

# Switching Costs and Equilibrium Prices

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## Abstract

In a competitive environment, switching costs have two effects. First, they increase the market power of a seller with locked-in customers. Second, they increase competition for new customers. The conventional wisdom is that the first effect dominates the second one, so that equilibrium prices are higher the greater switching costs are. I provide sufficient conditions for the dynamic competition effect to dominate, so that switching costs lead to more competitive markets. The set of sufficient conditions includes low levels of switching costs and high values of the discount factor.

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# 1 Introduction

Consumers frequently must pay a cost in order to switch from their current supplier to a different supplier (Klemperer, 1995; Farrell and Klemperer, 2007). These costs motivate some interesting questions: are markets more or less competitive in the presence of switching costs? Specifically, are prices higher or lower under switching costs? How do firm profits and consumer surplus vary as switching costs increase?

While there are many variations on the switching cost theme, one important distinction is whether sellers can or cannot discriminate between locked-in and not locked-in consumers. In this paper, I consider the former case. Examples of this scenario include magazine subscriptions and bank accounts. Klemperer (1995) suggests many more examples.

Most of the economics literature has addressed the leading motivating questions by solving some variation of a simple two-period model.<sup>1</sup> The equilibrium of this game typically involves a bargain-then-ripoff pattern: in the second period, the seller takes advantage of a locked-in consumer and sets a high price (rip-off). Anticipating this second-period profit, and having to compete against rival sellers, the first-period price is correspondingly lowered (bargain).

One limitation of two-period models is that potentially they distort the relative importance of bargains and ripoffs. In particular, considering the nature of many practical applications, two-period models unrealistically create game-beginning and game-ending effects. To address this problem, I consider an infinite-period model where the state variable indicates the firm to which a given consumer is currently attached. The dynamic counterpart of the bargain-then-ripoff pattern is given by two corresponding effects on a firm's dynamic pricing incentives: the harvesting effect (firms with locked-in customers are able to price higher without losing demand) and the investment effect (firms without locked-in customers are eager to cut prices in order to attract new customers).

The harvesting and investment effects work in opposite directions in terms of market average price. Which effect dominates? Conventional wisdom and the received economics literature suggest that the harvesting effect dominates (Farrell and Klemperer, 2007). However, recent research casts doubt on this assertion (Doganoglu, 2005; Dubé, Hitsch and Rossi, 2007). In this paper,

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1. See Section 2.3.1. in Farrell and Klemperer (2007) for a survey.

I follow this line of research. I provide sufficient conditions such that the dynamic competition effect dominates, so that switching costs lead to more competitive markets. The set of sufficient conditions includes low levels of switching costs and high values of the discount factor.

■ **Related literature.** The classic reference on infinite period competition with switching costs is Beggs and Klemperer (1992). They show that switching costs lead to higher equilibrium prices. My approach differs from theirs in two important ways. First, they assume infinite switching costs (that is, a locked-in customer never leaves its supplier). Second, unlike myself they consider the case when the seller cannot discriminate between locked-in and not locked-in consumers.<sup>2</sup>

In a recent paper, Dubé, Hitsch and Rossi (2007) show, by means of numerical simulations, that if switching costs are small then the investment effect dominates, that is, switching costs increase market competitiveness. I analytically solve a version of their model. Analytical solution has two advantages. First, it leads to more general results, that is, results that are not dependent on specific assumptions regarding functional forms and parameter values. Second, the process of solving the model leads to a better understanding of the mechanics underlying the result that the average market price declines when switching costs increase. It also shows why there is an important difference between small and large switching costs.

In a recent paper, Doganoglu (2005) considers the case of small switching costs and shows that, along the equilibrium path, locked-in customers switch to the rival seller with positive probability. Moreover, steady-state equilibrium prices are decreasing in switching costs. Doganoglu's (2005) approach differs from mine in various respects. He assumes uniformly distributed preferences and linear pricing strategies; by contrast, I make very mild assumptions regarding the distribution of buyer preferences and the shape of the seller's pricing strategies. Moreover, my analysis goes beyond the case of small switching costs.

