Understanding Tail Risk[1]

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[1] Based on work with Nic Kozeniauskas, Julian Kozlowski, Anna Orlik and Venky Venkateswaran.

Why Study Information Frictions?

- Every expectation, mean, variance, covariance is conditioned on some information set. Information assumptions pervade every model.
- Preferences, technologies and budget constraints have been exhaustively studied. Information is less explored.
- The finance sector is all about collecting, processing, transmitting and selling data (information).
- Problem: We can't observe information. How to discipline? Solutions:
  - Count news stories, analyst reports, textual analysis, big data processing.
  - Model information choice: Information differs from preferences because we have some control over what we learn.
  - Treat agents like econometricians (today). They see what we see.

Introduction to Tail Risk

- Asset prices reflect risk. Risk depends on underlying variance of an outcome and on how much one knows.
- Many models shock tail risk, uncertainty, firm-specific risk ... But where do these come from?
- Tail Risk: $Prob[y_{t+1} < \alpha | I_t]$
- Uncertainty: Stdev of a forecast (error) conditional on I_t.

$$U_t = Std[y_{t+1}|I_t] = \sqrt{E\left[\left(y_{t+1} - E\left(y_{t+1}|I_t\right)\right)^2 | I_t\right]}$$

- Firm-specific (micro) risk: $\int (y_{it} - \bar{y}_t)^2 di$.
- Higher-order uncertainty: $\int (E[y_t|I_{it}] - \bar{E}[y_t])^2 di$.

Two Possible Sources of Shocks

1. Actual variance of some data-generating distribution changes.
   - Jurado, Ludvigson, Ng (2015): Find two large increases in macro variance.
   - Tail risk? Hard to measure changes.

2. Conditional variance changes because our beliefs about the distribution change.
   - Why would beliefs change if the true distribution is the same?
   - We must not know the distribution and learn about it.

How Do We Learn About Distributions?

1. A Bayesian parametric approach
2. A classical econometrics, non-parametric approach.

In both cases,

- We'll use macro data and standard econometric tools to estimate a distribution and then re-estimate it each period with new data. Our agents do the same.
- Changes in variance and tails of this distribution are a key source of shocks.

Bayesian Approach: What Distribution to Estimate?

- Key feature: Agents estimate tail probabilities.

  A normal distribution fixes these $\rightarrow$ no U_t action.
  Need parameters that govern higher moments (skewness).
  - Can capture skewness in GDP data (-0.3)
  - Key for our forecasts to resemble SPF forecast data

- Solution: Take a linear hidden state model (Kalman filter system) and do an exponential twist.

- A form of g-and-h transformation used in statistics for Bayesian distribution fitting (Headrick '10).

Forecasting Exercise

- We estimate this:

$$\begin{aligned} y_t &= c + b\exp(-S_t - \sigma\varepsilon_t) \\ S_t &= \rho S_{t-1} + \sigma^S \xi_t \end{aligned}$$

where ε_t and $\xi_t \sim iid\ N(0,1)$. $y_t =$ GDP growth.

Use real-time GDP data (1968-2013, Philly Fed) to estimate.

- Begin with prior beliefs estimated from 1947-68 data.
- Observe each quarter of data and apply Bayes' Law.
  - Metropolis-Hastings + change-of-measure
    $\rightarrow$ distributions of parameters.

- How big are uncertainty changes? $U_t = Std[y_{t+1}|I_t]$.


