Dynamic Demand for New and Used Durable Goods without Physical Depreciation

by

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Abstract

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This thesis studies the interaction between new and used durable goods without physical depreciation. In product categories such as CDs/DVDs and video games, the competition from used goods markets has been viewed as a serious problem by producers. These products physically depreciate negligibly, but owners' consumption values could depreciate quickly due to satiation. Consequently, used goods that are almost identical to new goods may become available immediately after a new product release. However, the existence of used goods markets also provides consumers with a selling opportunity. If consumers are forward-looking and account for the future resale value of a product in their buying decision, used goods markets could increase the sales of new goods. Thus, whether used good markets are harmful or beneficial to new-good producers is an empirical question.

To tackle this question, I extend the previous literature in three ways. First, I assemble a new data set from the Japanese video game market. This unique data set includes not only the sales and prices of new and used goods, but also the resale value of used copies, the quantity of used copies retailers purchased from consumers, and the inventory level of used copies at retailers. Second, I develop a structural model of forward-looking consumers that incorporates (i) new and used goods buying decisions, (ii) used goods selling decisions, (iii) consumer expectations about future prices of new and used goods as well as resale values of used goods, and (iv) the depreciation of both owners' and potential

buyers' consumption values. Third, I develop a new Bayesian estimation method to estimate my model. In particular, my method can alleviate the computational burden of estimating non-stationary discrete choice dynamic programming models with continuous state variables that evolve stochastically over time.

The estimation results suggest that consumers are forward-looking in the Japanese video game market and the substitutability between new and used video games is quite low. Using the estimates, I quantify the impact of eliminating the used video game market on new-game revenues. I find that the elimination of used video game market could reduce the revenue for a new game.

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Introduction

The existence of used goods markets has been viewed as a serious problem by producers in categories such as books, CDs/DVDs, and video games. Producers and their supporters argue that used goods retailing significantly lowers their profits and limits the development of new products. For instance, book publishers and authors expressed their annoyance to Amazon over used books sold on its websites (Tedeschi 2004). Video game publishers in Japan attempted to kill off used video game retailing by suing used video game retailers (Hirayama 2006). Their main argument is that products like books and video games physically depreciate negligibly, but owners' consumption values can decline very quickly due to satiation. As a result, unlike products that physically depreciate more considerably (such as cars), producers of books, video games, and other digital products may face competition from used goods that are almost identical to new goods immediately after a new product release.

However, their argument focuses only on one aspect of used goods markets (substitution effect). The existence of used goods markets provides consumers with a selling opportunity. If consumers are forward-looking and account for the future resale value at the time of making a buying decision, the effective price consumers pay for a product is lower than the price of the product (resale effect). This feature implies that the existence of used goods markets could increase the sales of new goods. Thus, whether the existence of used goods markets hurts or benefits new-good producers is an empirical question, and the answer depends on which effect, substitution or resale effect, dominates.

To tackle this question, I extend the previous literature in three dimensions. First, I assemble a new data set from the Japanese video game market, which includes weekly aggregate level data for 41 video game titles released in Japan between 2004 and 2008. The novel aspect of this data set is that in addition to the sales and price of new and used video game copies, it also includes the resale value of used copies,[1] the quantity of used copies retailers purchased from consumers, and the inventory level of used copies at retailers. Previous studies on new and used durable goods have not exploited these new dimensions before. This new data set allows me to significantly advance our knowledge about used goods market trading. In particular, the data set includes necessary variation for evaluating whether consumers are forward-looking, a factor that is crucial in my analysis.

Second, I develop a new empirical structural framework that utilizes the new data set. In my model, the demand for new goods and the demand for and the supply of used goods are generated by a dynamic discrete choice model of forward-looking consumers that incorporates (i) new and used goods buying decisions, (ii) used goods selling decision, (iii) consumer expectations about future prices of new and used goods and resale values of used goods, and (iv) the depreciation of both owners' and potential buyers' consumption values. To the best of my knowledge, this is the first dynamic model of demand that incorporates these four important features of consumer decision making in durable goods markets. In my model, the expected present discounted value of future payoffs from purchasing a product is determined by the dynamic consumer selling decision problem, which depends on the depreciation rate of owners' consumption values and future resale values. Furthermore, although the previous literature has identified consumer expectations about future prices as a critical element in dynamic durable goods purchase decisions (e.g., Nair 2007, Gordon 2009), this is the first study that uses data

[1] In this paper, resale value is defined as the amount of money consumers receive when they sell their used video games to retailers.

on resale values and studies the role of consumer expectations about future resale values in a dynamic durable good purchase decision. Finally, my model allows owners' consumption values and potential buyers' consumption values to depreciate differently over time. In the previous literature, the depreciation rate is always assumed to be identical for owners and buyers (e.g., Esteban and Shum 2007, Chen et al. 2010). However, this feature is imperative in product categories such as CDs/DVDs and video games because for product owners, the depreciation of consumption values is mainly due to satiation (satiation-based depreciation). Thus, potential buyers who have not been satiated at all could derive a higher consumption value than owners who are satiated. But for potential buyers, the freshness of the product may disappear as a product ages, causing consumption values to depreciate (freshness-based depreciation). My new data set allows me to measure these two concepts of depreciation separately.

Third, in order to estimate my model, I extend the Bayesian Markov chain Monte Carlo (MCMC) algorithm proposed by Imai, Jain and Ching (2009a) (IJC algorithm) to a non-stationary discrete choice dynamic programming model. Similar to the IJC algorithm, this new algorithm solves and estimates the model simultaneously. In conventional approaches to estimating a finite-horizon model, value functions need to be computed at all or a subset of pre-determined grid points in all time periods by backward induction. However, when a model has multiple continuous state variables that evolve stochastically, computing value functions at a large enough number of grid points can be computationally very demanding. My new algorithm alleviates the computational burden by partially solving value functions at only one randomly drawn state in each time period, storing them, and then approximating the expected future values by the weighted average of those partially solved value functions evaluated at different states in past iterations. I then combine the proposed algorithm with the pseudo-policy function approach (Ching 2010a; 2010b) to address potential price endogeneity problems. The method I propose augments the unobserved shocks based on the joint-likelihood of the demand-side model

and the pseudo-policy functions. Unlike the standard GMM approach (Berry et al. 1995), there is no inner loop for inverting the market share to recover the mean utility level. Also, unlike the simulated maximum likelihood method, I do not need to integrate out the unobserved shocks during the estimation.

I apply this new empirical structural framework to the Japanese video game market. The estimation results suggest that consumers are forward-looking in the Japanese video game market, and that substitutability between new and used video games is very low. Using the estimated model, I quantify the impact of eliminating the used video game market on new-game sales and revenues. On average, the elimination of the used video game market shifts down the demand for new copies of video games in the earlier part of the product lifecycle. This effect is mainly driven by the resale effects that dominate the substitution effects in the earlier part of video games' lifecycle. In the next experiment, I re-simulate the model by computing the optimal prices for new copies of video games. I find that under the optimal prices, the elimination of the used video game market increases the revenues for new games significantly.

The rest of the thesis is organized as follows. Chapter 1 presents a descriptive analysis of the Japanese video game market. I describe the new data set, and provide a descriptive analysis. Chapter 2 proposes an empirical framework for studying the dynamic demand for new and used durable goods with consumption satiation. I first describe the discrete choice dynamic programming model of consumer buying and selling decisions. Then, I explain how one can estimate the model. In particular, I describe the new Bayesian MCMC algorithm for non-stationary discrete choice dynamic programming models with continuous state variables that evolve stochastically. I show some Monte Carlo evidence using a simple dynamic model. I also discuss the identification issue. Chapter 3 presents a structural analysis of the Japanese video game market. I apply the empirical framework proposed in Chapter 2 to the Japanese video game market, and examine the impact of the used good market on the new good market.

Chapter 1

A Descriptive Analysis of the Japanese Video Game Market

1.1 Introduction

One of the important contributions of this thesis is to assemble a new data set that captures new and used video game trading. This data set contains not only the sales and price of new and used goods, but also the resale value of used goods, the quantity of used copies retailers bought from consumers, and the inventory of used copies at retailers. This chapter presents the new data set and conducts a descriptive analysis. Before presenting the data set, I will first briefly discuss the previous studies on new and used goods markets, and describe the advantages of my new data set.

Previous literature on new and used durable goods have largely focused on car and housing markets. For example, Esteban and Shum (2007) build a dynamic equilibrium model of durable goods oligopoly and study the impact of car durability and used car markets on equilibrium car manufacturers' behavior. Chen et al. (2010) extend Esteban and Shum (2007) and allow for transaction costs. Schiraldi (2010) estimates a dynamic discrete choice model of automobile replacement decisions, and studies the impact of

scrappage subsidies. Tanaka (2009) studies the market power of condominium developers in Tokyo in a dynamic durable goods oligopoly model.

One common feature of the models in these studies is that the depreciation rate of consumption values is assumed to be common across potential buyers and product owners. This assumption is motivated by two challenges: (i) when physical depreciation is present, it is difficult to separately measure the decline of owners' consumption values due to satiation from that due to physical depreciation; (ii) even if researchers can control for physical depreciation, they still need to observe both consumer buying and selling decisions in order to separately measure the depreciation of owners' consumption values and potential buyers' consumption values. The data sets used in these studies include only time-series variation in the quantities sold and price for new and used goods. They lack crucial information on the quantities sold by consumers to retailers and the associated resale values. Time-series variation in the price of new and used goods alone is not sufficient for disentangling the depreciation rates of owners' consumption values and potential buyers' consumption values even in the absence of physical depreciation. These challenges have limited previous studies, forcing them to assume that consumption value depreciation is identical for product owners and potential buyers.

Another important difference from these studies is that these papers assume that consumers are forward-looking and use the interest rate to calibrate consumers' discount factor. In my research, whether or not consumers are forward-looking is a crucial factor that determines the potential net benefit of eliminating a used goods market for producers. If consumers are myopic, the future selling opportunity will have no impact on the purchase decision today and eliminating used goods markets is potentially beneficial to producers. Thus, it is important to investigate whether consumers are forward-looking. Important variation that allows me to investigate is the resale value of used copies and the declining rate of video game sales. Thus, by examining how consumer expectation about the future resale value affects the current demand for video games, I will be able

to identify whether consumers are forward-looking. This identification strategy is similar to Chevalier and Goolsbee (2009), who study whether students are forward-looking in their textbook purchase decision. However, they do not observe whether students actually keep the textbook or sell it at the end of each semester. Thus, the resale value of a textbook is assumed to affect all students' utility for buying a textbook. In my research, I observe how many owners sell their games to retailers and at what resale value. Thus, I can model the expected future payoff from buying a video game as a function of not only the future resale value, but also the value of keeping the video game. This allows me to better control for the impact of the resale value on buying decisions.

Now I turn to discuss my new data set. All the data are taken from publicly available sources, which are archived at the National Diet Library in Tokyo, Japan. As the data pertain to the Japanese video game market, I begin the discussion by providing some background on the Japanese video game industry.

1.2 Japanese video game industry

The video game industry in Japan took off in the mid-80s, and since then, the industry has been growing rapidly. The size of the industry in 2009 had reached $5.5 billion on a revenue basis (including sales of hardware, software, other equipments). This is about three times larger than the theatrical movie revenue in Japan, making it one of the most important sectors in the Japanese entertainment industry.[1]

Used video game retailing in Japan has been an issue for video game publishers since 90s. In 1998, several video game publishers sued used video game retailers for used video game retailing and attempted to completely kill off the used video game market. However, they lost the lawsuit in 2002 and used video game retailing continues to thrive today. In 2009, the sales of used video game software alone amounts to $1.0 billion on a

[1] In the US, the size of the video game industry has grown to $20.2 billion in 2009, which is now twice as large as the theatrical movie revenues.

revenue basis.

No clear explanation has been provided for why the used video game market has been historically large in Japan as compared to North America. Although this question is beyond the scope of this thesis, I will provide two plausible answers that come from the differences between Japanese and North American markets. One reason could be that unlike North America, video game renting by third-party companies is prohibited by law in Japan.[2] Another reason documented in Hirayama (2006) is the flat-pricing strategy commonly adopted by video game publishers - the price of new games is maintained at the Manufacturer's Suggested Retail Price (MSRP) at least one year after the release.[3] In contrast, price-skimming is a common strategy for new games in North America. For example, Nair (2007) reports that on average, the price of new games dropped by 4.2% monthly in the U.S. during his sample period 1998-2000. However, it can also be argued that the used video game market has induced publishers in Japan to adopt the flat-pricing strategy. Liang (1999) uses a theoretical model to show that when used goods markets are present, durable goods monopolists may be able to credibly commit to a high price over time (avoiding the Coase conjecture).

1.3 Data description

I collected a data set of 41 video games that were released in Japan between 2004 and 2008. They are listed in Table 1.1. In what follows, I briefly describe each type of data and the source. I will use the following three video games when graphically showing the data pattern: i) Dragon Quest VIII: Journey of the Cursed King, ii) Pokémon Diamond and Pearl, and iii) Wii Sports. These three video games are chosen because they exhibit different sales and price patterns over time.

[2]In principle, video game publishers can run rental business for their own video games. However, only one publisher has attempted to operate it in the history and did not succeed and closed the operation.

[3]Note that in Japan, resale price maintenance is illegal for video games while it is still legal for books, magazines, newspapers and music.

