Available online at www.sciencedirect.com
SCIENCE DIRECT®
Journal of Banking & Finance 30 (2006) 811–821
Journal of BANKING & FINANCE Applied Section
www.elsevier.com/locate/jbf

Hedging volatility risk

Menachem Brenner [a,*], Ernest Y. Ou [b], Jin E. Zhang [c]

[a] *Stern School of Business, New York University, New York, NY 10012, USA*
[b] *Archeus Capital Management, New York, NY 10017, USA*
[c] *School of Business and School of Economics and Finance, The University of Hong Kong, Pokfulam Road, Hong Kong*

Received 11 April 2005; accepted 5 July 2005
Available online 9 December 2005

Abstract

Volatility risk plays an important role in the management of portfolios of derivative assets as well as portfolios of basic assets. This risk is currently managed by volatility "swaps" or futures. However, this risk could be managed more efficiently using options on volatility that were proposed in the past but were never introduced mainly due to the lack of a cost efficient tradable underlying asset.

The objective of this paper is to introduce a new volatility instrument, an option on a straddle, which can be used to hedge volatility risk. The design and valuation of such an instrument are the basic ingredients of a successful financial product. In order to value these options, we combine the approaches of compound options and stochastic volatility. Our numerical results show that the straddle option is a powerful instrument to hedge volatility risk. An additional benefit of such an innovation is that it will provide a direct estimate of the market price for volatility risk.

JEL classification: G12; G13

Keywords: Volatility options; Compound options; Stochastic volatility; Risk management; Volatility index

* Corresponding author. Tel.: +1 212 998 0323; fax: +1 212 995 4731.
E-mail addresses: mbrenner@stern.nyu.edu (M. Brenner), ernie@archeuscap.com (E.Y. Ou), jinzhang@hku.hk (J.E. Zhang).

0378-4266/$ - see front matter
doi:10.1016/j.jbankfin.2005.07.015

1. Introduction

Risk management is concerned with various aspects of risk, in particular, price risk and volatility risk. While there are various efficient instruments (and strategies) to deal with price risk, exhibited by the volatility of asset prices, there is practically only one type of derivative which deals with volatility risk, namely "volatility swaps", which is basically a forward/futures contract either on realized volatility or on implied volatility. In this paper we are introducing a new volatility instrument, an option on a *forward-start straddle*, which in our opinion dominates the usefulness of existing alternatives, including "volatility swaps".

While option traders, in general, are subject to volatility risk, as well as other risks, the main concern of delta-neutral volatility traders is the risk that volatility may change. It is true that one can bet on volatility changes (or hedge them) with a strategy that combines holding of static options and dynamically trade the underlying asset as shown by Carr and Madan (1998). Such a strategy, however, may be very costly and not practical for many users.

Given the large and frequent shifts in volatility in the recent past[1] especially in periods like the summer of 1997, the fall of 1998 and the fall of 2001, there is a growing need for instruments to hedge volatility risk. Past proposals of such instruments included futures and options on a volatility index. The idea of developing a volatility index was suggested by Brenner and Galai (1989, 1993).[2] In 1993 the Chicago Board Options Exchange (CBOE) has introduced a volatility index, named VIX, which was based on implied volatilities from options on the S&P100 index. Recently, the CBOE had changed the methodology, to calculate VIX, which is now based on the general principle of spanning the second moment. The new forward-looking volatility index uses current option prices to predict the next 30 days realized volatility. This volatility index uses the S&P500 at-the-money put and call options and all out-of-the-money options weighted by the inverse of the square of their strike prices. This approach is based on the work by Derman et al. (1996) and Carr and Madan (1998) while a more general treatment can be found in Bakshi and Madan (2000) and in Bakshi et al. (2003) (Fig. 1).

Only in March 2004 did the Chicago Futures Exchange (CFE) launch its first product, a futures contract on VIX. Options on VIX have been planned for some time now but have not been introduced yet. The main reason, in our opinion, that it has taken so long to introduce such derivatives is the lack of a cost-efficient tradable underlying asset which market makers could use to hedge their positions and to price them.