I'M UNCERTAIN ABOUT THE UNCERTAINTY OF THE ECONOMY. THIS, I AM CERTAIN OF...
ALDINGER
ple's Weekly World
6/2008

Result 1: Large Uncertainty Shocks

	known parameters		*estimated params*	
	normal	skewed	normal	skewed
Std dev of U_t	0	0.05%	0.48%	1.50%

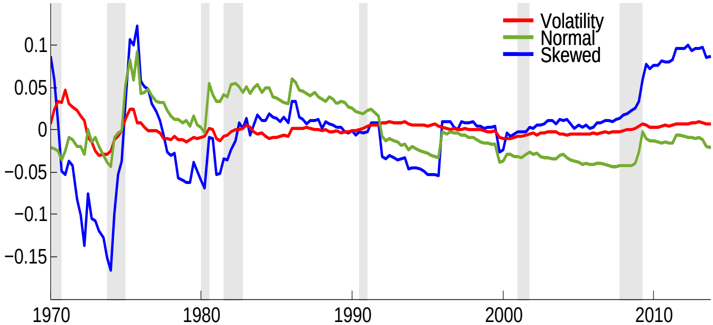
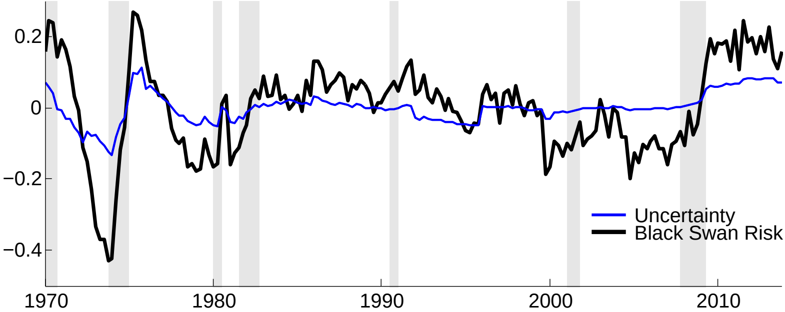


Figure: Uncertainty (U_t) in linear and skewed models, in mean-zero, log deviations from trend.

Parameter learning + Skewness = Large uncertainty shocks.

What Explains Large Shocks? Tail Risk.

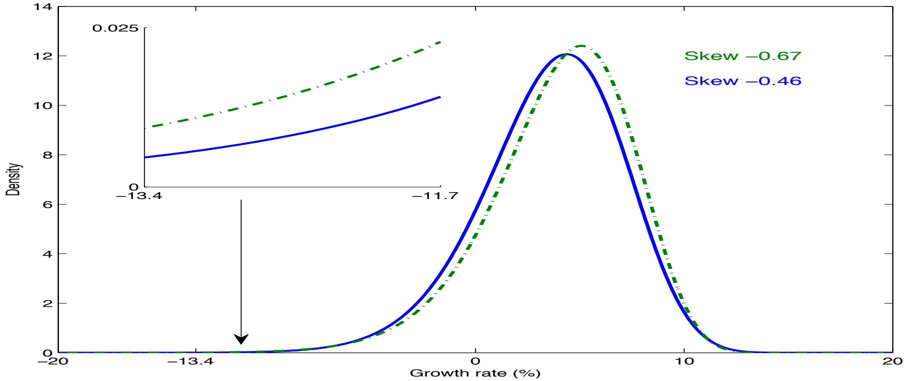
$Tail\ Risk_t \equiv Prob[y_{t+1} \leq -6.8\% | y^t]$ (1-in-100 year event)
Correlation(BSw, U_t) is 75% (both detrended).



Most changes in uncertainty come from re-estimating tail risk.

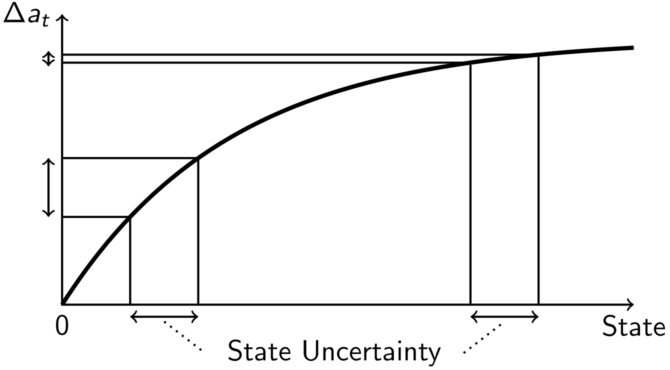
Why Is Tail Risk Volatile?

