..

■ **16.1** “Perfect competition is not only impossible but inferior, and has no title to being set up as a model of ideal efficiency.” Do you agree? Why or why not?

Solution: In a static sense, perfect competition is the most efficient industry structure, since it maximizes social welfare. In a dynamic sense, however, perfect competition is not necessarily the ideal model, since it may be less conducive to technological progress than situations that allow for some (temporary) degree of market power. The latter would not necessarily be monopoly markets (in a static sense) but rather forms of oligopoly, in which firms compete not only in quantity (or price) but also in R&D, so that they can outpower rivals in the future and gain some market power.

■ **16.2** “Competition implies a dynamic system whereby industries tend to become more and more concentrated.” Do you agree? Why or why not?

Solution: There is no clear answer to this question. The reason why industries would move towards a higher concentration is the presence of a steep learning curve (see the case of the wide-body aircraft manufacturing industry). If one of the firms is moving faster down the learning curve than its rivals, it can end up in a position where its competitive advantage is big enough so that it remains the sole major player in the industry.

On the other hand, we saw that if there is uncertainty regarding the threat of entry, outsiders have a greater incentive to perform R&D, which implies a higher likelihood of entry and the industry becomes more competitive.

■ **16.3** Two firms are engaged in Bertrand competition. There are 10,000 people in the population, each of whom is willing to pay at most 10 for at most one unit of the good. Currently, both firms have a constant marginal cost of 5.

(a) What is the equilibrium in this market? What are the firms' profits?

(b) Suppose that one firm can adopt a new technology that lowers its marginal cost to 3. What is the equilibrium now? How much would this firm be willing to pay for this new technology?

(c) Suppose the new technology mentioned in (b) is available to both firms. The cost to a firm of purchasing this technology is 10,000. The game is now played in two stages. First, the firms simultaneously decide whether to adopt the new technology or not. Then, in the second stage, the firms set prices simultaneously. Assume that each firm knows whether or not its rival acquired the new technology when choosing its prices. What is (are) the Nash equilibrium (equilibria) of this game? (What does your answer suggest about why firms engage in patent races?)⁴⁴

Solution:

⁴⁴Adapted from Haas School of Business economics problem sets.

- (a) Both firms charge $p = 5$ and earn $\pi = 0$.
- (b) The firm with the lower cost technology charges a fraction of a cent less than $p = 5$ and sells to all 10,000 customers. Its profits are $\pi = [(5 - 3) \cdot 10,000] = 20,000$. It would be willing to pay up to 20,000 for this technology.
- (c) There are two pure-strategy equilibria: (1) firm 1 invests in the low-cost technology and firm 2 does not, and (2) firm 2 invests in the low-cost technology and firm 1 does not. It is not an equilibrium for both firms to invest or for neither firm to invest. (There is also a mixed strategy equilibrium in which each firm invests with probability 0.5.)

■ **16.4*** In 1984, the U.S. Congress passed legislation that allowed generic-drug makers to receive fast marketing approval from the Food and Drug Administration (FDA). Since then, the market share of generic-drug companies has increased considerably (in volume). Branded-drug companies have attempted different tactics to protect their market share. In some cases, large pharmaceutical firms have paid generic firms to keep off the market. Ivax Corp. and Novartis AG, for example, have agreed not to market a generic competitor to Abbott Laboratories' hypertension drug Hytrin. In exchange, Abbott pays quarterly fees totaling several million dollars.⁴⁵

Compare this example to the discussion on the persistence of monopoly power.

Solution: Suppose for simplicity that Abbott Laboratories is a monopolist on the hypertension drug market. From Section 16.2, we know that a monopolist has a greater incentive to maintain its monopoly power than a rival has to enter. In other words, the monopolist has more to lose from competition than the rival has to gain (the efficiency effect). There are therefore potential gains from an agreement like to one described above.

■ **16.5** Patent life is 17 years in the U.S. and 20 years in Europe. From the perspective of social welfare, do you find this period too short or too long?

Solution: The discussion in Section 16.3 suggests that it is optimal to provide relatively weak patents for relatively long periods of time.