Rather than an option on an implied volatility index or an option on an index computed from the prices of many options, some of which hardly trade, we present here a proposal to introduce an option on a straddle (STO). The key feature of the straddle option is that the underlying asset is an at-the-money-forward (ATMF) straddle. The ATMF straddle is a traded asset priced in the market place and well understood by market participants. Since it is ATMF, its relative value (call + put)/stock is mainly affected by volatility. Changes in volatility translate almost linearly into changes in the value of the underlying,

[1] The volatility of volatility can be observed from the behavior of a volatility index, VIX (based on the new methodology), provided in Fig. 1.

[2] Gastineau (1977) and Galai (1979) have proposed an index of option prices which corresponds to an implied volatility index. Such an index is also described in Cox and Rubinstein (1985).

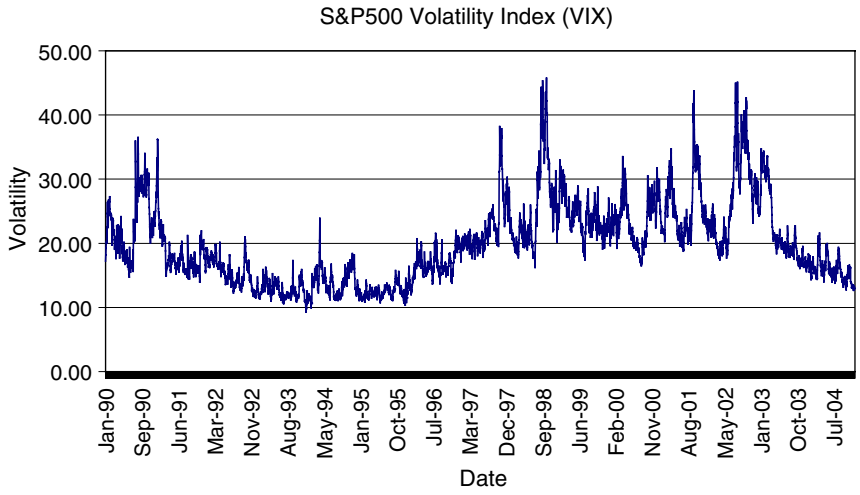

Fig. 1. Closing level on the S&P500 Volatility Index (VIX). The sample period is January 3, 1990–December 3, 2004. *Source:* CBOE.

the ATMF straddle.[3] Thus options on the ATMF straddle are options on volatility. We believe that such an instrument will be very attractive to market participants, especially to market makers.

An additional benefit of such an innovation is that it will provide a market price for volatility risk. A few recent papers have examined this issue empirically (e.g., Coval and Shumway, 2001, Bakshi and Kapadia, 2003; Buraschi and Jackwerth, 2001). Examining different strategies they conclude that the volatility risk premium is negative. Currently there is no market that calibrates this premium as, for example, the risk premium in the stock market. The return on the underlying of the proposed product, an always ATMF straddle, will provide such a calibration.

In the next section we describe in detail the design of the instrument. In Section 3 we derive the value of such an option. Section 4 compares this instrument to other alternatives and Section 5 provides the conclusions.

2. The design of the straddle option

One obvious class of users of these options are volatility speculators who buy and sell volatility using standard call and put options which are affected by changes in the underlying asset and by interest rates in addition to changes in volatility. They simply may not be interested in such a package, especially if it costs more than a direct bet on volatility. The other potential classes of users are hedgers who mainly trade in the options market, like market makers in options, and portfolio managers who allocate funds between stocks and bonds using a mean-variance analysis. Since their allocation, and performance, may be affected by an unexpected change in volatility they may want to insure against volatility risk. Again, this can be done using standard straddles but this approach is inefficient since

[3] Strictly speaking this is true in a B–S world (see Brenner and Subrahmanyam (1988)) but here, with stochastic volatility, it may include other parameters (e.g., vol. of volatility).

it insures against both: volatility changes (vega) and changes in delta (gamma). The price of the straddle reflects the broader coverage which is not sought after. To isolate the volatility risk one could dynamically trade the straddle such that it always is ATMF but such a strategy entails transactions costs that become very high depending on the frequency of rebalancing which in turn depends on volatility itself. Thus, the desired instrument, presented next, should be a hedge against volatility risk only and should cost less than the alternatives including transactions costs.

To manage the market volatility risk, say of the S&P500 index, a new instrument, a straddle option or STO (K_{STO}, T_1, T_2) with the following specifications, could be introduced. At the maturity date T_1 of this contract, the buyer has the *option* to buy a then at-the-money-forward straddle with a pre specified exercise price K_{STO}. The buyer receives both, a call and a put, with a strike price equal to the forward price, given the index level at time T_1. The straddle matures at time T_2.