- Extreme event probabilities are very sensitive to small revisions in skewness.
- Skewness keeps fluctuating **because it is hard to learn**.



Tail Risk amplifies uncertainty in bad times

Skewness can be represented as a concave function of a normal.

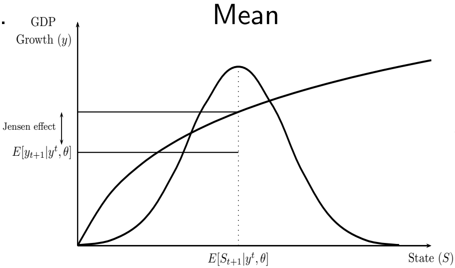


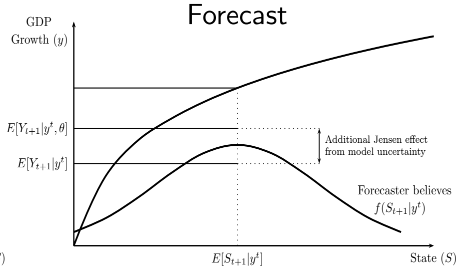
Skewness, which governs tail risk, amplifies macro uncertainty in bad states.

Tail Risk Also Creates Forecast Bias

$E[y_{t+1}|y^t,\theta]$ is mean GDP growth $= 2.68\%$.
$E[y_{t+1}|y^t]$ is average growth forecast $= 2.29\%$ in data, $= 2.27\%$ in model.

Lemma: Suppose $y = g(x)$ where g is concave and $x \sim N(\mu,\sigma)$. μ and σ are unknown, with unbiased beliefs. Then mean $>$ forecast.





When we estimate parameter uncertainty and skewness, we match the bias in professional forecasts.

Approach 2: Non-parametric, Classical Estimation

Consider an iid shock, ϕ_t, with unknown pdf g

Information set: finite history of shock realizations $\{\phi_{t-s}\}_{s=0}^{n_t-1}$

The Gaussian kernel density estimator

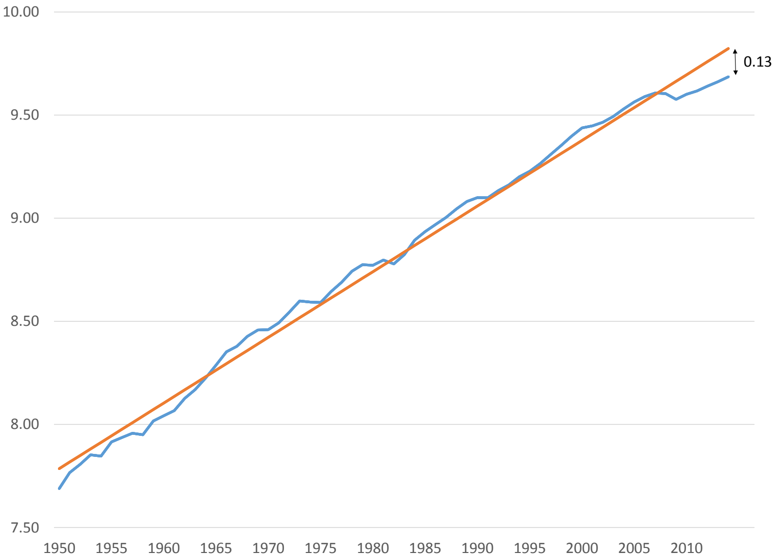
$$\hat{g}_t(\phi) = \frac{1}{n_t \kappa} \sum_{s=0}^{n_t-1} \Omega\left(\frac{\phi - \phi_{t-s}}{\kappa}\right)$$

Key property: Beliefs are martingales

$$\mathbb{E}_t[\hat{g}_{t+1} | \mathcal{I}_t] \approx \hat{g}_t \qquad \rightarrow \qquad \text{Persistence}$$

Next: use this mechanism to create persistence (long run risk).