■ **16.6** Should firms be allowed to enter into agreements regarding R&D?

Solution: R&D agreements between firms help alleviate the free-rider problem occurring due to the inevitable spillovers from R&D activity. Moreover, such agreements help reduce the risk associated with an R&D project. There might also be important synergies stemming

⁴⁵ *The Wall Street Journal Europe*, November 19, 1998.

from the combination of experience, intellectual and mental resources and so on. On the other hand, if spillovers are low and the gain from R&D to a firm is a loss to another, then R&D joint ventures may lead to an undesirable reduction in R&D expenditures. See Section 16.3.

■ **17.1*** You have created a business-to-business (B2B) Internet venture directed at an industry with exactly fifty (50) identical firms. Your services allow these firms to do business with each other more efficiently as members of your trading network. You plan to sell access to your service for a price p per member firm. Each firm's benefit from the service is given by $2n$, where n is the number of other firms joining the B2B network as a member. So, if 21 firms join your service, each places a value of 2×20 or 40 on membership in your network.

Suppose for part (a) that you set the price, p , and then firms simultaneously and independently decide whether or not to join as members.

(a) Show that, for a price greater than zero and lower than 98, there exist exactly two Nash equilibria in the simultaneous-move game played by firms deciding whether or not to join the network as members.

Suppose for part (b) that you are able to persuade 10 firms to join your network at an initial stage as "Charter Members." At a second stage, you set a price for the remaining 40 firms. These 40 firms then simultaneously decide (as in part (a)) whether to join your network as regular members.

(b) For each price p , determine the equilibria of the game played between the remaining 40 firms in the second stage.

Finally, for part (c), consider the same situation as in part (c), but suppose that, when there are multiple Nash equilibria, firms behave conservatively and conjecture that the low-adoption Nash equilibrium will be played. (Note that, by the definition of Nash equilibrium, this conjecture is self-fulfilling.)

(c) How much would you be willing to pay (in total to all 10 early adopters) in order to persuade the first 10 firms to join the network as Charter Members?

Solution:

- (a) Suppose that no firm joins the network. Then the benefit for an individual firm to join the network is zero. If price is positive, the net benefit is negative, which implies that it is a best response not to join the network, which in turn confirms the conjecture that no firm joins the network. We thus have a Nash equilibrium where no firm joins the network for any positive price.

Suppose now that each firm conjectures that all of the other firms will join the network. The expected benefit from joining the network is therefore $98 = 2 \times 49$. If price is less than 98, the net benefit is positive, which in turn confirms the conjecture that all firms join the network. We thus have a Nash equilibrium where all firms join the network for a price less than 98.

Suppose that you are able to persuade 10 firms to join the network at an initial stage. At a second stage, you set a price for the remaining 40 firms. These 40 firms then simultaneously decide whether to join the network (as in (a)).

- (b) Each of the second-mover firms knows that the number of adopters is at least 10. It follows that the benefit from joining the network is at least $2 \times 10 = 20$. Therefore, if price is less than 20, then the zero-adoption equilibrium is no longer a Nash equilibrium. Only the full-adoption equilibrium remains. For higher prices, however, the two equilibria are possible, for the same reasons as in (a).

Suppose that, when there are multiple Nash equilibria, firms behave conservatively and conjecture that the low-adoption equilibrium will be played. (Note that, by definition of Nash equilibrium, this conjecture is self-fulfilling.)

- (c) If no firm joins the network in the first stage, then the game in the second stage is as described in (a). Since for any positive price there are two equilibria and firms behave “conservatively”, it follows that no firm joins the network in the second stage and profits are zero. If however the 10 firms do join the network at the initial stage, then, in the second stage, you can set a price of up to 20 and know that all firms will join the network (since this is the only Nash equilibrium and firms know that; in fact, joining the network would be a dominant strategy). For a price $P = 20$, this leads to profits $20 \times 40 = 600$. We conclude that you should be willing to spend up to 600 to persuade the first 10 firms to join the network.

■ **17.2** Empirical evidence suggests that, between 1986 and 1991, consumers were willing to pay a significant premium for spreadsheets that were compatible with the Lotus platform, the dominant spreadsheet during that period.⁴⁶ What type of network externalities is this evidence of?

