Asset Pricing with Liquidity Risk*

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Abstract

This paper solves explicitly an equilibrium asset pricing model with liquidity risk — the risk arising from unpredictable changes in liquidity over time. In our liquidity-adjusted capital asset pricing model, a security's required return depends on its expected liquidity as well as on the covariances of its own return and liquidity with market return and market liquidity. In addition, the model shows how a negative shock to a security's liquidity, if it is persistent, results in low contemporaneous returns and high predicted future returns. The model provides a simple, unified framework for understanding the various channels through which liquidity risk may affect asset prices. Our empirical results shed light on the total and relative economic significance of these channels.

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1 Introduction

Liquidity is risky and has commonality: it varies over time both for individual stocks and for the market as a whole (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2000), and Huberman and Halka (1999)). Liquidity risk is often noted in the press, for instance:

> *The possibility that liquidity might disappear from a market, and so not be available when it is needed, is a big source of risk to an investor.*
> — *The Economist* September 23, 1999

and in the financial industry:

> *there is also broad belief among users of financial liquidity — traders, investors and central bankers — that the principal challenge is not the average level of financial liquidity ... but its variability and uncertainty*
> — Persaud (2003)

This paper presents a simple theoretical model that helps explain how asset prices are affected by liquidity risk and commonality in liquidity. The model provides a unified theoretical framework that can explain the empirical findings that return sensitivity to market liquidity is priced (Pastor and Stambaugh (2003)), that average liquidity is priced (Amihud and Mendelson (1986)), and that liquidity comoves with returns and predicts future returns (Amihud (2002), Chordia, Roll, and Subrahmanyam (2001), Jones (2001), and Bekaert, Harvey, and Lundblad (2003)).

In our model, risk averse agents in an overlapping-generations economy trade securities whose liquidity varies randomly over time. We solve the model explicitly and derive a liquidity-adjusted capital asset pricing model (CAPM). Our model of liquidity risk complements the existing theoretical literature on liquidity and transactions costs, which deals with deterministic trading costs (for instance, Amihud and Mendelson (1986), Constantinides (1986), Vayanos (1998), Vayanos and Vila (1999), Gârleanu and Pedersen (2000), Huang (2002)). In the liquidity-adjusted CAPM, the expected return of a security is increasing in its expected illiquidity and its "net beta," which is proportional to the covariance of its return, r^i, net of illiquidity costs, c^i, with the market portfolio's net return, $r^M - c^M$. The net beta can be decomposed into the standard market beta and three betas representing different forms of liquidity risk. These liquidity risks are associated with *(i)* commonality in liquidity with the market liquidity, $\operatorname{cov}(c^i, c^M)$; *(ii)* return sensitivity to market liquidity, $\operatorname{cov}(r^i, c^M)$; and *(iii)* liquidity sensitivity to market returns, $\operatorname{cov}(c^i, r^M)$.

We explore the cross-sectional predictions of the model using NYSE and AMEX stocks over the period 1963–1999. We use the illiquidity measure of Amihud (2002) to proxy for c^i. We find that the liquidity-adjusted CAPM fares better than the standard CAPM in terms of R^2 for cross-sectional returns and p-values in specification tests, even though both models employ exactly one degree of freedom. The model has a good fit for portfolios sorted on liquidity, liquidity variation, and size, but the model cannot explain the cross-sectional returns associated with the book-to-market effect.

An interesting result that emerges from our empirical exercises based on Amihud's illiquidity measure is that illiquid securities also have high liquidity risk. In particular, a security that has high average illiquidity c^i also tends to have high commonality in liquidity with the market liquidity, high return sensitivity to market liquidity, and high liquidity sensitivity to market returns. While this collinearity is itself interesting, it also complicates the task of distinguishing statistically the relative return impacts of liquidity, liquidity risk, and market risk. There is, however, some evidence that the total effect of the three liquidity risks matters over and above market risk and the level of liquidity.

It is interesting to consider the total and relative economic significance of liquidity level and each of the three liquidity risks by evaluating their contribution to cross-sectional return differences. It is, however, difficult to accurately distinguish the relative economic effects because of the inherent collinearity in the data. One of the benefits of having an economic model is that it provides a restrictive structure under which the identification problem is alleviated. Under the model's restrictions, liquidity risk contributes on average about 1.1% annually to the difference in risk premium between stocks with high expected illiquidity and low expected illiquidity. We decompose the effect of liquidity risk into the contribution from each of the three kinds of risk, recognizing that these estimates are subject to error and rely on the validity of the model:

First, we estimate that the return premium due to commonality in liquidity, $\mathrm{cov}(c^i, c^M)$, is 0.08%. Hence, while the model shows that investors require a return premium for a security that is illiquid when the market as a whole is illiquid, this effect appears to be small. The commonality in liquidity has been documented by Chordia, Roll, and Subrahmanyam (2000), Huberman and Halka (1999), and Hasbrouck and Seppi (2000), but these papers do not study the implications for required returns.

Second, we estimate that the return premium due to $\mathrm{cov}(r^i, c^M)$ is 0.16%. This model-implied premium stems from investors' preference for securities with high returns when the market is illiquid. Pastor and Stambaugh (2003) find empirical support for this effect using monthly data over 34 years with a measure of liquidity that they construct based on the return reversals induced by order flow.

Third, we estimate that the return premium due to $\mathrm{cov}(c^i, r^M)$ is 0.82%. Intuitively, investors are willing to pay a premium for a security that is liquid when the market return is low. We note that $\mathrm{cov}(c^i, r^M)$ appears to be the most important source of liquidity risk although it has not previously been considered in the academic literature. It is, however, reflected in industry practices such as legal disclaimers for certain asset management firms, e.g.

> *Risks of investing in smaller companies include ... the potential difficulty of selling these stocks during market downturns (illiquidity).*
> — Legal Disclaimer, Investec Asset Management, 2004.[1]

The return premium due to the level of liquidity is calibrated based on the average turnover to be 3.5% so the combined effect of the differences in liquidity risks and differences in the level of liquidity is estimated to be 4.6% per year. These estimates of the relative importance of liquidity level and the liquidity risks depend on the model-implied restrictions of a single risk premium and a level effect consistent with the turnover. If we depart from the model restrictions and estimate each liquidity risk premium as a free parameter then the economic effect of liquidity risk appears to be larger, but the unrestricted premia are estimated with little precision. Pastor and Stambaugh (2003) find a large (7.5%) effect of liquidity risk ($\mathrm{cov}(r^i, c^M)$) using an unrestricted liquidity risk premium and without controlling for the level of liquidity.

[1] Source: http://www2.investecfunds.com/US/LegalDisclaimer/Index.cfm

Finally, the model also shows that, since liquidity is persistent,[2] liquidity predicts future returns and liquidity co-moves with contemporaneous returns. This is because a positive shock to illiquidity predicts high future illiquidity, which raises the required return and lowers contemporaneous prices. This may help explain the empirical findings of Amihud, Mendelson, and Wood (1990), Amihud (2002), Chordia, Roll, and Subrahmanyam (2001), Jones (2001), and Pastor and Stambaugh (2003) in the US stock market, and of Bekaert, Harvey, and Lundblad (2003) in emerging markets.

In summary, we offer a simple theoretical framework that illustrates several channels through which liquidity risk can affect asset prices. The model is a useful first step in understanding how a number of recent empirical findings fit together. Finally, our empirical analysis suggests that the effects of liquidity level and liquidity risk are separate, although the analysis is made difficult by collinearity, and that one channel for liquidity risk that has not been treated in the prior literature, $\mathrm{cov}(c^i, r^M)$, may be of empirical importance.

The paper is organized as follows. Section 2 describes the economy. Section 3 derives the liquidity-adjusted capital asset pricing model and outlines how liquidity predicts and co-moves with returns. Section 4 contains an empirical analysis. Section 5 concludes. Proofs are in the Appendix.

2 Assumptions

The model assumes a simple overlapping generations economy in which a new generation of agents is born at any time $t \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (Samuelson (1958)). Generation t consists of N agents, indexed by n, who live for two

[2] Amihud (2002), Chordia, Roll, and Subrahmanyam (2000, 2001), Hasbrouck and Seppi (2000), Huberman and Halka (1999), Jones (2001), and Pastor and Stambaugh (2003).

periods, t and $t+1$. Agent n of generation t has an endowment at time t and no other sources of income, trades in periods t and $t+1$, and derives utility from consumption at time $t+1$. He has constant absolute risk aversion A^n so that his preferences are represented by the expected utility function $-E_t \exp(-A^n x_{t+1})$, where x_{t+1} is his consumption at time $t+1$.

There are I securities indexed by $i = 1, \ldots, I$ with a total of S^i shares of security i. At time t, security i pays a dividend of D_t^i, has an ex-dividend share price of P_t^i, and has an illiquidity cost of C_t^i, where D_t^i and C_t^i are random variables.[3] The illiquidity cost, C_t^i, is modeled simply as the per-share cost of selling security i. Hence, agents can buy at P_t^i but must sell at $P_t^i - C_t^i$. Short-selling is not allowed.

Uncertainty about the illiquidity cost is what generates the liquidity risk in this model. Specifically, we assume that D_t^i and C_t^i are autoregressive processes of order one, that is:

$$
\begin{aligned}
D_t &= \bar{D} + \rho^D (D_{t-1} - \bar{D}) + \varepsilon_t \\
C_t &= \bar{C} + \rho^C (C_{t-1} - \bar{C}) + \eta_t \,,
\end{aligned}
$$

where[4] $\bar{D}$, $\bar{C} \in \mathbb{R}_+^I$ are positive real vectors, $\rho^D, \rho^C \in [0,1]$, and (ε_t, η_t) is an independent identically distributed normal process with mean $E(\varepsilon_t) = E(\eta_t) = 0$ and variance-covariance matrices $\mathrm{var}(\varepsilon_t) = \Sigma^D$, $\mathrm{var}(\eta_t) = \Sigma^C$, and $E(\varepsilon_t \eta_t^\top) = \Sigma^{CD}$.

We assume that agents can borrow and lend at a risk-free real return of $r^f > 1$, which is exogenous. This can be interpreted as an inelastic bond market, or a

[3]All random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, and all random variables indexed by t are measurable with respect to the filtration $\{\mathcal{F}_t\}$, representing the information commonly available to investors.

[4]For notational convenience we assume that all securities have the same autocorrelation of dividends and liquidity (ρ^D and ρ^C) although our results apply more generally.

generally available production technology that turns a unit of consumption at time t into r^f units of consumption at time $t+1$.

The assumptions with respect to agents, preferences, and dividends are strong. These assumptions are made for tractability, and, as we shall see, they imply natural closed-form results for prices and expected returns. The main result (Proposition 1) applies more generally, however. It holds for arbitrary utility functions defined on $(-\infty, \infty)$ as long as conditional expected net returns are normal,[5] and also for arbitrary return distribution and quadratic utility. Furthermore, it can be viewed as a result of near-rational behavior, for instance, by using a Taylor expansion of the utility function (see Huang and Litzenberger (1988), Markowitz (2000), and Cochrane (2001)). Our assumptions allow us, additionally, to study return predictability caused by illiquidity (Proposition 2) and the co-movements of returns and illiquidity (Proposition 3), producing insights that also seem robust to the specification.

Perhaps the strongest assumption is that investors need to sell all their securities after one period (when they die). In a more general setting with endogenous holding periods, deriving a general equilibrium with time-varying liquidity is an onerous task. While our model is mostly suggestive, it is helpful since it provides guidelines concerning the first-order effect of liquidity risk, showing which risks are priced. The assumption of overlapping generations can capture investors' life-cycle motives for trade (as in Vayanos (1998), and Constantinides, Donaldson, and Mehra (2002)), or can be viewed as a way of capturing short investment horizons (as in De Long, Shleifer, Summers, and Waldmann (1990)) and the large turnover observed empirically in many markets.

[5]The normal returns assumption is an assumption about endogenous variables that is used in standard CAPM analysis (for instance, Huang and Litzenberger (1988)). This assumption is satisfied in the equilibrium of the model of this paper.

It should also be noted that a narrow interpretation of the illiquidity cost, C_t^i, is that it is a transaction cost such as broker fees and bid-ask spread, in line with the literature on exogenous transactions costs. More broadly, however, the illiquidity cost could represent other the real costs, for instance, arising from delay and search associated with trade execution as in Duffie, Gârleanu, and Pedersen (2000). The novelty in our model arises from the fact that we allow this cost to be time-varying. While research on endogenous time-variation in illiquidity is sparse, in a recent paper Eisfeldt (2004) presents a model in which liquidity fluctuates with real-sector productivity and investment.

3 Liquidity-Adjusted Capital Asset Pricing Model

This section derives a liquidity-adjusted version of the Capital Asset Pricing Model (CAPM) and studies its asset pricing implications.