## 2 Model

Consider an industry where two sellers compete over an infinite number of periods for sales to  $n$  infinitely lived buyers. Each buyer purchases one unit

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2. Related papers include Farrell and Shapiro (1988), To (1995), and Padilla (1995).

each period from one of the sellers. A buyer's valuation for seller  $i$ 's good is given by  $z_i$ , which I assume is stochastic and i.i.d. across sellers and periods.<sup>3</sup> Moreover — and this a crucial element in the model — if the buyer previously purchased from seller  $j$ , then his utility from buying from seller  $i$  in the current period is reduced by  $s$ , the cost of switching between sellers.

In each period, sellers set prices simultaneously and then each buyer chooses one of the sellers. I assume that sellers are able to discriminate between locked-in and not locked-in buyers (that is, buyers who are locked in to the rival seller). Without further loss of generality, I hereafter focus on the sellers' competition for a particular buyer. I focus on symmetric Markov equilibria where the state indicates which seller made the sale in the previous period. I denote the seller who made a sale in the previous period (the "incumbent" seller) with the subscript 1, and the other seller (the "challenger" seller) with the subscript 0.

Symmetry implies that the buyer's continuation values from being locked in to seller  $i$  or seller  $j$  are the same. This greatly simplifies the analysis. In particular, in each period the buyer chooses the incumbent seller if and only if

$$z_1 - p_1 \geq z_0 - p_0 - s$$

Define

$$\begin{aligned} x &\equiv z_1 - z_0 \\ P &\equiv p_1 - p_0 - s \end{aligned} \tag{1}$$

In words,  $x$  is the relative preference for the incumbent's seller product, whereas  $P$  is the price difference corrected for the switching cost. It follows that the buyer chooses the incumbent if and only if  $x > P$ .

Define by  $q_1$  and  $q_0$  the probability that the buyer chooses the incumbent or the entrant, respectively. If  $x$  is distributed according to  $F(x)$ , then we have

$$\begin{aligned} q_1 &= 1 - F(x) \\ q_0 &= F(x) \end{aligned}$$

I make the following assumptions regarding the c.d.f.  $F$  and the corresponding density  $f$ :

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3. In this sense, my model differs from the literature on customer recognition, where sellers learn about their buyers' valuations. See Villas-Boas (1999, 2006), Fudenberg and Tirole (2000), Doganoglu (2005).

**Assumption 1** (i)  $F(x)$  is continuously differentiable; (ii)  $f(x) = f(-x)$ ; (iii)  $f(x) > 0, \forall x$ ; (iv)  $f(x)$  is unimodal; (v)  $F(x)/f(x)$  is strictly increasing.

In the Appendix, I present a Lemma that proves a series of properties of  $F(x)$  that are derived from Assumption 1.

In this paper, I will focus on symmetric Markov equilibria. My first result shows that there exists only one such equilibrium. The proof of this and the remaining results in the paper may be found in the Appendix.

**Proposition 1** *There exists a unique symmetric Markov equilibrium.*

In the next two sections, I offer two sets of sufficient conditions for the main result in the paper, namely that an increase in switching costs implies a decrease in average equilibrium price. In the next section, I consider the case of small switching costs. In Section 3, I consider the case of a high discount factor.

### 3 Small switching costs

My main goal is to characterize equilibrium pricing as a function of switching costs  $s$ . In this section, I consider the case of small switching costs and prove that average price is decreasing in  $s$ . Let  $\bar{p}$  be the average price paid by the buyer, that is,

$$\bar{p} = q_1 p_1 + q_0 p_0.$$

**Proposition 2** *If  $s$  is small, then  $\bar{p}$  is decreasing in  $s$ .*

To understand the intuition for Proposition 2, it is useful to look at the sellers' first-order conditions. The incumbent seller's value function is given by

$$v_1 = (1 - F(P))(p_1 + \delta v_1) + F(P) \delta v_0$$

where  $v_i$  is seller  $i$ 's value. In words: with probability  $1 - F(P)$ , the incumbent seller makes a sale. This yields a short-run profit of  $p_1$  and a continuation value of an incumbent,  $v_1$ . With probability  $F(P)$ , the incumbent loses the sale, makes zero short run profits, and earns a continuation value  $v_0$ .