Solution: One can think of this both as a type of direct and indirect externality. For example, exchanging files with users of the Lotus package, one has to own a compatible spreadsheet. This is a case of direct network externality. On the other hand, developments in the Lotus software can (potentially) be easily adapted and adopted by compatible software products, which is a case of indirect network externality.

■ **17.3** People are more likely to buy their first home computer in areas where a high fraction of households already own computers or where a large share of their

⁴⁶See

GANDAL, NEIL (1994), “Hedonic Price Indexes for Spreadsheets and an Empirical Test for Network Externalities,” *Rand Journal of Economics* **25**, 160–170..

friends and family own computers: a ten percent greater penetration in the surrounding city is associated with a one percent higher adoption rate.⁴⁷ How can this be explained by network externalities? What alternative explanations are there?

Solution: The larger the number of households/friends that own computers, the larger the the possibilities for direct communication among users and the greater utility one derives from owning a computer. Communication among users may consist of email exchange, learning how to use a given software, exchange of files, and so forth.

An alternative explanation is that certain areas attract more “sophisticated” users than other areas. If this were the case, those areas would have a higher penetration rate and a higher adoption rate. But then the correlation between penetration and adoption would be just that—correlation, not causality. Another situation of correlation-not-causality is when a higher penetration implies a greater degree of competition in the computer market, lower prices and a higher adoption rate.

See the cited reference for additional alternative explanations.

■ **17.4**** In the early days of Automated Teller Machines (ATMs), there were very few interbank networks, that is, each bank’s network was incompatible with the other banks’. Empirical evidence shows that banks with a larger network of branches adopted ATMs earlier. To what extent can network externalities explain this observation?⁴⁸

Solution: Network effects imply that the value of using a given ATM system is increasing in the number of ATM machines compatible with that system. Part of the value created by a network is gained by the consumer, part by the banks. Assuming (as is empirically observed) that the cost of adoption is decreasing over time, the above observation implies that the critical moment in time at which it pays to adopt ATMs is earlier the greater the network effect. Since the number of branches is a good proxy for the number of ATMs, this implies that banks with a greater number of ATM machines are likely to adopt first, as the evidence shows was indeed the case.

■ **17.5** How would you respond to the following quotation:

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GOOLSBEE, AUSTAN, AND PETER J KLENOW (1998), “Evidence on Learning and Network Externalities in the Diffusion of Home Computers,” University of Chicago..

⁴⁸See

SALONER, GARTH, AND ANDREA SHEPHARD (Adoption of Technologies With Network Effects: An Empirical Examination of the Adoption of Automated Teller Machines), “Rand Journal of Economics,” 26 479–501, ..

Apple Computer, the company that brought you the idiot-friendly Macintosh, is staring at bankruptcy. Meanwhile, the great army of technocrats at Microsoft, which only last year managed to reproduce the look and feel of a 1980's Mac, lumbers on, invincible.

A bad break for Apple? A rare exception to the Darwinian rules in which the best products win the hearts and dollars of consumers?⁴⁹

Solution: This is an instance of how network externalities work and how “the best technology” does not always win. As in Section 17.2, it is possible that the lock-in to the Microsoft operating system resulted from a series of “small historical events,” not from the inherent superiority of the Microsoft operating system.

■ **17.6*** Consider the model of technology adoption presented in Section ?? . Suppose that the utility derived by an A fan from technology A is given by $u + n_A$ if n_A is less than $\bar{n}_A$, and $u + \bar{n}_A$ for values of n greater than $\bar{n}_A$. Likewise utility from buying technology B is as before except that for n_B greater than $\bar{n}_B$ we get $\bar{n}_B$. Analogous expressions apply for B fans. In other words, network externalities are bounded: once the network reaches a certain size, no additional benefits are gained from a larger network.

Show that, under these circumstances and for certain values of u, v , three different outcomes are possible: (a) the industry becomes locked-in to technology A ; (b) the industry becomes locked-in to technology B ; (c) the two technologies survive in the long run.