We are interested in how an asset's expected (gross) return,

$$r_t^i = \frac{D_t^i + P_t^i}{P_{t-1}^i},$$

depends on its relative illiquidity cost, defined as

$$c_t^i = \frac{C_t^i}{P_{t-1}^i},$$

on the market return,

$$r_t^M = \frac{\sum_i S^i(D_t^i + P_t^i)}{\sum_i S^i P_{t-1}^i},$$

and on the relative market illiquidity,

$$c_t^M = \frac{\sum_i S^i C_t^i}{\sum_i S^i P_{t-1}^i} \ .$$

In a competitive equilibrium of the model (henceforth referred to simply as equilibrium), agents choose consumption and portfolios so as to maximize their expected utility taking prices as given, and prices are determined such that markets clear.

To determine equilibrium prices, consider first an economy with the same agents in which asset i has a dividend of $D_t^i - C_t^i$ and no illiquidity cost. In this imagined economy, standard results imply that the CAPM holds (Markowitz (1952), Sharpe (1964), Lintner (1965), and Mossin (1966)). We claim that the equilibrium prices in the original economy with frictions are the same as those of the imagined economy. This follows from two facts: *(i)* the net return on a long position is the same in both economies; *(ii)* all investors in the imagined economy hold a long position in the market portfolio, and a (long or short) position in the risk-free asset. Hence, an investor's equilibrium return in the frictionless economy is feasible in the original economy, and is also optimal, given the more limited investment opportunities due to the short-selling constraints.[6]

These arguments show that the CAPM in the imagined frictionless economy translates into a CAPM in net returns for the original economy with illiquidity costs. Rewriting the one-beta CAPM in net returns in terms of gross returns, we get a liquidity-adjusted CAPM for gross returns. This is the main testable[7]

[6] This argument applies more generally since positive transactions costs imply that a short position has a worse payoff than minus the payoff of a long position. We impose the short-sale constraint because C can be negative in our normal setting.

[7] Difficulties in testing this model arise from the fact that it makes predictions concerning conditional moments as is standard in asset pricing. See Hansen and Richard (1987), Cochrane (2001),

implication of this paper:

Proposition 1 *In the unique linear equilibrium, the conditional expected net return of security i is*

$$E_t(r^i_{t+1} - c^i_{t+1}) = r^f + \lambda_t \frac{\mathrm{cov}_t(r^i_{t+1} - c^i_{t+1}, r^M_{t+1} - c^M_{t+1})}{\mathrm{var}_t(r^M_{t+1} - c^M_{t+1})} \tag{1}$$

where $\lambda_t = E_t(r^M_{t+1} - c^M_{t+1} - r^f)$ is the risk premium. Equivalently, the conditional expected gross return is

$$\begin{aligned} E_t(r^i_{t+1}) \quad = \quad & r^f + E_t(c^i_{t+1}) + \lambda_t \frac{\mathrm{cov}_t(r^i_{t+1}, r^M_{t+1})}{\mathrm{var}_t(r^M_{t+1} - c^M_{t+1})} + \lambda_t \frac{\mathrm{cov}_t(c^i_{t+1}, c^M_{t+1})}{\mathrm{var}_t(r^M_{t+1} - c^M_{t+1})} \\ & -\lambda_t \frac{\mathrm{cov}_t(r^i_{t+1}, c^M_{t+1})}{\mathrm{var}_t(r^M_{t+1} - c^M_{t+1})} - \lambda_t \frac{\mathrm{cov}_t(c^i_{t+1}, r^M_{t+1})}{\mathrm{var}_t(r^M_{t+1} - c^M_{t+1})} \, . \end{aligned} \tag{2}$$

Equation (2) is simple and natural. It states that the required excess return is the expected relative illiquidity cost, $E_t(c^i_{t+1})$, as found theoretically and empirically[8] by Amihud and Mendelson (1986)), plus four betas (or covariances) times the risk premium. These four betas depend on the asset's payoff and liquidity risks. As in the standard CAPM, the required return on an asset increases linearly with the market beta, that is, covariance between the asset's return and the market return. This model yields three additional effects which could be regarded as three forms

and references therein. An unconditional version of (2) applies under stronger assumptions as discussed in Section 3.3.

[8]Empirically, Amihud and Mendelson (1986, 1989) find the required rate of return on NYSE stocks to increase with the relative bid-ask spread. This result is questioned for NYSE stocks by Eleswarapu and Reinganum (1993), but supported for NYSE stocks (especially for amortized spreads) by Chalmers and Kadlec (1998), and for Nasdaq stocks by Eleswarapu (1997). Gârleanu and Pedersen (2000) find that adverse-selection costs are priced only to the extent that they render allocations inefficient. The ability of a market to allocate assets efficiently may be related to market depth, and, consistent with this view, the required rate of return has been found to decrease with measures of depth (Brennan and Subrahmanyam (1996) and Amihud (2002)). Easley, Hvidkjær, and O'Hara (2002) find returns to increase with a measure of the probability of informed trading.

of liquidity risks.

3.1 Three Liquidity Risks

1. $\mathrm{cov}_t(c^i_{t+1}, c^M_{t+1})$: The first effect is that the return increases with the covariance between the asset's illiquidity and the market illiquidity. This is because investors want to be compensated for holding a security that becomes illiquid when the market in general becomes illiquid. The potential empirical significance of this pricing implication follows from the presence of a time-varying common factor in liquidity, which is documented by Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2000), and Huberman and Halka (1999). These papers find that most stocks' illiquidities are positively related to market illiquidity, so the required return should be raised by the commonality-in-liquidity effect. The effect of commonality in liquidity on asset prices is, however, not studied by these authors; We study this effect empirically in Section 4.

In this model, the risk premium associated with commonality in liquidity is caused by the wealth effects of illiquidity. Also, this risk premium would potentially apply in an economy in which investors can choose which securities to sell. In such a model, an investor who holds a security that becomes illiquid (that is, has a high cost c^i_t) can choose not to trade this security and instead trade other (similar) securities. It is more likely that an investor can trade other (similar) securities, at low cost, if the liquidity of this asset does not co-move with the market liquidity. Hence, investors would require a return premium for assets with positive covariance between individual and market illiquidity.

2. $\mathrm{cov}_t(r^i_{t+1}, c^M_{t+1})$: The second effect on expected returns is due to covariation between a security's return and the market liquidity. We see that $\mathrm{cov}_t(r^i_{t+1}, c^M_{t+1})$

affects required returns negatively because investors are willing to accept a lower return on an asset with a high return in times of market illiquidity. Related effects also arise in the theoretical models of Holmstrom and Tirole (2000), who examine implications of corporate demand for liquidity, and Lustig (2001), who studies the equilibrium implications of solvency constraints. Empirical support for this effect is provided by Pastor and Stambaugh (2003), who find that "the average return on stocks with high sensitivities to [market] liquidity exceeds that for stocks with low sensitivities by 7.5% annually, adjusted for exposures to the market return as well as size, value, and momentum factors." Sadka (2002) and Wang (2002) also present consistent evidence for this effect using alternative measures of liquidity.

3. $\mathrm{cov}_t(c^i_{t+1}, r^M_{t+1})$: The third effect on required returns is due to covariation between a security's illiquidity and the market return. This effect stems from investors' willingness to accept a lower expected return on a security that is liquid in a down market. When the market declines, investors are poor, and the ability to sell easily is especially valuable. Hence, an investor is willing to accept a discounted return on stocks with low illiquidity costs in states of poor market return. We find consistent evidence for this liquidity risk in the stock market in Section 4, and the effect seems economically important. Also, anecdotal evidence[9] suggests that private equity is illiquid during down markets, which, together with our model, may help explain the high average return documented by Ljungqvist and Richardson (2003).

Outside our model, intuition suggests that a low market return causes wealth problems for some investors, who then need to sell. If a selling investor holds securities that are illiquid at this time, then his problems are magnified. Con-

[9]E.g., the Institute for Fiduciary Education (2002) characterizes private equity as an "illiquid asset class" and points out that "In down equity markets, exits are more difficult and little cash is returned." Source: http://www.ifecorp.com/Papers-PDFs/Wender1102.PDF

sistent with this intuition, Lynch and Tan (2003) find that the liquidity premium is large if the transactions costs covary negatively with wealth shocks, among other conditions. This is consistent with our effect of $\mathrm{cov}_t(c^i_{t+1}, r^M_{t+1})$ to the extent that r^M proxies for wealth shocks. Lynch and Tan (2003) complement our paper by showing through calibration that, even if an investor chooses his holding period endogenously, the liquidity premium can be large (3.55% in one calibration). They follow Constantinides (1986) in using a partial-equilibrium framework and defining the liquidity premium as the decrease in expected return that makes an investor indifferent between having access to the asset without transaction costs rather than with them.

The three covariances thus provide a characterization of the liquidity risk of a security. We note that all these covariances can be accounted for by simply using the conditional CAPM in net returns as in (1). It is useful, however, to use gross returns and illiquidity as the basic inputs for several reasons: First, computing the net return is not straightforward since it depends on the investor's holding period, and the holding period may be different from the econometrician's sampling period. We explain in Section 4 how we overcome this problem. Second, the empirical liquidity literature is based on measures of gross return and illiquidity costs, and the model provides a theoretical foundation for the empirical relations between these security characteristics. Third, a pricing relation for gross returns and illiquidity, which is similar in spirit to (2), may hold in richer models in which net returns are not sufficient state variables. As argued above, additional liquidity effects outside the model suggest risk premia of the same sign for the covariance terms in (2). These additional liquidity effects also suggest that the size of the risk premia need not be identical across the covariance terms. To accommodate the possibility of a richer liquidity framework, we also consider a generalization of

(2) in our empirical work in Section 4.

3.2 Implications of Persistence of Liquidity

This section shows that persistence of liquidity implies that liquidity predicts future returns and co-moves with contemporaneous returns.

Empirically, liquidity is time-varying and *persistent*,[10] that is, $\rho^C > 0$. This model shows that persistent liquidity implies that returns are *predictable*. Intuitively, high illiquidity today predicts high expected illiquidity next period, implying a high required return.

Proposition 2 *Suppose that* $\rho^C > 0$, *and that* $q \in \mathbb{R}^I$ *is a portfolio*[11] *with* $E_t(P_{t+1}^q + D_{t+1}^q) > \rho^C P_t^q$. *Then, the conditional expected return increases with illiquidity,*

$$\frac{\partial}{\partial C_t^q} E_t(r_{t+1}^q - r^f) > 0. \tag{3}$$

Proposition 2 relies on a mild technical condition, which is satisfied, for instance, for any portfolio with positive price and with $E_t(P_{t+1}^q + D_{t+1}^q)/P_t^q \geq 1$. The proposition states that the conditional expected return depends positively on the current illiquidity cost, that is, the current liquidity predicts the return.

Jones (2001) finds empirically that the expected annual stock market return increases with the previous year's bid-ask spread and decreases with the previous year's turnover. Amihud (2002) finds that illiquidity predicts excess return both for the market and for size-based portfolios, and Bekaert, Harvey, and Lundblad (2003) find that illiquidity predicts returns in emerging markets.

[10]See Amihud (2002), Chordia, Roll, and Subrahmanyam (2000, 2001), Hasbrouck and Seppi (2000), Huberman and Halka (1999), Jones (2001), and Pastor and Stambaugh (2003).

[11]For any $q \in \mathbb{R}^I$, we use the obvious notation $D_t^q = q^\top D_t$, $r_t^q = \frac{\sum_i q^i (D_t^i + P_t^i)}{\sum_i q^i P_{t-1}^i}$ and so on.

Predictability of liquidity further implies a negative conditional covariance between contemporaneous returns and illiquidity. Naturally, when illiquidity is high, the required return is high also, which depresses the current price, leading to a low return. This intuition applies as long as liquidity is persistent ($\rho^C > 0$) and innovations in dividends and illiquidity are not too correlated ($q^\top \Sigma^{CD} q$ low for a portfolio q) as is formalized in the following proposition.

Proposition 3 *Suppose $q \in \mathbb{R}^I$ is a portfolio such that $\rho^C(r^f\, q^\top \Sigma^{CD} q + (r^f - \rho^D) q^\top \Sigma^C q) > (r^f)^2\, q^\top \Sigma^{CD} q$. Then, returns are low when illiquidity increases,*

$$\operatorname{cov}_t(c_{t+1}^q, r_{t+1}^q) < 0 \tag{4}$$

Consistent with this result, Chordia, Roll, and Subrahmanyam (2001), Jones (2001), and Pastor and Stambaugh (2003) find a negative relation between the market return and measures of market illiquidity, Amihud (2002) finds a negative relation between the return on size portfolios and their corresponding unexpected illiquidity, and Bekaert, Harvey, and Lundblad (2003) find a negative relationship between illiquidity and returns for emerging markets.