The proposed contract has two main features: first, the value of the contract at maturity depends on the volatility expected in the interval T_1 to T_2 and therefore it is a tool to hedge future volatility. It is sensitive to changes in volatility but not to changes in interest rates or to large changes in the spot. Second, the asset underlying this option is the contemporaneously traded straddle.[4] We believe that, unlike the volatility options, this design will greatly enhance its acceptance and use by the investment community. Compared to the available alternatives it is the most cost effective one (see Section 4).

The proposed instrument is conceptually related to two known exotic option contracts: compound options and forward start options.[5] Unlike the conventional compound option this proposed option is an option on a straddle with a strike price, unknown at time 0, to be set at time T_1 to the forward value of the index level. In general, in valuing compound options it is assumed that volatility is constant (see, for example, Geske (1979)). Given that the objective of the instrument presented here is to manage volatility risk, we need to introduce stochastic volatility.

3. Valuation of the straddle option

In this section we first describe briefly the related valuation literature and then provide our valuation model for the straddle option (STO). We apply a specific stochastic volatility (SV) model and illustrate its properties.

The first theoretical paper[6] to value options on a volatility index is by Grünbichler and Longstaff (1996). They specify a mean reverting square root diffusion process for volatility. Their framework is similar to that of Hull and White (1987) and Stein and Stein (1991) and others. Since volatility is not trading they assume that the premium for volatility risk is proportional to the level of volatility. This approach is in the spirit of the equilibrium approach of Cox et al. (1985) and Longstaff and Schwartz (1992). A more recent paper

[4] Theoretically there is no difference if the delivered option is a call, a put or a straddle since they are all ATMF. Practically, however, there may be some difference in prices due, for example, to transactions costs. A straddle would provide a less biased hedge vehicle.

[5] Forward start options are paid for now but start at some time T_1 in the future. A forward start option with maturity T_2 can be regarded as a special case of the straddle option in which the strike price K_{STO} is zero.

[6] Brenner and Galai (1993) use a binomial framework to value such options where tradability is assumed implicitly.

by Detemple and Osakwe (2000) also uses a general equilibrium framework to price European and American style volatility options. They emphasize the mean-reverting in log volatility model.

3.1. The valuation model

We assume a risk-neutral diffusion process and a stochastic volatility (SV) model similar to the one by Stein and Stein (1991):

$$\mathrm{d}S_t = rS_t\,\mathrm{d}t + \sigma_t S_t\,\mathrm{d}B_t^1, \tag{1}$$

$$\mathrm{d}\sigma_t = \delta(\theta - \sigma_t)\,\mathrm{d}t + k\,\mathrm{d}B_t^2. \tag{2}$$

Eq. (1) describes the dynamics of an equity index S_t with a stochastic volatility σ_t. Eq. (2) describes the dynamics of volatility itself which is reverting to a long run mean θ where δ is the adjustment rate and k is the volatility of volatility. r is the risk-free rate. B_t^1 and B_t^2 are two independent Brownian motions.

It should be noted that the conditional joint density of return and volatility in the Stein and Stein (1991) model also assumes zero correlation. In principle it should be possible to derive the joint characteristic function for the non-zero correlation using the Fourier methods given in Bakshi and Madan (2000). The joint density will be written as a double inverse Fourier transformation. The valuation of the ATMF option will require the computation of quadruple integration which is beyond the scope of our paper.

Our model, which is based on the above assumptions and derived next, is relevant for several non-equity markets, in particular the FX market and could be used as the benchmark case for other markets where non-zero correlation is the prevailing case.

Given the above, the conditional probability density function of S_T is given by

$$f(S_T|S_t,\sigma_t;r,T-t,\delta,\theta,k) = \mathrm{e}^{-r(T-t)} f_0\left(S_T \mathrm{e}^{-r(T-t)}\right), \tag{3}$$

where

$$f_0(S_T) = \frac{1}{2\pi}\left(\frac{S_t}{S_T}\right)^{3/2}\frac{1}{S_t}\int_{-\infty}^{\infty} I\left(\left(\eta^2+\frac{1}{4}\right)\frac{T-t}{2}\right)\cos\left(\eta \ln\frac{S_T}{S_t}\right)\mathrm{d}\eta$$

in which the function $I(\lambda)$ is given by Eq. (8) of Stein and Stein (1991).