For each video game, weekly aggregate sales of new copies and its manufacturer suggested retail price (MSRP) are obtained from the weekly top 30 sales ranking published in Weekly Famitsu Magazine.[4] In Figure 1.1, I plot the sales of new copies for the three video games, along with the average sales of new copies across 41 video games in the right-bottom sub-figure. On average, I observe the sales of new copies for 18 weeks. In general, all 41 video games exhibit a declining sales pattern, with the sharpest decline in the first few weeks. Any increases in the sales after release are driven by holiday seasons. In my data set, the median percentage of new game copies sold in the release week relative to the total sales after one year is 54%, and the median percentage of new game copies sold within the first month (4 weeks) after release is 82%. Thus, the sales of new copies is heavily concentrated within the first month. Although all games show a declining sales pattern, the declining rate varies across games. For example, while both Dragon Quest VIII and Pokémon Diamond and Pearl show a sharp decline in sales in the first few weeks, Wii Sports exhibits a moderate declining pattern even after accounting for the holiday season effect in weeks 4-6. In the next section, I will examine the declining rates in detail.

For the price of new copies, as mentioned above, it has been an industry practice in Japan for video game publishers to maintain the price at the initial level (MSRP). Table 1.1 shows the MSRP for each game. It ranges from JPY 3,800 ($\simeq$ USD 38) to JPY 9,240 ($\simeq$ USD 92). In general, console games are priced higher than handheld games.[5]

Next, weekly aggregate trading volumes of used copies at retailers (both buying and selling volumes), weekly average retail price of used copies, and weekly average retail resale value of used copies are taken from the Annual Video Game Industry Report

[4]Weekly Famitsu Magazine is a major weekly video game magazine in Japan published by Enterbrain, Inc. The first issue was published in June 1986. Enterbrain Inc. is a Japanese magazine publisher and a consulting company that focuses on video games and computer entertainment.

[5]In my data set, video games released on GameBoyAdvance, Nintendo DS, and PlayStation Portable are handheld games, and PlayStation 2, PlayStation 3, Nintendo GameCube, and Nintendo Wii are console games.

published by Media Create Co., Ltd.[6] The data are collected from retailers, where about 85% of used game trading took place during my sample period.[7] Weekly aggregate trading volumes of used copies consist of i) the quantity demanded for used copies (i.e., the quantity of used copies sold from retailers to consumers), and ii) the quantity supplied of used copies (i.e., the quantity of used copies sold from consumers to retailers). Used video game retailers maintain a list of resale values for games, and procure game copies from consumers who wish to sell their games at those resale values. Thus, used video game retailers maintain two prices: the price of used copies, which is the amount consumers pay to retailers when consumers buy a used copy, and the resale value of used copies, which is the amount retailers pay to consumers when they buy a used copy from consumers. On average, I observe the used game trading volumes and prices for 34 weeks.

Figure 1.2 shows the weekly trading volumes of used copies for the same set of three games and the average trading volume. First, more than half of video games in my data follow a pattern similar to Dragon Quest VIII, i.e., both quantities demanded and supplied start low, increase and reach the peak in about week 5, followed by a decline. This pattern is reflected in the right-bottom sub-figure for the average pattern. However, there are also video games that shows a relatively flat pattern after the first few weeks (see Pokémon Diamond and Pearl and Wii Sports). What is common across all video games is that both quantities demanded for and supplied of used copies are low in the first few weeks. One potential reason is because owners who bought a video game right after its release may still enjoy it in the first few weeks, and thus fewer used copies are supplied to retailers. However, as time goes by, more consumers become satiated and start selling their games. In reality, even if consumers wish to buy a used copy, they cannot do so unless used copies are made available by retailers. As few copies are supplied in the first

[6]The Annual Video Game Industry Report was first published in 1998, and since then, it has been the major video game industry report in Japan. Media Create Co. is a Japanese company that gathers and analyzes data from the digital entertainment industry, especially focusing on the video game market.

[7]Less than 10% of used game trading occurred on online auction websites during my sample period.

few weeks, it could be quite difficult for consumers to find a used copy at retailers. Thus, potential search costs associated with finding a used copy at retailers could drive down the quantity demanded for used copies in the first few weeks.

Second, for most of the 41 video games, I observe an excess supply of used copies, i.e., retailers buy many more used copies from consumers than they actually sell to consumers. The only exception among the 41 games is Wii Sports, which shows a very little excess supply. One might think that this excess supply is a simple flow issue; the quantity supplied in a week must always be larger than the quantity demanded at the time of data collection. Alternatively, it may be because retailers try to reduce the stockout and thus always carry a positive level of inventory. To see this more clearly, I compute the inventory level of used copies at retailers' shelves at the beginning of each week. This level is calculated by subtracting the cumulative quantity demanded for used copies from the cumulative quantity supplied of used copies. Figure 1.3 shows the weekly inventory level of used copies. By construction, the inventory level at the beginning of week 2 is zero for all video games. It can be seen that on average, the inventory of used copies at retailers keeps increasing at least up to week 20. For about a half of the 41 video games, the inventory reaches its peak and starts declining within the sample period (similar pattern to Dragon Quest VIII). There are a few video games that have a low level of inventory (similar to Wii Sports, but Wii Sports exhibits the lowest inventory level over time). The remaining games show an increasing pattern until the end of the sample period (similar to Pokémon Diamond and Pearl). These increasing patterns of inventory reject the hypotheses that the excess supply is either due to a flow issue or a simple stockout-avoiding behavior by retailers. For the latter, we can compare the level of inventory with the actual quantity demanded for used copies over time. Figure 1.5 plots the ratio of the number of used copies available at retailers to the actual quantity demanded for used copies in each week. For most of the video games, the quantity demanded starts to decline after weeks 5-10, yet the inventory level continues to increase. Consequently,

the ratio keeps growing at least until the inventory level starts to decline. Thus, a simple stockout-avoiding behavior by retailers cannot fully explain the data pattern for inventory, suggesting that contrary to the assumption of static perfect competition made in previous studies, used video game retailers exhibit forward-looking behavior when managing the inventory level. This is an interesting question to investigate. However, as the main focus of this thesis is the demand-side dynamics, I will only briefly discuss this issue as a future research direction in Chapter 2. In this thesis, I make use of this inventory level data as a proxy for the potential effect of used-copy availability on consumers' used-copy buying decision. As I discussed earlier, when the inventory level is low, consumers may have difficulty in finding a used copy at retailers even if they wish to buy a used copy rather than a new copy. I approximate this potential search cost associated with finding a used copy at retailers using the inventory level of used copies.

Finally, I compute the relative size of used goods markets to new goods markets based on the quantity demanded for new and used copies. It is calculated by the ratio of the cumulative used copy sales to the cumulative new copy sales at the end of the sample period. Note that I do not observe the sales of new copies for a video game once the video game drops out of the top 30 sales ranking. Thus, for some video games, the number of weeks for which I observe the sales of new copies is smaller than that for the sales of used copies. However, in any given week, I observe the upper-bound of new-copy sales (i.e., the sales of the 30th game in the ranking in each week), and the lower-bound (which is zero). For weeks in which I do not observe the sales of new copies, I apply the upper- and lower-bound sales to compute the cumulative sales of new copies. Table 1.1 shows the relative market size for each game (labeled as "size of used sales"). The average relative market size across the 41 video games falls in between 21% and 27%. This percentage is lower than the 30% industry-wide relative size of used to new goods markets reported in the Annual Video Game Industry Report between 2004 and 2008. However, since my data set contains at most one year of sales data, and that the sales of used copies typically

lasts longer than the sales of new copies, the actual relative market size is expected to be larger if the data set has a longer time-series. The most important observation here is that I have some variation in the relative size across the 41 video games. The smallest relative size is 4% while the largest relative size is between 48% and 77%. In my policy simulation, I eliminate the used video game markets and simulate the consumer demand for new copies. This simulation exercise could be over-stretching if the parameters were estimated using only a set of video games with a large relative market size of used to new game markets.

Next, I plot weekly prices and resale values of used copies in Figure 1.4. As a reference, I also plot the price of new copies. In general, both price and resale value of used copies decline over time for most of the 41 video games (Dragon Quest VIII, Pokémon Diamond and Pearl). However, there are a few video games for which price and resale value are relatively flat over time (Wii Sports). Second, although both price and resale value generally decline over time, the declining rates differ. The average weekly declining rate of the used-copy price is 1.1%, and the average weekly declining rate of the resale value is 2.1%. The initial price and resale value of used copies (i.e., in week 2) are on average 81.7% and 67.4% of the price of new copies, respectively. In week 20, those numbers become 64.2% and 39.6%. The difference in declining rates between the price and the resale value of used copies suggests that the depreciation rate of owners' consumption value could be different from that of potential buyers. In the next section, I will examine the declining rates in more detail.

In addition to the sales and price data of new and used copies, I also collected several video game characteristics from Weekly Famitsu Magazine and Famitsu Game Hakusho,[8] including average critic rating, average user rating, dummies for handheld game, sequel games, story-based games,[9] and multi-player games. The average critic rating is based

[8]Famitsu Game Hakusho is an annual video game industry report published by Enterbrain, Inc. It was first published in 2005

[9]Story-based games (e.g., role-playing games) have an ending of the story, while non-story-based

on Weekly Famitsu Magazine's "Cross Review" ratings, where four video game critics examine each video game and provide a 10-point scale rating one week prior to the video game release. The average user rating is based on a consumer survey conducted by Enterbrain, Inc. and is standardized against a set of video games that were released in the same year by Enterbrain, Inc.

Finally, the potential market size for a video game is measured by the installed base of the platform on which the video game was released. The platforms of the 41 games include four consoles (PlayStation 2, PlayStation 3, Nintendo GameCube, Nintendo Wii) and three handhelds (GameBoyAdvance, Nintendo DS, PlayStation Portable). I collected weekly sales of all seven platforms above from their release week to calculate the cumulative sales.

Table 1.2 summarizes some of the important statistics about the data I described above.

1.4 Descriptive analysis

In this section, I use a simple regression and briefly summarize the data pattern for i) sales of new copies, ii) sales of used copies, iii) relative size of used to new goods markets, and iv) price and resale value of used copies. The primary purpose is to explore the underlying patterns.

1.4.1 Sales of new copies

We have seen that the sales of new copies generally declines over time, with a sharp decline in the first few weeks. Also, the declining rate varies across video games. In this subsection, I will further investigate the declining rate using a simple regression. In particular, I look at the impact of observed product characteristics on the declining

games (e.g., sports games) have no clear definition for ending.

rates. Table 1.3 shows some regression results. The dependent variable is logged sales of new copies, and the independent variables include the price of new copies, holiday season dummies, observed product characteristics, the number of weeks in release, and the interactions between observed product characteristics and the number of weeks in release. I explore three specifications. Model 1 includes only the price of new copies, holiday season dummies, and the number of weeks in release as independent variables. Model 2 adds observed product characteristics to Model 1. Model 3 allows the coefficient on the number of weeks in release to depend on observed product characteristics. By examining the adjusted R-squared, I observe a reasonable improvement when I include the interaction terms between the number of weeks in release and observed product characteristics.

In Model 3, I find that handheld games, non-sequel games, non-story-based games, and multi-player games tend to have lower declining rates than their counterparts. Also, video games with a higher critic or user rating tend to have lower declining rates. As for sequel games, this finding is consistent with Haviv and Moorthy (2010), who examine the diffusion paths of sequel versus non-sequel movies and find that the box office revenues for sequel movies are more "front-loaded," i.e., the box office revenues are concentrated in the earlier part of movies' lifecycle rather than scattered over time. One reason behind this is because consumers may already know what they can expect from a sequel movie, and thus the movie-going decision depends less on the word-of-mouth that comes out after a movie release. Suggestive of word-of-mouth, I find that video games with a higher user rating tend to have lower declining rates. Although the average user rating is a proxy for word-of-mouth, it does seem to capture some post-release quality information. Note that in the regression, I also include the average critic rating, which is another quality measure. This critic rating information is available for consumers one week prior to a video game release (published in Weekly Famitsu Magazine). The coefficients on the linear terms for critic and user ratings suggest that an increase in the average critic

rating shifts up the initial sales of new copies, but an increase in the user rating makes the initial sales of new copies lower. However, the coefficients on the interaction terms suggest that an increase in the average user rating dampens the declining sales of new copies.

Another interesting observation is that story-based games tend to have higher declining rates than non-story-based games. Note that as I will show below, the resale value of used copies for story-based games tends to decline faster than that for non-story-based games. If consumers are forward-looking and expect that the resale value for story-based games declines quickly, they may purchase sooner so as to sell the game at a higher resale value. This behavior could make the declining rate of the sales of new copies higher. Thus, the finding is consistent with forward-looking consumers.

1.4.2 Sales of used copies

One important difference between the sales patterns of new and used copies is that the sales of used copies initially increases, and then decreases. As I argued above, the initial increase could be due to a shortage of used copies at retailers. Thus, in the analysis for the sales of used copies, I also include the inventory level of used copies as an independent variable to capture the potential search costs associated with buying a used copy. Other independent variables are similar to the analysis for new-copy sales. Table 1.4 shows the regression results. The dependent variable is logged sales of used copies. Again, I run three specifications. Model 1 includes only the price and inventory of used copies, holiday season dummies, and the number of weeks in release as independent variables. Model 2 adds observed product characteristics to Model 1. Model 3 allows the coefficient on the number of weeks in release to depend on observed product characteristics. The coefficient on used-copy price is positive in Models 1 and 2, but it became negative in Model 3 where I control for the impact of observed product characteristics on the declining rate of used-copy sales. The coefficient associated with the inventory level is positive and significant,

which helps explain the initial increase in the sales of used copies. One difference from the results in Table 1.3 is that neither critic nor user ratings have a significant impact on the declining rate. Also, the dummy for story-based games does not have a significant impact. One reason could be that the sales of used copies has an increasing and then decreasing pattern for most video games, and thus the linear specification in the number of weeks in release does not fully capture the declining rate.