3.3 An Unconditional Liquidity-Adjusted CAPM

To estimate the liquidity-adjusted CAPM, we derive an unconditional version. An unconditional result obtains, for instance, under the assumption of independence over time of dividends and illiquidity costs. Empirically, however, illiquidity is persistent. Therefore, we rely instead on an assumption of constant conditional covariances of innovations in illiquidity and returns.[12] This assumption yields the unconditional result that,

$$E(r_t^i - r_t^f) = E(c_t^i) + \lambda\beta^{1i} + \lambda\beta^{2i} - \lambda\beta^{3i} - \lambda\beta^{4i} \ , \tag{6}$$

where

$$\beta^{1i} = \frac{\operatorname{cov}(r_t^i, r_t^M - E_{t-1}(r_t^M))}{\operatorname{var}\left(r_t^M - E_{t-1}(r_t^M) - [c_t^M - E_{t-1}(c_t^M)]\right)} \tag{7}$$

$$\beta^{2i} = \frac{\operatorname{cov}(c_t^i - E_{t-1}(c_t^i), c_t^M - E_{t-1}(c_t^M))}{\operatorname{var}\left(r_t^M - E_{t-1}(r_t^M) - [c_t^M - E_{t-1}(c_t^M)]\right)} \tag{8}$$

$$\beta^{3i} = \frac{\operatorname{cov}(r_t^i, c_t^M - E_{t-1}(c_t^M))}{\operatorname{var}\left(r_t^M - E_{t-1}(r_t^M) - [c_t^M - E_{t-1}(c_t^M)]\right)} \tag{9}$$

$$\beta^{4i} = \frac{\operatorname{cov}(c_t^i - E_{t-1}(c_t^i), r_t^M - E_{t-1}(r_t^M))}{\operatorname{var}\left(r_t^M - E_{t-1}(r_t^M) - [c_t^M - E_{t-1}(c_t^M)]\right)} \ , \tag{10}$$

and $\lambda = E(\lambda_t) = E(r_t^M - c_t^M - r^f)$. Next, we describe the empirical tests of this unconditional relation.

4 Empirical Results

In this section, we estimate and test the liquidity-adjusted CAPM as specified in Equation (6). We do this in five steps:

(i) We estimate, in each month t of our sample, a measure of illiquidity, c_t^i, for each individual security i. (Section 4.1.)

[12]Alternatively, the same unconditional model can be derived by assuming a constant risk premium λ, and by using the fact that for any random variables X and Y, it holds that

$$E(\operatorname{cov}_t(X, Y)) = \operatorname{cov}(X - E_t(X), Y) = \operatorname{cov}(X - E_t(X), Y - E_t(Y)). \tag{5}$$

We note that the possible time-variation of risk premium is driven by constant absolute risk aversion in our model, but with constant relative risk aversion the risk premium is approximately constant. See Friend and Blume (1975).

(ii) We form a "market portfolio" and sets of 25 test portfolios sorted on the basis of illiquidity, illiquidity variation, size, and book-to-market by size, respectively. For each portfolio and each month, we compute its return and illiquidity. (Section 4.2.)

(iii) For the market portfolio as well as the test portfolios, we estimate the innovations in illiquidity, $c_t^p - E_{t-1}(c_t^p)$. (Section 4.3.)

(iv) Using these illiquidity innovations and returns, we estimate and analyze the liquidity betas. (Section 4.4.)

(v) Finally, we consider the empirical fit of the (unconditional) liquidity-adjusted CAPM by running cross-sectional regressions. To check the robustness of our results, we do the analysis with a number of different specifications. (Section 4.5.)

4.1 The Illiquidity Measure

Liquidity is (unfortunately) not an observable variable. There exist, however, many proxies for liquidity. Some proxies, such as the bid-ask spread, are based on market microstructure data, which is not available for a time series as long as is usually desirable for studying the effect on expected returns. Further, the bid-ask spread measures well the cost of selling a small number of shares, but it does not necessarily measure well the cost of selling many shares. We follow Amihud (2002) in estimating illiquidity using only daily data from the Center for Research in Security Prices (CRSP). In particular, Amihud (2002) defines the illiquidity of stock i in month t as

$$ILLIQ_t^i = \frac{1}{Days_t^i} \sum_{d=1}^{Days_t^i} \frac{|R_{td}^i|}{V_{td}^i}, \tag{11}$$

where R^i_{td} and V^i_{td} are, respectively, the return and dollar volume (in millions) on day d in month t, and $Days^i_t$ is the number of valid observation days in month t for stock i.

The intuition behind this illiquidity measure is as follows. A stock is illiquid — that is, has a high value of $ILLIQ^i_t$ — if the stock's price moves a lot in response to little volume. In our model, illiquidity is the cost of selling and, as discussed in Section 2, real markets have several different selling costs including broker fees, bid-ask spreads, market impact, and search costs. Our empirical strategy is based on an assumption that $ILLIQ$ is a valid instrument for the costs of selling, broadly interpreted. Consistent with this view, Amihud (2002) shows empirically that $ILLIQ$ is positively related to measures of price impact and fixed trading costs over the time period in which he has the microstructure data. Similarly, Hasbrouck (2002) computes a measure of Kyle's lambda using micro-structure data for NYSE, AMEX and NASDAQ stocks, and finds that its Spearman (Pearson) correlation with $ILLIQ$ in the cross-section of stocks is 0.737 (0.473). Hasbrouck (2002) concludes that "[a]mong the proxies considered here, the illiquidity measure [ILLIQ] appears to be the best." Furthermore, $ILLIQ$ is closely related to the Amivest measure of illiquidity, which has often been used in the empirical microstructure literature.[13]

There are two problems with using $ILLIQ$. First, it is measured in "percent per dollar," whereas the model is specified in terms of "dollar cost per dollar invested." This is a problem because it means that $ILLIQ$ is not stationary (e.g., inflation is ignored). Second, while $ILLIQ$ is an instrument for the cost of selling, it does not directly measure the cost of a trade. To solve these problems, we define a

[13]The Amivest measure of liquidity is the average ratio of volume to absolute return.

normalized measure of illiquidity, c_t^i, by

$$c_t^i = \min\left(0.25 + 0.30\, ILLIQ_t^i\, P_{t-1}^M\,, 30.00\right)\ , \tag{12}$$

where P_{t-1}^M is the ratio of the capitalizations of the market portfolio at the end of month $t-1$ and of the market portfolio at the end of July 1962. The P_{t-1}^M adjustment solves the first problem mentioned above, and it makes this measure of illiquidity relatively stationary. The coefficients 0.25 and 0.30 are chosen such that the cross-sectional distribution of normalized illiquidity (c_t^i) for size-decile portfolios has approximately the same level and variance as does the effective half spread[14] reported by Chalmers and Kadlec (1998). This normalized illiquidity is capped at a maximum value of 30% in order to ensure that our results are not driven by the extreme observations of $ILLIQ_t^i$. Furthermore, a per-trade cost greater than 30% seems unreasonable and is an artifact of the effect of low volume days on $ILLIQ_t^i$.

Chalmers and Kadlec (1998) report that the mean effective spread for size-decile portfolios of NYSE and AMEX stocks over the period 1983–1992 ranges from 0.29% to 3.41% with an average of 1.11%. The normalized illiquidity, c_t^i, for identically formed portfolios has an average of 1.24%, a standard deviation of 0.37%, and matches the range as well as the cross-sectional variation reported by Chalmers and Kadlec (1998). This means that we can interpret the illiquidity measure c_t^i as directly related to (a lower bound of) the per-trade cost.

Admittedly, this is a noisy measure of illiquidity. This makes it harder for us to find an empirical connection between return and illiquidity, and it can enhance omitted-variable problems. The noise is reduced by considering portfolios rather

[14]The effective half spread is the difference between the transaction price and the midpoint of the prevailing bid-ask quote, see Chalmers and Kadlec (1998), Table 1.

than individual stocks.

4.2 Portfolios

We employ daily return and volume data from CRSP from July 1st, 1962 until December 31st, 1999 for all common shares (CRSP sharecodes 10 and 11) listed on NYSE and AMEX.[15] Also, we use book-to-market data based on the COMPUSTAT measure of book value.[16]

We form a market portfolio for each month t during this sample period based on stocks with beginning-of-month price between 5 and 1000, and with at least 15 days of return and volume data in that month.

We form 25 illiquidity portfolios for each year y during the period 1964 to 1999 by sorting stocks with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 100 days.[17] We compute the annual illiquidity for each eligible stock as the average over the entire year $y-1$ of daily illiquidities, analogously to monthly illiquidity calculation in (11). The eligible stocks are then sorted into 25 portfolios, $p \in \{1, 2, \ldots, 25\}$, based on their year $y-1$ illiquidities.

Similarly, we form 25 illiquidity-variation portfolios (denoted "σ(illiquidity)

[15]Since volume data in CRSP for Nasdaq stocks is available only from 1982 and includes interdealer trades, we employ only NYSE and AMEX stocks for sake of consistency in the illiquidity measure.

[16]We are grateful to Joe Chen for providing us with data on book-to-market ratios. The book-to-market ratios are computed as described in Ang and Chen (2002): [For a given month] the book-to-market ratio is calculated using the most recently available fiscal year-end balance sheet data on COMPUSTAT. Following Fama and French (1993), we define "book value" as the value of common stockholders' equity, plus deferred taxes and investment tax credit, minus the book value of preferred stock. The book value is then divided by the market value on the day of the firm's fiscal year-end.

[17]Amihud (2002) and Pastor and Stambaugh (2003) employ similar requirements for the inclusion of stocks in their samples. These requirements help reduce the measurement error in the monthly illiquidity series.

portfolios") by ranking the eligible stocks each year based on the standard deviation of daily illiquidity measures in the previous year, and 25 size portfolios by ranking stocks based on their market capitalization at the beginning of the year.

Finally, we form portfolios sorted first in 5 book-to-market quintiles and then in 5 size quintiles within the book-to-market groups as in Fama and French (1992) and Fama and French (1993). This sample is restricted to stocks with book-to-market data in year $y - 1$. When considering the portfolio properties, we use the year-y book-to-market, averaging across stocks with available book-to-market data in that year.

For each portfolio p (including the market portfolio), we compute its return in month t, as

$$r_t^p = \sum_{i \text{ in } p} w_t^{ip} r_t^i, \tag{13}$$

where the sum is taken over the stocks included in portfolio p in month t, and where w_t^{ip} are either equal weights or value-based weights, depending on the specification.[18]

Similarly, we compute the normalized illiquidity of a portfolio, p, as

$$c_t^p = \sum_{i \text{ in } p} w_t^{ip} c_t^i, \tag{14}$$

[18]The returns, r_t^i, are adjusted for stock delisting to avoid survivorship bias, following Shumway (1997). In particular, the last return used is either the last return available on CRSP, or the delisting return, if available. While a last return for the stock of -100% is naturally included in the study, a return of -30% is assigned if the deletion reason is coded in CRSP as 500 (reason unavailable), 520 (went to OTC), 551–573 and 580 (various reasons), 574 (bankruptcy) and 584 (does not meet exchange financial guidelines). Shumway (1997) obtains that -30% is the average delisting return, examining the OTC returns of delisted stocks. Amihud (2002) employs an identical survivorship bias correction.

where, as above, w_t^{ip} are either equal weights or value-based weights, depending on the specification.

The model's results are phrased in terms of value-weighted returns and value-weighted illiquidity for the market portfolio. Several studies, however, focus on equal-weighted return and illiquidity measures, for instance Amihud (2002) and Chordia, Roll, and Subrahmanyam (2000). Computing the market return and illiquidity as equal-weighted averages is a way of compensating for the over-representation in our sample of large liquid securities, as compared to the "true" market portfolio in the economy. In particular, our sample does not include illiquid assets such as corporate bonds, private equity, real estate, and many small stocks, and these assets constitute a significant fraction of aggregate wealth.[19] Therefore, we focus in our empirical work on an equal-weighted market portfolio, although we also estimate the model with a value-weighted market portfolio for robustness. Also, we use both equal- and value-weighted averages for the test portfolios.

4.3 Innovations in Illiquidity

Illiquidity is persistent. The auto-correlation of the market illiquidity, for instance, is 0.87 at a monthly frequency. Therefore, we focus on the innovations, $c_t^p - E_{t-1}(c_t^p)$, in illiquidity of a portfolio when computing its liquidity betas as explained in Section 3.3.