Maximizing with respect to  $p_1$ , we get the incumbent seller's first-order condition:

$$p_1 = \frac{1 - F(P)}{f(P)} - \delta V \tag{2}$$

where  $V \equiv v_1 - v_0$  is the difference, in terms of continuation value, between winning and losing the current sale. In other words,  $-\delta V$  is the “cost,” in terms of discounted continuation value, of winning the current sale.

Since  $q_1 = 1 - F(P)$  and  $P = p_1 - p_0 - s$ , we have  $\frac{dq_1}{dp_1} = f(P)$ . It follows that (2) may be re-written as

$$\frac{p_1 - (-\delta V)}{p_1} = \frac{1}{\epsilon_1}$$

where  $\epsilon_1 \equiv \frac{dq_1}{dp_1} \frac{p_1}{q_1}$ . This is simply the “elasticity rule” of optimal pricing, with one difference: the future discounted value from winning the sale appears as a negative cost (or subsidy) on price.

We thus have two forces on optimal price, which might be denoted by “harvesting” and “investing.” If the seller is myopic ( $\delta = 0$ ), then optimal price is given by the first term in the right-hand side of (2). The greater the value of  $s$ , the smaller the value of  $P$  (as shown in the proof of Proposition 2), and therefore the greater the value of  $p_1$ . We thus have harvesting, that is, a higher switching cost implies a higher price (by the incumbent seller, which is the more likely seller).

Suppose however that  $\delta > 0$ . Then we have a second effect, investing, which leads to lower prices. The greater the value of  $s$ , the greater the difference between being an incumbent and being an entrant, that is, the greater the value of  $V$ .

What is the relative magnitude of the harvesting and the investment effects on average price? First notice that harvesting leads to a higher price by the incumbent but lower by the entrant. In fact, by symmetry, the effects are approximately of the same absolute value when  $s$  is close to zero. This implies that, for  $s$  close to zero and in terms of average price, the harvesting effects approximately cancel out, since for  $s = 0$  incumbent and challenger sell with equal probability.

Not so with the dynamic effect. In fact, the entrant’s first-order condition is given by

$$p_0 = \frac{F(P)}{f(P)} - \delta V$$

that is, the “subsidy” resulting from the value of winning is the same as for the incumbent. It follows that the effect on average price is unambiguously negative, and of first-order importance.

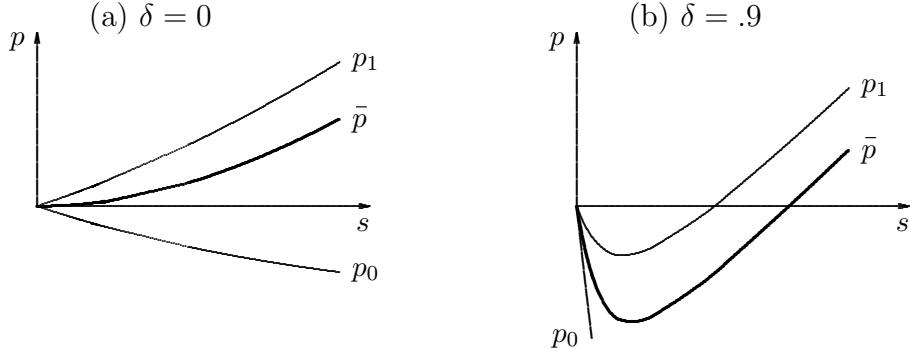


Figure 1: Switching cost and equilibrium price.

In other words, the harvesting effect is symmetric: the amount by which the incumbent increases its price is the same as the amount by which the outsider lowers its price. However, the investment effect is equal for both sellers — and negative.

