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THE LEARNING CURVE, MARKET DOMINANCE, AND PREDATORY PRICING

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Strategic implications of the learning curve hypothesis are analyzed in a model of a price-setting, differentiated duopoly selling to a sequence of heterogeneous buyers with uncertain demands. A unique and symmetric Markov perfect equilibrium is characterized, and two concepts of self-reinforcing market dominance investigated. One is increasing dominance (ID), whereby the leading firm has a greater probability of winning the next sale; the other is increasing increasing dominance (IID), whereby a firm's probability of winning the next sale increases with the length of its lead. Sufficient conditions for IID (and thus for ID) are that the discount factor is sufficiently low or sufficiently high. Other sufficient conditions for ID and IID are given in the case of two-step learning, in which a firm reaches the bottom of its learning curve after just two sales. However, examples are also constructed for the two-step learning case in which neither ID nor IID holds. It is also shown that, in equilibrium, IID implies that learning is privately disadvantageous, although it is socially advantageous. Finally, introducing avoidable fixed costs and possible exit into the model yields a new theory of predatory pricing based on the learning curve hypothesis.

KEYWORDS: Learning curve, predatory pricing, dynamic oligopoly, Markov perfect equilibrium.

1. INTRODUCTION

THERE ARE SEVERAL STRATEGIC IMPLICATIONS of the hypothesis that a firm's unit cost declines with its cumulative production—the learning curve hypothesis. First, by moving down the learning curve faster than its rivals a firm gains a strategic advantage.[2] Second, recognizing this potential for strategic advantage, firms compete aggressively, and perhaps even unprofitably, to move down their learning curves. Third, even a mature firm might compete aggressively to prevent a rival from moving down its learning curve. Fourth, the strategic advantage conferred by learning may drive rivals from the market, creating an incentive for predatory pricing. We study these strategic issues in a dynamic duopoly model.

The learning curve hypothesis is not new, and has been studied in many industries. These include airframes (Wright (1936), Asher (1956), Alchian (1963)), machine tools (Hirsch (1952)), metal products (Dudley (1972)), nuclear power plants (Zimmerman (1982), Joskow and Rozanski (1979)), chemical

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[2]This idea formed the basis of the Boston Consulting Group's (1970, p. 54) advice to produce a lot early on.

processing (Lieberman (1984)), shipbuilding (Searle and Goody (1945), Argote, Beckman, and Epple (1990)), and semiconductors (Webbink (1977)). There is also casual evidence that learning by doing matters for how firms compete. For example, in discussing the semiconductor market, *The Economist* (October 13, 1990) states that chip makers "almost bankrupted themselves" selling early generations of memory chips. Newhouse (1982) makes similar remarks about airframe manufacturers in discussing competition between the Boeing 747, the Lockheed L1011, and the McDonnell-Douglas DC10 beginning in the late 1960's.

The industrial organization literature on the strategic implications of the learning curve is sparse.[3] Lee (1975) argued that learning might raise entry barriers in a dynamic limit pricing model. Spence (1981) showed with numerical examples how, in a Cournot quantity-setting model, a learning curve creates entry barriers against late entrants; Ghemawat and Spence (1985) elaborated how industry spillovers mitigate this effect. An extreme form of spillovers, industry-wide learning, was analyzed by Stokey (1986). Fudenberg and Tirole (1983) showed that, in a linear two-period Cournot model: (i) firms' outputs might decrease with learning, (ii) learning is socially beneficial, and (iii) a balanced-budget tax-subsidy scheme can improve welfare. Ross (1986) demonstrated numerically how learning enhanced a Stackelberg advantage, and Dasgupta and Stiglitz (1988) showed how learning might enhance other (possibly small) exogenous strategic advantages. Gilbert and Harris (1981) discuss how a learning curve for investment enables an incumbent to preempt entrants repeatedly by installing lumpy new capacity anticipating demand growth. Mookherjee and Ray (1991) analyzed how learning curves facilitate oligopoly collusion in a dynamic Bertrand model.

The Dasgupta-Stiglitz article is perhaps closest in theme to our own approach, being concerned with how learning influences the evolution of market structure. Adapting Fudenberg and Tirole's linear Cournot model, Dasgupta and Stiglitz showed how granting one firm a small initial cost advantage can lead to increasing market concentration as learning proceeds. Indeed, they demonstrated that, if firms are sufficiently myopic and learning proceeds fast enough, then an oligopoly with initially asymmetric costs eventually becomes monopolized. They also showed how in a homogeneous product Bertrand model a learning curve yields a monopoly market structure if one firm has a customer loyalty advantage.

Our approach to the strategic implications of the learning curve is distinct in several ways. First, our model of dynamic oligopoly is different. We assume a price setting duopoly selling to a sequence of heterogeneous buyers with uncertain preferences for one product over the other. The previous literature has focused mostly on homogeneous products, deterministic demand, and

[3] Antecedents are Arrow (1962), who considered the learning curve hypothesis in a growth model, and Rosen (1972) who analyzed the implications of learning by doing for a competitive firm. See Mookherjee and Ray (1989) for a detailed literature review.

quantity-setting.[4] We think our model better captures the market for commercial airframes, for example. This market operates on the basis of closed competitive price negotiations, and product differentiation and demand uncertainty clearly matter (Newhouse (1982)). A virtue of our approach is that it enables us to study the endogenous evolution of market shares arising from asymmetries generated by demand realizations.

Second, our modeling approach relies on general functional forms, although admittedly it is specialized in other respects. In contrast, most previous oligopoly learning curve models have relied on special functional forms and specific parameter values. Our approach characterizes equilibrium by a two-dimensional system of difference equations. The two dimensions define the "state of the system" which is given by the cumulative previous sales of both firms.

Third, we address questions new to the learning curve literature. It is a familiar idea in industrial organization (e.g., Flaherty (1980), Gilbert and Newbery (1982), Vickers (1986), Budd, Harris, and Vickers (1993)), and quite clear from the theory of races (e.g., Harris and Vickers (1985, 1987)), that, once ahead, a leader might tend to stay ahead. In our model, this "increasing dominance" property means that a leader has a greater probability of selling to the next buyer. We provide sufficient conditions for increasing dominance and show it implies learning is privately disadvantageous, i.e. symmetric firms would be better off if no learning were possible. We also obtain sufficient conditions for an even stronger property of "increasing increasing dominance" meaning that a leader's probability of winning the next competition increases with the length of its lead.[5]

Furthermore, we extend our model to include avoidable fixed costs and develop a new theory of predatory pricing based on the learning curve hypothesis. In contrast to previous formal models (Ordover and Saloner (1989), Roberts (1987)), ours relies neither on financial market imperfections nor on asymmetric information. We argue that: (i) entry and subsequent exit can be an equilibrium outcome, (ii) the possibility of a rival's exit induces more aggressive pricing,

[4] Mookherjee and Ray (1991) analyze a dynamic price-setting oligopoly with deterministic demand and learning by doing. They focus on "folk theorem" results, thus indicating the potential scope for cooperative behavior. In contrast, our restriction to Markov perfect equilibria focuses on noncooperative strategic interaction. After completing our analysis, we learned of an article by Habermeier (1992) containing a model with a structure similar to our own. Habermeier analyzed the model numerically for special cases, focusing on the question of whether one firm will permanently dominate the market. This question does not arise in our model because of a technical assumption on demand. See note 6.

[5] Independent work by Salant (1991) shows market dominance is self-reinforcing in a model with deterministic demand and stochastic learning. Budd, Harris, and Vickers (1993) analyze increasing dominance in a duopoly in which short run profits are a function of a one-dimensional state variable that measures the extent to which one firm is "leading" the other. Their abstract model is more about R&D competition than learning curve competition, but it does have similarities: the market evolves uncertainly and the equilibrium concept is Markov perfection. They prove that if the future is discounted heavily, market evolution is governed by a "joint profit effect," i.e., increasing dominance obtains if greater asymmetry between firms increases joint profits. They also identify other effects that are of lower order importance when the future is discounted heavily but may be significant otherwise.

which, in turn, increases the probability of exit, and (iii) such predatory pricing might be socially beneficial. We also show that predatory and nonpredatory equilibria can coexist, and prices below marginal cost need not indicate a predatory intent except for a mature incumbent.

The sequel is organized as follows. Section 2 presents our model and characterizes a unique and symmetric Markov perfect equilibrium. Section 3 presents our results about market dominance, and Section 4 presents our results about predatory pricing. Section 5 discusses some normative implications of the learning curve hypothesis, and Section 6 concludes by summarizing and indicating some open directions for future research.

2. THE GENERAL MODEL

Consider a price-setting, differentiated duopoly selling to an infinite sequence of heterogeneous buyers. In each period, a buyer demands at most one unit of the good from one of the two firms. We assume a sale always occurs and denote by x the premium a buyer is willing to pay for firm 2's product. The "preference parameter" x varies across buyers and is not observed by the firms. However, it is common knowledge that buyers' preference parameters are independently and identically distributed according to a differentiable cumulative distribution function $F(x)$. Let $f(x)$ denote the corresponding density function. We make several simplifying assumptions.