To compute these innovations, we first define the *un-normalized* illiquidity,

[19]Heaton and Lucas (2000) report that stocks constitute only 13.6% of national wealth, while non-corporate (i.e. private) equity is 13.8%, other financial wealth is 28.2%, owner-occupied real estate is 33.3%, and consumer durables is 11.1%.

truncated for outliers, of a portfolio p as

$$\overline{ILLIQ}_t^p := \sum_{i \text{ in } p} w_t^{ip} \min\left(ILLIQ_t^i \,, \frac{30.00 - 0.25}{0.30\, P_{t-1}^M} \right) , \tag{15}$$

where w_t^{ip} is the portfolio weight. As explained in Section 4.1, we normalize illiquidity to make it stationary and to put it on a scale corresponding to the cost of a single trade.

To predict market illiquidity, we run the following regression:

$$\begin{aligned} \left(0.25 + 0.30\,\overline{ILLIQ}_t^M\, P_{t-1}^M\right) = a_0 \quad &+ \quad a_1 \left(0.25 + 0.30\,\overline{ILLIQ}_{t-1}^M\, P_{t-1}^M\right) \\ &+ \quad a_2 \left(0.25 + 0.30\,\overline{ILLIQ}_{t-2}^M\, P_{t-1}^M\right) \\ &+ \quad u_t \ . \end{aligned} \tag{16}$$

Note that the three terms inside parentheses in this specification correspond closely to c_t^M, c_{t-1}^M, and c_{t-2}^M, respectively, as given by (12) and (14), with the difference that the same date is used for the market index (P_{t-1}^M) in all three terms. This is to ensure that we are measuring innovations only in illiquidity, not changes in P^M. Our results are robust to the specification of liquidity innovations and, in particular, employing other stock-market variables available at time $t-1$ did not improve significantly the explanatory power of the regression. Pastor and Stambaugh (2003) employ a specification to compute market liquidity innovations that is similar in spirit to the AR(2) specification in (16).

The residual, u, of the regression in (16) is interpreted as the standardized market illiquidity innovation, $c_t^M - E_{t-1}(c_t^M)$, that is,

$$c_t^M - E_{t-1}(c_t^M) := u_t \ . \tag{17}$$

and innovations in portfolio illiquidity are computed in the same way, using the same AR coefficients.

For the market illiquidity series, the AR(2) specification has a R^2 of 78%. The resulting innovations in market illiquidity, $c_t^M - E_{t-1}(c_t^M)$, have a standard deviation of 0.17%. Figure 1 plots the time-series of these innovations, scaled to have unit standard deviation. The auto-correlation of these illiquidity innovations is low (-0.03) and, visually, they appear stationary. Employing AR(1) specification produces a significantly greater correlation of innovations (-0.29), whereas employing AR(3) specification produces little improvement in the explanatory power.

The measured innovations in market illiquidity are high during periods that anecdotally were characterized by liquidity crisis, for instance, in 5/1970 (Penn Central commercial paper crisis), 11/1973 (oil crisis), 10/1987 (stock market crash), 8/1990 (Iraqi invasion of Kuwait), 4,12/1997 (Asian crisis), and 6–10/1998 (Russian default and Long-Term Capital Management crisis). The correlation between this measure of innovations in market illiquidity and the measure of innovations in liquidity used by Pastor and Stambaugh (2003) is -0.33.[20] (The negative sign is due to the fact that Pastor and Stambaugh (2003) measure liquidity, whereas we follow Amihud (2002) in considering *il*liquidity.)

4.4 Liquidity Risk

In this section, we present the descriptive statistics of liquidity risk, measured through the betas β^{2p}, β^{3p} and β^{4p}. We focus on the value-weighted illiquidity portfolios whose properties are reported in Table 1. Similar conclusions are drawn from examining the properties of equal-weighted illiquidity portfolios or

[20]We thank Pastor and Stambaugh for providing their data on innovations in market liquidity.

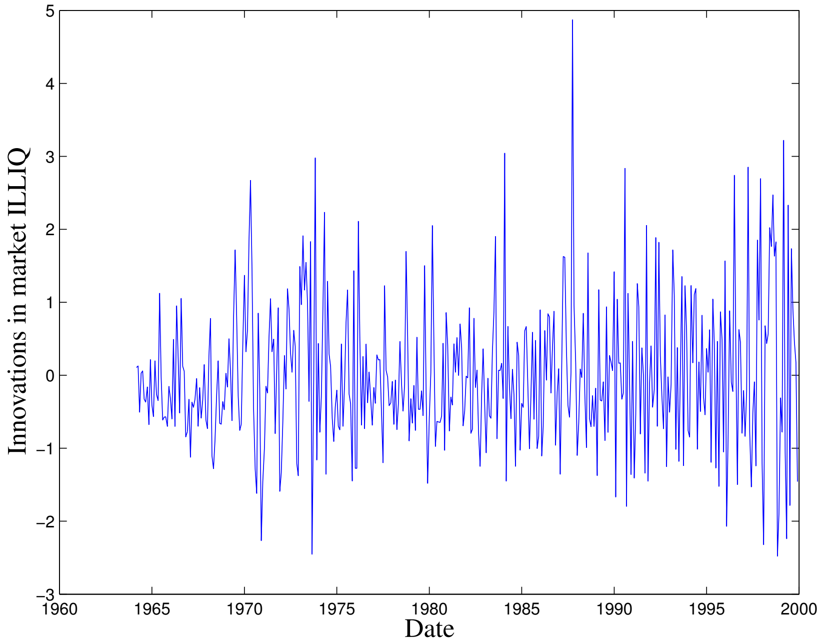

Figure 1: Standardized innovations in market illiquidity from 1964-1999.

size portfolios (not reported). The four betas, β^{1p}, β^{2p}, β^{3p} and β^{4p}, for each portfolio are computed as per Equation (7) using the entire time-series, that is, using all monthly return and illiquidity observations for the portfolio and the market portfolio from the beginning of year 1964 till end of year 1999. Similarly, average illiquidity $E(c^p)$ for a portfolio is computed using the entire time-series of monthly illiquidity observations for the portfolio. This approach of using the entire time-series in computing the portfolio characteristics is similar to the one adopted in Black, Jensen, and Scholes (1990) and Pastor and Stambaugh (2003).

Table 1 shows that the sort on past illiquidity successfully produces portfolios with monotonically increasing average illiquidity from portfolio 1 through portfolio 25. Not surprisingly, we see that illiquid stocks — that is, stocks with high values of $E(c^p)$ — tend to have a high volatility of stock returns, a low turnover, and a small market capitalization. Furthermore, we find that illiquid stocks also have high liquidity *risk*: they have large values of β^{2p} and large negative values of β^{3p} and β^{4p}. This is an interesting result on its own. It says that a stock, which is illiquid in absolute terms (c^p), also tends to have a lot of commonality in liquidity with the market ($\text{cov}(c^p, c^M)$), a lot of return sensitivity to market liquidity ($\text{cov}(r^p, c^M)$), and a lot of liquidity sensitivity to market returns ($\text{cov}(c^p, r^M)$). We note that all of the betas are estimated with a small error (i.e., a small asymptotic variance). Indeed, almost all of the betas are statistically significant at conventional levels.

A liquidity beta is proportional to the product of the correlation between its respective arguments and their standard deviations. As noted before, more illiquid stocks have greater volatility of returns. Furthermore, since illiquidity is bounded below by zero, it is natural that more illiquid stocks also have more volatile illiquidity innovations. This is verified in Table 1 which shows that the standard deviation of portfolio illiquidity innovations, $\sigma(\Delta c^p)$, increases monotonically in portfolio illiquidity. The higher variability of returns and illiquidity innovations are, however, not the sole drivers of the positive relationship between illiquidity and liquidity risk. The correlation coefficients between c^p and c^M (r^p and c^M) are also increasing (decreasing) in portfolio illiquidity. The correlation coefficients between c^p and r^M are decreasing in illiquidity between portfolios $1 - 15$ and are gradually increasing thereafter. Nevertheless, the variability of c^p ensures that the covariances between c^p and r^M are decreasing in illiquidity.[21]

The co-linearity of measures of liquidity risk is confirmed by considering the correlation among the betas, reported in Table 2. The co-linearity problem is not just a property of the liquidity-sorted portfolios; it also exists at an individual

[21] These correlations are not reported in the table for sake of brevity.

stock level as is seen in Table 3. The co-linearity at the stock level is smaller, which could be due in part to larger estimation errors. While this co-linearity is theoretically intriguing, it makes it hard to empirically distinguish the separate effects of illiquidity and the individual liquidity betas.[22]

4.5 How Liquidity Risk Affects Returns

In this section, we study how liquidity risk affects expected returns. We do this by running cross-sectional regressions on our test portfolios using a GMM framework that takes into account the pre-estimation of the betas (as in Cochrane (2001)). Standard errors are computed using the Newey and West (1987) method with 2 lags.[23]

Illiquidity and σ(illiquidity) Portfolios

The potential effect of liquidity and liquidity risk is, of course, detected by considering portfolios that differ in their liquidity attributes. Hence, we consider first the liquidity-adjusted CAPM (6) for portfolios sorted by illiquidity and the illiquidity variation.

To impose the model-implied constraint that the risk premia of the different

[22] We have not been able to construct portfolios which allow us to better identify the separate beta effects. For instance, we have considered portfolios based on predicted liquidity betas, similar to the approach taken by Pastor and Stambaugh (2003). These results are not reported as these portfolios did not improve statistical power: The liquidity betas after portfolio formation turned out to be better sorted for illiquidity and size portfolios than for the portfolios sorted using predicted liquidity betas. We attribute this, in part, to the large estimation errors associated with predicting liquidity betas at the individual stock level.

[23] Our point estimates are the same as those derived using OLS (either in a pooled regression or using the Fama and MacBeth (1973) method). Our standard errors correspond to those of Shanken (1992) except that the GMM method also takes serial correlation into account.

betas is the same, we define the "net beta" as

$$\beta^{net,p} := \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}. \tag{18}$$

With this definition, the liquidity-adjusted CAPM becomes

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda \beta^{net,p}\,, \tag{19}$$

where we allow a non-zero intercept, α, in the estimation, although the model implies that the intercept is zero. In our model, investors incur the illiquidity cost exactly once over their holding period. The coefficient κ adjusts for the difference between the monthly period used in estimation, and the typical holding period of an investor (which is the period implicitly considered in the model). More precisely, κ is the ratio of the monthly estimation period to the typical holding period.[24] The average holding period is proxied by the period over which all shares are turned over once. Hence, we calibrate κ as the average monthly turnover across all stocks in the sample.[25] In the sample of liquidity portfolios, κ is calibrated to 0.034, which corresponds to a holding period of $1/0.034 \cong 29$ months. The expected illiquidity, $E(c_t^p)$, is computed as the portfolio's average illiquidity. Note that the structure of the liquidity-adjusted CAPM and its calibration using

[24] If the estimation period is equal to the holding period, then the model implies (19) with $\kappa = 1$. If the estimation period is κ times the holding period, then $E(r_t^p - r_t^f)$ is (approximately) κ times the expected holding period return, and $\beta^{net,p}$ is assumed to be approximately κ times the holding-period net beta. This is because a κ-period return (or illiquidity innovation) is approximately a sum of κ 1-period returns (or illiquidity innovations), and because returns and illiquidity innovations have low correlation across time. The illiquidity, $E(c^p)$, however, does not scale with time period because it is an average of daily illiquidities (not a sum of such terms). Therefore, the $E(c^p)$ term is scaled by κ in (19).

[25] To run the regression (19) with a fixed κ, we treat the net return, $E(r_t^p - r_t^f) - \kappa E(c_t^p)$, as the dependent variable. All R^2 are, however, based on the same dependent variable namely $E(r_t^p - r_t^f)$.

κ equal to the average monthly turnover for stocks make the estimation different from the typical cross-sectional regression study in which the asset-pricing relationship is backed out from the return series and data on security characteristics such as beta, size, book-to-market, etc.

The liquidity-adjusted CAPM (19) has only *one* risk premium, λ, that needs to be estimated as in the standard CAPM. Here, the risk factor is the net beta instead of the standard market beta. Hence, the empirical improvement in fit relative to the standard CAPM is not achieved by adding factors (or otherwise adding degrees of freedom), but simply by making a liquidity adjustment.

The estimated results for Equation (19) are reported in line 1 of Table 4, with illiquidity portfolios in Panel A and σ(illiquidity) portfolios in Panel B. With either portfolio, the risk premium λ is positive and significant at a 1% level and α is insignificant, both results lending support to our model. The R^2 of the liquidity-adjusted CAPM is high relative to the standard CAPM, reported in line 3. In line 2, we estimate the liquidity-adjusted CAPM with κ as a free parameter, which results in only modest changes in κ and λ.

While the improvement in fit of the liquidity-adjusted CAPM over the CAPM is encouraging, it does not constitute a test of the effect of liquidity risk. To isolate the effect of liquidity risk (β^2, β^3, and β^4) over liquidity level ($E(c)$) and market risk (β^1), we consider the relation

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda \beta^{net,p} \tag{20}$$

In line 4, this relation is estimated with κ at its calibrated value. We see that β^{net} is insignificant for illiquidity portfolios, but significant for σ(illiquidity) portfolios. In line 5, the relation is estimated with κ as a free parameter. In this regression, the support for the model is stronger in that β^{net} is significant with either portfolio.