Figure 1 illustrates Proposition 2. (In this and in the remaining numerical illustrations, I assume  $x$  is distributed according to a standardized normal.) On the horizontal axis, the value of switching cost varies from zero to positive values. On the vertical axis, three prices are plotted: the incumbent's price, the challenger's price, and average price. The left-hand panel shows the case when  $\delta = 0$ . Since there is no future, only the harvesting effect applies. The incumbent seller sets a price that is higher the higher the value of  $s$ . The challenger sets a price that is lower the higher  $s$  is. This results in an average price which is increasing in  $s$ . In other words, in a static world switching costs imply higher prices.<sup>4</sup> Notice however that the derivative of average price with respect to  $s$  equals zero when  $s = 0$ . That is, for small values of  $s$  the impact of  $s$  on  $\bar{p}$  is of second-order magnitude.

The investment effect, by contrast, is of first-order magnitude even for small values of  $s$ . This implies that, for a positive value of  $\delta$ , the investment effects dominates the harvesting effect for small values of  $s$ . This is illustrated by the right-hand panel, where it can be seen that  $\bar{p}$  is declining in  $s$  for small values of  $s$ .

Dubé, Hitsch and Rossi (2006) claim that, for various products, the value of switching cost lies in the region when the net effect of switching costs is

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4. This corresponds to the “ripoff effect” in two-period models. See Farrell and Klemperer (2007).

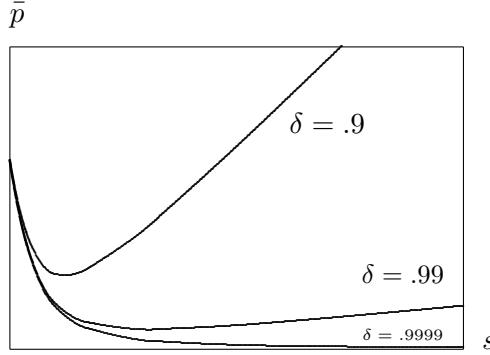


Figure 2: Switching cost and equilibrium price as  $\delta \rightarrow 1$ .

to decrease average price. In the next section, I show that, if the discount factor is sufficiently close to one, then switching costs lead to lower prices for *any* positive value of switching costs.

## 4 High discount factor

In the previous section, I showed that, for any preference distribution  $F$  satisfying Assumption 1 and any positive discount factor  $\delta$ , if the switching cost  $s$  is small enough then average price is decreasing in  $s$ . In this section, I provide an alternative sufficient condition for competitive switching costs. I show that, for any preference distribution  $F$  satisfying Assumption 1 and for any positive value of the switching cost  $s$ , if the discount factor is sufficiently close to 1 then average price is decreasing in  $s$ .

**Proposition 3** *If  $\delta$  is close to 1, then  $\bar{p}$  is decreasing in  $s$ .*

Figure 2 illustrates Proposition 3. It plots average price as a function of  $s$  for various values of  $\delta$ . The curve corresponding to  $\delta = .9$  is identical to Figure 1. It is U shaped: for small values of  $s$ , average price is decreasing in  $s$  (Proposition 2). However, for high values of  $s$ , average price becomes increasing in  $s$ . As we consider higher values of  $\delta$ , the U shape becomes more and more extended, so that, for a given range  $[0, \bar{s}]$  of values of  $s$ , average price eventually becomes uniformly decreasing in  $s$  (Proposition 3).

I next attempt to provide an intuitive explanation for Proposition 3. In discussing Proposition 2, we saw that equilibrium prices are given by

$$\begin{aligned} p_1 &= \frac{1 - F(P)}{f(P)} - \delta V \\ p_0 &= \frac{F(P)}{f(P)} - \delta V \end{aligned} \tag{3}$$

(As I mentioned earlier, this is just the “elasticity rule” with the added element that sellers “subsidize” their cost by  $\delta V$ .) We also saw that the seller value functions are given by

$$\begin{aligned} v_1 &= (1 - F(P))(p_1 + \delta v_1) + F(P) \delta v_0 \\ v_0 &= F(P)(p_0 + \delta v_1) + (1 - F(P)) \delta v_0 \end{aligned}$$

Substituting equilibrium prices into the value functions and simplifying we get

$$\begin{aligned} v_1 &= \frac{(1 - F(P))^2}{f(P)} + \delta v_0 \\ v_0 &= \frac{F(P)^2}{f(P)} + \delta v_0 \end{aligned} \tag{4}$$

If  $\delta = 0$ , seller value is given by short-run profit, the first term on the right-hand side of the value functions. In a dynamic equilibrium, seller value is given by these short-term profits plus  $\delta v_0$ , *regardless of whether the seller wins or loses the current sale*.