ASSUMPTION 1: (a) *$f(x)>0$ on the real line*;[6] (b) *$f(x)$ is differentiable*;[7] (c) *$f(x)$ is symmetric about zero*; (d) *$H(z)\equiv F(x)/f(x)$ is increasing.*

Assumption 1(d) has an interpretation in the context of a one-period, asymmetric-cost Bertrand model. Let c_1 be the cost of firm 1. If $P=p_2-p_1$ is the price differential, then $F(P)$ is the probability that firm 1 makes a sale, and $p_1/H(P)$ is the price elasticity of expected demand for firm 1's product. The first-order condition for firm 1 is $p_1-H(P)=c_1$. The assumption implies that, given p_2, firm 1's expected marginal revenue decreases as a lower p_1 increases expected sales.[8]

It will be useful to introduce some additional notation which also has an interpretation in the one-shot model. Let $C\equiv c_2-c_1$ be the cost difference between firm 2 and firm 1. Subtracting firm 1's first-order condition from the corresponding expression for firm 2 yields $P+G(P)=C$, where $G(x)=H(x)-H(-x)$. This is an equilibrium condition determining the price difference as a function of the cost difference. Moreover, substituting firm 1's

[6] With a bounded support, it is possible that one firm will dominate the market permanently (Habermeier (1992)). Assumption 1(a) eliminates this possibility.

[7] The assumption that $f(x)$ is differentiable matters for Lemmas B.2 and E.2.

[8] More precisely, define for firm 1 the expected quantity $q=F(P)$, the inverse demand curve $\psi(q)\equiv p_2-F^{-1}(q)$, and the revenue function $R(q)\equiv\psi(q)\cdot q$. Marginal revenue, $R'(q)=\psi(q)+\psi'(q)\cdot q$, is decreasing if $0\geq 2\cdot\psi'(q)+\psi''(q)\cdot q=-[1+H'(P)]/f(P)$.

first-order condition back into its objective function, $(p_1 - c_1) \cdot F(P)$, yields an equilibrium expected profit $\Pi(P) \equiv H(P) \cdot F(P)$ in the one shot game. These two functions, $G(\cdot)$ and $\Pi(\cdot)$, reappear later. We refer to $\Pi(\cdot)$ as the "one-shot profit function." Note that $G(x) = \Pi(x) - \Pi(-x)$, $\Pi(x)$ is increasing, and $\Pi'(0) = 1$.

The key assumption of our model is that a firm's unit cost is a decreasing function of cumulative past sales, $c(s)$. We further assume that learning is finite, in that a firm reaches the bottom of its learning curve upon making m sales.

ASSUMPTION 2: (a) $c(s) > c(s+1)$ *for* $s < m$; (b) $c(s) = c(m)$ *for* $s \geqslant m$.

We maintain Assumptions 1 and 2 throughout.

Firms maximize expected discounted profits, and our solution concept is Markov perfect equilibrium (MPE). This is a subgame perfect equilibrium with the property that each firm's strategy depends only on the state of the game. The state of the game is defined by a pair (i, j), where i and j are the cumulative sales of firm 1 and firm 2 respectively. Since learning is bounded, state $(i, m+k)$ is equivalent to state (i, m) for $k > 0$. Therefore, a strategy for firm 1 is a mapping $p(i, j)$ that gives its price for each possible state of the game and has the property that $p(i, m+k) = p(i, m)$ for $k \geqslant 0$. A strategy for firm 2 is defined analogously, although we prove that a MPE must be symmetric.[9]

Given a strategy for each firm, we define recursively a value function $v(i, j)$, giving the value of the game for firm 1 in state (i, j). There is, of course, an analogous value function for firm 2, but we do not need to introduce notation for it explicitly. The following characterization of a MPE relates these value functions and the equilibrium strategies. It employs the following notation. First, $P(i, j)$ is the difference between firm 1 and firm 2's price, so firm 2's strategy is described by $p(i, j) - P(i, j)$, and a symmetric strategy satisfies $p(j, i) = p(i, j) - P(i, j)$. Second, $w(i, j) = v(i+1, j) - v(i, j+1)$ is firm 1's "prize" from winning a sale, and $W(i, j)$ is the difference between firm 1 and firm 2's prize. Third, $C(i, j) = c(i) - c(j)$ is the cost difference between the firms. Finally, δ is the discount factor, satisfying $0 < \delta < 1$.

THEOREM 2.1: *A MPE has the following properties*:

$$p(i,j) - H(-P(i,j)) = c(i) - \delta w(i,j), \tag{2.1}$$

$$P(i,j) + G(P(i,j)) = C(i,j) - \delta W(i,j), \tag{2.2}$$

$$v(i,j) = \Pi(-P(i,j)) + \delta v(i,j+1), \tag{2.3}$$

$$v(i,m+1) = v(i,m). \tag{2.4}$$

[9] We allow prices to be negative. However, the equilibrium prices characterized by Theorem 2.1 shift by a if the learning curve is also shifted by a. We can therefore assure positive equilibrium prices by making a high enough.

THEOREM 2.2: *There exists a unique and symmetric MPE.*

PROOFS: See Appendix A.

These equilibrium conditions have clear interpretations. Equation (2.1) is the first-order condition for the asymmetric cost one-shot game discussed before, except that the discounted "prize" from winning enters as a production subsidy. Accordingly, the difference in prices characterized by equation (2.2) equals the one-shot equilibrium price difference corresponding to these subsidized costs.

Equation (2.3) indicates that the value function can be decomposed into the sum of the one-shot profit corresponding to the equilibrium price difference and the discounted value of losing. This decomposition follows from the interpretation of the prize as a production subsidy, i.e., the value of winning is implicit in the one-shot profit function. The reader is cautioned that this profit function does not characterize short run profit since it includes the prize from winning.

Our main point is that the learning curve creates implicit production subsidies. The significant complication arising in the dynamic model is that these subsidies are determined endogenously by the dynamic interaction of the firms. They depend on future possible paths of equilibrium prices. More specifically, equations (2.2), (2.3), and the boundary condition (2.4) determine the path of equilibrium price differences, and fully describe the evolution of market structure and the value functions of the firms. The implicit production subsidies are calculated by solving these equations.

3. MARKET DOMINANCE

Once ahead, does a firm tend to stay ahead? Does this tendency increase with the length of the lead? Since $F(P(i,j))$ is the probability that firm j wins the next sale, these questions are equivalent to asking: Is $P(i,j)$ negative for $i>j$? Is $P(i,j)$ decreasing in i? We call a positive answer to the first question increasing dominance (ID) and a positive answer to the second increasing increasing dominance (IID). Note that IID implies ID because $P(i,i)=0$ by symmetry (Theorem 2.2).

Lest these market dominance properties seem obvious, consider the following plausible intuition from Scherer and Ross (1990, p. 372):

> "As the leading firm approaches the bottom of its learning curve, its incentive to constrain price in the hope of gaining future cost advantages weakens, and it will be tempted to price less aggressively and reap the profit fruits of its prior pricing restraint."

This intuition is incomplete. A firm at the very bottom of its learning curve maintains a strategic advantage as long as its rival has a higher cost. The desire to keep the rival from moving down its learning curve too quickly creates an incentive for even a mature leading firm to price aggressively.

The equilibrium characterization of the previous section indicates that market dominance properties depend on a "cost effect" and a "prize effect." The cost effect is easy to understand. As cost differences between firms widen so should price differences. The cost effect by itself suggests IID should hold, and indeed it does if the future is sufficiently unimportant.

THEOREM 3.1: *For δ sufficiently close to zero, $P(i+1, j) < P(i, j)$ for $i < m$ and $j \leqslant m$.*

PROOF: Since $G(\cdot)$ is an increasing function by Assumption 1(d), the result follows from Assumption 2 and equation (2.2) as $\delta \to 0$. *Q.E.D.*

The prize effect is more interesting. It refers to the prize from winning that constitutes an implicit production subsidy. If the lagging firm has a sufficiently larger prize, then the prize effect could dominate the cost effect, and IID or even ID could fail.

An initial observation is that for $\delta > 0$ but sufficiently small, the prize effect reinforces the cost effect if the one-shot profit function is convex, as it is for many distribution functions, including normal or uniform distribution functions. To see this, observe that the prize difference, $W(i, j)$ equals the difference between the joint payoff if i wins and the joint payoff if j wins. For small δ, the joint payoff approximately equals the joint profit of a corresponding one-shot game, and the joint profit in the one-shot game increases as the cost gap widens if and only if $\Pi(\cdot)$ is convex. Therefore, the prize effect widens price differences when the future is not too important and one-shot joint profits increase with the cost gap.[10]

This observation is interesting but not really consequential, because the cost effect always dominates when the future is discounted heavily. It is much more important to understand the prize effect when δ is large and the future weighs heavily. We begin by considering equilibrium pricing in the limit as δ goes to unity. The result is that firms always price as if at the bottom of their learning curves.