We note that κ is estimated to be negative in Panel A, although it is statistically insignificant. Since the model implies that κ should be positive, we estimate in line 6 with the restriction that $\kappa = 0$. With this specification, β^{net} remains significant in both panels. In conclusion, there is some evidence that liquidity risk matters over and above market risk and liquidity level. The collinearity problems imply, however, that this evidence is weak.

We note that a negative coefficient on β^1 does not imply a negative risk premium on market risk since β^1 is also contained in β^{net}. Rather, a negative coefficient suggests that liquidity risk may have a higher risk premium than market risk. For instance, line 4 of Table 4A means that

$$\begin{aligned} E(r_t^p - r_t^f) &= -0.333 + 0.034E(c_t^p) - 3.181\beta^{1p} + 4.334\beta^{net,p} \\ &= -0.333 + 0.034E(c_t^p) + 1.153\beta^{1p} + 4.334\left(\beta^{2p} - \beta^{3p} - \beta^{4p}\right) \end{aligned}$$

Finally, in line 7 we allow all of the betas to have different risk premia λ^i, and in line 8 we further let κ be a free parameter. That is, lines 7–8 estimate the generalized relation

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1\beta^{1p} + \lambda^2\beta^{2p} + \lambda^3\beta^{3p} + \lambda^4\beta^{4p} \tag{21}$$

without the model restrictions that $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$. We see that the multicollinearity problems are severe, and, hence, statistical identification of the separate effects of the different liquidity risks is difficult. Of course, we must also entertain the possibility that not all these risk factors are empirically relevant.

The empirical fit of the standard CAPM is illustrated in the top panel of Figure 2 for illiquidity portfolios and in Figure 3 for σ(illiquidity) portfolios. The middle and bottom panels show, respectively, the fit of the constrained and unconstrained liquidity-adjusted CAPM, that is, lines 1 and 8, respectively, from Table 4. We see that the liquidity adjustment improves the fit especially for the illiquid portfolios, consistent with what our intuition would suggest. We note that the number of free parameters is the same in top and middle panels, so the improvement in fit is not a consequence of more degrees of freedom.

Economic Significance of Results

It is interesting to consider the economic significance of liquidity risk. To get a perspective on the magnitude of the effect, we compute the annual return premium required to hold illiquid rather than liquid securities. This is computed as the product of the risk premium and the difference in liquidity risk across liquidity portfolios. If we use the unrestricted model in line 8 of Table 4A then our estimates are very noisy because of the multicollinearity problem. Instead, the benefit of having an economic model is that we can impose its structure and can get relatively tight estimates. Hence, we use the calibrated value of κ and the common risk premium, $\lambda = 1.512$, from line 1. Of course, when interpreting the results, one must bear in mind that they rely on the validity of the model.

The difference in annualized expected return between portfolio 1 and 25 that can be attributed to a difference in β^2, the commonality between the portfolio illiquidity and market illiquidity, is

$$\lambda(\beta^{2,p_{25}} - \beta^{2,p_1}) \cdot 12 = 0.08\%.$$

Similarly, the annualized return difference stemming from the difference in β^3, the sensitivity of the portfolio return to market illiquidity, is

$$-\lambda(\beta^{3,p_{25}} - \beta^{3,p_1}) \cdot 12 = 0.16\%,$$

and the effect of β^4, the sensitivity of the portfolio illiquidity to market return, is

$$-\lambda(\beta^{4,p_{25}} - \beta^{4,p_1}) \cdot 12 = 0.82\%.$$

The total effect of liquidity risk is therefore 1.1% per year. Using the standard error of the estimates of λ and the betas, the 95% confidence interval for the total effect of $\beta^2 - \beta^3 - \beta^4$ is $[0.24\%, 1.88\%]$. Hence, under the model restrictions and using the calibrated κ, the effect of liquidity risk is significantly different from zero.

Interestingly, of the three liquidity risks the effect of β^4, the covariation of a security's illiquidity to market returns, appears to have the largest economic impact on expected returns. (Also, it has the highest t-statistics in the unrestricted regression of lines 7–8 in Table 4.) This liquidity risk has not been studied before either theoretically or empirically.

The difference in annualized expected return between portfolio 1 and 25 that can be attributed to a difference in the expected illiquidity, $E(c)$, is 3.5%, using the calibrated coefficient. The overall effect of expected illiquidity and liquidity risk is thus 4.6% per year.

While the magnitude of liquidity risk is economically significant, it is lower than the magnitude estimated by Pastor and Stambaugh (2003). This could be due to the fact that they employ a different measure of liquidity, or due to the fact that they sort portfolios based on liquidity risk (in their case, β^3) whereas we sort based on the level of liquidity. Also, this could be because they do not control for the level of illiquidity which has been shown to command a significant premium in a number of studies including Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Brennan, Chordia, and Subrahmanyam (1998), Datar, Naik, and Radcliffe (1998), Swan (2002), and Dimson and Hanke (2002). Finally, the difference could also arise because we restrict the risk premia on different liquidity betas to be the same. For instance, the magnitude of the risk premium related to β^4 is estimated to be higher in line 7–8 of Table 4A. This higher risk premium results in a per year effect of about 9% from β^4 alone.[26]

The collinearity between liquidity and liquidity risk implies that the most robust number is their overall effect. Further, our results suggest that studies that focus on the separate effect of liquidity (or liquidity risk) can possibly be reinterpreted as providing an estimate of the overall effect of liquidity and liquidity risk.

Robustness, Size, and Book-to-Market

To check the robustness of our results, we consider different specifications and portfolios. First, we consider whether our results are robust to the choice of value weighting versus equal weighting. Table 5A reports the results with equal-weighted illiquidity portfolios and equal-weighted market, and Table 5B with value-weighted illiquidity portfolios and value-weighted market. The results and their significance are similar to those of Table 4A. First, β^{net} is borderline significant at a 5% level in line 1 of Table 5A, but insignificant at this level in Table 5B. In both tables, the liquidity-adjusted CAPM has a higher R-square than the standard CAPM. In particular with value-weighted portfolios in Table 5B, the standard CAPM has an R-square of 0.0%, whereas the liquidity-adjusted CAPM has an R-square of 48.6%. There is further evidence that liquidity risk matters over and

[26]In another recent paper, Chordia, Subrahmanyam, and Anshuman (2001) find that expected returns in the cross-section are higher for stocks with low variability of liquidity, measured using variables such as trading volume and turnover. They examine the firm-specific variability of liquidity. By contrast, our model and tests suggest that it is the co-movement of firm-specific liquidity with market return and market liquidity that affects expected returns.

above liquidity level and market risk. In particular, β^{net} is significant in line 5 of Table 5A, and in all of lines 4–6 in Table 5B. (Also, β^{net} is significant in line 6 of Table 5A, but this line is not relevant since the coefficient on $E(c^p)$ has the correct sign in line 5.)

As a further robustness check, we re-estimate our model with size-based portfolios and portfolios sorted first in 5 book-to-market quintiles and then in 5 size quintiles within the book-to-market groups (as in Fama and French (1992)).

Small-sized stocks are illiquid (in absolute terms as measured by $E(c)$) and also have high liquidity risk (as measured by the three betas β^{2p}, β^{3p} and β^{4p}). Table 6A shows that the cross-sectional regressions have coefficients that are similar to our earlier results, but the statistical significance is reduced. The coefficient of β^{net} is estimated to be positive and the liquidity-adjusted CAPM still has a higher R^2 than the standard CAPM. Figure 4 shows graphically the fit for size portfolios of the standard CAPM, and the liquidity-adjusted CAPM, with constrained and unconstrained risk premia. We see that the liquidity adjustment improves the fit, particularly for the smaller size portfolios.

Table 6B and Figure 5 show the models' fit of the B/M-by-size portfolios. We recover the well-known[27] result that CAPM does relatively poorly for B/M-by-size portfolios (adjusted $R^2 = 22.9\%$) since market beta is relatively "flat" across these portfolios. The liquidity-adjusted CAPM in line 1 provides a moderate improvement in the fit (adjusted $R^2 = 40.6\%$) whereas the model with unconstrained risk premia produces a significant improvement in the fit (adjusted $R^2 = 73.3\%$). It should be noted, however, that the unconstrained specification may be "over fitted" in the sense that some of the risk premia estimated have incorrect sign and they are all insignificant. The negative coefficient on β^{net} in line 5 suggest that

[27] See Fama and French (1992) and Fama and French (1993).

the model is misspecified for these portfolios.

To further consider the model's ability to explain the size and book-to-market effects, we run our regressions while controlling for size and book-to-market (Table 7). We do this both for illiquidity portfolios (Panel A) and for B/M-by-size portfolios (Panel B). The results with illiquidity portfolios are similar to the earlier results, although the standard errors increase because of the additional variables. The coefficient on β^{net} is significant in the liquidity-adjusted CAPM of line 1. The coefficient on size is always insignificant and the coefficient on book-to-market is insignificant in all specifications except line 2. (Including volatility does not change the results, and volatility is not significant. These results are not reported.) With B/M-by-size portfolios (Table 7B) the model performs poorly. Indeed, the coefficient on β^{net} is negative, although insignificant, and the coefficient on B/M is significant in most specifications. To summarize, the results with illiquidity portfolios suggest that liquidity risk matters while controlling for book to market, while the results with B/M-by-size portfolios suggest that liquidity risk does not explain the book-to-market effect. (Pastor and Stambaugh (2003) reach a similar conclusion.) Hence, our simple model fails to explain the entire investment universe.

Specification Tests

We perform several specification tests of the liquidity-adjusted CAPM. First, we note that we fail to reject at conventional levels the model-implied restriction that $\alpha = 0$ in the liquidity-adjusted CAPM (lines 1–2 and 4–8 of Table 4), whereas this restriction is rejected for the standard CAPM (at a 10% level in line 3 Table 4A, and at a 5% level in Table 4B). Second, in context of the model with unrestricted risk premia in line 8 of Table 4, a Wald test[28] fails to reject the five model-implied restrictions $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$, $\alpha = 0$, and $\kappa = k$, where k is the calibrated value. The p-value is 47% in Table 4A and 28% in Table 4B. The CAPM restrictions $\lambda^2 = \lambda^3 = \lambda^4 = 0$, $\alpha = 0$, and $\kappa = 0$ have p-values of 15% and 8.7%, respectively. The CAPM is rejected in lines 5 and 6 since β^{net} is significant.

Another testable restriction implied by the model is that the risk premium equals the expected net return on the market in excess of the risk-free rate. The point estimate of the risk premium, λ, is larger than the sample average of the excess return of the market net of transaction costs, $E(r_t^M - r_t^f - \kappa c_t^M)$ and the p-value is 6.6% in regression 1 of Table 4A and 7.3% in Table 4B. In comparison, the test that the standard CAPM risk premium equals the $E(r_t^M - r_t^f)$ has p-values of 1.2% and 0.8%, respectively.

Lastly, we test that the linear model has zero average pricing error for all of the portfolios, a stringent test since it requires that the model is pricing all portfolios correctly. (We use a GMM test as in Cochrane (2001) page 241, which corresponds to the test of Shanken (1985).) With illiquidity portfolios, the p-values for regressions 1, 5, and 8 are, respectively, 8.5%, 9.9%, and 6.8%. In comparison, the standard CAPM has a p-value of 0.5%. With σ(illiquidity) portfolios the p-values for the liquidity-adjusted CAPM are, respectively, 16%, 42%, and 65%, and the p-value for the standard CAPM is 6.6%. The specification tests for size portfolios are similar, and lends further support to the model. This confirms the visual evidence from Figures 2–4 that the model fit for these portfolios is good.

With B/M-by-size portfolios, the Wald test of the liquidity-adjusted CAPM has a p-value of 47% and the test of zero pricing errors for regressions 1, 5, and

[28]We compute the joint variance-covariance test of the parameters in a GMM framework and derive a standard Wald test with an asymptotic chi-square distribution.

8 are, respectively, 15.7%, 38%, and 85%. The standard CAPM has a p-value of 23% for the Wald test and 3.2% for the test of zero pricing errors. The failure to reject the liquidity-adjusted CAPM using B/M-by-size portfolios may be due to low power since, as discussed above, the model fit is not good for these portfolios.