Solution: One can distinguish 3 cases:

- a) If $u + \bar{n}_A \leq \bar{n}_B$ then all future adopters will prefer technology B ;
- b) If $u + \bar{n}_B \leq \bar{n}_A$ then all future adopters will prefer technology A ;
- c) If $u + \bar{n}_B = \bar{n}_A$ or $u + \bar{n}_A \leq \bar{n}_B$ then adopters that would prefer technology B (and respectively A) are indifferent, therefore both technologies will survive.

■ **17.7** Explain why the market adoption of a new technology may be too fast or too slow.

Solution: The adoption of new a technology may feature excess inertia because of incomplete information about the preferences of future potential adopters. Even a slight probability that future users are conservative and therefore are better off not adopting the new technology, can act as a deterrent for early users that would prefer the new technology.

On the other hand, excess momentum may appear when sticking to the old technology is not a dominant strategy. That is, if initial users prefer the new technology, future users

⁴⁹ *The New York Times Magazine*, May 5, 1996.

will also choose the new technology, despite the fact that the utility from sticking to the old technology if the initial users were also conservative, is much higher (see the example in section 17.3).

■ **17.8** Company *A* has just developed a new technology. Company *B* approaches Company *A*, stating it has developed its own version of the technology and proposing a compromise that would make the two technologies compatible with each other. What advice would you give Company *A*?

Solution: If the firms decide to go alone, standards competition reduces the product-market size for both of them, and they may end up losing, since users have the easy option of staying with the old technology (see the example in box 17.3). Therefore, compatibility is the preferred action.

On the other hand, if product competition is fierce when the products are compatible, then the two firms will get duopoly profits which are lower than the expectation of profits from refusing the compromise (case in which each firm receives monopoly profits with probability 0.5). In this case, the decision to go alone is better.

■ **17.9** A standardization battle is currently under way in the recordable DVD industry, with Philips and Sony on one side, Matsushita and Toshiba on the other side. In an effort to coordinate on a standard, an industry group was set up: the DVD Forum. On April 1997, the forum's ten members voted eight-to-two to standardize around the Matsushita-backed format, leaving Philips and Sony stranded with their losing format. Within a few weeks, Philips and Sony announced they would start selling their own format.

What role can you see for public policy in this case?

Solution: A standardization war can have as an effect a delay in the introduction of a product, which is bad both for the consumers and producers. Hence, there is scope for public policy in settling the issue of which standard should be chosen. On the other hand, the cost of having only one standard is lower product variety and lower competition. Public policy has to weigh which effect is more important from the point of view of social welfare.

■ **17.10*** You are marketing a new wireless information device (WID). Consumers differ in their willingness to pay for the device. (No one needs more than one.) All consumers value owning a WID more highly, the larger is the total number of consumers using such devices. Denote the expected total number of WID users by n^e , which we also can call the “expected size of the WID network.”

If all consumers expect the size of the WID network to be n^e , and the price of the device is p , then the number of users who will want to buy the device (i.e., the total quantity demanded) is given by $n = 100 - p + vn^e$, where $0 < v < 1$. (Note that this is a standard linear relationship between price and unit sales for any given level of expected network size, n^e .)

(a) Interpret the parameter v . What factors influence v ?

Suppose that your marginal cost per WID is 20. Suppose also that consumers are quite sophisticated and form accurate expectations about the size of the WID network, for any price p that you might set, so that n must equal n^e .

(b) What is the profit-maximizing price of WIDs? How many are sold, and what profits do you earn?

Suppose that you could improve the performance of your WID communications network and thus enhance the network effects, raising v from $1/3$ to $1/2$.

(c) How much would you pay to develop this enhancement?

Solution:

- (a) The parameter v is the network benefit contributed by each additional WID sold. One way to think about this is as follows. When an additional WID is sold, n^e increases by one. This raises the benefit to all consumers from owning a WID since now there is one more WID out there with which they might communicate. Adding up this small improvement in the value of a WID for all consumers gives us the parameter v .

Suppose that your marginal cost per WID is 20. Suppose also that consumers are quite sophisticated and form accurate expectations about the size of the WID network, for any price p that you might set.