1.4.3 Relative size of used to new goods markets

In the previous section, I briefly discussed the relative market size of used to new goods. It may be useful to examine how the relative size is related to observed product characteristics. Here, I will use a similar regression to the above analyses and investigate it. In Table 1.1, I reported the relative market size for each video game at the end of each video game's sample period for used video game trading. To show the evolution of the relative market size graphically, I plot the relative market size over time for the three video games and the average pattern in Figure 1.6. Recall that the relative market size is computed using the upper- and lower-bound of the sales of new copies for weeks in which I do not observe the sales of new copies. For Dragon Quest VIII, I observe the sales of new copies up to the 15th week while I observe the trading volumes of used copies up to the 27th week. Thus, from the 16th to 27th week, I use the upper- and lower-bound of new-copy sales to compute the cumulative sales of new copies. But Figure 1.6 shows that the difference between the upper- and lower-bound of the relative market size is very negligible for Dragon Quest VIII.

The regressions are based on the lower- and upper-bound of the relative market sizes as dependent variables. The first two columns (Models 1 and 2) of Table 1.5 present the results based on the lower-bound of the relative market size, and the next two columns (Models 3 and 4) present the results based on the upper-bound of the relative market size. Two results are worth noting here. First, story-based games tend to have a faster

growth of the relative market size. This result is in line with the steeper declining rate of the sales of new copies for story-based games. If the demand for used copies remains strong even after the sales of new copies becomes very small, the relative market size of used to new copies becomes higher and higher over time. Another observation here is that video games with a higher user rating tend to have a slower growth of the relative market size. One reason could be that better quality games tend to be enjoyed by consumers for a longer period, which limits the supply of used copies in the used good market. In the structural estimation, I allow observed product characteristics to influence the satiation-based depreciation to capture this.

1.4.4 Price and resale value of used copies

One interesting aspect of my data is that I observe not only the price of used copies, but also the resale value of used copies, which decline over time differently. In this subsection, I examine the price and resale value of used copies in two ways. First, I will investigate how the initial price and resale value of used copies are related to the price of new copies, along with observed product characteristics. I will then examine how the declining rates are related to observed product characteristics.

The first two columns in Table 1.6 show the regression results for the initial price and resale value of used copies. Most of the observed product characteristics are not significant, partly because the sample has only 41 video games. However, I find that critic ratings have a positive and significant impact on both initial price and resale value. This might not be surprising as retailers observe the critic ratings prior to the release of each video game, and could rely on the critic ratings to determine the initial price and resale value of used copies.

The third and fourth columns in Table 1.6 show the results based on the entire sample. Most of the observed product characteristics have significant impacts on the declining rates of both price and resale value of used copies. Handheld games and multi-player

games tend to decline slower, in contrast to sequel games and story-based games. Both critic and user ratings makes the declining rate lower, but user ratings are only significant for the price of used copies while critic ratings are only significant for the resale value of used copies. One might argue that the used-copy price of video games with a higher user rating declines at slower rates because they are demanded by consumers for longer periods. But the resale value may be influenced more by other product characteristics such as whether a video game is story-based. For those video games, consumers may be satiated quickly. Thus, even if a story-based video game has a high user rating, the resale value of its used copies can decline quickly.

1.5 Conclusion

This chapter introduces a new data set assembled from the Japanese video game market, and investigates the empirical regularities of (i) the sales of new and used copies, (ii) relative size of used to new video game markets, and (iii) the price and the resale value of used copies.

One important finding is that the inventory of used copies keeps increasing even after the quantity demanded for used copies starts to fall. This pattern suggests that the used video game market in Japan is not perfectly competitive in a static sense, i.e., the quantity demanded for used copies is equal to the quantity supplied of used copies in every time period. This finding is important as no prior study has examined the evolution of the inventory of used goods due to the data limitation.

In the descriptive analysis, I find that different observed product characteristics have different impacts on the declining rate of the sales of new and used copies and the price and resale value of used copies. For example, handheld games and multi-player games tend to have lower declining rates for the sales of both new and used copies as well as the price and resale value of used copies, while sequel games tend to have higher declining

rates for these four types of data. Also, I find that story-based games tend to have higher declining rates for the sales of new copies, and the price and the resale value of used copies, but not for the sales of used copies. Average critic and user rating tend to make the declining rate lower, but they do not have a significant impact on the declining rate of used-copy sales. All these observed product characteristics will be used in the structural analysis in Chapter 3 to control for the satiation-based depreciation for owners.

Furthermore, I examine the relative size of used to new video game markets. One important observation is that video games in my data set exhibit variation in the relative market size, which helps me investigate the impact of the used video game market on new-copy sales and revenues. The regression results suggest that story-based games tend to have a faster growth in the relative size of used to new video game markets than non-story-based games, consistent with the faster declining rates of the price and resale value of used copies for story-based games. Also, games with a higher average user rating tend to have a slower growth.

This new data set allows me to develop a new empirical framework for studying the dynamic demand for new and used durable goods, which will be the subject of Chapter 2.

Table 1.1: List of video game titles

Game ID	Game Title	Publisher	Platform	Release Date	MSRP in JPY	Size of Used Sales	
						Upper bound	Lower bound
1	Dragon Quest V: Hand of the Heavenly Bride	SQUARE-ENIX	PS2	3/25/04	8,190	0.24	0.23
2	Samurai Warriors	KOEI	PS2	2/11/04	7,140	0.27	0.24
3	Onimusha 3	CAPCOM	PS2	2/26/04	7,329	0.35	0.30
4	Dragon Ball Z2	BANDAI-NAMCO	PS2	2/05/04	7,140	0.30	0.25
5	Grand Theft Auto: Vice City	CAPCOM	PS2	5/20/04	7,140	0.39	0.32
6	Pokémon FireRed and LeafGreen	NINTENDO	GBA	1/29/04	5,040	0.04	0.04
7	Dragon Quest VIII: Journey of the Cursed King	SQUARE-ENIX	PS2	11/27/04	9,240	0.18	0.17
8	Dynasty Warriors 4	KOEI	PS2	2/24/05	7,140	0.23	0.22
9	Metal Gear Solid 3: Snake Eater	KONAMI	PS2	12/16/04	7,329	0.30	0.27
10	Dragon Ball Z3	BANDAI-NAMCO	PS2	2/10/05	7,140	0.27	0.24
11	Gran Turismo 4	SCE	PS2	12/28/04	7,665	0.15	0.14
12	Dynasty Warriors	KOEI	PSP	12/16/04	5,544	0.27	0.21
13	Resident Evil 4	CAPCOM	GC	1/27/05	8,190	0.33	0.22
14	Pokémon Emerald	NINTENDO	GBA	9/16/04	4,800	0.05	0.05
15	Super Mario 64 DS	NINTENDO	DS	12/02/04	4,800	0.06	0.06
16	Final Fantasy XII	SQUARE-ENIX	PS2	3/16/06	8,990	0.26	0.23
17	New Super Mario Bros.	NINTENDO	DS	5/25/06	4,800	0.11	0.11
18	Animal Crossing: Wild World	NINTENDO	DS	11/23/05	4,800	0.09	0.09
19	Kingdom Hearts II	SQUARE-ENIX	PS2	12/22/05	7,770	0.28	0.22
20	Winning Eleven 10	KONAMI	PS2	4/27/06	7,329	0.26	0.21
21	Yakuza	SEGA	PS2	12/08/05	7,140	0.77	0.48
22	Monster Hunter Freedom	CAPCOM	PSP	12/01/05	5,040	0.20	0.20
23	Gundam Seed: O.M.N.I. vs Z.A.F.T.	BANDAI-NAMCO	PS2	11/17/05	7,140	0.46	0.24
24	Dragon Quest Monsters: Joker	SQUARE-ENIX	DS	12/28/06	5,040	0.41	0.34
25	Monster Hunter Freedom 2	CAPCOM	PSP	2/22/07	5,229	0.19	0.19
26	Pokémon Diamond and Pearl	NINTENDO	DS	9/28/06	4,800	0.05	0.05
27	Yakuza 2	SEGA	PS2	12/07/06	7,140	0.65	0.37
28	Wii Sports	NINTENDO	Wii	12/02/06	4,800	0.11	0.11
29	Professor Layton and the Curious Village	LEVEL5	DS	2/15/07	4,800	0.32	0.32
30	Dragon Quest Swords: The Masked Queen and the Tower of Mirrors	SQUARE-ENIX	Wii	7/12/07	6,800	0.48	0.34
31	Mario Party 8	NINTENDO	Wii	7/26/07	5,800	0.16	0.16
32	Crisis Core: Final Fantasy VII	SQUARE-ENIX	PSP	9/13/07	6,090	0.23	0.20
33	Dynasty Warriors: Gundam	BANDAI-NAMCO	PS3	3/01/07	7,800	0.50	0.20
34	Monster Hunter Freedom Unite	CAPCOM	PSP	3/27/08	4,800	0.25	0.24
35	Super Smash Bros. Brawl	NINTENDO	Wii	1/31/08	6,800	0.22	0.20
36	Dragon Quest V: Hand of the Heavenly Bride	SQUARE-ENIX	DS	7/17/08	5,490	0.27	0.24
37	Mario Kart Wii	NINTENDO	Wii	4/10/08	5,800	0.12	0.12
38	Professor Layton and the Diabolical Box	LEVEL5	DS	11/29/07	4,800	0.32	0.25
39	Rhythm Heaven Gold	NINTENDO	DS	7/31/08	3,800	0.12	0.12
40	Winning Eleven 2008	KONAMI	PS3	11/22/07	7,980	0.39	0.17
41	Persona 4	ATLUS	PS2	7/10/08	7,329	0.31	0.19

Notes:

1. Platform: PS2 - PlayStation 2, PS3 - PlayStation 3, GC - Nintendo GameCube, Wii - Nintendo Wii, GBA - GameBoyAdvance, DS - Nintendo DS, PSP - PlayStation Portable

2. Size of used sales is defined as a ratio of the cumulative used copy sales to the cumulative new copy sales at the end of the video game's sample period for used copy sales. For some games, I do not observe new copy sales for as many weeks as used copy sales, thus I use the upper- and lower-bound of new copy sales (which I observe) to create the upper- and lower-bound of the size of used sales.

Table 1.2: Summary statistics

Variables	Average	S.D.	Max	Min
Price of new copies (in JPY)	6,388.6	1,381.7	9,240	3,800
Price of used copies (in JPY)	4,154.0	931.3	7,433	1,461
Resale value of used copies (in JPY)	2,724.5	963.2	6,547	813
Sales of new copies	68,312.1	165,143.5	2,236,881	2,772
Sales of used copies	7,135.6	5,497.3	62,734	18
Quantity sold by owners	8,486.0	7,314.0	55,830	105
Inventory of used copies at retailers	38,096.2	42,260.0	208,776	31
Market size (installed base)	12,418,879.6	7,051,686.4	25,391,982	247,429
Weekly # new game introduction	6.91	3.94	17	0
Dummy for handheld games	0.39	0.49	1	0
Dummy for sequel games	0.93	0.26	1	0
Dummy for story-based games	0.63	0.49	1	0
Dummy for multiplayer games	0.56	0.5	1	0
Critic rating (in 10-point scale)	8.77	0.68	10	7
User rating*	54.6	10.6	72	18

Notes: USD 1 ≈ JPY 100

* User rating is a standardized score againg a set of video games released in the same year (by Enterbrain, Inc.)