5 Conclusion

This paper derives a model of liquidity risk. The model in its simplest form shows that the CAPM applies for returns net of illiquidity costs. This implies that investors should worry about a security's performance and tradability both in market downturns and when liquidity "dries up." Said differently, the required return of a security i is increasing in the covariance between its illiquidity and the market illiquidity, $\mathrm{cov}_t(c^i_{t+1}, c^M_{t+1})$, decreasing in the covariance between the security's return and the market illiquidity, $\mathrm{cov}_t(r^i_{t+1}, c^M_{t+1})$, and decreasing in the covariance between its illiquidity and market returns, $\mathrm{cov}_t(c^i_{t+1}, r^M_{t+1})$. The model further shows that a positive shocks to illiquidity, if persistent, are associated with a low contemporaneous returns and high predicted future returns.

Hence, the model gives an integrated view of the existing empirical evidence related to liquidity and liquidity risk, and it generates new testable predictions. We find, in a variety of specifications, that the liquidity-adjusted CAPM explains the data better than the standard CAPM, while still exploiting the same degrees of freedom. Further, we find weak evidence that liquidity risk is important over and above the effects of market risk and the level of liquidity. The model has a reasonably good fit for portfolios sorted by liquidity, liquidity variation, and size, but it fails to explain the book-to-market effect.

The model provides a framework in which we can study the economic significance of liquidity risk. We find that liquidity risk explains about 1.1% of cross-sectional returns when the effect of average liquidity is calibrated to the typical holding period in the data and the model restriction of a single risk premium is imposed. About 80% of this effect is due to the liquidity sensitivity, $\mathrm{cov}_t(c^i_{t+1}, r^M_{t+1})$, to the market return, an effect not previously studied in the literature. Freeing up risk premia leads to larger estimates of the liquidity risk premium, but these results are estimated imprecisely because of collinearity between liquidity and liquidity risk.

While the model gives clear predictions that seem to have some bearing in the data, it is obviously simplistic. The model and the empirical results are suggestive of further theoretical and empirical work. In particular, it would be of interest to explain the time-variation in liquidity, and why stocks that are illiquid in absolute terms also are more liquidity risky in the sense of high values of all three liquidity betas. Another interesting topic is the determination of liquidity premia in a general equilibrium with liquidity risk and endogenous holdings periods. We note that if investors live several periods, but their probability of living more than one period approaches zero, then our general-equilibrium economy is approached (assuming continuity). Hence, our effects would also be present in the more general economy, although endogenous holding periods may imply a smaller effect of liquidity risk (as in Constantinides (1986)). The effect of liquidity risk is strengthened, however, if investors have important reasons to trade frequently. Such reasons include return predictability and wealth shocks (as considered in the context of liquidity by Lynch and Tan (2003)), differences of opinions (e.g. Harris and Raviv (1993)), asymmetric information (e.g. He and Wang (1995)), institutional effects (e.g. Allen (2001)), taxes (e.g. Constantinides (1983)), etc. It would be interesting to determine the equilibrium impact of liquidity risk in light of these trading motives.

A Appendix

Proof of Proposition 1:

We first solve the investment problem of any investor n at time t. We assume, and later confirm, that the price at time $t+1$ is normally distributed conditional on the time t information. Hence, the investor's problem is to choose optimally the number of shares, $y^n = (y^{n,1}, \ldots, y^{n,I})$, to purchase according to

$$\max_{y^n \in \mathbb{R}^I_+} \left(E_t(W^n_{t+1}) - \frac{1}{2} A^n \operatorname{var}_t(W^n_{t+1}) \right),$$

where

$$W^n_{t+1} = (P_{t+1} + D_{t+1} - C_{t+1})^\top y^n + r^f (e^n_t - P_t^\top y^n),$$

and e^n_t is this agent's endowment. If we disregard the no-short-sale constraint, the solution is

$$y^n = \frac{1}{A^n} \left(\operatorname{var}_t(P_{t+1} + D_{t+1} - C_{t+1})\right)^{-1} \left(E_t(P_{t+1} + D_{t+1} - C_{t+1}) - r^f P_t\right).$$

We shortly verify that, in equilibrium, this solution does not entail short selling. In equilibrium, $\sum_n y^n = S$, so equilibrium is characterized by the condition that

$$P_t = \frac{1}{r^f} \left[E_t(P_{t+1} + D_{t+1} - C_{t+1}) - A \operatorname{var}_t(P_{t+1} + D_{t+1} - C_{t+1}) S \right],$$

where $A = \left(\sum_n \frac{1}{A^n}\right)^{-1}$. The unique stationary linear equilibrium is

$$P_t = \Upsilon + \frac{\rho^D}{r^f - \rho^D} D_t - \frac{\rho^C}{r^f - \rho^C} C_t, \tag{A.1}$$

where

$$\Upsilon = \frac{1}{r^f - 1}\left(\frac{r^f(1-\rho^D)}{r-\rho^D}\bar{D} - \frac{r^f(1-\rho^C)}{r^f-\rho^C}\bar{C} - A\mathrm{var}_t\left[\frac{r^f}{r-\rho^D}\varepsilon_t - \frac{r^f}{r^f-\rho^C}\eta_t\right] S\right)$$

and $S = (S^1, \ldots, S^I)$ is the total supply of shares.

With this price, conditional expected net returns are normally distributed, and any investor n holds a fraction $A/A^n > 0$ of the market portfolio $S > 0$ so he is not short selling any securities. Therefore, our assumptions are satisfied in equilibrium.

Finally, since investors have mean-variance preferences, the conditional CAPM holds for net returns. See, for instance, Huang and Litzenberger (1988). Rewriting in terms of net returns yields the result stated in the proposition.

□

Proof of Proposition 2:

The conditional expected return on a portfolio q is computed using (A.1):

$$\begin{aligned} E_t(r^q_{t+1}) &= E_t\left(\frac{P^q_{t+1} + D^q_{t+1}}{P^q_t}\right) \\ &= \frac{E_t(\Upsilon^q + \frac{r^f}{r^f-\rho^D}D^q_{t+1} - \frac{\rho^C}{r^f-\rho^C}C^q_{t+1})}{\Upsilon^q + \frac{\rho^D}{r^f-\rho^D}D^q_t - \frac{\rho^C}{r^f-\rho^C}C^q_t} \end{aligned}$$

so we have that

$$\begin{aligned} &\frac{\partial}{\partial C^q_t}E_t(r^q_{t+1} - r^f) \\ &= \frac{1}{(P^q_t)^2}\left(-\frac{(\rho^C)^2}{r^f-\rho^C}P^q_t + \frac{\rho^C}{r^f-\rho^C}E_t(P^q_{t+1} + D^q_{t+1})\right) \end{aligned}$$

This partial derivative is greater than 0 under the conditions given in the proposition. $\square$

Proof of Proposition 3:

The conditional covariance between illiquidity and return for a portfolio q is:

$$\begin{aligned}
\mathrm{cov}_t(c^q_{t+1}, r^q_{t+1}) &= \frac{1}{(P^q_t)^2}\,\mathrm{cov}_t(C^q_{t+1}\,,\; P^q_{t+1} + D^q_{t+1}) \\
&= \frac{1}{(P^q_t)^2}\,\mathrm{cov}_t(C^q_{t+1}\,,\; \frac{r^f}{r^f - \rho^D} D^q_{t+1} - \frac{\rho^C}{r^f - \rho^C} C^q_{t+1}) \\
&= \frac{1}{(P^q_t)^2}\left(\frac{r^f}{r^f - \rho^D}\, q^\top \Sigma^{CD} q - \frac{\rho^C}{r^f - \rho^C}\, q^\top \Sigma^C q\right)
\end{aligned}$$

which yields the proposition. $\square$

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Table 1: **Properties of illiquidity portfolios**.

This table reports the properties of the odd–numbered portfolios of 25 value-weighted illiquidity portfolios formed each year during 1964–1999 as described in Section 4.2. The four betas (β^{ip}) are computed for each portfolio using all monthly return and illiquidity observations for a portfolio, and an equal-weighted market portfolio. In particular, these betas based on (7), where the innovations in portfolio illiquidity and market illiquidity are computed using the AR(2) specification in (16) for the standardized illiquidity series, and the innovations in market portfolio return is computed using an AR(2) specification for the market return series that also employs available market characteristics at the beginning of the month (return, volatility, average illiquidity, log of average dollar volume, log of average turnover, all measured over past six months, and log of one-month lagged market capitalization). The t-statistics, reported in parenthesis, are estimated using GMM. The standard deviation of the portfolio illiquidity innovations is reported under the column $\sigma(\Delta c^p)$. The average illiquidity, $E(c^p)$, the average excess return, $E(r^{e,p})$, the turnover (trn), the market capitalization (size), and book-to-market (BM) are computed for each portfolio as time-series averages of the respective monthly characteristics. Finally, $\sigma(r^p)$, is the average of the standard deviation of daily returns for the portfolio's constituent stocks computed each month.

	β^{1p} $(\cdot 100)$	β^{2p} $(\cdot 100)$	β^{3p} $(\cdot 100)$	β^{4p} $(\cdot 100)$	$E(c^p)$ (%)	$\sigma(\Delta c^p)$ (%)	$E(r^{e,p})$ (%)	$\sigma(r^p)$ (%)	trn (%)	size (bl$)	BM
1	55.10	0.00	−0.80	−0.00	0.25	0.00	0.48	1.43	3.25	12.50	0.53
	(14.54)	(0.08)	(−5.90)	(−0.10)							
3	67.70	0.00	−1.05	−0.03	0.26	0.00	0.39	1.64	4.19	2.26	0.72
	(16.32)	(0.58)	(−7.14)	(−0.62)							
5	74.67	0.00	−1.24	−0.07	0.27	0.01	0.60	1.74	4.17	1.20	0.71
	(20.44)	(1.27)	(−7.43)	(−1.36)							
7	76.25	0.00	−1.27	−0.10	0.29	0.01	0.57	1.83	4.14	0.74	0.73
	(20.63)	(2.18)	(−7.49)	(−2.03)							
9	81.93	0.01	−1.37	−0.18	0.32	0.02	0.71	1.86	3.82	0.48	0.73
	(33.25)	(3.79)	(−8.00)	(−3.74)							
11	84.59	0.01	−1.41	−0.33	0.36	0.04	0.73	1.94	3.87	0.33	0.76
	(34.21)	(5.07)	(−7.94)	(−5.85)							
13	85.29	0.01	−1.47	−0.40	0.43	0.05	0.77	1.99	3.47	0.24	0.77
	(34.15)	(6.84)	(−8.01)	(−7.46)							
15	88.99	0.02	−1.61	−0.70	0.53	0.08	0.85	2.04	3.20	0.17	0.83
	(42.88)	(6.87)	(−8.35)	(−8.45)							
17	87.89	0.04	−1.59	−0.98	0.71	0.13	0.80	2.11	2.96	0.13	0.88
	(27.54)	(8.16)	(−8.18)	(−9.30)							
19	87.50	0.05	−1.58	−1.53	1.01	0.21	0.83	2.13	2.68	0.09	0.92
	(40.74)	(7.63)	(−8.75)	(−8.77)							
21	92.73	0.09	−1.69	−2.10	1.61	0.34	1.13	2.28	2.97	0.06	0.99
	(37.85)	(7.33)	(−8.34)	(−6.11)							
23	94.76	0.19	−1.71	−3.35	3.02	0.62	1.12	2.57	2.75	0.04	1.09
	(39.71)	(6.85)	(−8.68)	(−5.91)							
25	84.54	0.42	−1.69	−4.52	8.83	1.46	1.10	2.87	2.60	0.02	1.15
	(20.86)	(6.40)	(−8.23)	(−3.35)							

Table 2: **Beta correlations for illiquidity portfolios.**

This table reports the correlations of the four covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, for the 25 value-weighted illiquidity portfolios formed for each year during 1964–1999 as described in Section 4.2. The four betas are computed for each portfolio as per (7) using all monthly return and illiquidity observations for the portfolio and the market portfolio. The monthly innovations in portfolio illiquidity and market illiquidity are computed using the AR(2) specification in (16) for the standardized illiquidity series. The monthly innovations in market portfolio return are computed using an AR(2) specification for the market return series that also employs available market characteristics at the beginning of the month.

	β^{1p}	β^{2p}	β^{3p}	β^{4p}
β^{1p}	1.000	0.441	−0.972	−0.628
β^{2p}		1.000	−0.573	−0.941
β^{3p}			1.000	0.726
β^{4p}				1.000

Table 3: **Beta correlations for individual stocks.**

This table reports the correlations of the four covariances, β^{1i}, β^{2i}, β^{3i} and β^{4i}, for the common shares listed on NYSE and AMEX during the period 1964–1999. The correlations are computed annually for all eligible stocks in a year as described in Section 4.2 and then averaged over the sample period. The four betas are computed for each stock as per (7) using all monthly return and illiquidity observations for the stock and the market portfolio. The monthly innovations in market illiquidity are computed using the AR(2) specification in (16) for the standardized market illiquidity series. The innovations in stock illiquidity are computed using a similar AR(2) specification with coefficients estimated for the market illiquidity. The monthly innovations in market portfolio return are computed using an AR(2) specification for the market return series that also employs available market characteristics at the beginning of the month.