The transition probability density function of σ_T is normal with mean $\theta + (\sigma_t - \theta)\,\mathrm{e}^{-\delta(T-t)}$ and variance $k^2(1-\mathrm{e}^{-2\delta(T-t)})/2\delta$,

$$f(\sigma_T|\sigma_t;T-t,\delta,\theta,k) = \frac{1}{\sqrt{\frac{\pi k^2}{\delta}(1-\mathrm{e}^{-2\delta(T-t)})}}\exp\left[-\frac{\left(\sigma_T-\theta-(\sigma_t-\theta)\mathrm{e}^{-\delta(T-t)}\right)^2}{\frac{k^2}{\delta}(1-\mathrm{e}^{-2\delta(T-t)})}\right]. \tag{4}$$

The joint distribution of S_T and σ_T is

$$f(S_T,\sigma_T) = f(S_T)f(\sigma_T) \tag{5}$$

since the two Brownian motions are assumed independent.

Using risk-neutral valuation with the above joint distribution, the value of the straddle ST at time T_1 is

$$\begin{aligned} \mathrm{ST}_{T_1} &= 2\mathrm{e}^{-r(T_2-T_1)} \int_{S_{T_1}\mathrm{e}^{r(T_2-T_1)}}^{\infty} \left(S_{T_2} - S_{T_1}\mathrm{e}^{r(T_2-T_1)}\right) f(S_{T_2}|S_{T_1})\,\mathrm{d}S_{T_2} \\ &\equiv 2S_{T_1}F(\sigma_{T_1}; T_2 - T_1, r, \delta, \theta, k), \end{aligned} \tag{6}$$

where the strike price K_{ST} is $S_{T_1}\mathrm{e}^{r(T_2-T_1)}$.

Given the values of the straddle ST, the price of the straddle option, STO, at time $t = 0$ can be computed as

$$\mathrm{STO}_0 = \int_0^{\infty} G(\sigma_{T_1}) f(\sigma_{T_1}|\sigma_0)\,\mathrm{d}\sigma_{T_1}, \tag{7}$$

where

$$G(\sigma_{T_1}) = 2F(\sigma_{T_1})\mathrm{e}^{-rT_1} \int_{\frac{K_{\mathrm{STO}}}{2F(\sigma_{T_1})}}^{\infty} \left(S_{T_1} - \frac{K_{\mathrm{STO}}}{2F(\sigma_{T_1})}\right) f(S_{T_1}|S_0)\,\mathrm{d}S_{T_1}. \tag{8}$$

We next present the results of the special case where volatility changes only once and is known at time zero. We assume a constant volatility σ_1 between time 0 and T_1 (expiry date of STO) and a constant volatility σ_2 between T_1 and T_2 (maturity date of the straddle ST). Assuming that the Black–Scholes assumptions hold, and using the Brenner and Subrahmanyam (1988) approximation for ATMF options, we have

$$\mathrm{ST}_{T_1} \equiv \alpha \cdot S_{T_1} = 2(2N(d_1) - 1)S_{T_1} \cong \frac{2S_{T_1}}{\sqrt{2\pi}}\sigma_2\sqrt{T_2 - T_1}, \tag{9}$$

where $d_1 = \frac{1}{2}\sigma_2\sqrt{T_2 - T_1}$. The straddle is practically linear in volatility. The relative value of the straddle, $\alpha = \mathrm{ST}_{T_1}/S_{T_1}$ is solely determined by volatility to expiration.

Thus the price of the STO at any time t, $0 \leqslant t < T_1$, using the B–S model, is

$$\mathrm{STO}_t = \alpha \cdot S_t \cdot N(d) - K_{\mathrm{STO}}\mathrm{e}^{-r(T_1-t)} \cdot N(d - \sigma_1\sqrt{T_1 - t}). \tag{10}$$

$$d = \frac{\ln(\alpha S_t/K_{\mathrm{STO}}) + (r + \frac{1}{2}\sigma^2)(T_1 - t)}{\sigma_1\sqrt{T_1 - t}} \tag{11}$$

The values obtained from (10) can provide some insight in interpreting the results of the stochastic volatility (SV) case given in Eqs. (7) and (8) which are presented next.