Table 1.3: Regressions for sales of new copies

Dependent Variable: Logged sales of new copies Independent Variables	Model 1	Model 2	Model 3
New-copy price	-2.21e-4***	-5.07e-4***	-6.03e-4***
	(3.37e-5)	(6.77e-5)	(6.46e-5)
Dummy for handheld games		-0.719***	-1.02***
		(0.156)	(0.192)
Dummy for sequel games		-0.152	1.06***
		(0.153)	(0.216)
Dummy for story-based games		0.141	0.389**
		(0.096)	(0.144)
Dummy for multi-player games		0.509***	-0.037
		(0.105)	(0.150)
Critic rating		0.460***	0.388***
		(0.069)	(0.103)
User rating		0.011*	-0.021**
		(0.005)	(0.007)
Weeks in release	-0.054***	-0.063***	-0.371***
	(0.004)	(0.004)	(0.058)
Dummy for handheld games			0.033***
× Weeks in release			(0.009)
Dummy for sequel games			-0.093***
× Weeks in release			(0.014)
Dummy for story-based games			-0.018*
× Weeks in release			(0.009)
Dummy for multi-player games			0.069***
× Weeks in release			(0.012)
Critic rating			0.018*
× Weeks in release			(0.007)
User rating			2.57e-3***
× Weeks in release			(4.83e-4)
Dummy for holiday seasons	1.12***	1.07***	1.22***
	(0.133)	(0.128)	(0.114)
Constant	12.1***	9.40***	12.0***
	(0.223)	(0.566)	(0.819)
Adjusted R-squared	0.299	0.376	0.508
# observations	742	742	742

Notes: Standard errors are in parentheses; * p<0.05, ** p<0.01, *** p<0.001

Table 1.4: Regressions for sales of used copies

Dependent Variable: Logged sales of used copies Independent Variables	Model 1	Model 2	Model 3
Used-copy price	1.18e-4***	5.68e-4*	-1.12e-4***
	(2.45e-5)	(2.75e-5)	(3.05e-5)
Logged inventory of used copies	0.179***	0.190***	0.174***
	(0.008)	(0.009)	(0.009)
Dummy for handheld games		0.085	-0.457***
		(0.065)	(0.080)
Dummy for sequel games		-0.363***	0.318*
		(0.067)	(0.126)
Dummy for story-based games		-0.263***	-0.179*
		(0.048)	(0.084)
Dummy for multi-player games		-0.022	-0.384***
		(0.049)	(0.087)
Critic rating		0.307***	0.361***
		(0.031)	(0.055)
User rating		-0.005*	-0.002
		(0.002)	(0.004)
Weeks in release	-0.015***	-0.022***	-0.023
	(0.002)	(0.002)	(0.023)
Dummy for handheld games			0.022***
× Weeks in release			(0.003)
Dummy for sequel games			-0.030***
× Weeks in release			(0.005)
Dummy for story-based games			-0.003
× Weeks in release			(0.004)
Dummy for multi-player games			0.021***
× Weeks in release			(0.004)
Critic rating			2.86e-4
× Weeks in release			(2.41e-3)
User rating			-1.28e-5
× Weeks in release			(1.69e-4)
Dummy for holiday seasons	0.109	0.105	0.151*
	(0.068)	(0.128)	(0.064)
Constant	6.76***	5.06***	5.16***
	(0.156)	(0.278)	(0.481)
Adjusted R-squared	0.272	0.337	0.399
# observations	1384	1384	1384

Notes: Standard errors are in parentheses; * p<0.05, ** p<0.01, *** p<0.001

Table 1.5: Regressions for relative market size of used to new goods

Dependent Variable:	Lower-bound of relative market size		Upper-bound of relative market size	
Independent Variables	Model 1	Model 2	Model 3	Model 4
New-copy price	2.96e-5***	1.66e-5***	1.68e-5***	1.08e-5**
	(4.45e-6)	(4.37e-6)	(3.19e-6)	(3.19e-6)
Used-copy price	-2.38e-5***	6.71e-6	-2.35e-5***	-1.38e-5***
	(4.00e-6)	(4.46e-6)	(2.86e-6)	(3.25e-6)
Dummy for handheld games	-0.34**	0.012	-0.024**	-0.053***
	(0.011)	(0.014)	(0.008)	(0.010)
Dummy for sequel games	-0.061***	-0.072***	-0.056***	-0.037**
	(0.010)	(0.018)	(0.007)	(0.013)
Dummy for story-based games	0.026***	-0.062***	0.015**	-0.032***
	(0.007)	(0.012)	(0.005)	(0.008)
Dummy for multi-player games	-0.052***	-0.047***	-0.052***	-0.041***
	(0.007)	(0.012)	(0.005)	(0.009)
Critic rating	-0.010*	-0.018*	-7.58e-4	-0.015**
	(0.005)	(0.008)	(0.003)	(0.006)
User rating	-9.74e-5	0.001	8.77e-5	0.001***
	(2.83e-4)	(4.99e-4)	(2.03e-4)	(3.64e-4)
Weeks in release	0.006***	0.008**	0.004***	0.001
	(2.60e-4)	(0.003)	(1.85e-4)	(0.002)
Dummy for handheld games		-0.003***		0.001***
× Weeks in release		(4.37e-4)		(3.64e-4)
Dummy for sequel games		9.71e-4		-8.73e-4
× Weeks in release		(6.70e-4)		(4.89e-4)
Dummy for story-based games		0.005***		0.003***
× Weeks in release		(5.38e-4)		(3.92e-4)
Dummy for multi-player games		-7.79e-4		-8.15e-4
× Weeks in release		(5.97e-4)		(4.35e-4)
Critic rating		2.79e-4		7.54e-4**
× Weeks in release		(3.33e-4)		(2.43e-4)
User rating		-1.00e-4***		-8.26e-5***
× Weeks in release		(2.40e-5)		(1.75e-5)
Dummy for holiday seasons	-0.029**	-0.045***	-0.027***	-0.035***
	(0.009)	(0.009)	(0.007)	(0.006)
Constant	0.139**	0.108	0.142***	0.206***
	(0.040)	(0.067)	(0.029)	(0.049)
Adjusted R-squared	0.608	0.654	0.608	0.641
# observations	1384	1384	1384	1384

Notes: Standard errors are in parentheses; * p<0.05, ** p<0.01, *** p<0.001

Table 1.6: Regressions for prices and resale values of used copies

	Initial period		Entire sample	
Dependent Variable:	Used-copy price	Resale value	Used-copy price	Resale value
Independent Variables	Model 1	Model 2	Model 3	Model 4
New-copy price	0.737***	0.638***	0.453***	0.270***
	(0.046)	(0.077)	(0.022)	(0.026)
Inventory of used copies			-0.009***	-0.011***
			(4.51e-4)	(5.24e-4)
Dummy for handheld games	43.6	22.5	-485.3***	-437.5***
	(116.7)	(197.5)	(71.7)	(83.8)
Dummy for sequel games	-128.7	166.1	158.0	395.7***
	(116.7)	(197.5)	(94.5)	(110.4)
Dummy for story-based games	8.25	72.9	101.0	132.7
	(71.3)	(120.5)	(62.3)	(72.8)
Dummy for multi-player games	51.3	200.6	-150.3*	54.7
	(69.2)	(117.0)	(64.8)	(75.7)
Critic rating	132.8*	210.3*	116.6**	127.1**
	(48.8)	(82.5)	(41.1)	(48.0)
User rating	-8.27**	-8.22	-2.00	8.61**
	(2.89)	(4.89)	(2.67)	(3.12)
Weeks in release			-105.5***	-105.4***
			(16.7)	(19.5)
Dummy for handheld games			50.1***	55.0***
× Weeks in release			(2.51)	(2.93)
Dummy for sequel games			-28.2***	-39.5***
× Weeks in release			(3.56)	(4.16)
Dummy for story-based games			-14.0***	-18.1***
× Weeks in release			(2.89)	(3.38)
Dummy for multi-player games			31.4***	20.9***
× Weeks in release			(3.23)	(3.78)
Critic rating			2.43	7.19**
× Weeks in release			(1.80)	(2.10)
User rating			0.825***	0.283
× Weeks in release			(0.127)	(0.147)
Dummy for holiday seasons			199.1***	359.9***
			(46.8)	(54.6)
Constant	-156.1	-1491.4	1435.3***	275.7***
	(406.0)	(686.7)	(354.6)	(414.1)
Adjusted R-squared	0.974	0.915	0.754	0.699
# observations	41	41	1384	1384

Notes: Standard errors are in parentheses; * p<0.05, ** p<0.01, *** p<0.001

Figure 1.1: Weekly sales of new copies

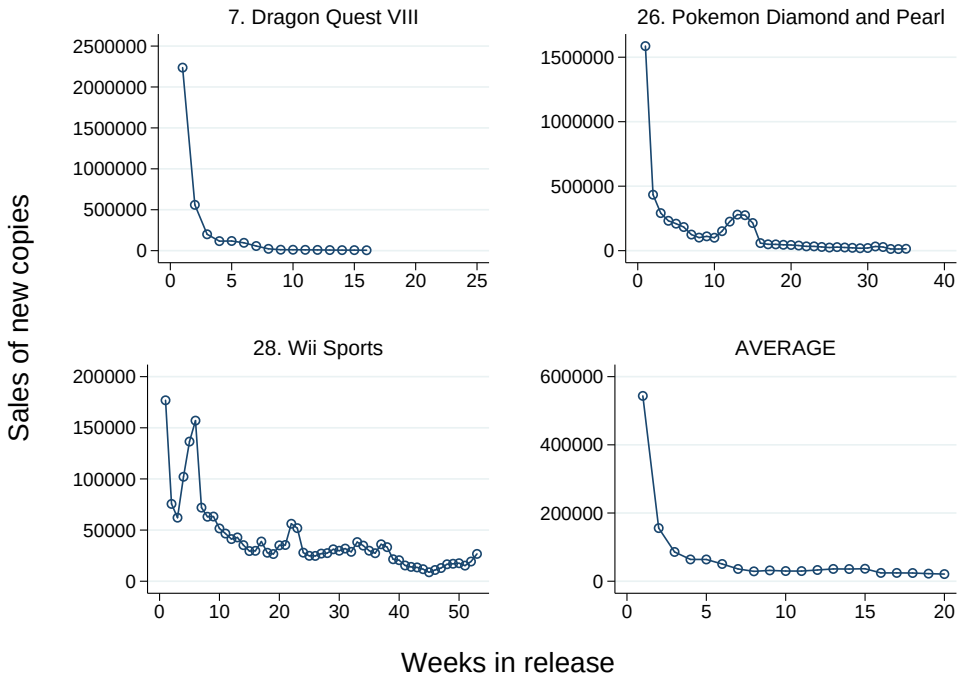


Figure 1.2: Weekly trading volumes of used copies

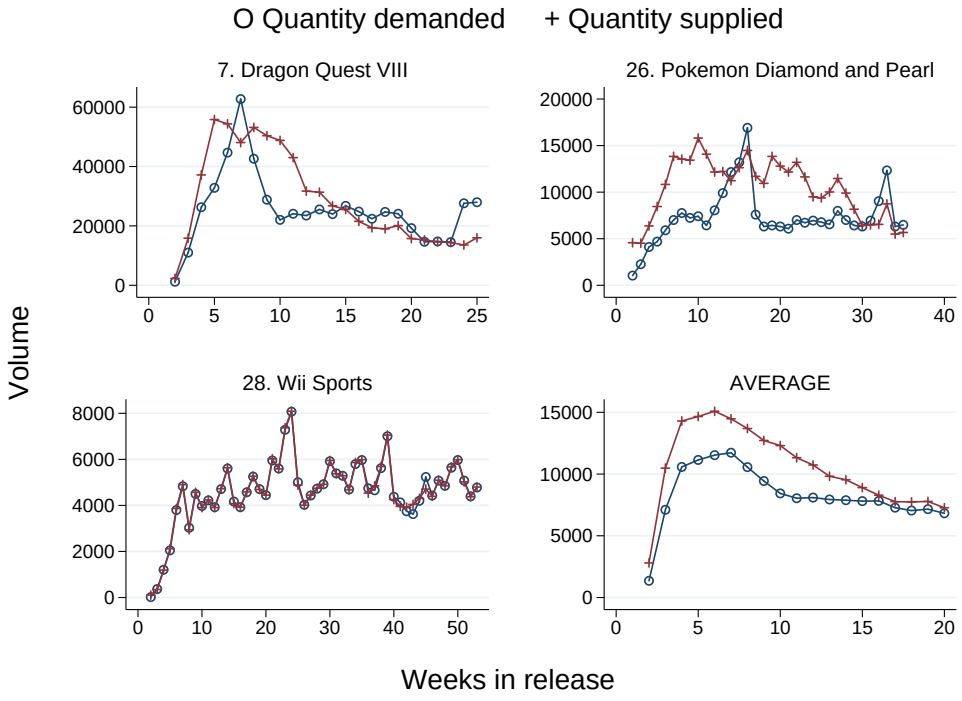


Figure 1.3: Weekly inventory of used copies

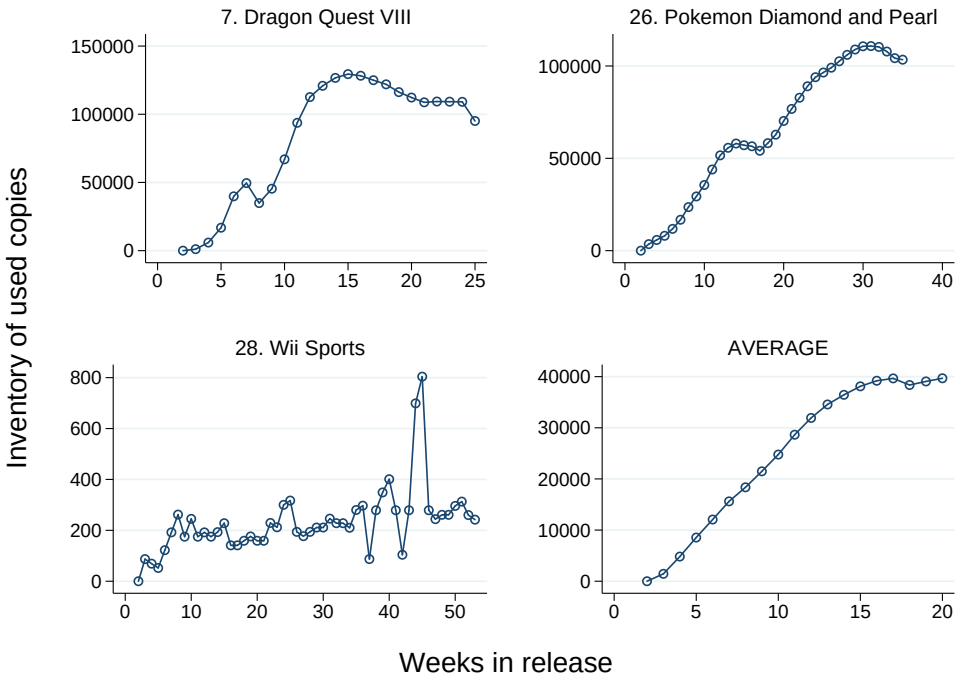


Figure 1.4: Weekly price of new and used copies and weekly resale value of used copies