THEOREM 3.2:

$$\lim_{\delta \to 1} p(i, j) = c(m) + H(0). \tag{3.1}$$

PROOF: See Appendix B.

There is a clear explanation for this result. When δ equals unity, production is "timeless." Moreover, since there are an infinite number of buyers, each firm must reach the bottom of its learning curve eventually. Thus, in each period, the relevant marginal cost is $c(m)$ regardless of experience (cf. Spence (1981)), and

[10]This is similar to the joint profit effect identified by Budd, Harris, and Vickers (1993) when the future is discounted heavily.

each firm prices accordingly. It is precisely as if each firm plays repeatedly the one-shot Bertrand equilibrium with marginal cost $c(m)$.

Theorem 3.2 implies that price differences are negligible when the future is very important, i.e.

$$\lim_{\delta \to 1} P(i,j) = 0. \tag{3.2}$$

Therefore, each firm has an approximately equal chance of winning each sale and the market evolves accordingly.

Equations (2.1), (3.1), and (3.2) together imply that $w(i,j) \to C(i,m)$ as $\delta \to 1$. Therefore, $W(i,j) \to C(i,j)$. The cost effect and prize effect have opposite signs, but they exactly offset each other. A heuristic intuition is as follows. Firms get to the bottom of the learning curve eventually, but incur higher costs along the way. The difference between actual cost and cost at the bottom of the learning curve is a "learning cost." The sum of prospective learning costs for a firm with i previous sales is

$$\sum_{k=i}^{m-1} C(k,m).$$

Therefore, the difference in total prospective learning costs for a firm with cumulative sales $i+1$ and a firm with cumulative sales i is simply $C(i,m)$. Since prices are approximately constant for δ close to unity, this is also approximately the difference in expected payoffs. That is $w(i,j) \to C(i,m)$ and $W(i,j) \to C(i,j)$.[11]

The observation that the prize effect exactly offsets the cost effect when the future matters as much as the present raises the following question. In the neighborhood of $\delta = 1$, does the cost effect or the prize effect dominate? If the cost effect dominates, then ID holds for δ less than but close to unity. We obtain a very sharp answer. Not only ID but IID holds in the neighborhood of $\delta = 1$.

THEOREM 3.3: *For δ less than but sufficiently close to unity, $P(i+1,j) < P(i,j)$ if $i < m$ and $j \leqslant m$.*

PROOF: Follows from Lemmas B.1 and B.2 in Appendix B.

Here is an intuitive explanation. If firms don't discount the future, then each prices as if cost is $c(m)$. In other words, each firm has a "subsidized cost" equal to $c(m)$. In the near term, both firms incur learning costs, and these are larger for the lagging firm. A firm that discounts the future a little bit is tempted to raise price slightly to gain a better margin of price over subsidized cost in the short run. Of course this reduces the probability of making the sale, but this disadvantage is offset somewhat by shifting learning costs into the future. This

[11]We thank John Sutton for suggesting this intuition to us.

offset is greater for the lagging firm with the larger learning costs. Therefore, the lagging firm has a greater incentive to raise price, and this incentive increases with the size of the lag. In fact, as we prove in Appendix B,

$$(3.3) \qquad \frac{dp(i,j)}{d\delta} \to -\sum_{k=i}^{m-1} C(k,m)$$

and

$$(3.4) \qquad \frac{dP(i,j)}{d\delta} \to -\sum_{k=i}^{j-1} C(k,m) \quad \text{for} \quad i<j.$$

As $\delta \to 1$, the mark-up of prices above $c(m)+H(0)$ is approximately proportional to prospective learning costs, and therefore the equilibrium price difference is approximately proportional to the difference in prospective learning costs. We conclude that IID holds for δ near unity. Another implication of (3.3) is that firm i's price is decreasing as it moves down its learning curve.

It remains to ask whether ID and IID hold for intermediate values of δ. To do so, we focus on the case of two-step learning, i.e. $m=2$. Obviously, this also characterizes the final stages of learning for the general model with an arbitrary value of m, i.e. for $(i,j) \geq (m-2, m-2)$.

THEOREM 3.4: *If $m=2$ and the one-shot profit function is convex, then ID holds. If, in addition, the learning curve is convex and either $\delta \leqslant 1/2$ or $c(m-1)$ is not too large, then IID holds.*

PROOF: See Appendix C.

In the two-step learning model, ID means that $P(m-1,m)$, $P(m-2,m)$, and $P(m-2,m-1)$ are all positive, and IID means that $P(m-2,m) > P(m-1,m)$ and $P(m-2,m) > P(m-2,m-1)$. It is true that $P(m-1,m)>0$ and $P(m-2,m)>0$ hold even without the convexity assumptions. However, we use convexity of the learning curve to prove $P(m-2,m) > P(m-1,m)$, and convexity of the profit function to prove $P(m-2,m-1)>0$. Finally, the additional restrictions on δ and $c(m-1)$ assure $P(m-2,m) > P(m-2,m-1)$. Details and counterexamples are in Appendix C. Here we give some heuristic explanations.

The reason why $P(m-1,m)>0$ is instructive. It is explained by a simple fixed point argument shown in Figure 1. The "price locus" defined by $P+G(P)=C-\delta W$, describes how the equilibrium price difference (P) depends on the difference in subsidized costs ($C-\delta W$). However, the subsidized cost difference in state $(m-1,m)$ itself depends on the price differential because

$$W = -[\Pi(P)-\Pi(0)] + [\Pi(0)-\Pi(-P)]/(1-\delta).$$

Thus the "subsidized-cost locus" graphs $C-\delta W$ as a function of P. $P(m-1,m)$

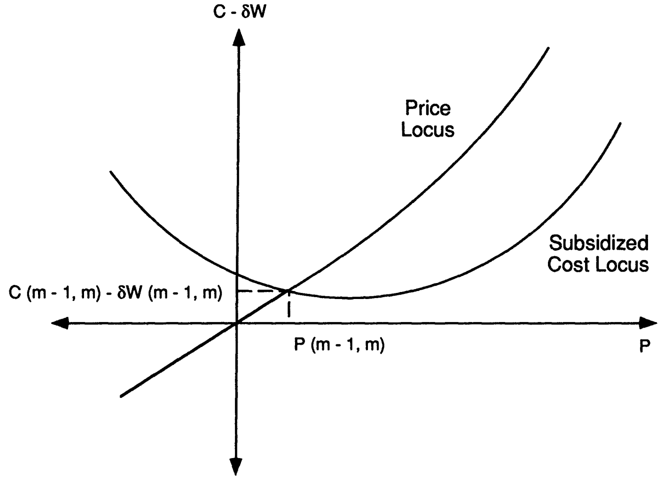

FIGURE 1—Equilibrium in state $(m-1, m)$.

and $W(m-1,m)$ are determined jointly by the intersection of the two curves. The two curves have the general shapes illustrated and must intersect where $P(m-1,m)>0$.[12] However, $W(m-1,m)$ could be either positive or negative depending on the size of δ.

The reason why $P(m-2,m)>0$ is similar, except that the subsidized-cost locus may be higher or lower. The reason for the shift is twofold: $C(m-2,m)$ is different from $C(m-1,m)$, and $W(m-2,m)$ depends not only on $P(m-2,m)$ but on $P(m-1,m)$ also. If the locus shifts downwards, as it will if $C(m-2, m-1)$ is small but $P(m-1,m)$ is large, then $P(m-2,m)<P(m-1,m)$ contradicting IID.[13] The convexity of the learning curve assures that $C(m-2,m-1)$ is not small enough for this failure of IID to happen.

A heuristic explanation for $P(m-1,m)>P(m-2,m)$ is as follows. If $c(m-1)$ is large then the leading firm will have a large cost advantage in state $(m-1,m)$ and the price difference $P(m-1,m)$ will be correspondingly large. The leading firm is not unhappy with this situation and so does not worry too much if the lagging firm wins in state $(m-2,m)$, particularly if $C(m-2,m-1)$ is small, in which case the lagging firm does not very much lower its cost by winning.

The possibility of $P(m-2,m)<P(m-1,m)$ is fascinating because it implies the lagging firm is worse off by moving down its learning curve! To see this,

[12]The price locus is upward sloping and intersects the origin. The subsidized cost locus has a positive intercept (assuming $C>0$) and intersects the price locus once in the positive orthant.

[13]To be precise the subsidized cost locus shifts up by an amount

$$C(m-2,m-1)-(1-\delta)[\Pi(P(m-1,m))-\Pi(0)]-\delta[\Pi(-P(m-1,m))-\Pi(0)]/(1-\delta).$$

With this modification the point of intersection between the price locus and the subsidized cost locus determines $P(m-2,m)$.

observe that Theorem 2.1 implies

$$v(m-i,m)=\frac{\Pi(-P(m-i,m))}{1-\delta}.$$

Therefore $P(m-2,m)<P(m-1,m)$ translates directly to $v(m-2,m)>v(m-1,m)$.