The values of STO are computed numerically in Table 1a and 1b using a range of parameter values. In choosing the parameter values we rely on the same empirical studies that Stein and Stein (1991) do. In these studies, for example, the long-run volatility is between 15% and 20%, the parameter k can have values as high as 0.5 and the mean reversion parameter can range from 4 to 20.

Next to the values from the SV model, in 1a, we present the values using the BS model ($k = 0$). As expected, the value of the compound option using the SV model is larger than the value of this option using the BS model. The difference between the two depends on the values of the other parameters in the SV model and the strike price K_{STO}. For relatively low strike prices, K_{STO}, the effect of stochastic volatility is rather small and the values are not that different from a BS value, ignoring stochastic volatility. For higher strike prices (out of the money) the effect of k, the volatility of volatility, is much larger. For

Table 1a
The value of the straddle option, STO, at $t = 0$ for a combination of strike price K_{STO} and volatility of volatility k

K_{STO}	$k = 0$ (BS)	$k = 0.10$	$k = 0.20$	$k = 0.30$	$k = 0.40$	$k = 0.50$
0	11.274	11.352	11.580	11.841	12.146	12.564
1	10.274	10.352	10.583	10.874	11.231	11.699
2	9.274	9.352	9.585	9.904	10.311	10.829
3	8.274	8.352	8.587	8.933	9.388	9.957
4	7.274	7.352	7.590	7.962	8.465	9.083
5	6.274	6.352	6.592	6.990	7.542	8.210
6	5.274	5.352	5.594	6.020	6.619	7.338
7	4.274	4.352	4.601	5.054	5.700	6.467
8	3.277	3.360	3.629	4.111	4.793	5.602
9	2.308	2.408	2.713	3.222	3.919	4.754
10	1.439	1.564	1.907	2.428	3.113	3.942
11	0.774	0.908	1.254	1.757	2.406	3.195
12	0.355	0.470	0.771	1.223	1.812	2.538
13	0.140	0.218	0.446	0.820	1.331	1.981
14	0.048	0.092	0.245	0.531	0.957	1.521
15	0.014	0.035	0.129	0.335	0.674	1.152
16	0.004	0.013	0.065	0.206	0.466	0.861
17	0.001	0.004	0.032	0.124	0.318	0.636
18	0.000	0.001	0.016	0.074	0.215	0.466
19	0.000	0.000	0.007	0.044	0.144	0.339
20	0.000	0.000	0.003	0.026	0.096	0.245

$S_0 = 100$, $r = 0$, σ_0, initial volatility $= 0.20$, θ, long-run volatility $= 0.20$, δ, reversion parameter $= 4.00$, $T_1 = 0.5$, $T_2 = 1.0$.

Table 1b
The value of STO at $t = 0$ for a combination of the initial volatility σ_0 and the volatility of volatility k

σ_0	$k = 0.00$	$k = 0.10$	$k = 0.20$	$k = 0.30$	$k = 0.40$	$k = 0.50$
0.00	0.095	0.221	0.548	1.023	1.633	2.389
0.10	0.273	0.408	0.745	1.230	1.850	2.611
0.20	0.535	0.663	0.992	1.474	2.095	2.854
0.30	0.849	0.965	1.277	1.748	2.364	3.117
0.40	1.194	1.299	1.592	2.048	2.654	3.397
0.50	1.558	1.654	1.930	2.369	2.961	3.692
0.60	1.937	2.026	2.285	2.707	3.284	4.000
0.70	2.325	2.408	2.654	3.059	3.621	4.321
0.80	2.722	2.800	3.034	3.423	3.969	4.654
0.90	3.127	3.200	3.423	3.798	4.328	4.997
1.00	3.537	3.606	3.818	4.181	4.695	5.349

$S_0 = 100$, $r = 0$, $\theta = 0.20$, $\delta = 4.00$, $T_1 = 0.5$, $T_2 = 1.0$, $K_{STO} = 11.5$.

$K_{STO} = 12$, slightly out-of-the-money, the value of STO at $k = 0.3$ is about 1.6 times larger than STO at $k = 0.1$ (1.22 vs. 0.47) while the BS value is only 0.36.