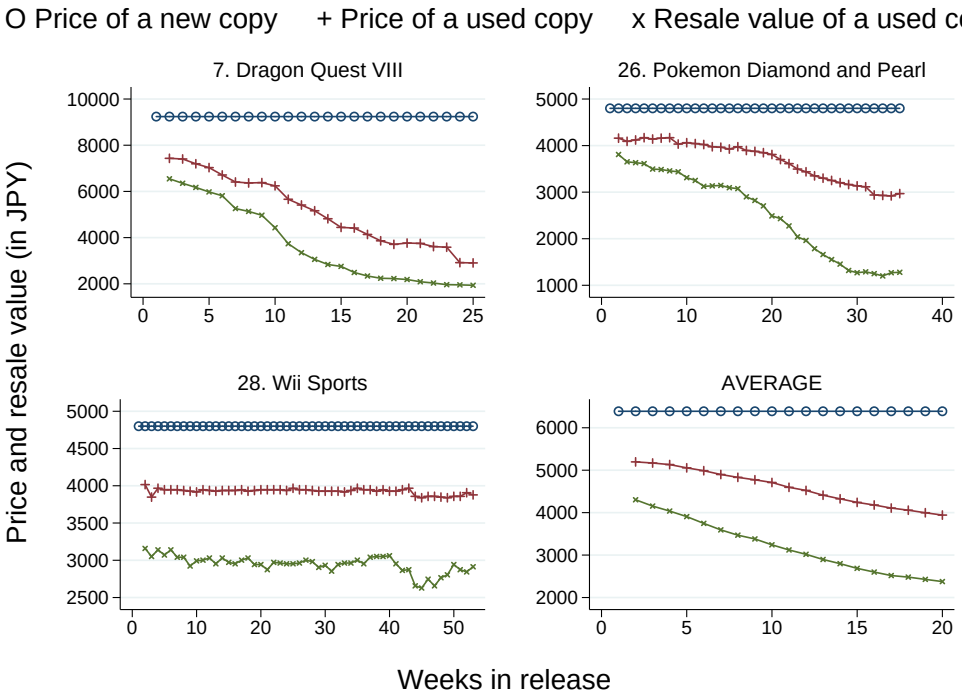


Figure 1.5: Size of inventory relative to quantity demanded for used copies

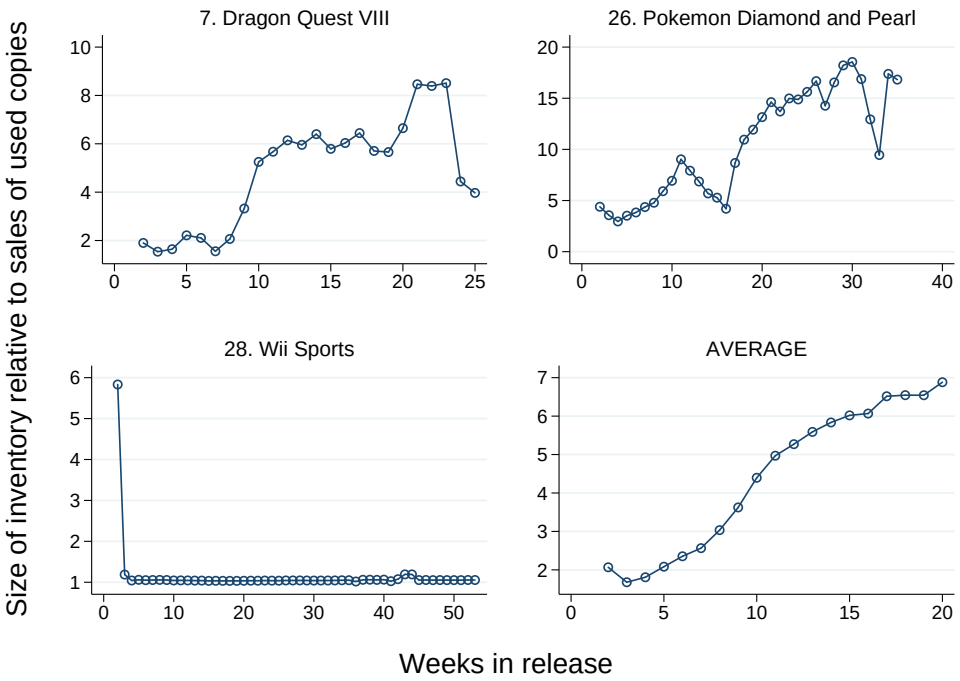
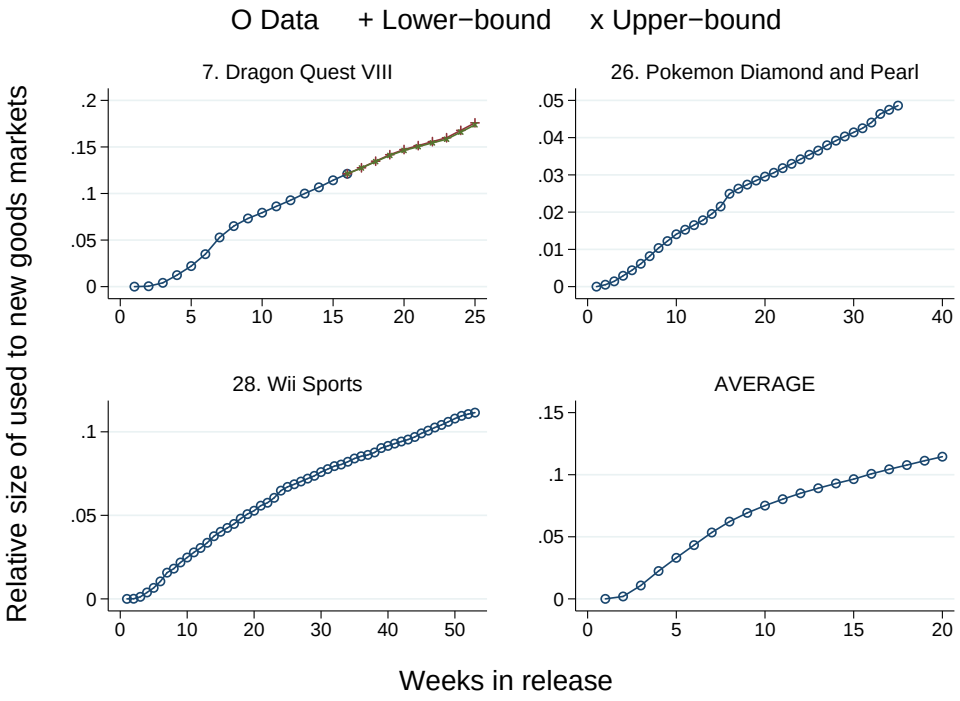


Figure 1.6: Size of used goods markets relative to new goods markets



Chapter 2

An Empirical Framework for Studying Dynamic Demand for New and Used Durable Goods without Physical Depreciation

2.1 Introduction

This chapter develops a new empirical framework for studying dynamic demand for new and used durable goods with consumption satiation. It mainly consists of two sections. First, I develop a formal model of forward-looking consumers' buying and selling decisions. Second, I develop a new Bayesian MCMC algorithm to estimate my model. Before turning to the discussion about the new empirical framework, I will briefly describe the new features of my model, and the advantages of using my new Bayesian MCMC algorithm.

My modeling framework is built on the previous literature on the dynamic purchase decisions in consumer durable goods markets (e.g., Melnikov 2000, Song and Chintagunta

2003, Nair 2007, Gordon 2009, Goettler and Gordon 2010, Carranza 2010, Gowrisankaran and Rysman 2009). In particular, my research is closely related to Nair (2007). He studies the intertemporal price discrimination in the US video game industry, and examines the role of consumer price expectation. However, during his sample period, the U.S. used video game market was very small. Thus, he did not account for the impact of the used video game market in his analysis.

I extend the previous literature by explicitly modeling the dynamic consumer selling decisions and allowing a role of consumer expectation about the future resale value in both buying and selling decision problems. This is new in the literature as previous studies are unable to use data on how many consumers sell their games to retailers and the associated resale values (e.g., Esteban and Shum 2007, Chen et al. 2010, Schiraldi 2010). Furthermore, my model incorporates the two concepts of consumption value depreciation: freshness-based depreciation for potential buyers and satiation-based depreciation for product owners. Purohit (1992) examines the depreciation of used cars, measured by used car prices, in response to feature changes incorporated in new model cars in primary markets. Engers et al. (2009) study how much variation in used car prices can be explained by the net flow of benefits to car owners. They provide evidence that the net flow of benefits, which is similar to owners' consumption values in my research, can explain used car prices. However, these two papers do not separately measure the two types of consumption value depreciation.

My thesis also extends the previous literature on the estimation of non-stationary single-agent discrete choice dynamic programming (DDP) models. In my modeling framework, I adopt a finite-horizon DDP model to allow for an impact of time on consumption values. A popular estimation methodology for a finite-horizon model is Keane and Wolpin (1994). Their algorithm computes the value functions at only a subset of state points in each time period, and use the interpolation to recover the value functions at the rest of state points. This method has significantly reduced the computational burden of esti-

mating a non-stationary DDP models, and has been widely used in both Marketing and Economics literature.

However, when the state space consists of multiple continuous variables, the computational burden could still be prohibitive. To alleviate the computational burden, I extend the Bayesian MCMC algorithm proposed by Imai, Jain and Ching (2009a) (IJC algorithm) to a non-stationary discrete choice dynamic programming model. Similar to the IJC algorithm, the new algorithm solves and estimates the model simultaneously, and thus has the potential to reduce the computational burden significantly.

In the estimation of my model, I combine this new algorithm with the pseudo-policy function approach by Ching (2010b) to control for the potential price endogeneity problem.[1] Recently, the estimation of DDP models with market-level data and potential endogeneity problems has received much attention. Gowrisankaran and Rysman (2009) introduce an estimation algorithm that combines the nested fixed point (NFP) algorithm by Rust (1987) with the market share inversion by Berry et al. (1995), and then uses the Generalized Method of Moments (GMM) to estimate this class of models. Also, Dubé et al. (2009) introduce an estimation algorithm that is based on the Mathematical Program with Equilibrium Constraints (MPEC). This method maximizes the likelihood function subject to a system of nonlinear constraints characterized by the Bellman equations. One advantage of this method is that it eliminates the two inner loops entirely. These two algorithms have expanded our toolbox for estimating this class of models.

My approach is related to Dubé et al. (2009) in the sense that it also eliminates the two inner loops completely. The NFP loop is eliminated as the IJC algorithm only applies the Bellman operator once in each iteration. For the market share inversion, Dubé et al. (2009) treat the unobserved product characteristics as parameters that

[1] If the model is stationary, we can combine the original IJC algorithm with the pseudo-policy function approach. Also, in principle, other limited information approach such as Villas-Boas and Winer (1999) or Petrin and Train (2010) can be combined with the IJC algorithm. I will discuss some advantages of using the pseudo-policy function approach later in this chapter.

maximize the objective function. My approach augments the unobserved shocks based on the joint-likelihood of the demand-side model and the pseudo-policy functions. This approach is similar to Yang et al. (2003) and Musalem et al. (2009) in the context of static discrete choice models with potential endogeneity problems. Moreover, my approach takes advantage of the MCMC method. Geweke et al. (2001) document that in estimating a DDP model, the high dimension of the parameter vector could make it difficult to find the global maxima of the simulated likelihood function unless the initial parameter values are set very close to the true values. However, the MCMC method is robust to initial parameter values.

Now I turn to discuss my model, and then the estimation strategy.

2.2 Model

In this section, I describe a new discrete choice dynamic programming model of consumer buying and selling decisions. Throughout the presentation of the model, I will use video games to describe the model. But the model can be applied to other markets with similar features as well.

2.2.1 Model setup

I assume that consumers make buying and selling decisions separately for each game denoted by g.[2] Let $i = 1, 2, \ldots, I$ index discrete consumer type, and let t be the time index ($t = 1$ indicates the release period). In period $t = 1$, no consumers own game g and there are no used copies available. Thus, each consumer's decision problem is to decide

[2]I do not explicitly model the competition among different video games since my focus in this paper is the competition between new and used copies of the same game. I control for the impact of the availability of other games on the purchase decision of game g by including the cumulative number of newly introduced games since game g's release. Note that Nair (2007) finds evidence that the substitutability between two different video games is very low in the U.S. market, and thus does not model the competition among differen video games.

whether to buy a new copy or not. In period $t > 1$, consumers who have not purchased the game up to $t-1$ observe the prices of new and used copies, the resale value, and the inventory level of used copies at retailers (all the information is available for consumers in the public domain), and decide whether to purchase a new or used good, or not to purchase anything. Let $j = 0, 1, 2$ denote no purchase option, new good purchase, and used good purchase, respectively. If consumers have already bought game g prior to time t and have not sold it yet, then they decide whether or not to keep the game given the resale value in period t. Let $k = 0, 1$ denote keeping and selling options, respectively. If consumers sell their game, they will exit the market. In Figure 2.1, I present the consumers' problems as a decision tree.