It is also interesting that $P(m-2,m-1)<0$ is possible if $\Pi(\cdot)$ is not convex. Details are in Appendix C, but the rough idea behind this failure of ID is as follows. The lagging firm prices aggressively to avoid state $(m-2,m)$ which is highly unprofitable. This concern dominates the thinking of the lagging firm when (i) $C(m-2,m-1)$ is close to 0, so there is virtually no cost effect in state $(m-2,m-1)$, (ii) δ is small, so what happens beyond next period is irrelevant, and (iii) $\Pi(\cdot)$ is concave in the relevant range so the laggard loses more than the leader gains by moving to state $(m-2,m)$ rather than $(m-1,m-1)$.[14] In these circumstances ID fails. Even if $\Pi(\cdot)$ is convex, so ID holds, IID may fail, i.e., $P(m-2,m-1)>P(m-2,m)$ for related reasons.

We close this section by showing that ID implies learning is privately disadvantageous, in the sense that symmetric firms would be better off if no further learning were possible. All of the benefits of learning, and more, are competed away.

THEOREM 3.5: *If ID holds, then $v(i,i)$ is lower than it would be in an environment with no learning beyond (i,i), (i.e., $m=i$).*

PROOF: From equation (2.3),

$$v(i,i)=\Pi(0)+\sum_{j=i+1}^{\infty}\delta^{j-1}\Pi(-P(i,j))<0. \tag{3.5}$$

ID implies $\Pi(-P(i,j))<\Pi(0)$ for $j>i$. Therefore $v(i,i)<\Pi(0)/(1-\delta)$ which equals the present value of profits in the absence of further learning. *Q.E.D.*

Anecdotal evidence supports the idea that a learning curve does indeed have a negative impact on industry profits. A first example is given by commercial aircraft construction, which is known to be subject to significant learning economies. Seitz et al. (1985) state that out of 22 commercial aircraft developed, only three—the Boeing 707, 727, and the DC-8—have been profitable. Returns on assets and on sales have been significantly below those for all manufacturing.

A second example comes from the semiconductor industry. "The chip business is a matter of yields, learning from experience and forward pricing. The greater the investment a manufacturer makes in a semiconductor plant, the more chips it can turn out. The higher its output, the lower its unit costs and the greater its operating experience. That translates, in turn, into higher yields and still lower unit costs" (*The Economist*, July 14, 1990). Competition for sales in the previous generation of memory chips (1-megabit DRAMs) was such that

[14]Thus, in this case, the joint profit effect, identified by Budd, Harris, and Vickers (1993), dominates.

chip makers "almost bankrupted themselves." Now that a new generation of memory chips (4-megabit DRAMs) is overtaking the previous one, "Japan's giant chip makers are rushing into what looks like a suicidal expansion of 4-megabit chip production" (*The Economist*, October 13, 1990).

4. PREDATORY PRICING

Predation refers to actions that are unprofitable but for their possible contribution to a rival's exit (Ordover and Willig (1981)). By introducing an avoidable fixed cost into our model, we demonstrate how a learning curve can create equilibrium incentives for predatory pricing.

Specifically, we amend our model by assuming that firms incur a positive fixed cost each period they remain in the market. At the beginning of each period, firms decide simultaneously whether to remain or exit. Exiting, a firm gets a payoff of zero. Staying in, the firm pays a fixed cost in the amount $(1-\delta)A$. For expositional simplicity, we assume firms cannot re-enter after exiting.

If both firms stay in the market, then competition in that period proceeds as before, except that the possible future exit of a rival affects equilibrium pricing, as we shall demonstrate. If its rival exits, a firm with experience i gets an assumed monopoly payoff of $v^{\#}(i)-A$ for the subgame beginning at that date.[15] We assume that $v^{\#}(i) \geqslant v(i,j)+v(j,i)$ for all j; monopoly profit is greater than the sum of duopoly profits.

In what follows, we adopt the following notation. As in Section 2, $v(i,j)$ denotes the equilibrium value function for the subgame beginning in state (i,j), assuming that $A=0$. $v'(i,j)$ denotes the corresponding value function for the case $A>0$, calculated before entry decisions are made. $P(\cdot)$ and $P'(\cdot)$ are distinguished similarly, as are $p(\cdot)$ and $p'(\cdot)$.

As our purpose here is to establish "possibility" results, we proceed by analyzing special cases and examples. We begin with the case of one-step learning, $m=1$. In this case we know from Lemma C.1 in Appendix C and equation (2.3) that

$$v(1,0)>v(1,1)>v(0,0)>v(0,1).$$

If $v(0,1)=\Pi(-P(0,1))/(1-\delta)\geqslant A$, there obviously exists a MPE with no exit, but we show there might exist also a MPE with exit in state $(0,1)$. Obviously, if $v(0,1)<A$, exit must occur in state $(0,1)$.

THEOREM 4.1: *If* $m=1$ *and* $(1-\delta)A=\Pi(-P(0,1))-\varepsilon$, *then, for* $|\varepsilon|>0$ *sufficiently small, there exists a MPE in which*:

(*a*) *both firms enter initially*;
(*b*) *the firm losing the first sale exits*; *and*
(*c*) $p'(0,0)<p(0,0)$.

PROOF: See Appendix D.

[15] With more structure on the model we could characterize $v^{\#}(i)$ explicitly, but we don't need to.

Thus, if $\Pi(-P(0,1)) = (1-\delta)A + \varepsilon$, where ε is a small positive number, there exist multiple equilibria, one involving predatory pricing and exit. The intuition behind the predation result is clear. The possibility of the rival's exit increases the "prize" from winning the first sale, inducing more aggressive pricing. On the other hand, if ε is a small negative number, it cannot be an equilibrium for both firms to remain in the market in state $(0,1)$, and equilibrium necessarily involves predatory pricing in state $(0,0)$ as the firms compete to survive.

In cases of multiple equilibria, the predatory one might be considered a "bootstrap" equilibrium. The leading firm sets a predatory price expecting that winning the next sale will induce exit, and this pricing behavior makes that expectation self-fulfilling. It is perhaps surprising that Markov perfection does not eliminate bootstrap equilibria of this sort.[16] Moreover, it is tempting to think that an appropriate forward induction argument would eliminate such equilibria, but that is not clear. Staying in rather than exiting might "signal" that the lagging firm expects to play the nonpredatory equilibrium, but a predatory response might in turn signal that the leader expects to play the predatory one. Which signal is more convincing?[17]

A possible policy drawback of the $m=1$ predation result is that there is no distinction between predator and prey. Both firms are predating against each other in state $(0,0)$. However, for more complicated learning curves, asymmetric market positions can emerge in which the lower cost firm predates. We show this for the case of two step learning.

THEOREM 4.2: *Let* $m=2$; *there exists parameters supporting an equilibrium with*

(*a*) *the lagging firm exiting in state* $(0,2)$;
(*b*) $P'(0,1) > P(0,1)$;
(*c*) $p'(1,0) < p(1,0)$.

PROOF: See Appendix D.

These results provide a very satisfactory theory of predatory pricing, with the following features. First, an incumbent firm perceives that a lower price increases the probability that a rival will exit the market. Second, such exit is rational for the rival. Third, it was rational for the rival to have entered in the first place. Fourth, the possibility of the rival's exit leads the firm to price lower than it would were the rival committed not to exit. Fifth, such pricing increases the probability of exit.

Notice that prices below marginal cost are not necessarily predatory (cf. Areeda and Turner (1975)). Theorem 3.2 states that $p(i,j) \to c(m) + H(0)$ as

[16] Maskin and Tirole (1988) demonstrate multiple Markov perfect equilibria with a similar character in a dynamic quantity setting model with no learning.

[17] While formal forward induction concepts do not apply directly to our framework, we conjecture that a suitable adaptation of Van Damme's criterion would destroy both the accommodating equilibrium and the predatory one (see Fudenberg and Tirole (1992, Chapter 11)).

$\delta \to 1$, and our assumptions certainly allow $c(i) > c(m) + H(0)$, in which case $p(i,j) < c(i)$ for δ sufficiently large. Therefore, below cost pricing can happen even when $A = 0$ precludes the possibility of exit. However, we next prove that once a firm reaches the bottom of its learning curve, there is no explanation for below marginal cost pricing.

THEOREM 4.3: *$A = 0$ implies $p(m,i) > c(m)$.*

PROOF: From equation (2.3), $w(m,i) = \Pi(P(i,m)) - (1-\delta)v(m,i+1)$. Therefore, from equation (2.1) and the definition of $H(\cdot)$,

$$\begin{aligned} p(m,i) &= c(m) + H(P(i,m)) - \delta\Pi(P(i,m)) + \delta(1-\delta)v(m,i+1) \\ &\geqslant c(m) + (1-\delta)\Pi(i,m) + \delta(1-\delta)\,v(m,i+1) \\ &> c(m). \end{aligned}$$

Q.E.D.