This is an important point and one worth exploring in greater detail. To understand the intuition, it may be useful to think of an auction with two bidders with the same valuations. Specifically, each bidder gets  $w$  if he wins the auction and  $l$  if he loses. The Nash equilibrium is for both bidders to bid  $w - l$ . It follows that equilibrium value is  $l$  for both bidders (winner or loser). In other words, the extra gain a bidder receives from being the winner,  $w - l$ , is bid away, so that a bidder can't expect more than  $l$ .

In the dynamic game at hand the analog of  $l$  is the continuation value if the seller loses the current sale,  $\delta v_0$ . So the idea is that all of the extra gain in terms of future value,  $\delta V = \delta(v_1 - v_0)$ , is bid away in terms of lower prices.

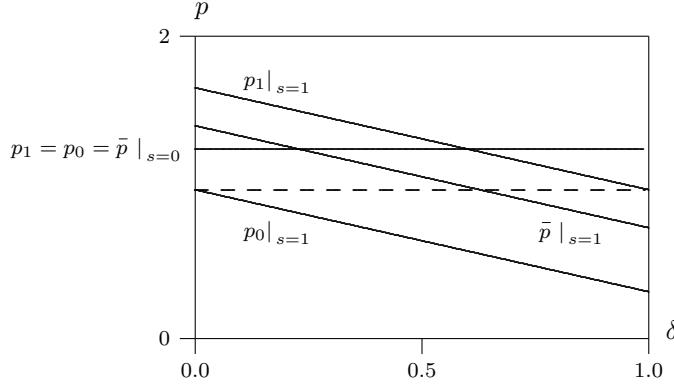


Figure 3: High price, low price, and average price as a function of the discount factor, when  $s = 0$  and when  $s = 1$ .

What does this imply in terms of equilibrium prices? From (4), we get

$$V = v_1 - v_0 = \frac{(1 - F(P))^2}{f(P)} - \frac{F(P)^2}{f(P)} = \frac{1 - F(P)}{f(P)} - \frac{F(P)}{f(P)} \quad (5)$$

Substituting for  $V$  in (3) we get

$$\lim_{\delta \rightarrow 1} p_1 = \frac{1 - F(P)}{f(P)} - V = \frac{F(P)}{f(P)} \quad (6)$$

But, as we can see from (3), the right-hand side of (6) is simply the equilibrium value of  $p_0$  when  $\delta = 0$ . In words, as the discount factor tends to 1, the incumbent's price level converges to the entrant's static price level (i.e., when  $\delta = 0$ ). But we know that, in a static model, increasing vertical product differentiation (in particular, increasing the switching cost) leads to a lower price by the "entrant" seller. That is, as we increase  $s$  from zero to a positive value, keeping  $\delta = 0$ , then the incumbent's price increases and the entrant's price decreases. If  $s = 0$ , then equilibrium price is the same regardless of the value of  $\delta$ . Finally, putting it all together, we conclude that, as  $\delta \rightarrow 1$  and  $s > 0$ , the high price is at the level of the lower price when  $s = 0$ ; and so switching costs lead to lower average price.

The above argument is illustrated in Figure 3. If  $s = 0$ , then equilibrium price is the same for incumbent and entrant; moreover, it is independent of the value of  $\delta$ . Now consider a positive switching cost, say  $s = 1$ . If  $\delta = 0$ , then we have a standard problem of vertical product differentiation. The

incumbent's price is higher than under no switching costs, whereas the entrant's price is lower than under no switching cost. Average price increases with switching costs, for two reasons: first, the increase in the incumbent's price is greater than the decrease in the entrant's price. Second, the incumbent sells with a higher probability.