The state space of the consumer decision model consists of the following variables:

(1) price of new and used goods (p_1, p_2),

(2) resale value (r),

(3) inventory level of used copies at retailers (Y), which controls for the impact of the availability of used copies on consumer buying decisions,

(4) time since release (t), which characterizes the single-period consumption value to potential buyers,

(5) time since purchase (τ), which affects the single-period consumption value to owners,

(6) unobserved demand shocks for new and used copies (ξ_1, ξ_2),

(7) unobserved supply shocks for used copies (ξ_s),

(8) cumulative number of newly introduced games since the release of the focal game (C).

As I will describe later, (1), (6), and (8) appear only in the consumer buying decision problem, (5) and (7) appear only in the consumer selling decision problem, and (2)-(4)

appear in both consumer buying and selling decision problems.

I will first describe the single-period utility functions for buying and selling decisions, and then move to the description of the value functions.

2.2.2 Single-period utility functions

In each period, consumers derive a value from owning game g (consumption values). I assume that once consumers purchase a good, they will derive the same consumption value regardless of whether it is new or used. This is to capture the idea that the goods considered in this research have no physical depreciation. However, the decision to purchase used goods may be influenced by other factors such as the availability of used goods, psychological costs associated with used goods,[3] etc. Let $v_i^g(t,\tau)$ be type-i consumer's single-period consumption value of owning game g at time t if he has owned game g for τ periods prior to time t.[4] If he hasn't purchased game g, he will receive $v_i^g(t,0)$ if he purchases it at time t. I capture the two types of consumption value depreciation, freshness-based depreciation for potential buyers and satiation-based depreciation for owners, as follows. To capture the freshness-based depreciation for potential buyers, I allow $v_i^g(t,0)$ to depreciate as t increases. That is, the initial single-period consumption value decreases as video games age. To capture the satiation-based depreciation for owners, I allow $v_i^g(t,\tau)$ to depreciate as τ increases, i.e., as the duration of ownership (τ) increases, the single-period consumption value goes down.[5]

Suppose that a consumer has not bought game g by time t. Type-i consumer's single-

[3]My assumption here is that once consumers overcome this psychological costs when they make a purchase decision, then the consumption value they receive in subsequent periods is not affected by the psychological costs

[4]That is, this consumer purchased the game at time $t-\tau$.

[5]The actual functional form of $v_i^g(t,\tau)$ used in the empirical application will be discussed in Chapter 3.

period utility for buying decisions at time t is given by:

$$u_{ijt}^{g} = \begin{cases} v_i^g(t,0) - \alpha p_{1t}^g + \xi_{1t}^g + \epsilon_{i1t}^g & \text{if buying a new copy } (j=1) \\ v_i^g(t,0) - \alpha p_{2t}^g - l_Y(Y_t^g) + \xi_{2t}^g + \epsilon_{i2t}^g & \text{if buying a used copy } (j=2) \\ l_C(C_t^g) + \epsilon_{i0t}^g & \text{if no purchase } (j=0), \end{cases} \tag{2.1}$$

where p_{1t}^g and p_{2t}^g are the prices of new and used copies of game g at time t, respectively, and α is the price-sensitivity. I restrict the price-sensitivity for new and used goods to be identical; ξ_{1t}^g and ξ_{2t}^g are *i.i.d.* unobserved demand shocks to new and used copies, respectively.[6] I assume they are normally distributed with zero mean and the standard deviation σ_{ξ_j}; Y_t^g is the inventory level of used copies for game g at retailers in period t; $l_Y(Y_t^g)$ is the one-time transaction cost that consumers incur when buying a used good (search costs, psychological costs for pre-owned games, etc.); C_t^g is the cumulative number of newly introduced games at time t since the introduction of game g (including the games released in the same week as game g).

In Equation (2.1), I allow ϵ_{ijt}^g to be correlated across options j. I model the correlation in a nested logit framework. Let $\epsilon_{ijt}^g = \zeta_{iht}^g + (1-\eta)\upsilon_{ijt}^g$ where h indexes nest and takes two possible values: $h=1$ groups the buying options (i.e., buying a new or used copy), and $h=0$ is the no purchase option. Thus, the consumer buying decision problem here is equivalent to a two-stage decision making where consumers first decide whether or not to buy, and if buying, then consumers choose a new or used copy. In this setup, the parameter $\eta \in [0,1)$ measures the within-nest correlation. If $\eta = 0$, the within-group correlation is zero and the model boils down to the multinomial logit. As η approaches one, the within-group correlation goes to one.

Next, consider consumers' selling decision. Suppose that a consumer has bought game g and kept it for τ periods prior to time t. Type-i consumer's single-period utility for

[6] Unobserved demand shocks may capture variations in publicity about game g, sales of game g-related products (e.g., movies, cartoons, animations, etc.), economic recession, and so on.

selling decisions at time t is given by

$$w_{ikt}^g(\tau) = \begin{cases} \alpha r_t^g - \mu + \xi_{st}^g + e_{i1t}^g & \text{if selling } (k=1) \\ v_i^g(t,\tau) + e_{i0t}^g & \text{if keeping the game } (k=0) \end{cases}$$

where r_t^g is the resale value of game g at time t. Note that the price-sensitivity here is restricted to be the same as that in buying decisions; μ captures any additional cost of selling (transaction costs, endowment effects, etc.) that is common across video games and time; ξ_{st}^g is an *i.i.d.* unobserved shock to owners for selling decisions at time t;[7] e_{ikt}^g is an idiosyncratic error for consumer i, option k, time t, and game g. I assume that ξ_{st}^g is normally distributed with zero mean and the standard deviation σ_{ξ_s}, and e_{ikt}^g is *i.i.d.* extreme value distributed with the scale parameter equal to one.

2.2.3 Value functions

Since the dynamic consumer selling decision is nested within the dynamic consumer buying decision through the expected future payoff, I start off by describing the dynamic consumer selling decision, and then describe the dynamic buying decision. To simplify the notation, I will drop the g superscript. Also, let $\xi_{dt} = (\xi_{1t}, \xi_{2t})$ be the vector of unobserved demand shocks (as opposed to ξ_{st}, which is the unobserved shocks for selling decisions), and β be the discount factor common across consumer types.

Let $W_{it}(r_t, Y_t, \xi_{st}, t, \tau)$ be the value function of the consumer selling decision problem for type-i consumer who has owned the video game for τ period. Note that $(p_{1t}, p_{2t}, C_t, \xi_{dt})$ do not enter W_{it}. The inventory level, Y_t, is included because it may affect the distribution of the future resale value. The Bellman equation is given by

$$\begin{aligned} W_{it}(r_t, Y_t, \xi_{st}, t, \tau) &= E_e \max_{k\in\{0,1\}} \{W_{ikt}(r_t, Y_t, \xi_{st}, t, \tau) + e_{ikt}\} \\ &= \ln\left(\sum_{k=0}^{1} \exp(W_{ikt}(r_t, Y_t, \xi_{st}, t, \tau))\right), \end{aligned}$$

[7] The unobserved shock to owners may capture variations in the economic situation, sales of newly introduced games and their related products, etc.

where the second equality follows from the assumption of the extreme value distribution on e_{ikt}, and W_{ikt}'s are type-i consumer's alternative-specific value functions given by

$$W_{ikt}(r_t, Y_t, \xi_{st}, t, \tau) = \begin{cases} \alpha r_t - \mu + \xi_{st} & \text{selling,} \\ v_i(t,\tau) + \beta E[W_{it+1}(r_{t+1}, Y_{t+1}, \xi_{st+1}, t+1, \tau+1)|(r_t, Y_t, \xi_{st}, t, \tau)] & \text{keeping.} \end{cases}$$

The expectation in $E[W_{it+1}(r_{t+1}, Y_{t+1}, \xi_{st+1}, t+1, \tau+1)|(r_t, Y_t, t, \tau)]$ is taken with respect to the future resale value (r_{t+1}), inventory level (Y_{t+1}), and unobserved shock for selling decision (ξ_{st+1}).

The probability of selling the game by type-i consumer at $(r_t, Y_t, \xi_{st}, t, \tau)$ is given by

$$\Pr(k = 1|r_t, Y_t, \xi_{st}, t, \tau; i) = \frac{\exp(W_{i1t}(r_t, Y_t, \xi_{st}, t, \tau))}{\sum_{k'=0}^{1} \exp(W_{ik't}(r_t, Y_t, \xi_{st}, t, \tau))}.$$

Next, consider the consumer buying decision. Let $V_{it}(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t)$ be the value function for type-i consumer who have not bought the game until time t. The Bellman equation is given by

$$\begin{aligned} & V_{it}(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t) \\ &= E_\epsilon \max_{j\in\{0,1,2\}} \{V_{ijt}(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t) + \epsilon_{ijt}\} \\ &= \ln\left(\exp(V_{i0t}(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t)) + \left[\sum_{j'=1}^{2} \exp\left(\frac{V_{ij't}(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t)}{1-\eta}\right)\right]^{1-\eta}\right), \end{aligned}$$

where the second equality follows from the distributional assumption on ϵ_{ijt}, and V_{ijt}'s are type-i consumer's alternative-specific value functions given by

$$\begin{aligned} & V_{ijt}(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t) \\ &= \begin{cases} v_i(t,0) - \alpha p_{1t} + \xi_{1t} + \beta E[W_{it+1}(r_{t+1}, Y_{t+1}, \xi_{st+1}, t+1, 1)|(r_t, Y_t, \xi_{st}, t, 0)] & \text{new copy,} \\ v_i(t,0) - \alpha p_{2t} - l_Y(Y_t) + \xi_{2t} + \beta E[W_{it+1}(r_{t+1}, Y_{t+1}, xi_{st+1}, t+1, 1)|(r_t, Y_t, \xi_{st}, t, 0)] & \text{used copy,} \\ l_C(C_t) + \beta E[V_{it+1}(p_{1t+1}, p_{2t+1}, r_{t+1}, Y_{t+1}, C_{t+1}, \xi_{dt+1}, t+1)|(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t)] & \text{no purchase,} \end{cases} \end{aligned}$$

where the expectation in $E[V_{it+1}(p_{1t+1}, p_{2t+1}, r_{t+1}, Y_{t+1}, C_{t+1}, \xi_{dt+1}, t+1)|(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t)]$ is taken with respect to the future prices of new and used goods (p_{1t+1}, p_{2t+1}), resale value (r_{t+1}), inventory level (Y_{t+1}),the cumulative number of competing games (C_{t+1}), and the unobserved shocks for the buying decision (ξ_{dt+1}).

The probability of choosing $j > 0$ by type-i consumers at $(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t)$ is given by

$$\Pr(j|p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t; i) = \Pr(h = 1|p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t; i) \cdot \Pr(j|h = 1, p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t; i),$$

where

$$\begin{aligned}
\Pr(h = 1|p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t; i) &= \frac{\left[\sum_{j'=1}^{2} \exp\left(\frac{V_{ij't}}{1-\eta}\right)\right]^{1-\eta}}{\exp(V_{i0t}) + \left[\sum_{j'=1}^{2} \exp\left(\frac{V_{ij't}}{1-\eta}\right)\right]^{1-\eta}}, \\
\Pr(j|h = 1, p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t; i) &= \frac{\exp\left(\frac{V_{ijt}}{1-\eta}\right)}{\sum_{j'=1}^{2} \exp\left(\frac{V_{ij't}}{1-\eta}\right)}.
\end{aligned}$$

Given the finite time horizon, value functions for both buying and selling decisions can be computed by backward induction from the terminal period $t = T$.

2.2.4 Evolution of consumer type and aggregate sales

Let ψ_i be the population proportion of type-i consumers and $\sum_{i=1}^{I} \psi_i = 1$. In order to derive the aggregate market share of new and used goods, and aggregate volume of used goods supplied by owners, I need to derive the evolution of the size for each consumer type. Let M_{it}^d be the size of type-i consumers who have not bought the video game. It evolves according to

$$M_{it+1}^d = M_{it}^d(1 - \sum_{j=1}^{2} \Pr(j|p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t; i)) + N_{it+1},$$

where N_{it+1} is the size of new type-i consumers who enter the market at time $t + 1$. In my application, this corresponds to the sales of the platform for the game at time $t + 1$, weighted by the population proportion of type-i consumers. I assume that the proportion of new type-i consumers follows the population proportion, ψ_i.

Next, let $M_{it}^s(\tau)$ be the size of type-i consumers who have bought and owned the video game for τ periods at time t. It evolves according to

$$M_{it+1}^s(\tau) = \begin{cases} M_{it}^d \sum_{j=1}^{2} \Pr(j|p_{1t}, p_{2t}, r_t, Y_t, \xi_{dt}, t; i) & \text{if } \tau = 1, \\ M_{it}^s(\tau - 1) \cdot \Pr(k = 0|r_t, Y_t, \xi_{st}, t, \tau - 1; i)) & \text{if } \tau > 1. \end{cases}$$

The aggregate sales across consumer types is then given by

$$Q_{jt}(p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t) = \sum_{i=1}^{I} M_{it}^{d} \Pr(j|p_{1t}, p_{2t}, r_t, Y_t, C_t, \xi_{dt}, t; i) + \varepsilon_{jt}, \tag{2.2}$$

where $j = 1$ is new goods, and $j = 2$ is used goods, and ε_{jt} represents a measurement error. The aggregate quantity sold by consumers across consumer types and different durations of ownerships is given by

$$Q_{st}(r_t, Y_t, \xi_{st}, t) = \sum_{i=1}^{I} \sum_{\tau=1}^{t-1} M_{it}^{s}(\tau) \Pr(k = 1|r_t, Y_t, \xi_{st}, t, \tau; i) + \varepsilon_{st}, \tag{2.3}$$

where ε_{st} represents a measurement error.