The proof of Theorem 4.3 suggests that a mature incumbent might want to price below cost in order to induce exit. However, as the next theorem shows, this cannot be true for subgames along the equilibrium path.

THEOREM 4.4: *If along the equilibrium path both firms remain active in state (m,i), then $p(m,i) \geqslant c(m)$.*

PROOF: Suppose not, that is, $p(m,i) < c(m)$. Then,

$$\begin{aligned} v(m,i) &= [p(m,i) - c(m,i) + \delta v(m,i)]F(P(i,m)) \\ &\quad + \delta[1 - F(P(i,m))]v(m,i+1) \\ &= [1 - \delta F(P(i,m))]^{-1} \\ &\quad \times \{(p(m,i) - c(m,i))F(P(i,m)) \\ &\qquad + [1 - F(P(i,m))]\delta v(m,i+1)\} \\ &< v(m,i+1) \end{aligned}$$

since $v(m,i+1) > 0 > p(m,i) - c(m,i)$. *Q.E.D.*

In other words, pricing below cost by a mature incumbent cannot occur on the equilibrium path even if $A > 0$. The reason is simple. If below cost pricing by the mature incumbent would drive the rival from the market, then the rival should have exited already, or never entered in the first place. However, we conjecture it is possible to explain equilibrium predatory pricing (in the sense of Areeda and Turner) by a mature incumbent by introducing declining demand into the model. Specifically, suppose that beyond a certain date, say t', buyers begin to arrive more slowly. This is captured by assuming that δ is lower after t'. Now suppose that the market is in state (i,m) at some date t prior to date t', and let $v(i,m,t)$ denote the value function for the lagging firm. Suppose that $v(i,m,t+1) < v(i,m,t)$ since, other things being equal, the future looks less favorable as time goes on. The firm might stay in at date t, hoping to move down the learning curve, but exit at date $t+1$ if it fails to do so. This gives the mature firm an incentive for predatory pricing at date t.

5. NORMATIVE RESULTS

We have focused on positive economic issues, but our theory has normative implications too. Appendix E characterizes a social optimum, and proves several results. If the future matters sufficiently, i.e. $\delta \to 1$, equilibrium learning is too slow from society's standpoint, i.e., it would be better for the leading firm to learn more quickly. The rough intuition is that quicker learning better exploits "dynamic economies of scale." Moreover, for the case of two-step learning (i.e. $m = 2$), equilibrium learning is socially desirable even though it is privately disadvantageous.

It is also true that predatory pricing may be socially desirable. We have shown that, for specific parameter values, there may exist two equilibria, one where predation and exit occurs, and one where it does not. Which equilibrium is better socially? It is well known that free entry into an industry with scale economies may lead to excessive entry (Mankiw and Whinston (1986)). Therefore, it is not surprising that there may exist parameter values such that the predation equilibrium is socially better than the equilibrium with no predation. In fact, when the two equilibria co-exist, a comparison involves a trade-off between several effects. The predation equilibrium features (i) less total production because of monopoly power; (ii) less product variety; (iii) lower production costs due to quicker learning; and (iv) lower fixed costs due to fewer firms in the market. It is not hard to construct examples where the last two effects dominate, making the predatory equilibrium better from society's standpoint (see Cabral and Riordan (1991)).

6. CONCLUSION

We modeled dynamic price competition with learning by doing for a duopoly facing a sequence of buyers with uncertain demands. We showed that market dominance can be increasingly self-reinforcing, that learning can be socially desirable but privately disadvantageous, and that equilibrium learning can be too slow from society's standpoint. We also developed a theory of predatory pricing arising from learning economies, and argued that predation might speed learning in a socially desirable way.

However, this agenda only scratches the surface. Learning economies give rise to many other interesting issues for strategic oligopoly interaction. For example, important issues surround the timing of production.[18] In our model, the timing of production was determined by the exogenous arrival of buyers. Certainly, this

[18] Majd and Pindyck (1989) analyze the optimal production plan of a price-taking firm with a learning curve, short run decreasing returns to scale, and facing uncertain future prices. Fershtman and Spiegel (1983) compare optimal production under monopoly and competition. See also Spence (1981), Fudenberg and Tirole (1983), and Gulledge and Womer (1986) on optimal production plans for a monopolist.

is very artificial. For example, in industries such as airframe manufacturing, where learning curves are known to matter (Wright (1936)), delivery dates are negotiated between buyers and sellers, and delivery lags are long. In general, the optimal timing of production trades off short run scale economies and long run learning economies. A very interesting issue is how strategic interaction affects this tradeoff.

There are also many policy issues surrounding learning economies. We addressed predatory pricing, but only briefly. Obviously, it would be desirable to have a much clearer delineation of when predatory pricing is likely to be harmful. Also, it would be interesting to address how enforcement rules against predatory conduct, such as the Areeda-Turner standard, affect dynamic oligopoly competition and the evolution of market structure. Still another area of policy application is the so-called infant industry argument for tariff protection in the presence of learning economies, discussed by Dasgupta and Stiglitz (1988) and Krugman (1984). Finally, oligopoly competition with learning economies might have implications for the optimal design of patent policy. The real advantage of patents may not be so much the temporary protection they afford, as the opportunity to move down the learning curve first.

Another issue for future research concerns the microeconomic foundations of the learning-by-doing hypothesis. This hypothesis might be interpreted to mean that process R&D is complementary with production, the simple learning curve being one extreme example. In this more general context, investments in cost reduction are endogenous, and firms' incentives to invest will depend on how much they produce and on their strategic positions. We conjecture (and can show in special cases) that firms will stop investing in process R&D at certain points. Thus, it seems possible that rival firms have different cost levels even in the long run, leading to a richer theory of long run market structure.

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APPENDIX A: EQUILIBRIUM

PROOF OF THEOREM 2.1: At (i, j) firm 1 solves the following maximization problem:

$$\max_p F(q-p)[p-c(i)+\delta v(i+1,j)]+[1-F(q-p)]\delta v(i,j+1) \tag{A.1}$$

where q is the equilibrium price of firm 2. The first-order condition for firm 1 is

$$F(q-p)-f(q-p)[p-c(i)+\delta v(i+1,j)]+f(q-p)\delta v(i,j+1)=0 \tag{A.2}$$

which simplifies to $p = c(i) + H(q-p) - \delta w(i,j)$. Substituting $p(i,j)$ and $P(i,j)$ for p and $(p-q)$ we obtain (2.1). Taking the difference between (2.1) and its symmetric counterpart for firm 2, and simplifying, we obtain (2.2). Furthermore, substituting $p(i,j)$, as given by (2.1), and $-P(i,j)$ for p and $(q-p)$ in the maximand of problem (A.1) and simplifying, we obtain (2.3). Finally, the boundary conditions (2.4) follow from the fact that (i,m) and $(i,m+1)$ are equivalent states by Assumption 2. *Q.E.D.*

PROOF OF THEOREM 2.2: First, consider state (m,m). The boundary condition (2.4) implies $w(m,m)=0$ and $W(m,m)=0$. Since $C(m,m)=0$ and $G(\cdot)$ is increasing, (2.2) implies $P(m,m)=0$. Equations (2.3) and (2.4) then imply $v(m,m)=\Pi(0)/(1-\delta)$, and equation (2.1) implies $p(m,m) = c(m)+H(0)$. This solution characterizes a unique and symmetric MPE for (m,m) subgames because Theorem 2.1 says (2.1)–(2.4) are necessary conditions, the firms are symmetric, and Assumption 1 implies the maximand of (A.1) has a unique interior maximum establishing sufficiency.

Next, consider state (i,m), $i<m$ and adopt the induction hypothesis that there exists a unique and symmetric MPE for the subgame beginning in state $(i+1,m)$. By symmetry, there must also exist a unique and symmetric equilibrium for the subgame beginning in state $(m,i+1)$. Using Theorem 2.1, firm 1's value function is

$$v(i,m) = \frac{\Pi(-P(i,m))}{1-\delta},$$

and the corresponding value function for firm 2 is

$$\bar{v}(m,i) = \Pi(P(i,m)) + \delta v(m,i+1).$$

Therefore, under the induction hypothesis,

$$W(i,m) = v(i+1,m) + (1-\delta)v(m,i+1) - \frac{\Pi(-P(i,m))}{1-\delta} - \Pi(P(i,m)), \tag{A.3}$$

and, using equation (2.3) and $G(x) = \Pi(X) - \Pi(-x)$, equation (2.2) implies that $P(i,m)$ must solve

$$P + (1-\delta)\Pi(P) - \frac{\Pi(-P)}{1-\delta} = C(i,m) - \delta v(i+1,m) - \delta(1-\delta)v(m,i+1) \tag{A.4}$$

for P. The right-hand side is bounded and independent of P, while the left-hand side is continuously increasing, ranging between $-\infty$ and $+\infty$. It follows that equation (A.4) yields a unique solution for $P(i,m)$, given values of $v(i+1,m)$ and $v(m,i+1)$ that exist uniquely under the induction hypothesis. Given $P(i,m)$, $p(i,m)$ is determined uniquely by (2.1), with

$$w(i,m) = v(i+1,m) - \frac{\Pi(-P(i,m))}{1-\delta}. \tag{A.5}$$

as implied by (2.3) and (2.4). A parallel argument for state (m,i) establishes $P(m,i) = -P(i,m)$ and $P(m,i) = p(i,m) - P(i,m)$. Thus, under the induction hypothesis, (2.1)–(2.4) characterize necessary conditions for a unique MPE in state (i,m) and a symmetric MPE in state (m,i). Sufficiency follows from Assumption 1 as above. Finally, a similar, but even simpler, induction argument establishes a unique and symmetric MPE for other subgames. *Q.E.D.*