US Real GDP: Stagnation

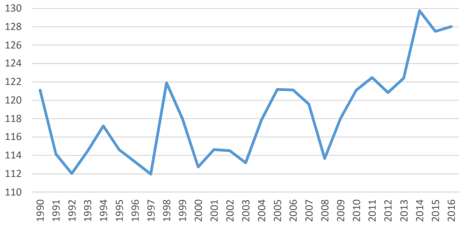


Tail Risk Stayed High

Why do some recessions have persistent effects ?

- Because they cause us to re-assess macro risk.

Suggestive evidence from financial markets: A tail risk index



Note: Constructed from out-of-the-money put options on S&P 500

Economic Model - based on Gourio (AER, 2012)

Representative household with Epstein-Zin preferences over, $C_t - \zeta \frac{L_t^{1+\gamma}}{1+\gamma}$

A continuum of firms, indexed by i

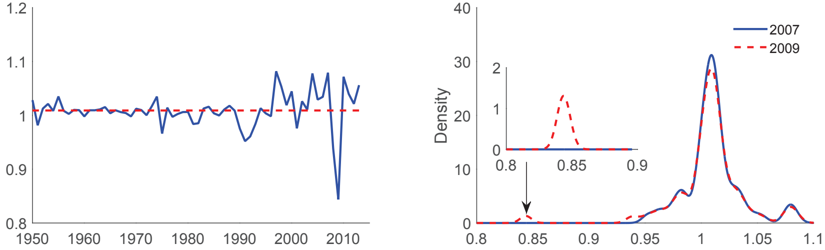
- Production: $y_{it} = A k_{it}^{\alpha} l_{it}^{1-\alpha}$
- Aggregate capital quality shocks: $k_{it} = \phi_t \hat{k}_{it} \qquad \phi_t \sim g(\cdot) \quad iid$
- Idiosyncratic shocks (iid): $\Pi_{it} = v_{it} \left[y_{it} + (1-\delta) k_{it} \right], \ \int v_{it} di = 1$

Debt has a tax advantage and a default cost.

Labor hired in advance, before observing shocks.

Capital Quality Shocks

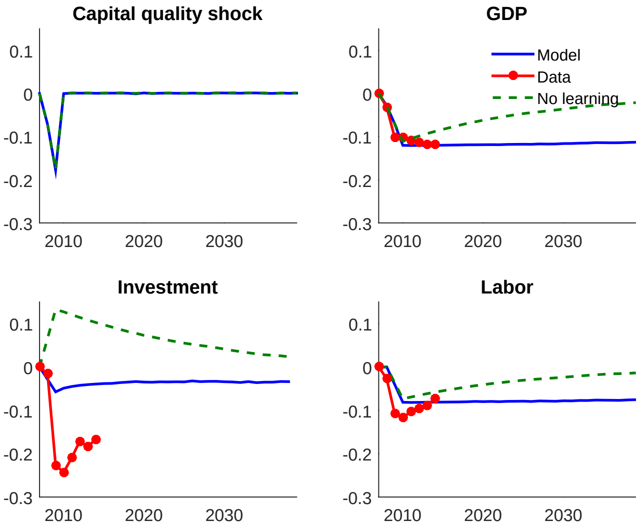
Key feature: Increase left tail risk, post-2009



We do:

- Calibrate model,
- feed in this data through 2007, normalize '07 outcome to 0.
- observe effects of 08-09 shocks,
- take random draws from the 2009 distribution (report avg outcome).

Stagnation: Model and Data



Without belief revisions, a steady recovery to initial level

Conclusions

- Obviously, no one knows the true distribution of shocks. Simple, displined tools to replace rational expectations hypothesis.
- New data permanently reshapes our assessment of macro risks, especially tail risks because data on tails is scarce.
- Changes in tail risk provide a unified theory of uncertainty, risk, sentiment shocks and belief biases.
- $\rightarrow$ A new persistence mechanism / source of long-run risk.
  $\rightarrow$ A new source of price fluctuations
  $\rightarrow$ A new risk factor

How much of business cycle fluctuations, asset pricing puzzles or other phenomena could learning about tail risk explain?