- (b) Since consumers have accurate expectations we can set n^e equal to n and invert the total demand for WIDs to get $p = 100 - (1 - v)n$ as the inverse demand curve and a marginal revenue curve of $MR = 100 - 2(1 - v)n$. The optimal number of WIDs to sell is that which equates this marginal revenue with marginal cost. That is, $100 - 2(1 - v)n = 20$ or $n = (80)/2(1 - v)$. The profit maximizing price is then $p = 100 - (1 - v)(80)/2(1 - v) = 60$ and profits are $(60 - 20)80/2(1 - v) = 1600/(1 - v)$.
- (c) When $v = 1/3$, the firm's profits are $1600/(1 - .333) = \$2,400$. When $v = 1/2$, the firm's profits are $1600/(1 - .5) = \$3,200$. Therefore, the most the firm would pay to develop this enhancement is $\$3,200 - \$2,400 = \$800$.

■ **17.11*** Two firms, Compress and Squeeze, offer incompatible software products that encrypt and shrink the size of large data files for safe storage and/or faster transmission. This software category exhibits strong network effects, since users seek to send files to each other, and a file saved in one format cannot be retrieved using the other format. The marginal cost of serving one customer is \$40 for either firm.

To keep things simple, suppose that there are only two customers, "Pioneer" and "Follower," and two time periods, "This Year" and "Next Year." As the name

suggests, Pioneer moves first, picking one format This Year. Pioneer cannot change her choice once it is made. In contrast, Follower picks Next Year. Follower will be aware of Pioneer's pick when the time comes for Follower to pick. The annual interest rate is 20% for both Compress and Squeeze and Pioneer.

Pioneer regards Compress and Squeeze as equally attractive products. Pioneer values either product at \$100 during This Year (before Follower enters the market), and at \$100 during Next Year if Follower does not pick the same product. If Follower does pick the same product Next Year, Pioneer's value during Next Year will be \$136. (In other words, the network effect is worth \$36 to Pioneer.)

Follower has very similar preferences. If Follower picks the same product Next Year as Pioneer did This Year, Follower values that product at \$136. Alternatively, if Follower picks a different product Next Year than Pioneer did This Year, the value to Follower of that product will be only \$100.

Finally, suppose that Compress and Squeeze simultaneously set prices This Year at which they offer their products to Pioneer. (One could just as well say that they bid for Pioneer's business.) Then Compress and Squeeze simultaneously set prices Next Year at which they will offer their products to Follower.

For simplicity, please assume that Pioneer will pick Compress if Pioneer is just indifferent between Compress and Squeeze, and that Follower will pick the same product as Pioneer if Follower is indifferent between Compress and Squeeze given the values they offer and the prices they charge.

(a) What prices will Compress and Squeeze set Next Year in bidding to win Follower's business if Compress wins Pioneer's business This Year?

(b) What prices will Compress and Squeeze set This Year in bidding to win Pioneer's business?

(c) What product will Pioneer buy, and what product will Follower buy?

(d) What are the resulting payoffs of Compress, Squeeze, Pioneer, and Follower?

(e) Describe in words the advantages of early or late adopters identified in this problem.

(f) How does all of this change if there is rapid technological progress so that costs Next Year are much lower than costs This Year?

(g) How does your analysis change if the (marginal) cost of serving a customer is only 20 rather than 40?

Solution:

- (a) If Compress wins during the first period, then Compress offers an extra \$36 value over Squeeze to Follower. Squeeze will compete as best possible by offering its product at cost, \$40, but Compress can win by charging \$76. (We could make this \$75.95, but the numbers are simplified by breaking ties in favor of Compress.)
- (b) The equilibrium derived in A generates profits Next Year to Compress of \$36, which are equal to \$30 in This Year dollars (given the 20% interest rate). By symmetry, Squeeze would also enjoy profits of \$36 Next Year if Pioneer picks Squeeze this year. This implies that both Compress and Squeeze are prepared to set a price as low as \$10 to win Pioneer's business: losing Pioneer's business means they will lose Follower's business as well and earn

zero; bidding \$10 means losing \$30 This Year and earning profits of \$36 Next Year, which gives zero in present discounted value.