APPENDIX B: MARKET DOMINANCE WITH HIGH DISCOUNT FACTORS

LEMMA B.1:

$$\lim_{\delta\to 1} P(i,j) = 0. \tag{B.1}$$

PROOF: Define

$$S(i) \equiv \lim_{\delta\to 1} \frac{\Pi(0) - \Pi(-P(i,m))}{1-\delta}. \tag{B.2}$$

From equation (2.3),

$$w(i,m)=\frac{\Pi(0)-\Pi(-P(i,m))}{1-\delta}-\frac{\Pi(0)-\Pi(-P(i+1,m))}{1-\delta} \tag{B.3}$$

and

$$w(m,i)=\Pi(P(i,m))-(1-\delta)\sum_{k=1}^{m-i-1}\delta^{k-1}\Pi(P(i+k,m))-\delta^{m-i-1}\Pi(0). \tag{B.4}$$

We prove by induction that $\lim_{\delta\to 1}P(i,j)=0$ and

$$S(i)=\sum_{k=i}^{m}C(k,m). \tag{B.5}$$

If our induction hypothesis holds for every $i'>i$, then (B.2)–(B.4) imply

$$\lim_{\delta\to 1}w(i,m)=S(i)=\sum_{k=i+1}^{m}C(k,m)$$

and

$$\lim_{\delta\to 1}w(m,i)=\lim_{\delta\to 1}\Pi(P(i,m))-\Pi(0).$$

Substituting $W(i,m)=w(i,m)-w(m,i)$, and $G(P(i,m))=\Pi(P(i,m))-\Pi(-P(i,m))$ in (2.2), taking limits, and rearranging, we get

$$\lim_{\delta\to 1}\left[P(i,m)+\Pi(0)-\Pi(-P(i,m))\right]+S(i)=\sum_{k=i}^{m}C(k,m). \tag{B.6}$$

It follows that $\lim_{\delta\to 1}P(i,m)=0$ and that $S(i)=\sum_{k=1}^{m}C(k,m)$. In fact, if $\lim_{\delta\to 1}P(i,m)\neq 0$, then $S(i)$, as defined by (B.2) would be unbounded, contradicting (B.6).

Now if $\lim_{\delta\to 1}P(i',j')=0$ for all $(i',j')>(i,j)$, equation (2.3) implies $\lim_{\delta\to 1}w(i,j)=S(i)-S(i+1)=C(i,m)$. It follows from (2.2) that

$$\lim_{\delta\to 1}\left[P(i,j)+G(P(i,j))\right]=0.$$

Since $G(0)=0$ and $G'>0$, it must be that $P(i,j)\to 0$. *Q.E.D.*

Proof of Theorem 3.2: From equation (2.1), and Lemma B.1 we have

$$\lim_{\delta\to 1}p(i,j)=c(i)+H(0)-\lim_{\delta\to 1}w(i,j).$$

From the proof of Lemma B.1, we have $\lim_{\delta\to 1}w(i,j)=C(i,m)$. Substituting and simplifying yields the desired result. *Q.E.D.*

We next consider the monotonicity properties of $P(i,j)$ in the neighborhood of $\delta=1$. In view of Lemma B.1, the next result implies that $P(i,j)$ is monotonic in its arguments for δ sufficiently close to one. Theorem 3.3 follows immediately.

Lemma B.2: For $i<j$,

$$\lim_{\delta\to 1}\frac{dP(i,j)}{d\delta}=-\sum_{k=i}^{j-1}C(k,m). \tag{B.7}$$

Proof: Differentiating equation (2.2) with respect to δ, we get

$$\frac{dP(i,j)}{d\delta}\left[1+G'(P(i,j))\right]+W(i,j)+\delta\frac{dW(i,j)}{d\delta}=0. \tag{B.8}$$

The proof of Lemma B.1 establishes $\lim_{\delta\to 1} W(i,j) = C(i,j)$. Therefore, Lemma B.1, $G'(0) = 2$, and (B.8) imply

$$R(i,j) \equiv -\lim_{\delta\to 1} \frac{dP(i,j)}{d\delta} = \frac{1}{3}\left(C(i,j) + \lim_{\delta\to 1} \frac{dW(i,j)}{d\delta}\right). \tag{B.9}$$

Differentiating (B.4), we have

$$\frac{dw(m,i)}{d\delta} = \Pi'(P(i,m))\frac{dP(i,m)}{d\delta} + \sum_{k=i+1}^{m-1} \delta^{k-i-1}\Pi(P(k,m))$$
$$-(1-\delta)\sum_{k=i+1}^{m-1} \frac{d(\delta^{k-i-1}\Pi(P(k,m)))}{d\delta} - (m-i-1)\delta^{m-i-2}\Pi(0).$$

Therefore, Lemma B.1 and $\Pi'(0) = 1$ imply

$$\lim_{\delta\to 1} \frac{dw(m,i)}{d\delta} = -R(i,m), \tag{B.10}$$

and (B.9) implies $3R(i,m) = C(i,m) + \lim_{\delta\to 1} dw(i,m)/d\delta + R(i,m)$. Rearranging this last expression, using the fact that $S(i) = R(i,m)$ by L'Hôpital's rule applied to (B.2), and using (B.5),

$$\begin{aligned} \lim_{\delta\to 1} \frac{dw(i,m)}{d\delta} &= 2R(i,m) - C(i,m) \\ &= 2S(i) - C(i,m) \\ &= S(i) + S(i+1). \end{aligned} \tag{B.11}$$

Again using equation (2.3), calculation establishes that

$$\lim_{\delta\to 1} \frac{dw(i,j)}{d\delta} = \sum_{k=j}^{m-1} R(i+1,k) - (m-j)S(i+1) + \lim_{\delta\to 1} \frac{dv(i+1,m)}{d\delta}$$
$$- \sum_{k=j+1}^{m-1} R(i,k) + (m-j-1)S(i) - \lim_{\delta\to 1} \frac{dv(i,m)}{d\delta}$$

or, substituting (B.11) for

$$\lim_{\delta\to 1} \frac{dv(i+1,m)}{d\delta} - \lim_{\delta\to 1} \frac{dv(i,m)}{d\delta},$$

and rearranging,

$$\begin{aligned} \lim_{\delta\to 1} \frac{dw(i,j)}{d\delta} &= \sum_{k=j}^{m-1} R(i+1,k) - \sum_{k=j+1}^{m-1} R(i,k) + (m-j)S(i) \\ &\quad -(m-j-1)S(i+1). \end{aligned} \tag{B.12}$$

Using $W(i,j) = w(i,j) - w(j,i)$ and equations (B.10) and (B.12), (B.9) defines a linear recursive system in $R(i,j)$. Since $R(i,m) = S(i)$, the system has a unique solution. It can be easily checked that $R(i,j) = S(i) - S(j)$ solves the system. Therefore, for $i < j$,

$$\lim_{\delta\to 1} \frac{dP(i,j)}{d\delta} \equiv -R(i,j) = -[S(i) - S(j)] = -\sum_{k=i}^{j-1} C(k,m).$$

Q.E.D.

LEMMA B.3:

$$\lim_{\delta\to 1} \frac{dp(i,j)}{d\delta} = -\sum_{k=i}^{m} C(k,m).$$

PROOF: Using Lemma B.2 and equation (B.5) we can simplify (B.12) to get

$$\text{(B.13)} \qquad \lim_{\delta \to 1} \frac{dw(i,j)}{d\delta} = R(i+1,j) + S(i).$$

Differentiating equation (2.1) and taking limits, we get

$$\begin{aligned} \lim_{\delta \to 1} \frac{dp(i,j)}{d\delta} &= -H'(0) \lim_{\delta \to 1} \frac{dP(I,j)}{d\delta} - \lim_{\delta \to 1} w(i,j) - \lim_{\delta \to 1} \frac{dw(i,j)}{d\delta} \\ &= R(i,j) - C(i,m) - R(i+1,j) - S(i) \\ &= -S(i) \end{aligned}$$

which follows from $H'(0) = 1$, (B.13), and the fact that

$$\lim_{\delta \to 1} w(i,j) = C(i,m),$$

as was shown in the proof of Lemma B.1. *Q.E.D.*

APPENDIX C: TWO STEP LEARNING

We begin by simplifying notation. Let $p \equiv P(m-1,m)$, $q \equiv P(m-2,m)$, and $r \equiv P(m-2,m-1)$. In the $m=2$ case, ID means $(p,q,r) > 0$ and IID is equivalent to $q > p > 0$ and $q > r > 0$.