As the value of  $\delta$  increases,  $p_1, p_0$  and  $\bar{p}$  decrease (linearly in  $\delta$ ).<sup>5</sup> When  $\delta = 1$ ,  $p_1$  is at the level of  $p_0$  when  $\delta = 0$  (by the argument presented above). We now have a series of inequalities: at  $\delta = 1$  and  $s = 1$ , average price is lower than the high price ( $\bar{p}|_{s=1,\delta=1} < p_1|_{s=1,\delta=1}$ ). This in turn is equal to the low price when  $\delta = 0$  ( $p_1|_{s=1,\delta=1} = p_0|_{s=1,\delta=0}$ ). This in turn is lower than average price when  $s = 0$ , regardless of the value of the discount factor ( $p_0|_{s=1,\delta=0} < p_0|_{s=0}$ ). And so, for  $\delta = 1$ , average price is lower with  $s = 1$  than with  $s = 0$ , an implication of Proposition 4 ( $\bar{p}|_{s=1,\delta=1} < \bar{p}|_{s=0,\delta=1}$ ).

## 5 Discussion

Figure 4 summarizes the main results in the paper. The SE curve represents the points at which the derivative of average price with respect to switching cost is zero. At points to the SE of this curve, an increase in switching cost implies a lower average price. Propositions 2 and 3 state two important properties of this curve: points with  $s$  sufficiently small (Proposition 2) or  $\delta$  sufficiently high (Proposition 3) below to region  $A$ . The figure suggests that this characterization is “tight,” that is, Propositions 2 and 3 describe the essential properties of the boundary of region  $A$ .

So far, I have examined how average price changes when switching costs increase. An alternative interesting comparison is between average price with  $s > 0$  and average price when  $s = 0$ . The NW-most curve in Figure 4 depicts points such that average price is the same as when  $s = 0$ . For points to the SE of this curve, average price is lower with switching costs than without switching costs.

**■ Profits and welfare.** Throughout the paper, I have looked at average price. What can we say about profits and welfare? First notice that, since

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5. To understand why prices vary linearly with  $\delta$ , notice that, from (5),  $V$  only depends on  $P$ . Moreover, subtracting the two equations (3), we get  $P$  as a function of  $s$  and  $V$ . It follows that the values of  $P$  and  $V$  depend on  $s$  but not on  $\delta$ . Finally, from (3)  $p_i$  is linear in  $\delta$ .

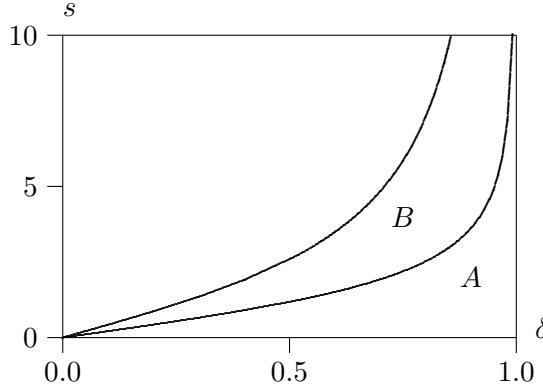


Figure 4: In region  $A$ , an increase in switching costs leads to a lower average equilibrium price. In regions  $A$  and  $B$ , average price is lower than it would be if switching costs were zero.

costs are the same for both sellers, industry value (on a per period basis) is given by average price. We thus conclude that, if  $s$  is small or if  $\delta$  is large, then switching costs lead to lower industry profits.

With respect to buyer welfare, we need to be a little more careful. In fact, in addition to price we must also take into account utility  $z_i$  and the switching cost  $s$ . However, for small values of  $s$ , market shares are approximately equal to 50%. It follows that the indirect effect on buyer welfare through the change in market shares is of second-order importance, and average price is a sufficient statistic for buyer welfare. We thus conclude that, if  $s$  is small, then switching costs lead to higher buyer welfare.