- (c) From the answer to B, we conclude that the Nash Equilibrium involves a bid of \$10 by each firm to serve Pioneer. Compress thus will win Pioneer's business, by our tie-breaking convention. Then Squeeze will bid \$40 to serve Follower, and Compress will bid \$76 to serve Follower. Compress will win, so both customers will buy from Compress.
- (d) Both firms earn zero profits in present discounted value. All of the profits are dissipated by bidding for Pioneer's business, since Pioneer "tips" the market towards one product or the other. Pioneer gets a surplus of \$90 This Year and \$136 Next Year. Follower gets surplus of \$60 Next Year.
- (e) Pioneer enjoys a nice strategic advantage by virtue of its ability to "tip" the market, i.e., to influence subsequent adopters.
- (f) With rapid technological progress, prices fall rapidly and Follower could well do better than Pioneer, simply because Follower can buy when the product is much cheaper to produce (or of higher quality). From the customer's perspective, waiting for products to improve must be balanced against the benefits of adopting early and thus enjoying very strong price competition between incompatible suppliers seeking to build their installed bases and thus gain competitive advantage.
- (g) If marginal costs are only \$20, then the price during This Year would be -\$10. The problem here is that many "phantom" customers could appear, take the \$10 subsidy to use the product, and then disappear. Actually paying customers to take your product can be a very dangerous strategy. Are you building an installed base of users or just giving away money to opportunistic "fake" customers?

■ **17.12*** Technological progress (of a sort) has led to the WalkDVD. As the name suggests, this is a miniature DVD player. It is attached to a pair of headphones and special viewing glasses which, together, allow for highly realistic sound and image effects, as well as easy mobility. Three firms, Son, Tosh and Phil, are planning to launch their WalkDVD players. There are two possible formats to choose from, S and T, and the three competitors have not agreed on which standard to adopt. Son prefers standard S, whereas Tosh prefers standard T. Phil does not have any strong preference other than being compatible with the other firms. Specifically, the payoffs for each player as a function of the standard they adopt and the number of firms that adopt the same standard are given by Table 2. For example, the value 200 in the Son row and S2 column means that if Son chooses the S standard and two firms choose the S standard, then Son's payoff is 200.

Suppose that all three firms simultaneously choose which standard to adopt.

(a) Show that "all firms choosing S" and "all firms choosing T" are both Nash equilibria of this game.

Table 2: Payoffs in standard setting game.

Firm	S1	S2	S3	T1	T2	T2
Son	100	200	250	40	80	110
Tosh	40	80	110	100	200	250
Phil	60	100	120	60	100	120

(b) Determine whether there are any other Nash equilibria in this simultaneous-move game.

Son has just acquired a firm that manufactures DVDs for the S format. For all practical purposes, this implies that Son is committed to the S format. It is now up to Tosh and Phil to simultaneously decide which format to choose.

(c) Write down the 2x2 payoff matrix for the game now played by Tosh and Phil. Find the Nash equilibrium of this game.

(d) Do you think Son's move was a good one? How would your answer differ if Phil had a slight preference for the T format (e.g., assume that payoffs for T1, T2 and T3 are 70, 110 and 130, respectively)?

Suppose now that all firms' payoffs are like Phil in the table above. You are Son.

(e) If you could choose, would you rather move before Tosh and Phil, or after them? Contrast your answer to what you have learned from the answers to parts (c) and (d).

Solution:

- (a) Suppose all firms choose S. By unilaterally deviating and choosing T instead, Son would get 40 instead of 250; Tosh would get 100 instead of 110; and Phil would get 60 instead of 120. Since all would stand to lose, we conclude that all with S constitutes a Nash equilibrium. By the same token, all choosing T is also a Nash equilibrium.
- (b) The only other possible (pure-strategy) Nash equilibria are for two firms to choose one standard and one to choose the other one. But such a situation cannot be a Nash equilibrium: the firm that is the sole adopter of one of the standards would be better off by joining the other firms. We conclude that there are no (pure-strategy) Nash equilibria in addition to the ones derived in the previous question.