Table 1b shows the combined effect of volatility and k, volatility of volatility, at the ATM strike of STO. As expected, the value of STO increases in both and is rather monotonic. *Stochastic volatility has a relatively bigger effect in a low volatility environment.*

The effect of the various parameters on the value of STO could be discerned from the previous tables but a better understanding of the complex relationships can be obtained from an examination of the various sensitivities presented in Fig. 2a and b. Fig. 2a

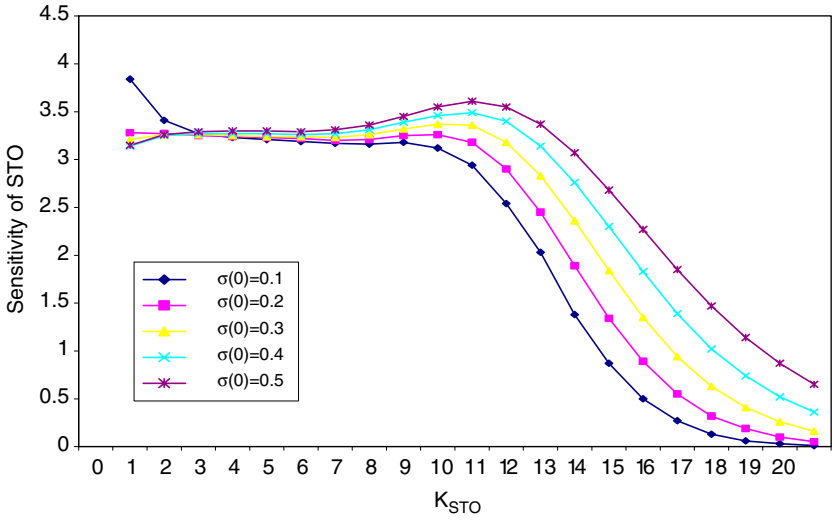

Fig. 2a. The sensitivity of STO to initial vol. at different K_{STO} and five levels of initial vol. $S_0 = 100$, $r = 0$, $k = 0.20$, $\theta = 0.20$, $\delta = 4.00$, $T_1 = 0.5$, $T_2 = 1.0$.

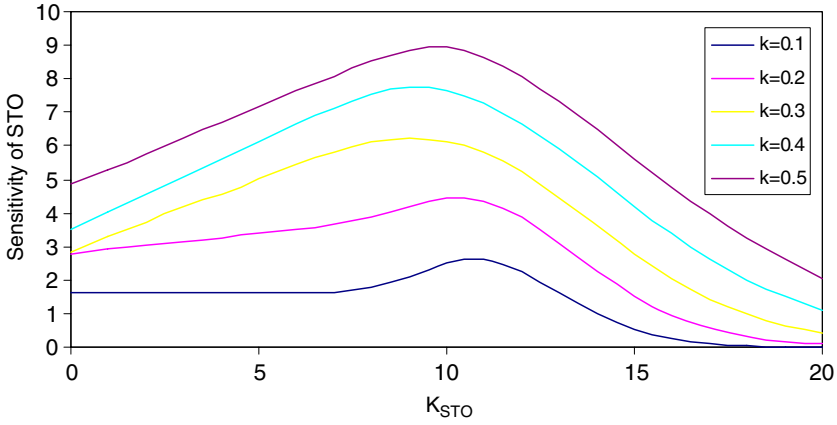

Fig. 2b. The sensitivity of STO to k (vol. of vol.) at different K_{STO} and five levels of k $S_0 = 100$, $r = 0$, $\sigma_0 = 0.20$, $\theta = 0.20$, $\delta = 4.00$, $T_1 = 0.5$, $T_2 = 1.0$.

provides the sensitivity of STO to changes in initial volatility, $\partial_{\mathrm{STO}}/\partial\sigma_0$ which is the main issue here. Fig. 2a provides these values at 5 levels of σ_0. The values are high at all levels of initial volatility, though they tend to decline as volatility decreases, indicating that changes in volatility could be effectively hedged by the straddle option. It becomes less effective as the strike price K_{STO} increases, the option is out-of-the-money. Fig. 2b provides values for the sensitivity of STO to k, volatility of volatility. The higher is k, the higher is the "vega" of STO. It is most sensitive at intermediate values of the strike price and approaches zero as the strike price increases.