2.3 Estimation Strategy

The estimation of consumer preference parameters is carried out in two steps. In the first step, I recover the processes of used game prices, resale values, inventory levels, and cumulative number of competing video games from the data.[8] These processes will then be used to form consumers' expectation about future price of used goods, resale value, inventory level, and cumulative number of competing video games in the second-step demand estimation, assuming that consumers have rational expectations. I model the processes of the price of used goods and the resale value to be a function of own lagged value, the lagged inventory level, and game characteristics, except for $t = 2$ where I assume that the initial price of used copies and resale value are a function of the price of new copies and game characteristics. I model the processes of the inventory level to be a function of its lagged value and game characteristics. The estimation results based on all 41 video games are reported in Tables 2.1 and 2.2. Note that Table 2.1 is essentially the same as the first two columns of Table 1.6. Finally, I model the process for the cumulative number of competing video games to be a function of own lagged value, and estimate the coefficients game by game.

In the second step, I estimate the dynamic discrete choice model with a finite time horizon.[9] Notice that the price of used goods and resale value may be correlated with unobserved shocks.

[8] I assume that consumers know that the price of new goods remains unchanged over time.

[9] In order to model the consumption value as a function of time since release and time since purchase, I use a finite-horizon model.

I propose a new estimation algorithm that extends the Bayesian MCMC algorithm in Imai et al. (2009a) (IJC algorithm) to a finite-horizon model, and combines it with the pseudo-policy function approach in Ching (2010a; 2010b) to control for the potential endogeneity problems. I will first describe the modified IJC algorithm, and then explain how I combine it with the pseudo-policy function approach.

2.3.1 Modified IJC algorithm

The IJC algorithm uses the Metropolis-Hastings algorithm to draw a sequence of parameter vectors from their posterior distributions. During the MCMC iterations, instead of fully solving for the value functions at each draw of parameter vectors as proposed in the nested fixed point algorithm (Rust 1987), the IJC algorithm partially solves for the value functions at each draw of parameter vectors (at the minimum, apply the Bellman operator only once), stores those partially solved value functions (they call them *pseudo*-value functions), and then uses them to nonparametrically approximate the expected value functions at the current trial parameter vector.[10] Imai et al. (2009b) show that as the MCMC iterations and the number of past pseudo-value functions for approximating the expected value functions increase, pseudo-value functions will converge to the true value functions, and the posterior parameter draws based on the pseudo-value functions will also converge to the true posterior distribution.[11]

One issue in applying the IJC algorithm in my framework is that the dynamic programming (DP) problem is non-stationary. However, the original IJC algorithm in Imai et al. (2009a) applies only to stationary DP problems. Their algorithm is essentially an extension of the contraction mapping procedure for solving the stationary DP problem. In general, the computational advantage of using IJC to estimate a finite horizon DP problem may be limited. This is because the IJC algorithm is essentially an extension of contraction mapping. In a finite horizon DP problem, there is no fixed-point computation of value functions and thus, one needs to solve for value functions by backward induction in each time period. However, if the dynamic

[10] Ching et al. (2010) provide a step-by-step guide for implementing the IJC algorithm.

[11] Imai et al. (2009b) shows that their algorithm converges under Gaussian kernel. Norets (2009) further shows that the IJC algorithm converges under nearest neighborhood kernel.

model has multiple continuous state variables, I argue that their idea for estimating DP models with continuous state variables (see Section 3.2 of Imai et al. 2009a) can be extended to help reduce the computational burden of integrating the continuation value even for a finite-horizon non-stationary dynamic model.

The main idea behind their algorithm for continuous state variables is to compute pseudo-value functions at one randomly drawn state vector in each iteration, which are then stored. The set of past pseudo-value functions used in approximating the expected future values will then be evaluated at different state points. Thus, one can simply adjust the weight assigned to each past pseudo-value function by the transition density from the current state to the state at which the past pseudo-value function is evaluated.

The main differences in implementation between my new algorithm and the original IJC algorithm are (1) pseudo-value functions are computed and stored for each time period, (2) in each MCMC iteration and in each time period, pseudo-value functions are computed only at one randomly drawn vector of continuous state variables, and (3) expected future values at time t are approximated using the set of pseudo-value functions at time $t+1$. Unlike conventional approaches, in which value functions need to be computed at all or a subset of pre-determined grid points in all periods (e.g., Rust 1997), the new algorithm computes pseudo-value functions at only one randomly drawn state point in each period, and the integration of the continuation value with respect to continuous state variables can be done by the weighted average of stored past pseudo-value functions. Thus, my approach has the potential to reduce the computational burden associated with the standard backward induction approach. One drawback of my proposed algorithm is that it is very memory-intensive: past pseudo-value functions need to be stored period by period. However, this caveat might not pose a serious problem as the cost of memory has been declining very quickly.

In what follows, I use a simple example to briefly explain the basic implementation of the proposed MCMC approach by focusing on the computation of value functions. The actual step-by-step implementation will be provided in Appendix C. I first describe a simple general model, and then discuss the conventional approach to highlight the difference in implementation between the conventional approach and the proposed Bayesian MCMC approach.

Description of the algorithm

Let S be a continuous state space (observable to econometricians), and let $f(s_{t+1}|s_t)$ be the transition probability for the observed state variables. I assume that the transition density does not depend on actions. Furthermore, let $U_{ijt}(s_t;\theta) = \bar{U}_{jt}(s_t;\theta) + \epsilon_{ijt}$ be the single-period utility function for alternative j at state s_t given the parameter vector, θ. For now, I assume that consumers differ only up to ϵ_{ijt}, the idiosyncratic error term. Thus, $\bar{U}_{jt}$ is the mean utility level. Given that there are J alternatives, the Bellman equation for $t < T$ can be written as

$$V_t(s_t;\theta) = E_\epsilon \max_j \{V_{jt}(s_t;\theta) + \epsilon_{ijt}\}. \tag{2.4}$$

The alternative-specific value function, V_{jt}, is given by

$$V_{jt}(s_t;\theta) = \bar{U}_{jt}(s_t;\theta) + \beta E[V_{t+1}(s_{t+1};\theta)|s_t] \tag{2.5}$$

where β is the discount factor, and

$$E[V_{t+1}(s_{t+1};\theta)|s_t] = \int V_{t+1}(s_{t+1};\theta) f(s_{t+1}|s_t) ds_{t+1}.$$

At $t = T$, we have

$$V_T(s_T;\theta) = E_\epsilon \max_j \{V_{jT}(s_T;\theta) + \epsilon_{ijT}\}$$

where

$$V_{jT}(s_T;\theta) = \bar{U}_{jT}(s_T;\theta).$$

Conventional Method

In a conventional method, the solution of the value functions at trial parameter vector θ is obtained as follows:

1. Discretize S into M points, $\{s_m\}_{m=1}^M$, which is fixed throughout the estimation.

2. Start with $t = T$. For all $m = 1, \ldots, M$, compute $V_T(s_T = s_m;\theta)$.

3. Move to $t = T-1$. For all m, compute $E[V_T(s_T;\theta)|s_{T-1} = s_m]$ using $\{V(s_T = s_m;\theta)\}_{m=1}^M$ and $f(s_T|s = s_m)$. That is,

$$E[V_T(s_T;\theta)|s_{T-1} = s_m] = \sum_{k=1}^{M} V_T(s_T = s_k;\theta) f(s_k|s_m).$$

   Then, compute $V_{T-1}(s_{T-1} = s_m;\theta)$ using Equations (2.4) and (2.5).

4. Repeat step 3 by backward induction until $t = 2$.

It can be shown that as M and/or T increase, the computational burden of solving the value functions at all m and t becomes very demanding. Keane and Wolpin (1994) propose to solve for the value functions only at a subset of $\{s_m\}_{m=1}^M$, and use the interpolation method to obtain the value functions at the remaining grid points. This technique helps reduce the computational burden significantly. However, one difficulty in implementation is that prior to the estimation, we do not know how many grid points and which grid points should be chosen so as to make the interpolation accurate. My proposed algorithm addresses this shortcoming because it randomly selects a state point in each MCMC iteration; thus, the past pseudo-value functions will be evaluated at randomly chosen state points. Also, the number of state points used for the approximation of the expected future values can also be increased by using a larger number of past pseudo-value functions for the approximation.

Another advantage of my proposed algorithm is that it solves and estimates the model simultaneously. Although the approach by Keane and Wolpin (1994) reduces the number of grid points at which value functions are solved, it still requires solving for the value functions exactly by backward induction at each trial parameter vector during the parameter search. My proposed algorithm only solves the pseudo-value functions at one randomly chosen state point per time period in each MCMC iteration, stores them, and then uses them to approximate the expected future values in the future MCMC iterations. Thus, my approach can potentially reduce the computational burden of the standard backward induction significantly.

Modified IJC algorithm

I now consider the implementation of my proposed algorithm. As argued above, the idea is that although we sequentially solve for the value functions from the terminal period, we do so

at only one randomly drawn sequence of state vectors.

Suppose we are in mth MCMC iteration. Let θ^{*m} be the candidate parameter vector in the Metropolis-Hastings algorithm in iteration m.

1. Draw one state vector for each $t > 1$, $(s_2^m, \ldots, s_T^m)$ from the uniform distribution $U[\underline{s}, \overline{s}]$, where $\underline{s}$ and $\overline{s}$ are the minimum and the maximum of the observed values, respectively.[12] This draw will change across MCMC iterations and thus depend on iteration m.

2. Start with $t = T$. Compute $V_T(s_T^m; \theta^{*m})$ and store it. This step is extremely fast (just store the static utility given s_T^m and θ^{*m}). Also note that these objects are "true value functions."

3. Move to $t = T - 1$. Given s_{T-1}^m, compute $\hat{E}[V_T(s_T; \theta^{*m})|s_{T-1}^m]$ by the weighted average of $\{V(s_T^l; \theta^{*l})\}_{l=m-N}^{m-1}$, where weights are given by the kernel for the parameter space and the transition probability from s_{T-1}^m to s_T^l, and N is the number of past pseudo-value functions used for the approximation of the expected future value. The hat on $\hat{E}$ indicates that this term represents a *pseudo*-expected future value:

$$\hat{E}[V_T(s_T; \theta^{*m})|s_{T-1}^m] = \sum_{l=m-N}^{m-1} V_T(s_T^l; \theta^{*l}) \frac{K_h(\theta^{*m} - \theta^{*l}) f(s_T^l | s_{T-1}^m)}{\sum_{k=m-N}^{m-1} K_h(\theta^{*m} - \theta^{*l}) f(s_T^l | s_{T-1}^m)}.$$

   Then, compute $\tilde{V}_{T-1}^m(s_{T-1}^m; \theta^{*m})$ using Equations (2.4) and (2.5). The tilde on $\tilde{V}_{T-1}^m$ indicates that this object is a *pseudo*-value function, and the m superscript on $\tilde{V}_{T-1}^m$ indicates that this value function is computed with the past pseudo-value functions available in iteration m.[13]

4. Repeat step 3 by backward induction until $t = 2$.

This process generates an output of $\{\theta^{*m}, \{s_t^m, \tilde{V}_t^m(s_t^m; \theta^{*m})\}_{t=2}^T\}$. We store this output in computer memory, and use it in the future MCMC iterations. When computing the likelihood value, we use this set of past pseudo-value functions for each time period to approximate the expected future value at any observed state vector.

[12] Note that $\underline{s}$ and $\overline{s}$ do not necessarily correspond to the minimum and the maximum of the observed values as long as they reasonably approximate the bounds for the possible values of s.

[13] There is no m superscript on V_{T-1} as this value function does not depend on any pseudo-value functions available in any iteration.

The MCMC draws obtained from this algorithm converge to the posterior distribution because of the following intuition. For an arbitrary parameter vector, θ, consider its neighborhood ($B(\theta)$) and use the past pseudo-value functions evaluated within the neighborhood to approximate the expected future values. As the number of the MCMC iterations increases, the number of times this neighborhood $B(\theta)$ is visited will also increase. At the same time, every time this neighborhood is visited, pseudo-value functions will be evaluated at a different random draw of state vector in each time period. Recall that the value functions at $t = T$ are the true ones. Now at $t = T-1$, the expected future value approximation will be more precise as $V_T(s_T; B(\theta))$ is evaluated at many different values of s_T. Thus, the pseudo-value functions $\tilde{V}_{T-1}(s_{T-1}; B(\theta))$ will improve as the number of pseudo-value functions used for the approximation of the expected future value increases. Also, as $V_{T-1}(s_{T-1}; B(\theta))$ is evaluated at different values of s_{T-1}, the expected future value approximation at $t = T-2$ conditional on s_{T-2} will improve. As the number of the MCMC iterations increases, the improvement continues backwards until $t = 2$.

I present Monte Carlo evidence based on a simple dynamic model in Section A.1 of Appendix A.