Son has just acquired a firm that manufactures DVDs for the S format. For all practical purposes, this implies that Son is committed to the S format. It is now up to Tosh and Phil to simultaneously decide which format to choose.

- (c) The game is as follows:

		Phil	
		S	T
Tosh	S	110 120	80 60
	T	100 100	200 100

Notice that Phil has a (weakly) dominant strategy: to choose S. Even if Phil assigns the lowest positive probability that Tosh is going along with S, it is strictly better off (in expected value) by choosing S. Knowing this, Tosh should choose S, since, conditional on Phil choosing S, payoff is greater with S (110) than with T (100). We conclude that both firms choose S.

Notice that (T,T) is also a Nash equilibrium. However, the discussion above implies that it would not be a very “reasonable” Nash equilibrium.

- (d) Son’s move was a brilliant one. In the simultaneous-move game, there are two Nash equilibria, one that is good for Son, one that is not so good. By moving ahead of the other players, Son is effectively able to lead the industry to adopt its preferred standard.

In the event Phil prefers the T standard, things are different. The game played between Tosh and Phil is now the following

		Phil	
		S	T
Tosh	S	110 120	80 70
	T	100 100	200 110

There are now two Nash equilibria: (S,S) and (T,T). Moreover, joint payoff are greater in the (T,T) equilibrium (310) than in the (S,S) equilibrium (230). It is not unlikely that Tosh and Phil choose T, in which case Son is worse off by choosing S than by choosing T as well.

Suppose now that all firms’ payoffs are like Phil in the table above. You are Son.

- (e) If all firms have payoffs as Phil, then a particular firm would prefer to move after the other firms have moved than to move at the same time. As in part A, there are two Nash equilibria in the simultaneous-move game. Moreover, both equilibria yield each firm the same payoff. If firms are able to coordinate perfectly on which equilibrium to choose, then moving at the same time or after does not make a difference. If however there is a small chance that coordination will fail, then moving later is (weakly) better, as it reduces the probability of mis-coordination.

■ **17.13***** Consider the market for a given piece of hardware—a photocopier of brand x , for example—that needs after-sale servicing. Suppose that there is free entry into this after market. Servicing photocopiers implies a fixed cost of κ and a marginal cost of γ per unit of service provided. Total demand for servicing is given by $D = \sigma(\alpha - p)$, where p is price and σ the number of photocopier owners. Finally, suppose that firms in the after market compete à la Cournot.

Show that consumer surplus (per consumer) in the after market is given by

$$U = \frac{1}{2} \left(\alpha - \gamma - \sqrt{\frac{\kappa}{\sigma}} \right)^2,$$

an increasing, concave function of σ . (Hint: apply the results on Cournot competition with free entry derived in Chapter ??). Take into account the fact consumer surplus per consumer is given by $(\alpha - p)^2/2$.)

Relate this result to the discussion on indirect network externalities (at the beginning of the chapter).

Solution: We have $p = a - \frac{Q}{S}$ and $q_i = \frac{(a-c)S}{n+1}$, $(\frac{a-c}{n+1})^2 = \frac{F}{S}$ and $n = \left[(a-c)\sqrt{\frac{S}{F}} - 1 \right]$ (see derivation in 14.10). Consumer surplus is given by $CS = \frac{(p(0)-p^*)Q^*}{2} = \frac{Q^{*2}}{2S}$. Therefore, consumer surplus per consumer is $U = \frac{CS}{S} = \frac{Q^{*2}}{2S^2} = \frac{n^2}{2S^2} \frac{(a-c)^2 S^2}{(n+1)^2} = \frac{n^2 F}{2S} = \frac{1}{2} [(a-c)\sqrt{\frac{S}{F}} - 1]^2 \frac{F}{S} = \frac{1}{2} (a-c - \sqrt{\frac{F}{S}})^2$.

This is a case of indirect network externalities. The greater the market size (S), the greater the need for after-sale services, and hence the greater competition in the after-sale market. This increase in competition lowers the price for after-sale services and increases consumer surplus.