	β^{1i}	β^{2i}	β^{3i}	β^{4i}
β^{1i}	1.000	0.020	−0.685	−0.164
β^{2i}		1.000	−0.072	−0.270
β^{3i}			1.000	0.192
β^{4i}				1.000

Table 4: **Illiquidity and σ(illiquidity) portfolios.**

This table reports the estimated coefficients from cross-sectional regressions of the liquidity-adjusted CAPM for 25 value-weighted portfolios using monthly data during 1964–1999 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p} \,,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using a GMM framework that takes into account the pre-estimation of the betas. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

Panel A: illiquidity portfolios

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−0.556	0.034					1.512	0.732
	(−1.450)	(—)					(2.806)	(0.732)
2	−0.512	0.042					1.449	0.825
	(−1.482)	(2.210)					(2.532)	(0.809)
3	−0.788		1.891					0.653
	(−1.910)		(3.198)					(0.638)
4	−0.333	0.034	−3.181				4.334	0.843
	(−0.913)	(—)	(−0.998)				(1.102)	(0.836)
5	0.005	−0.032	−13.223				13.767	0.878
	(0.013)	(−0.806)	(−1.969)				(2.080)	(0.861)
6	−0.160		−8.322				9.164	0.870
	(−0.447)		(−2.681)				(3.016)	(0.858)
7	−0.089	0.034	0.992	−153.369	7.112	−17.583		0.881
	(−0.219)	(—)	(0.743)	(−1.287)	(0.402)	(−1.753)		(0.865)
8	−0.089	0.033	0.992	−151.152	7.087	−17.542		0.881
	(−0.157)	(0.166)	(0.468)	(−0.280)	(0.086)	(−1.130)		(0.850)

Panel B: σ(illiquidity) portfolios

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−0.528	0.035					1.471	0.865
	(−1.419)	(—)					(2.817)	(0.865)
2	−0.363	0.062					1.243	0.886
	(−1.070)	(2.433)					(2.240)	(0.875)
3	−0.827		1.923					0.726
	(−2.027)		(3.322)					(0.714)
4	−0.014	0.035	−7.113				7.772	0.917
	(−0.037)	(—)	(−1.939)				(2.615)	(0.914)
5	0.094	0.007	−11.013				11.467	0.924
	(0.235)	(0.158)	(−2.080)				(2.480)	(0.914)
6	0.119		−11.914				12.320	0.924
	(0.305)		(−2.413)				(2.608)	(0.917)
7	0.464	0.035	−1.105	−83.690	−74.538	−14.560		0.940
	(0.913)	(—)	(−0.728)	(−0.663)	(−1.175)	(−1.662)		(0.931)
8	0.459	0.148	−1.125	−390.588	−73.552	−21.688		0.942
	(0.565)	(0.140)	(−0.485)	(−0.140)	(−1.943)	(−0.335)		(0.927)

Table 5: **Illiquidity portfolios: robustness of weighting method**

This table reports the estimated coefficients from cross-sectional regressions of the liquidity-adjusted CAPM for 25 liquidity portfolios using monthly data during 1964–1999. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p} ,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using a GMM framework that takes into account the pre-estimation of the betas. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

Panel A: equal-weighted illiquidity pf's, equal-weighted market

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−0.391	0.046					1.115	0.825
	(−0.889)	(—)					(1.997)	(0.825)
2	−0.299	0.062					0.996	0.846
	(−0.737)	(3.878)					(4.848)	(0.832)
3	−0.530		1.374					0.350
	(−1.082)		(2.085)					(0.322)
4	−0.088	0.046	−2.699				3.395	0.879
	(−0.249)	(—)	(−1.441)				(1.782)	(0.873)
5	0.105	0.008	−6.392				6.800	0.901
	(0.296)	(0.318)	(−2.238)				(2.427)	(0.886)
6	0.143		−7.115				7.467	0.900
	(0.397)		(−3.623)				(3.871)	(0.891)
7	−0.132	0.046	1.568	−141.416	47.823	−12.784		0.911
	(−0.633)	(—)	(1.295)	(−1.032)	(0.469)	(−1.553)		(0.898)
8	−0.053	0.117	1.207	−346.547	33.043	−17.356		0.913
	(−0.060)	(0.837)	(0.343)	(−0.796)	(0.186)	(−0.981)		(0.890)

Panel B: value-weighted illiquidity pf's, value-weighted market

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−1.938	0.034					2.495	0.486
	(−1.203)	(—)					(1.627)	(0.486)
2	−2.059	0.081					2.556	0.642
	(−1.755)	(2.755)					(2.107)	(0.609)
3	0.700		0.062					0.000
	(0.272)		(0.025)					(−0.043)
4	−1.536	0.034	−6.070				8.099	0.754
	(−2.033)	(—)	(−1.540)				(2.040)	(0.743)
5	−0.583	−0.076	−16.226				17.333	0.841
	(−0.718)	(−0.902)	(−2.978)				(3.453)	(0.819)
6	−1.241		−9.210				10.954	0.800
	(−1.271)		(−2.733)				(3.183)	(0.781)
7	−0.301	0.034	0.363	−4494.924	−370.840	−26.044		0.850
	(−0.285)	(—)	(0.268)	(−1.060)	(−0.806)	(−1.366)		(0.828)
8	0.039	−0.056	0.015	−116.450	−405.451	−13.135		0.865
	(0.031)	(−0.410)	(0.007)	(−0.010)	(−0.413)	(−0.270)		(0.829)

Table 6: **Size and B/M-by-size portfolios.**

This table reports the estimated coefficients from cross-sectional regressions of the liquidity-adjusted CAPM for 25 value-weighted size and B/M-by-size portfolios using monthly data during 1964–1999 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p} \, ,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using a GMM framework that takes into account the pre-estimation of the betas. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

Panel A: size portfolios

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−0.087	0.047					0.865	0.910
	(−0.274)	(—)					(1.864)	(0.910)
2	−0.059	0.056					0.823	0.912
	(−0.201)	(2.139)					(1.768)	(0.904)
3	−0.265		1.144					0.757
	(−0.789)		(2.270)					(0.747)
4	−0.043	0.047	−0.770				1.562	0.912
	(−0.151)	(—)	(−0.323)				(0.685)	(0.908)
5	−0.055	0.054	−0.168				0.984	0.912
	(−0.186)	(1.180)	(−0.050)				(0.266)	(0.900)
6	0.032		−4.633				5.278	0.902
	(0.112)		(−1.899)				(2.104)	(0.893)
7	−0.073	0.047	0.887	27.387	1.741	0.038		0.913
	(−0.122)	(—)	(0.304)	(0.342)	(0.009)	(0.006)		(0.901)
8	0.224	−0.408	−0.079	742.841	−42.800	7.933		0.929
	(0.552)	(−1.206)	(−0.047)	(1.157)	(−0.845)	(0.691)		(0.911)

Panel B: B/M-by-size portfolios

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	0.200	0.045					0.582	0.406
	(0.680)	(—)					(1.197)	(0.406)
2	0.453	0.167					0.182	0.541
	(1.657)	(3.452)					(0.377)	(0.499)
3	0.109		0.748					0.262
	(0.348)		(1.406)					(0.229)
4	0.529	0.045	−8.289				8.275	0.502
	(1.665)	(—)	(−2.013)				(2.198)	(0.481)
5	0.187	0.387	18.229				−17.458	0.571
	(0.626)	(3.061)	(2.344)				(−2.265)	(0.510)
6	0.574		−11.787				11.671	0.483
	(1.959)		(−3.102)				(2.902)	(0.436)
7	−0.425	0.045	4.606	203.397	198.027	−3.330		0.788
	(−0.254)	(—)	(0.483)	(0.200)	(0.526)	(−0.049)		(0.758)
8	−0.395	−0.031	4.545	397.770	195.128	0.380		0.789
	(−0.638)	(−0.028)	(1.722)	(0.115)	(1.612)	(0.004)		(0.733)

Table 7: **Controlling for size and book-to-market.**

This table reports the estimated coefficients from cross-sectional regressions of the liquidity-adjusted CAPM for 25 value-weighted illiquidity and B/M-by-size portfolios using monthly data during 1964–1999 with an equal-weighted market portfolio. We consider special cases of the relation:

$$E(r_t^p - r_t^f) = \alpha + \kappa E(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p} + \lambda \beta^{net,p} + \lambda^5 ln(size^p) + \lambda^6 BM^p \,,$$

where $\beta^{net,p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$, and the control variables $ln(size^p)$ and BM^p are, respectively, the time-series average of the natural log of the ratio of the portfolio's market capitalization at the beginning of the month to the total market capitalization, and BM is the time-series average of the average monthly book-to-market of the stocks constituting the portfolio. In some specifications, κ is set to be the average monthly turnover. The t-statistic, reported in the parentheses, is estimated using a GMM framework that takes into account the pre-estimation of betas. The R^2 is obtained in a single cross-sectional regression, and the adjusted R^2 is reported in the parentheses.

Panel A: liquidity portfolios

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	$ln(size^p)$	B/M	R^2
1	−1.358	0.034					2.158	0.142	1.076	0.865
	(−1.843)	(—)					(2.114)	(1.247)	(1.871)	(0.852)
2	−1.286	0.028					1.970	0.129	1.120	0.865
	(−1.501)	(1.129)					(1.869)	(0.950)	(2.215)	(0.838)
3	−0.818		0.798					0.043	1.350	0.850
	(−0.837)		(0.651)					(0.302)	(1.724)	(0.829)
4	−1.273	0.034	−3.740				6.145	0.155	0.679	0.869
	(−1.459)	(—)	(−0.576)				(0.891)	(1.054)	(0.814)	(0.850)
5	−0.441	−0.018	−12.278				13.565	0.068	0.159	0.882
	(−0.613)	(−0.227)	(−1.292)				(1.453)	(0.871)	(0.229)	(0.850)
6	−0.730		−9.313				10.988	0.098	0.339	0.880
	(−0.939)		(−1.884)				(2.106)	(0.788)	(0.598)	(0.856)
7	−0.491	0.034	1.253	−124.221	−18.359	−16.421		0.078	0.205	0.884
	(−0.369)	(—)	(0.714)	(−0.818)	(−0.180)	(−1.230)		(0.313)	(0.208)	(0.853)
8	−0.557	0.059	1.300	−183.466	−19.865	−17.238		0.087	0.253	0.884
	(−0.912)	(0.298)	(2.043)	(−0.325)	(−0.208)	(−0.922)		(0.773)	(0.376)	(0.836)

Panel B: B/M-by-size portfolios

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	$ln(size^p)$	B/M	R^2
1	0.310	0.045					−0.199	−0.084	0.251	0.924
	(1.040)	(—)					(−0.345)	(−1.415)	(2.892)	(0.917)
2	0.317	0.035					−0.236	−0.091	0.250	0.925
	(1.026)	(0.684)					(−0.311)	(−1.176)	(2.905)	(0.910)
3	0.365		−0.403					−0.119	0.246	0.920
	(1.177)		(−0.516)					(−2.155)	(2.749)	(0.909)
4	0.311	0.045	0.484				−0.696	−0.089	0.249	0.924
	(1.170)	(—)	(0.155)				(−0.262)	(−1.598)	(2.960)	(0.913)
5	0.340	−0.003	−3.145				2.850	−0.087	0.259	0.925
	(1.083)	(−0.039)	(−0.894)				(0.846)	(−1.224)	(3.108)	(0.906)
6	0.338		−2.930				2.639	−0.087	0.259	0.925
	(1.003)		(−1.366)				(0.613)	(−1.065)	(3.314)	(0.910)
7	0.237	0.045	0.490	−286.927	38.480	−14.711		−0.095	0.226	0.932
	(1.483)	(—)	(0.284)	(−1.063)	(0.615)	(−1.069)		(−1.613)	(2.868)	(0.915)
8	0.171	0.284	0.529	−916.982	42.353	−26.730		−0.100	0.233	0.937
	(0.249)	(0.308)	(0.232)	(−0.344)	(0.181)	(−0.391)		(−0.735)	(0.746)	(0.911)