■ **Robustness.** Although I make only very weak assumptions regarding the nature of product differentiation, I do make some important assumptions regarding the nature of pricing and the dynamics of buyer preferences. First, as mentioned in the introduction, I assume that sellers can discriminate between buyers who are locked in and buyers who are not. If sellers cannot discriminate, then we must simultaneously consider all buyers (not just one) and the seller's optimal price will strike a balance between harvesting locked-in buyers and investing on new buyers. Beggs and Klemperer (1992) argue that the balance tends to favor higher prices than without switching costs. The contrast between my result and that of Beggs and Klemperer (1992) bears some relationship to the literature of oligopoly price discrimination

(Corts, 1998). Oligopolists typically would like to commit not to price discriminate as this would soften overall price competition.

Secondly, I assume buyer preferences are i.i.d. across periods. Other models consider the possibility of serial correlation in buyer preferences. If the seller can discriminate between buyers, then we have a case of “customer recognition.” Basically, conditionally on having made a sale in the previous period, a seller should expect its locked-in buyer to have a higher  $z$  than the population distribution would suggest. Villas-Boas (1999), Fudenberg and Tirole (2000), Doganoglu (2005) consider this possibility. It is not clear what the combination of switching costs and customer recognition implies for average prices.

Thirdly, I assume symmetry, both in terms of costs and in terms of buyer preferences. This assumption is not innocuous. In fact, the argument underlying Proposition 2 depends crucially on symmetry: for low values of  $s$  and  $\delta$ , the increase in the incumbent’s price approximately cancels the decrease in price by the entrant. If market shares are approximately 50%, then average price changes by an amount that is of second-order magnitude. However, if one of the sellers is much greater than the other one (either because it has lower costs or a better product), then the same is no longer true. In other words, in an industry with a dominant seller, switching costs are likely to increase prices and reduce buyer welfare.

Having said that, I should also add that none of my results is knife-edged. In other words, my results are based on strict inequalities. This implies that I can slightly perturb the model and still get similar qualitative results.

## Appendix

The proofs of Propositions 1–3 will use repeatedly the following result, which characterizes several properties of  $F$  that follow from Assumption 1:

**Lemma 1** *Under Assumption 1, the following are strictly increasing in  $x$ :*

$$\frac{F(x)^2}{f(x)}, \quad \frac{F(x) - 1}{f(x)}, \quad \frac{2F(x) - 1}{f(x)}.$$

Moreover, the following is increasing in  $x$  iff  $x > 0$  (and constant in  $x$  at  $x = 0$ ):

$$\frac{(1 - F(x))^2 + (F(x))^2}{f(x)}.$$

**Proof of Lemma 1:** First notice that

$$\frac{F(x)^2}{f(x)} = F(x) \frac{F(x)}{f(x)}.$$

Since  $F(x)$  is increasing and  $\frac{F(x)}{f(x)}$  is strictly increasing (by Assumption 1), it follows that the product is strictly increasing.

Next notice that, by part (ii) Assumption 1,

$$\frac{F(x) - 1}{f(x)} = \frac{-F(-x)}{f(x)} = \frac{-F(-x)}{f(-x)}$$

Since  $\frac{F(x)}{f(x)}$  is strictly increasing,  $\frac{-F(-x)}{f(-x)}$  is strictly increasing too.

Next notice that

$$\frac{2F(x) - 1}{f(x)} = \frac{F(x) - 1}{f(x)} + \frac{F(x)}{f(x)}.$$

I have just proved that  $\frac{F(x) - 1}{f(x)}$  is strictly increasing. We thus has the sum of two strictly increasing functions, the result being a strictly increasing function.

Finally, taking the derivative of the fourth expression I get

$$\begin{aligned}
& \frac{d}{dx} \left( \frac{(1 - F(x))^2 + (F(x))^2}{f(x)} \right) = \\
&= \frac{(-2(1 - F(x))f(x) + 2F(x)f(x))f(x)}{(f(x))^2} - \\
&\quad - \frac{f'(x)((1 - F(x))^2 + (F(x))^2)}{(f(x))^2} \\
&= 4 \left( F(x) - \frac{1}{2} \right) - f'(x) \xi,
\end{aligned}$$

where  $\xi = ((1 - F(x))^2 + (F(x))^2) / (f(x))^2$  is positive. The result then follows from Assumption 1. ■