LEMMA C.1: $p > 0$.

PROOF: Define $X \equiv \Pi(p) - \Pi(0)$ and $x \equiv \Pi(-p) - \Pi(0)$. Notice that X is increasing and x is decreasing in p, both are zero when p is zero. Applying equation (2.3) we have $W(m-1,m) = -X - x/(1-\delta)$. Substituting into equation (2.2), and simplifying, we get

$$\text{(C.1)} \qquad p + (1-\delta)X - x/(1-\delta) = C(m-1,m).$$

The left-hand side is an increasing function of p, equaling zero when $p = 0$. Since the right-hand side is positive, so must be p. *Q.E.D.*

LEMMA C.2: $q > 0$.

PROOF: Let $Y \equiv \Pi(q) - \Pi(0)$ and $y \equiv \Pi(-q) - \Pi(0)$. It can be shown from equations (2.2) and (2.3) that

$$\text{(C.2)} \qquad q + (1-\delta)Y - y/(1-\delta) = C(m-2,m) - \delta[(1-\delta)X + x/(1-\delta)].$$

By an argument analogous to that in the previous proof, $q > 0$ if and only if the right-hand side is positive. But from equation (C.1) we know that

$$\text{(C.3)} \qquad p = C(m-1,m) - (1-\delta)X + x/(1-\delta) > 0.$$

Therefore,

$$\text{(C.4)} \qquad C(m-2,m) - \delta[(1-\delta)X + x/(1-\delta)] > p > 0,$$

and the proof is complete. *Q.E.D.*

LEMMA C.3: *If $c(\cdot)$ is convex, then $q > p$.*

PROOF: From equation (C.2), knowing that the left-hand side is increasing in q and that the right-hand side is not a function of q, we have the necessary and sufficient condition for $q > p$ that

$$\text{(C.5)} \qquad p + (1-\delta)X - x/(1-\delta) < C(m-2,m) - \delta[(1-\delta)X + x/(1-\delta)],$$

which results from substituting p for q. Substituting $2C(m-1,m)$ for $C(m-2,m)$ and equation

(C.1) for $C(m-1,m)$, and simplifying, inequality (C.5) becomes

$$p+(1-\delta)^2X-(1+\delta)x/(1-\delta)>0, \tag{C.6}$$

which is true, because the left-hand side is increasing in p and zero for $p=0$, and $p>0$. If $c(\cdot)$ is convex, then $C(m-2,m)>2C(m-1,m)$. Therefore (C.6) implies (C.5). *Q.E.D.*

REMARK C.1: These proofs suggest counterexamples to Lemma C.3. Suppose that $C(m-2,m-1)=0$, so that $C(m-2,m)=C(m-1,m)$. Comparing equations (C.1) and (C.2), we see that q is lower than p if and only if

$$(1-\delta)X+x/(1-\delta)>0. \tag{C.7}$$

If we make $C(m-1,m)$ very large, then, by equation (C.1), p will also be very large and so will X, whereas x is bounded below by $-\Pi(0)$. Therefore, given some δ, a sufficiently large $C(m-1,m)$ will imply condition (C.7) and $q>p$.

LEMMA C.4: *If $\Pi(\cdot)$ is convex, then $r>0$.*

PROOF: We consider two cases.
Case (i): $q>p$. Let $Z\equiv\Pi(r)-\Pi(0)$ and $z\equiv\Pi(-r)-\Pi(0)$. Calculation establishes

$$W(m-2,m-1)=-Y-\delta X-(y-2\delta x)/(1-\delta). \tag{C.8}$$

Therefore, equation (2.2) becomes

$$r+Z-z=C(m-2,m-1)+\delta Y+\delta^2X+\delta(y-2\delta x)/(1-\delta). \tag{C.9}$$

Since $C(m-2,m)=C(m-2,m-1)-C(m-1,m)$, we can use (C.1) and (C.2) to substitute for $C(m-2,m-1)$ and get

$$r+Z-z=(q+Y-y)-(p+X-x)+2\delta(X+x). \tag{C.10}$$

$r>0$ if and only if the right-hand side is positive. Since $q>p$ by assumption $(q+Y-y)>(p+X-x)$. Furthermore, $X+x\geqslant 0$ if $\Pi(\cdot)$ is convex, and the right-hand side of (C.10) is positive.
Case (ii): $q\leqslant p$. If there exist parameter values such that $r\leqslant 0$, then r will also be nonpositive if we set $c(m-2)$ equal to $c(m-1)$ and keep other parameters unchanged. This can be seen from (C.10): since $c(m-1)$ remains unchanged, so do the second and third terms on the right-hand side. Moreover, since $C(m-2,m)=c(m-2)-c(m)$ and $q+Y-y$ is increasing in q, the first term on the right-hand side becomes lower when we set $c(m-2)=c(m-1)$ as can be seen from equation (C.2). Finally, since $r+Z-z$ is increasing in r, the value of r remains negative.
We therefore assume $C(m-2,m-1)=0$. From equation (2.2), $r>0$ if and only if $W(m-2,m-1)>0$. Expanding and rearranging (C.8) we get

$$\begin{aligned}W(m-2,m-1)=&[\Pi(q)+\Pi(-q)-2\Pi(0)]\\&+\delta[\Pi(p)+\Pi(-q)-2\Pi(-p)]\\&+\delta^2[\Pi(0)+\Pi(-q)-2\Pi(-p)]/(1-\delta).\end{aligned}$$

Convexity of $\Pi(\cdot)$ implies that the first term on the right-hand side is nonnegative. Monotonicity of $\Pi(\cdot)$ and the assumption that $q\leqslant p$ imply that the second term is nonnegative, and the third term is positive. *Q.E.D.*

REMARK C.2: To construct a counterexample to Lemma C.4 let $C(m-2,m-1)=0$. Then $P(m-2,m-1)$ has the same sign as $-W(m-2,m-1)$. From equation (C.8), $W(m-2,m-1)>0$ if

$$-y-(1-\delta)Y-\delta(1-\delta)X+2\delta x>0. \tag{C.11}$$

As $\delta\rightarrow 0$, the left-hand side of (C.11) goes to $-y-Y$, which is positive if $\Pi(\cdot)$ is concave on the interval $[-q,q]$.
Notice that this counterexample is consistent with Theorem 3.1. There we are saying that for fixed $C(m-2,m-1)>0$ we can make δ small enough so that $r>0$. Here we are saying that, fixing $\delta>0$ sufficiently small, we can make $C(m-2,m-1)$ small enough that $r<0$.

LEMMA C.5: *$q > r$ if and only if*

$$p + (1 - 2\delta)X - (1 + 2\delta)x > 0. \tag{C.12}$$

PROOF: From equation (C.10) we have $q > r$ if and only if $-(p + X - x) + 2\delta(X + x) < 0$, which is equivalent to condition (C.12). *Q.E.D.*

REMARK C.3: Inequality (C.12) holds if $\delta \leqslant 1/2$. This follows from $p > 0$ and $\Pi(p) - \Pi(0) > 0 > \Pi(-p) - \Pi(0)$.

REMARK C.4: Inequality (C.12) holds if $c(m-1)$ is not too large. This follows from the facts that the derivative of the left-hand side of (C.12) evaluated at $p = 0$ equals 3, p is monotonically increasing in $c(m-1)$, and p goes to zero as $c(m-1)$ goes to zero.

REMARK C.5: It is possible to find counterexamples. For example, let $F(\cdot)$ be normal with a zero mean and variance σ^2. Choose $c(m)$ such that $P(m-1, m) = 1$ solves equations (2.2)–(2.4) for state $(m-1, m)$. Then, in condition (C.12), $p = 1$, and X and x are determined by σ^2. It can be shown numerically that this condition fails for σ^2 sufficiently small. Note that for these counterexamples $\Pi(\cdot)$ is convex and there is no restriction on the shape of the learning curve.

PROOF OF THEOREM 3.4: Follows from Lemmas C.1–C.5 and Remarks C.3–C.4.