An interesting observation regarding the value of STO emerges. Does STO have a higher value, relative to BS value, in markets with higher volatility? It seems that higher

σ , for a given k (volatility of volatility), tends to reduce the differences between SV values and BS values since σ is the dominant factor in the valuation. Thus, if higher σ is accompanied by higher k, STO valuation will not benefit much from using a stochastic volatility model. Though the findings are an outcome of the model proposed by Stein and Stein (1991), some of the results are consistent with general valuation concepts. For example, ATM option valuation, excluding jump processes, is largely dominated by volatility irrespective of model specifications.

4. The straddle option (STO) and other alternatives

The determinants of an efficient volatility-hedging instrument are, its sensitivity to volatility, its tradability and its cost effectiveness. The key features of STO make it the most efficient instrument when compared to the alternatives. As discussed earlier, the ATMF strike eliminates the sensitivity to interest rates. The compound option structure which includes setting the ATMF strike price at the delivery date achieves two things; One, it provides an instrument which is sensitive only to volatility. Two, it provides an underlying asset which is tradable.

Other alternatives are less efficient since they respond to other risks as well as volatility and/or they are more costly. We examined the two obvious alternatives; The current ATMF straddle and the compound option on a straddle. We have simulated the paths of the index and volatility and computed the costs of these alternatives comparing them to the one proposed here.

We assume the equity index S and its volatility σ to follow the stochastic processes given in Eqs. (1) and (2). The initial levels of S and σ are \$100 and 20%, respectively. The parameters in the above processes are: $r = 3\%$ (dividend yield $q = 0$), $\delta = 8$, $\theta = 20\%$ and $k = 20\%$. The total period is 1 year and we compare the values after 6 months. We randomly drew 400 simulation paths for S and σ . Along each simulated path of equity index S, we calculated the corresponding values for straddles and straddle options at each future time point. The values from these 400 paths give us means and variances.

The first alternative is an outright *straddle*. We calculate the values of a 1 year *straddle* whose time 0 strike price is set to be ATMF (i.e., $K = \$103.05$ given $S = 100$ and $r = 3\%$)

(a) the value of this straddle, at time 0, is \$16.287;
(b) the value of the same straddle after 6 months, (strike is still \$103.05) has a mean of 16.532 and a variance of 35.72.

The mean at 6 month time is just the value of the *same* straddle at time 0, plus the 3% carrying cost. The variance is large because the straddle can be deep in or out of the money at 6 month time, given its fixed strike level at \$103.05.

Now compare this to a straddle whose strike price will be reset to the ATMF level in 6 months time (i.e., if $S = 150$ in 6 month, then the strike of this straddle is set to be 152.27). The mean value of this straddle is *11.72* and its variance is *2.75*. This ATMF straddle has a much smaller mean and variance because its strike is reset to be at-the-money-forward. It still has some variance because the underlying S has a variance at 6 month time.

We now compute the value of a *straddle option* (call) which at maturity, in 6 month time, gives the holder the option to buy a then ATMF straddle. If the strike of this straddle option is 11.72, which is the mean value of the ATMF straddle, then at time 0 the value of

this straddle option is *0.654* (If the strike is 5, the value of the option 6.621). So the total cost of this alternative; straddle option and the ATMF straddle is much less than the simple straddle.

The second alternative is a *compound option* which at its maturity at 6 month time, gives the holder the option to buy a straddle whose strike is fixed at \$103.05 as in case 1. We know that its mean value in 6 month is \$16.53. If the strike of this compound option is 16.53, the mean value of its underling straddle, then the time 0 value of this *compound option* is *2.166* (If the strike is 5, the value of the option is 11.36; if the strike is 11.72, then the value of the option is 4.74). In other words at the same strike price as the STO, 11.72, the price of the straddle option (STO) is much lower than the compound option (0.654 vs. 4.74).

Finally, in a continuous and perfect market one could dynamically replicate the straddle option by resetting the strike price with every change in S but in reality even minimal transactions costs will make such a proposition very costly.