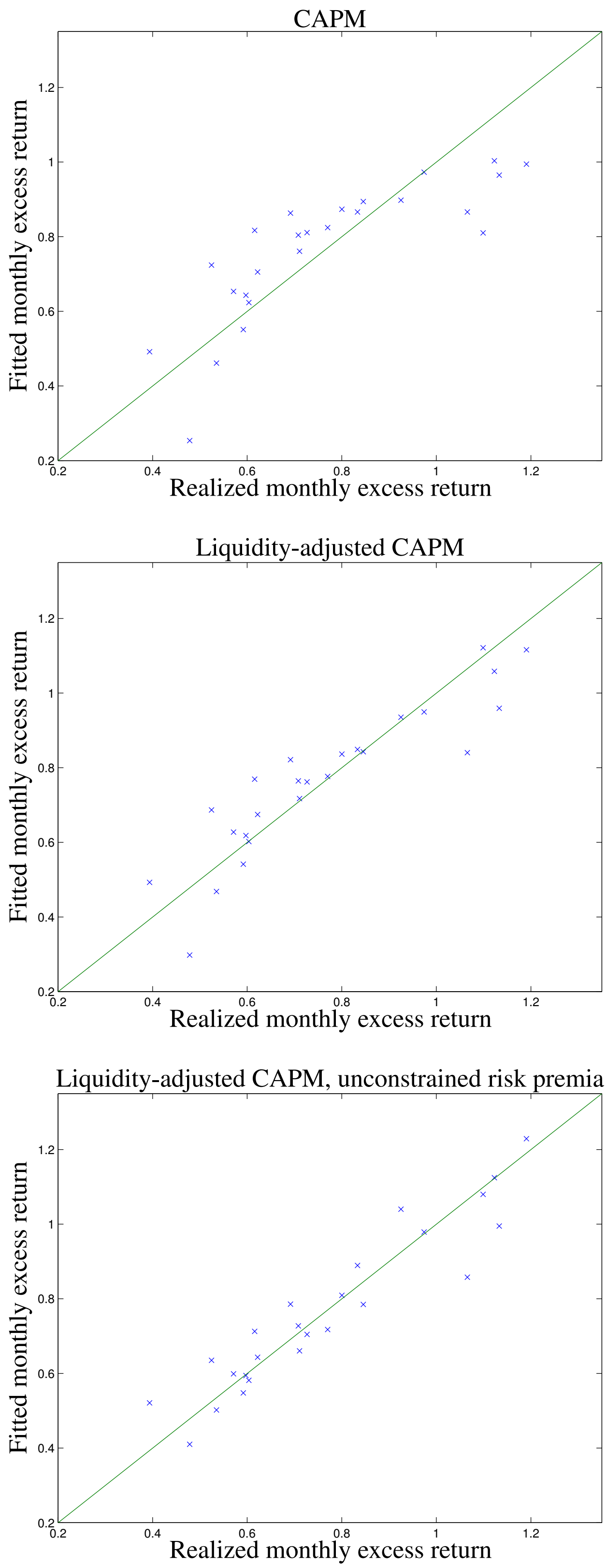

Figure 2: **Illiquidity portfolios:** The top panel shows the fitted CAPM returns vs. realized returns using monthly data 1964–1999 for value-weighted illiquidity portfolios. The middle panel shows the same for the liquidity-adjusted CAPM, and the lower panel shows the relation for the liquidity adjusted CAPM with unconstrained risk premia.

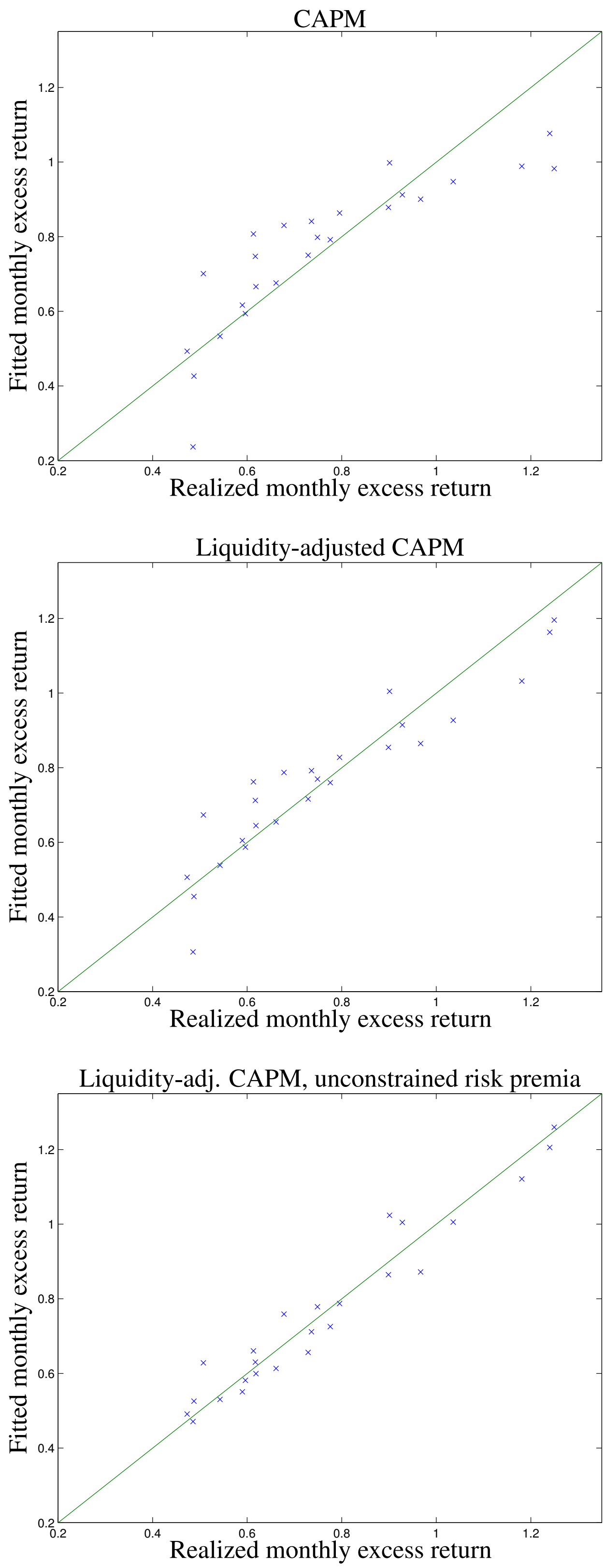

Figure 3: σ**(illiquidity) portfolios:** The top panel shows the fitted CAPM returns vs. realized returns using monthly data 1964–1999 for value-weighted σ(illiquidity) portfolios. The middle panel shows the same for the liquidity-adjusted CAPM, and the lower panel shows the relation for the liquidity adjusted CAPM with unconstrained risk premia.

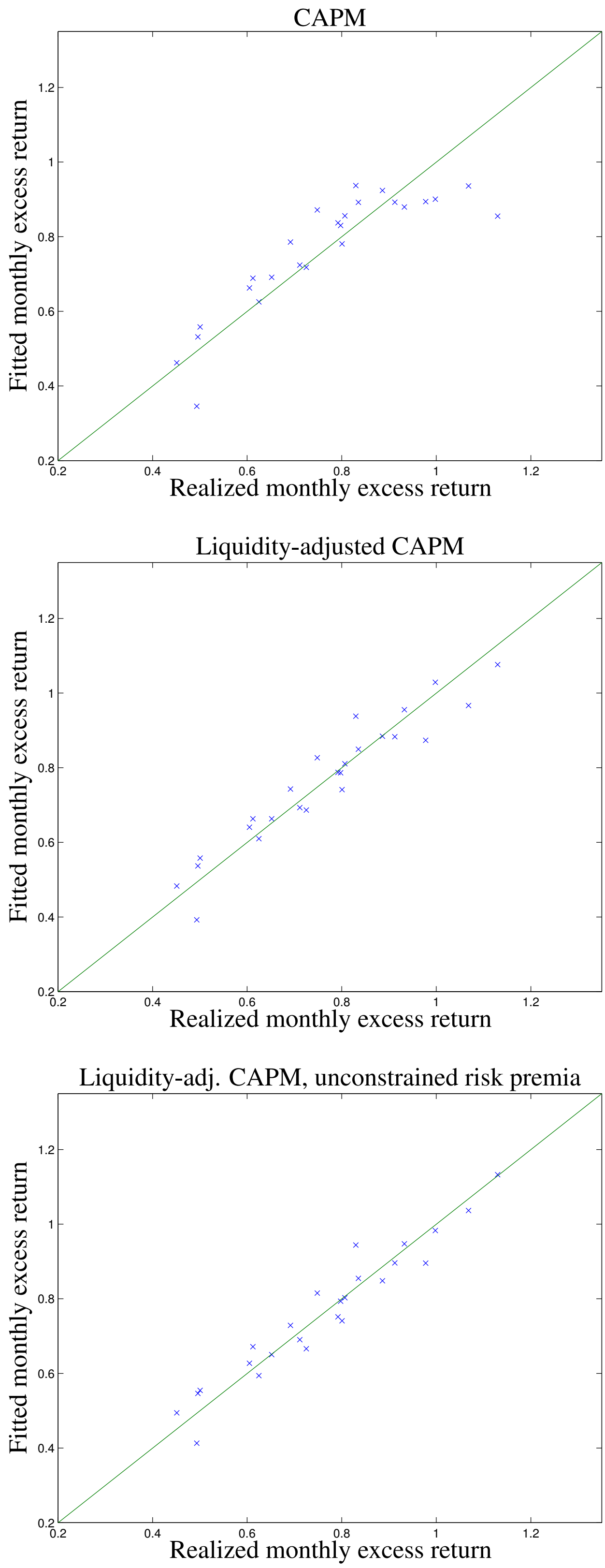

Figure 4: **Size portfolios:** The top panel shows the fitted CAPM returns vs. realized returns using monthly data 1964–1999 for value-weighted size portfolios. The middle panel shows the same for the liquidity-adjusted CAPM, and the lower panel shows the relation for the liquidity adjusted CAPM with unconstrained risk premia.

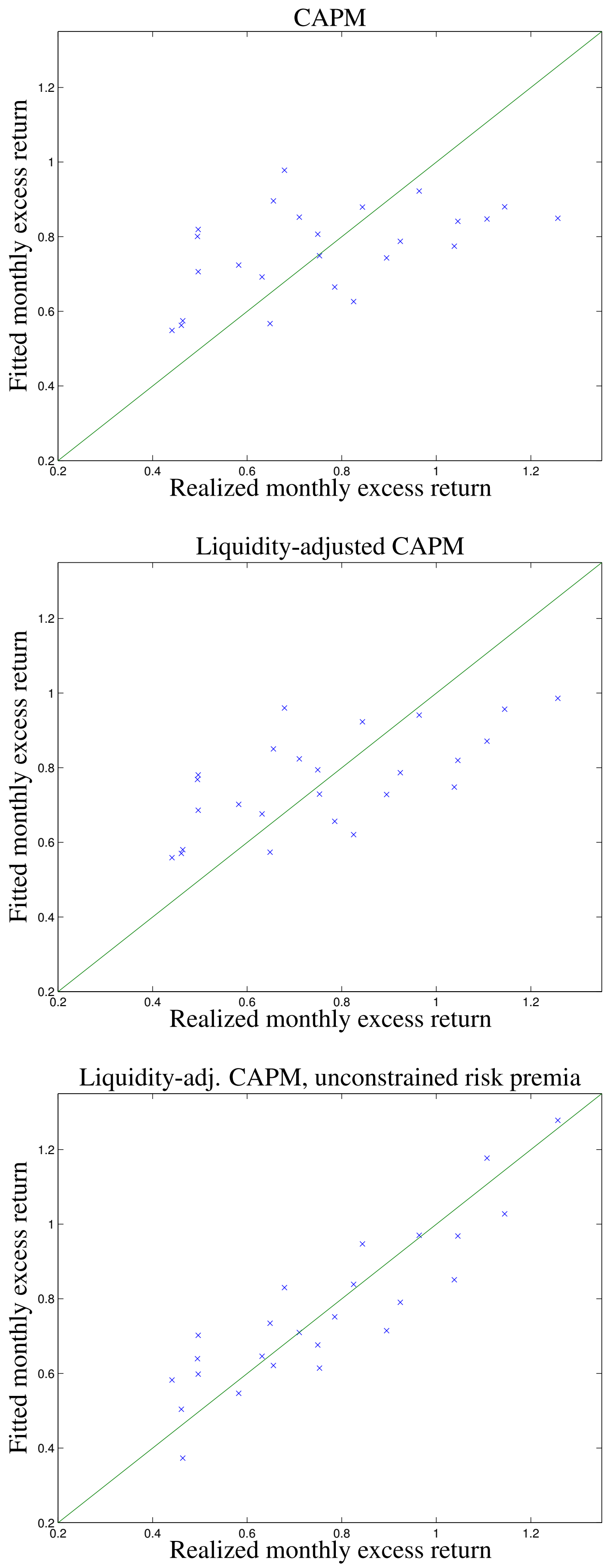

Figure 5: **Book-to-market by size portfolios:** The top panel shows the fitted CAPM returns vs. realized returns using monthly data 1964–1999 for value-weighted BM-size portfolios. The middle panel shows the same for the liquidity-adjusted CAPM, and the lower panel shows the relation for the liquidity adjusted CAPM with unconstrained risk premia.