APPENDIX D: PREDATORY PRICING

PROOF OF THEOREM 4.1: Adopt the equilibrium hypothesis that exit occurs only in state $(0,1)$. Thus, $v'(0,1) = 0$ and $v'(1,1) = v(1,1) - A$. Suppose the lagging firm were to deviate and stay in, and let q denote the rival's price. Under the hypothesis of equilibrium play in future periods, the deviant chooses p to maximize $[p - c(0) + \delta v'(1,1)]F(q - p)$. The first-order condition is

$$p + H(p - q) = c(0) - \delta[v(1,1) - A],$$

and the corresponding payoff (net of the entry cost) is $[\Pi(q - p) - (1 - \delta)A]$. On the other hand, the first-order condition for the rival is

$$q - H(p - q) = c(1) - \delta[v^{\#}(1) - v(1,1)].$$

Thus, the equilibrium price difference, $P = p - q$, satisfies

$$\begin{aligned} P + G(P) &= C(0,1) - \delta[v(1,1) - A] + \delta[v^{\#}(1) - v(1,1)] \\ &= C(0,1) - \delta W(0,1) + \delta A + \delta[v^{\#}(1) - v(1,0) - v(0,1)] \\ &> C(0,1) - \delta W(0,1), \end{aligned}$$

the last inequality following from $v^{\#}(1) > v(1,0) + v(0,1)$. Since $P > P(0,1)$, the deviant's payoff from staying in is strictly less than $[\Pi(-P(0,1)) - (1 - \delta)A]$ by an amount independent of ε. Therefore, it is an equilibrium for the lagging firm to exit in state $(0,1)$ if ε is sufficiently small.

Nevertheless, initial entry will be profitable if $v'(0,0) > 0$. By an analogue of equation (2.3), we have $v'(0,0) = \Pi(0) - (1 - \delta)A + \delta v'(0,1)$. Since $v'(0,1) = 0$ because of exit, it follows that initial entry occurs if $\Pi(0) > (1 - \delta)A$ which follows from $P(0,1) > 0$ (see Lemma C.1) if ε is sufficiently small.

It remains to show that exit in state $(0,1)$ influences pricing in state $(0,0)$. An analogue of equation (2.1) yields $p'(0,0) = c(0) + H(0) - \delta[v^{\#}(1) - A]$. Since $v^{\#}(1) > v(1,0)$ and $v(0,1)$ is approximately equal to A, $v\#(1) - A > w(0,0)$, and we conclude that $p'(0,0) < p(0,0)$. *Q.E.D.*

PROOF OF THEOREM 4.2: Define P° and A° to satisfy

$$P^{\circ} + G(P^{\circ}) = C(0,2) - \delta[v(1,2) + v(2,1) - v^{\#}(2) - A^{\circ}]$$

and $A^{\circ} = \Pi(-P^{\circ})/(1 - \delta)$. If $A = A^{\circ} + \varepsilon$ for $\varepsilon > 0$, then $P'(0,2) \equiv P^{\circ}$. A° is the lowest value of A

consistent with equilibrium exit in state (0, 2). Moreover, for ε sufficiently small and δ sufficiently close to one, an evaluation of first-order conditions establishes that this is the only state in which exit occurs. By an analogue of equation (2.2),

$$\begin{aligned} P'(0,1) + G(P'(0,1)) &= C(0,1) - \delta W'(0,1) \\ &= C(0,1) - \delta[v(1,1) - A] + \delta[v^{\#}(2) - v(1,1)] \\ &= C(0,1) - \delta W(0,1) + \delta A + \delta[v^{\#}(2) - v(2,0) - v(0,2)] \\ &> C(0,1) - \delta W(0,1). \end{aligned}$$

Therefore, $P'(0,1) > P(0,1)$. By an analogue of equation (2.1), we have

$$\begin{aligned} p'(1,0) &= c(0) + H(P'(0,1)) - \delta[v^{\#}(2) - v(1,1)] \\ &= c(0) + H(P'(0,1)) - \delta w(0,1) - \delta[v^{\#}(2) - v(2,0)] \\ &< c(0) + H(P(0,1)) - \delta w(0,1) = p(1,0). \end{aligned}$$

Q.E.D.

APPENDIX E: SOCIAL WELFARE

We define social welfare to be the sum of expected producer and consumer surpluses. We view the social planner as choosing a symmetric function $P^*(i,j)$ such that, in state (i,j), the buyer receives firm 2's product if his preference parameter (x) exceeds $P^*(i,j)$ and firm 1's product otherwise. Again, we assume that a transaction always takes place. In the following theorem, $\bar{y}$ is a buyer's expected value for firm 1's product.

THEOREM E.1: *The socially optimal solution has the following properties*:

(E.1) $\quad P^*(i,j) = C(i,j) - \delta W^*(i,j);$

(E.2) $\quad U^*(i,j) = \bar{y} - c(j) + \delta U^*(i,j+1) + \Gamma(P^*(i,j));$

where $W^*(i,j) \equiv U^*(i+1,j) - U^*(i,j+1)$, *and* $\Gamma(x) \equiv \int_x^\infty [1 - F(a)]da$; *and*

(E.3) $\quad U^*(i,m+1) = U^*(i,m).$

PROOF: By Bellman's principle, $U^*(i,j)$ obeys the following equation:

$$U^*(i,j) = \bar{y} - c(j) + \delta U^*(i,j+1) + \max_P \int_P^\infty [x - C(i,j) + \delta W^*(i,j)]\, dF(x).$$

The first-order condition for optimality yields (E.1). Substituting this first-order condition back into the Bellman equation, and integrating by parts, yields (E.2). Finally, the boundary conditions (E.3) follow from the fact that (i,m) and $(i,m+1)$ are equivalent states. *Q.E.D.*

We next contrast equilibrium outcomes with those corresponding to maximized social welfare. We first note that there are potential social benefits from learning, contrary to the absence of private benefits.

THEOREM E.2: *The maximized social benefits from learning are greater than in a world with no learning beyond* (i,i), $i < m$.

PROOF: Trivial.

We next further characterize optimum values for $P^*(i,j)$.

LEMMA E.1: $\lim_{\delta \to 1} P^*(i,j) = 0.$

PROOF: Analogous to proof of Lemma A.1.

LEMMA E.2:

$$\lim_{\delta \to 1} \frac{dP^*(i,j)}{d\delta} = 2 \lim_{\delta \to 1} \frac{dP(i,j)}{d\delta}.$$

PROOF: From (E.2), we have

$$U^*(i,m) = \frac{\bar{y} - c(m) + \Gamma(P^*(i,m))}{1-\delta}.$$

Therefore,

$$\text{(E.4)} \qquad W^*(i,m) = \frac{\Gamma(0) - \Gamma(P^*(i,m))}{1-\delta} - \frac{\Gamma(0) - \Gamma(P^*(i+1,m))}{1-\delta}.$$

By Lemma E.1 and equation (E.1), we have $\lim_{\delta \to 1} W^*(i,j) = C(i,j)$. Therefore, (E.4) can be solved recursively to get

$$\lim_{\delta \to 1} \frac{\Gamma(0) - \Gamma(P^*(i,m))}{1-\delta} = \sum_{k=i}^{m} C(k,m).$$

Finally, applying L'Hôpital's rule and $\Gamma'(0) = -1/2$ and results in Appendix B we get

$$\lim_{\delta \to 1} \frac{dP^*(i,m)}{d\delta} = -2S(i) = 2 \lim_{\delta \to 1} \frac{dP(i,m)}{d\delta}.$$

The remainder of the proof is similar to that of Lemma B.2. *Q.E.D.*

Lemmas B.1, E.1, and E.2 lead immediately to the following result.

THEOREM E.3: *For sufficiently large δ, the equilibrium probability of a sale by the leader is too low from society's point of view.*

Finally, we show that equilibrium two-step learning is socially advantageous if the learning curve is convex and IID holds. This contrasts with our previous conclusion that learning is privately disadvantageous if ID holds.

THEOREM E.4: *If $m = 2$, $c(\cdot)$ is convex and IID holds, then social welfare in equilibrium is greater than in a world without learning.*

PROOF: The equilibrium in the world with learning involves a trade-off with respect to the equilibrium in a world without learning. On the one hand, there is the positive effect of cost savings. On the other hand, there is the negative effect of price distortions. In state (i,j), $j \geqslant i$, expected cost savings are at least $F(P(i,j))C(i,j)$, the probability the low-cost firm makes a sale times the cost saving with respect to the high cost. (This is a lower bound of the cost savings because the highest cost ($c(0)$) can be higher than the high cost, that is, firm i's cost.) The loss due to price distortions is in turn given by

$$\int_0^{P(i,j)} x f(x)\, dx.$$

An upper bound for this loss is given by $[F(P(i,j)) - 1/2]P(i,j)$.

In all states (i,i) we have $C(i,j) = P(i,j) = 0$, so both bounds equal zero. Let us therefore consider the remaining states. From equation (C.4), we can see that $p = P(1,2) < C(1,2)$. As a result, $F(P(i,j))C(i,j) > (F(P(i,j)) - 1/2)P(i,j)$. A similar argument applies to showing $q = P(0,2) < C(0,2)$ from equation (C.5).

This leaves state (0,1) to consider. By IID, $P(0,1) < P(0,2)$. By the argument presented above, $P(0,2) < C(0,2)$. By convexity of the learning curve, $C(0,2) < 2C(0,1)$. Together, these imply that $P(0,1) < 2C(0,1)$. Therefore, $F(P(0,2))C(0,2) - [F(P(0,2)) - 1/2]P(0,2) > [1 - F(P(0,2))] C(0,2) > 0$. *Q.E.D.*

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