5. Conclusions

The stochastic behavior of volatility, which has always affected options premiums, has been, for the most part, ignored by market participants. However, any risk management system must cope with volatility risk and it can do so in several ways; using existing instruments, offered largely in the OTC market, and/or using a dynamic strategy. Recently, exchange traded futures on volatility have been added to the set of potential instruments. In this paper we present a derivative instrument, an option on a straddle, that can be used to hedge the risk inherent in stochastic volatility. This option could be traded on exchanges and used for risk management. As we show, it compares favorably with other possible alternatives; it is sensitive only to volatility, the underlying asset is tradable and it is a cost effective instrument. Also, such an instrument will provide a market price for volatility risk, which is currently estimated indirectly from existing instruments.

Since valuation is an integral part of using and trading such an option we derive the value of such an option using a stochastic volatility model. We compare the value of such an option to a benchmark value given by the BS model. We find that the value of such an option is very sensitive to changes in volatility and therefore cannot be approximated by the BS model.

Acknowledgements

The practical idea of using a straddle as the underlying rather than a volatility index was first suggested by Gary Gastineau in a discussion with one of the authors (M. Brenner). We understand that Gary has applied for a US patent on his volatility instrument, which is based on the same idea, the process used to implement it and related instruments. We would like to thank Steve Brown, Gary Gastineau, David Hait, Nikolay Halov, Pascal Nguyen, Marti Subrahmanyam, Alex Shapiro, Raghu Sundaram, Bruce Tuckman, David Weinbaum, and seminar participants at HKUST, Quantitative Method in Finance 2001 Conference, 2002 APFA/PACAP/FMA Finance Conference for their helpful comments.

We would also like to thank the anonymous referee for his valuable comments. JEZ has been supported by the Research Grants Council of Hong Kong under grant CERG-HKU 1068/01H and the University of Hong Kong.

References

Bakshi, G., Kapadia, N., 2003. Delta-hedged gains and the negative market volatility risk premium. Review of Financial Studies 16, 527–566.

Bakshi, G., Madan, D., 2000. Spanning and derivative-security valuation. Journal of Financial Economics 55, 205–238.

Bakshi, G., Kapadia, N., Madan, D., 2003. Stock return characteristics, skew laws, and differential pricing of individual equity options. Review of Financial Studies 16, 101–143.

Brenner, M., Galai, D., 1989. New financial instruments for hedging changes in volatility. Financial Analyst Journal (July/August), 61–65.

Brenner, M., Galai, D., 1993. Hedging volatility in foreign currencies. Journal of Derivatives 1, 53–59.

Brenner, M., Subrahmanyam, M., 1988. A simple formula to compute the implied standard deviation. Financial Analysts Journal, 80–82.

Buraschi, A., Jackwerth, J., 2001. The price of a smile: Hedging and spanning in option markets. Review of Financial Studies 14, 495–527.

Carr, P., Madan, D., 1998. Towards a theory of volatility trading. In: Jarrow, R. (Ed.), Volatility: New Estimation Techniques for Pricing Derivatives. Risk Books, London, pp. 417–427.

Coval, J., Shumway, T., 2001. Expected option returns. Journal of Finance 56, 983–1009.

Cox, J., Rubinstein, M., 1985. Appendix 8A: An Index of Option Prices, Options Markets. Prentice-Hall, New Jersey.

Cox, J.C., Ingersoll, J.E., Ross, S.A., 1985. A theory of the term structure of interest rates. Econometrica 53, 385–408.

Derman, E., Kamal, M., Kani, I., Zou, J., 1996. Valuing Contracts with Payoffs Based on Realized Volatility. Goldman Sach & Co. Manuscript.

Detemple, J., Osakwe, C., 2000. The valuation of volatility options. European Finance Review 4, 21–50.

Galai, D., 1979. A proposal for indexes for traded call options. Journal of Finance 34, 1157–1172.

Gastineau, G.L., 1977. An index of listed option premiums. Financial Analysts Journal 34, 1157–1172.

Geske, R., 1979. The valuation of compound options. Journal of Financial Economics 7, 63–81.

Grünbichler, A., Longstaff, F., 1996. Valuing futures and options on volatility. Journal of Banking and Finance 20, 985–1001.

Hull, J., White, A., 1987. The pricing of options on assets with stochastic volatilities. Journal of Finance 42, 281–300.

Longstaff, F.A., Schwartz, E.S., 1992. Interest rate volatility and the term structure. The Journal of Finance 47, 1259–1282.

Stein, E.M., Stein, J.C., 1991. Stock price distributions with stochastic volatility: An analytic approach. Review of Financial Studies 4, 727–752.