**Proof of Proposition 1:** The seller value functions are given by

$$\begin{aligned}
v_1 &= (1 - F(P))(p_1 + \delta v_1) + F(P)\delta v_0 \\
v_0 &= F(P)(p_0 + \delta v_1) + (1 - F(P))\delta v_0
\end{aligned} \tag{7}$$

The corresponding first-order conditions are

$$\begin{aligned}
-f(P)(p_1 + \delta v_1) + 1 - F(P) + f(P)\delta v_0 &= 0 \\
-f(P)(p_0 + \delta v_1) + F(P) + f(P)\delta v_0 &= 0
\end{aligned}$$

(Recall that, from (1),  $\frac{dP}{dp_1} = 1$  and  $\frac{dP}{dp_0} = -1$ .) Solving for optimal prices, I get

$$\begin{aligned}
p_1 &= \frac{1 - F(P)}{f(P)} - \delta V \\
p_0 &= \frac{F(P)}{f(P)} - \delta V
\end{aligned} \tag{8}$$

where

$$V \equiv v_1 - v_0$$

Substituting (8) for  $p_1, p_0$  in (7) and simplifying, I get

$$\begin{aligned} v_1 &= \frac{(1 - F(P))^2}{f(P)} + \delta v_0 \\ v_0 &= \frac{F(P)^2}{f(P)} + \delta v_0 \end{aligned}$$

It follows that

$$P = \frac{1 - F(P)}{f(P)} - \frac{F(P)}{f(P)} - s = \frac{1 - 2F(P)}{f(P)} - s \quad (9)$$

$$V = \frac{(1 - F(P))^2}{f(P)} - \frac{F(P)^2}{f(P)} = \frac{1 - 2F(P)}{f(P)} \quad (10)$$

Equation (9) may be rewritten as

$$P + \frac{2F(P) - 1}{f(P)} = -s \quad (11)$$

By Lemma 1, the left-hand side is strictly increasing in  $P$ , ranging from  $-\infty$  to  $+\infty$  as  $P$  itself ranges from  $-\infty$  to  $+\infty$ . This implies there exists a unique solution  $P$ . From (10), there exists a unique  $V$ . Finally, from (8) there exist unique  $p_0, p_1$ . ■

**Proof of Proposition 2:** Average price is given by

$$\bar{p} \equiv q_1 p_1 + q_0 p_0 = (1 - F(P))p_1 + F(P)p_0$$

Substituting (8) for  $p_1, p_0$  and (10) for  $V$ , and simplifying, I get

$$\begin{aligned} \bar{p} &= (1 - F(P)) \left( \frac{1 - F(P)}{f(P)} - \delta V \right) + F(P) \left( \frac{F(P)}{f(P)} - \delta V \right) \\ &= \frac{(1 - F(P))^2}{f(P)} + \frac{F(P)^2}{f(P)} - \delta V \\ &= \frac{(1 - F(P))^2 + F(P)^2}{f(P)} + \delta \left( \frac{2F(P) - 1}{f(P)} \right) \end{aligned} \quad (12)$$

Lemma 1 implies that, at  $s = 0$ , the first term on the right-hand side of (12) is constant in  $P$ . It also implies that the second term on the right-hand side of (12) is increasing in  $P$ . It follows that, if  $s$  is small, then  $\frac{d\bar{p}}{dP} > 0$ .

From (11) and the implicit function theorem, I get  $\frac{dP}{ds} < 0$ . Finally, if  $s$  is small then

$$\frac{d\bar{p}}{ds} = \left( \frac{d\bar{p}}{dP} \right) \left( \frac{dP}{ds} \right) < 0,$$

which concludes the proof. ■

**Proof of Proposition 3:** From (12),

$$\lim_{\delta \rightarrow 1} \bar{p} = 2 \frac{F(P)^2}{f(P)}$$

Lemma 1 then implies that, if  $\delta$  is sufficiently close to 1, then  $\bar{p}$  is increasing in  $P$ . From the proof of Proposition 2,  $P$  is decreasing in  $s$ . (Notice that that statement does not depend on  $s$  being small.) The result then follows by the chain rule of differentiation. ■

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