2.3.2 Pseudo-policy function approach

To control for the potential endogeneity problems of the price and resale value of used games, I follow the pseudo-policy function approach proposed by Ching (2000; 2010a; 2010b), who suggests approximating the pricing policy functions as a polynomial of observed and unobserved state variables of the equilibrium model.[14] Relative to other limited information approaches (e.g., Villas-Boas and Winer 1999) or the control function approach (Petrin and Train 2010), the pseudo-policy function approach allows researchers to model the relationship between the price and the unobserved shocks in a very flexible manner. One potential drawback is that as it requires a flexible functional form, the number of pseudo-policy function parameters to be estimated may become very large. However, the changes in pseudo-policy function parameters do not have any impact on the demand-side likelihood, which is the most computationally de-

[14]This approach can also be applied to control for the potential endogeneity of advertising/detailing (e.g., Ching and Ishihara 2010).

manding part in the estimation process. Thus, in either classical or Bayesian estimation, adding more parameters in pseudo-policy functions would not increase the computational burden significantly. As mentioned earlier, the MCMC approach is better at handling a high dimensional parameter space (Geweke et al. 2001). In contrast to classical estimation, where it is very difficult to optimize the objective function with a high dimensional parameter space, my MCMC algorithm should be more feasible in recovering the parameters of the pseudo-policy functions.

In my model, the state space of the equilibrium model includes unobserved shocks $(\xi_{1t}, \xi_{2t}, \xi_{st})$, consumption values $(v_i(t,\tau))$, inventory level (Y_t), cumulative number of newly introduced games (C_t), the size of potential buyers for each consumer type (M^d_{it}), and the size of owners for each consumer type and each duration of ownership $(M^s_{it}(\tau))$.

After experimenting several functional forms, I decided to use the following specification for the price of used goods (for $t \geq 2$):

$$\begin{aligned} \ln p_{2t} &= \omega_{10} + \omega_{11}\frac{1}{I}\sum_{i=1}^{I} v_i(t,0) + \omega_{12}\frac{1}{I(t-1)}\sum_{i=1}^{I}\sum_{\tau=1}^{t-1} v_i(t,\tau) + \omega_{13}(\xi_{2t}-\xi_{1t}) \\ &\quad + \omega_{14}\xi_{st} + \omega_{15}\frac{1}{I}\sum_{i=1}^{I} M^d_{it} + \omega_{16}\frac{1}{I(t-1)}\sum_{i=1}^{I}\sum_{\tau=1}^{t-1} M^s_{it}(\tau) + \omega_{17}Y_t + \nu^p_t, \end{aligned} \tag{2.6}$$

where ν^p_t is the prediction error. The pseudo-policy function for the resale value is specified as (for $t \geq 2$):

$$\begin{aligned} \ln r_t &= \omega_{20} + \omega_{21}\frac{1}{I}\sum_{i=1}^{I} v_i(t,0) + \omega_{22}\frac{1}{I(t-1)}\sum_{i=1}^{I}\sum_{\tau=1}^{t-1} v_i(t,\tau) + \omega_{23}(\xi_{2t}-\xi_{1t}) \\ &\quad + \omega_{24}\xi_{st} + \omega_{25}\frac{1}{I}\sum_{i=1}^{I} M^d_{it} + \omega_{26}\frac{1}{I(t-1)}\sum_{i=1}^{I}\sum_{\tau=1}^{t-1} M^s_{it}(\tau) + \omega_{27}Y_t + \nu^r_t, \end{aligned} \tag{2.7}$$

where ν^r_t is the prediction error.

Note that Y_t in the two equations plays a role of an instrument. Y_t is the inventory level of used games at retailers at the beginning of period t. Thus, it is uncorrelated with ξ_t's. However, it is plausible that the price and resale value of used games at time t depend on the inventory level. Similarly, C_t could be included as an instrument.[15] C_t is the cumulative number of newly introduced games since the release of the focal game. The release of a game is rarely postponed once the release week has been reached. This is because copies of games are already

[15] At this point, I have not included C_t in the empirical application in Chapter 3.

manufactured before the release week. Thus, C_t should be uncorrelated with ξ_t's. However, C_t would likely affect p_2 and r. On one hand, C_t could influence p_2 because it affects the demand for used copies of the focal game. Alternatively, C_t could affect r because owners of the focal game may be attracted to newly introduced games and choose to sell the focal game, which affect the supply of used copies of the focal game. Inclusion of these two exogenous observed state variables should help reduce the reliance of functional form restrictions for identification (Ching 2010b).

Assuming measurement errors in the sales of new and used copies as well as the quantities sold by consumers to retailers, and prediction errors in the pseudo-policy functions, I derive the joint likelihood of the demand-side model and the pseudo-policy functions. In the IJC algorithm described in the previous subsection, the joint likelihood will be used to compute the acceptance probabilities in the Metropolis-Hastings algorithm. Appendix B describes the construction of the likelihood function.

In Section A.2 of Appendix A, I show that this proposed estimation algorithm can handle the price endogeneity problems. Finally, I describe the step-by-step procedure for estimating my entire model in Appendix C.

2.3.3 Remark

Note that in the estimation, the consumers' expectation process for future prices and resale values of used copies is not modeled by pseudo-pricing policy functions. An alternative approach is to assume that consumers have rational expectations. That is, one can assume that consumers observe all the state variables and understand exactly how the equilibrium prices are generated, and then use the process implied by pseudo-pricing policy functions to form expectations about the future prices and resale values. I chose my current approach for the following two reasons. First, it seems unlikely that consumers are aware of all the state variables of the equilibrium model. In particular, it is difficult for consumers to obtain information about the size of potential buyers and owners for each consumer type and each duration of ownership at the time consumers make a buying or selling decision. In this scenario, consumers may adopt a simple Markov process to forecast the future prices and resale values. This approach to

modeling consumer expectation by a Markov process has been recently adopted in the field (e.g., Hendel and Nevo 2006, Gowrisankaran and Rysman 2009, Schiraldi 2010, Lee 2010a; 2010b). Moreover, Février and Wilner (2010) find evidence that consumers' expectation about future price reductions is best described by a simple Markov process while firms are more sophisticated and use a pricing policy that depends on other state variables as well. This finding suggests that the potential inconsistency issue in my estimation may not be as problematic as one might consider. Second, if pseudo-policy functions are used to form the consumer price expectation, the state space for the dynamic consumer buying and selling decision model must be specified to be that of the equilibrium model. Because there are many continuous state variables in the equilibrium model, including them will increase the computational burden of estimating the dynamic consumer buying and selling decision model dramatically. My current approach allows me to reduce the computational burden of estimating this model. Thus, I adopt it as the first step.

2.3.4 Identification

The identification of the parameters is mostly standard. Two new features of my forward-looking consumer model are the two types of consumption value depreciation for owners and potential buyers (satiation-based depreciation and freshness-based depreciation). The satiation-based depreciation is identified by the time-series variations in quantity sold by consumers to retailers across games, controlling for the variation in resale value and the size of owners (both observed). The freshness-based depreciation is identified by the average declining rate of new and used game sales. Heterogeneity in the freshness-based depreciation is identified by the time-series variations in the relative market share of new and used games, after controlling for the impact of the inventory level and the prices of new and used games on the relative market share. The relative market share over time tells us the strength of used-copy demand after the sales of new games declines. If the demand for used games is still relatively strong after the decline in the sales of new games, this may suggest that the consumption value remains higher for some consumers as opposed to others.

Finally, the discount factor is also identified in my framework. Recall that the resale value

does not enter the single-period utility function for buying decisions, but enters the expected continuation payoff for buying decisions. Thus, the resale value acts as an exclusion restriction that allows me to identify the standard discount factor (Rust 1994, Magnac and Thesmar 2002). An important identification assumption here is that consumers form an expectation about future resale values based on the current (and possibly past) resale value. As argued above, I model the consumer expectation process for resale values as a first-order Markov process, and this process is estimated from the data. Thus, if the true consumer expectation process is very different from the estimated process, the estimated discount factor may be biased. However, my estimated consumer processes give me very high R^2 (¿0.9). This suggests that the estimated processes should be able to approximate the actual consumer expectation processes reasonably well.

2.4 Conclusion

This chapter develops a new empirical framework for studying dynamic demand for new and used durable goods with consumption satiation. To the best of my knowledge, this is the first dynamic demand model that incorporates (i) new and used goods buying decisions, (ii) used goods selling decision, (iii) consumer expectations about future prices of new and used goods and resale values of used goods, and (iv) the depreciation of both owners' and potential buyers' consumption values. I then develop a new Bayesian MCMC algorithm for the estimation of my model. Two Monte Carlo exercises demonstrate that the new algorithm is able to recover the true parameter values reasonably well, even when prices are endogenous.

One limitation of the framework is that competition among different video games is not explicitly modeled. In my model, the competition is captured in a reduced-form fashion through $l_C(C_t^g)$, which is a function of the cumulative number of newly introduced games since the release of the focal game (C_t^g). However, if the substitutability between two games is mainly determined through product characteristics, my approach suffers from mis-specification bias. There are two reasons why I take the reduced-form approach. First, Nair (2007) finds that the substitutability between different games is generally low in the US video game market. This

assumption is also used in Lee (2010a; 2010b). Second, the focus of this paper is to examine the substitutability between new and used versions of the same game. Thus, I believe that the substitutability across different games is of second-order importance.

A natural extension of my dynamic demand model is to combine it with the supply-side dynamic decision problem. In particular, in order to fully quantify the benefits from eliminating the used video game market, we need to compare producers' maximized profit in the presence of the used video game market to the case where the used video game market is absent. This analysis requires us to explicitly model the competition between video game publishers and used video game retailers. This task could be very challenging, especially because as we saw in Chapter 1, my data on the inventory of used copies suggest that used video game retailers are forward-looking. Previous studies that model the dynamic competition between new and used goods markets always assume that used goods markets are perfectly competitive in a static sense (Esteban and Shum 2007, Chen et al. 2010, Tanaka 2009). Thus, we need to develop a new structural equilibrium model where video game publishers, used video game retailers, and consumers are all forward-looking. Although challenging, taking a step towards this direction is very important for providing managerial implications regarding the optimal pricing strategy in the product categories with active used goods markets.

Figure 2.1: Consumers' decision problems

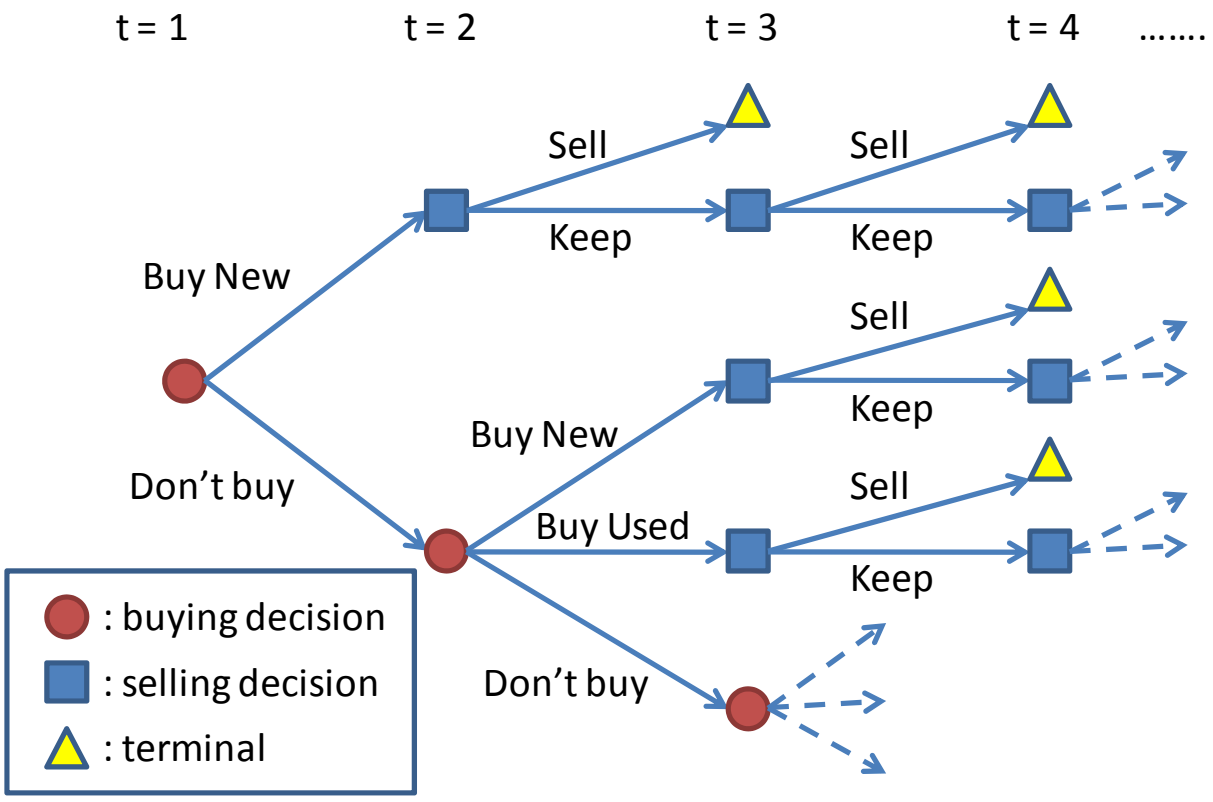


Table 2.1: Regressions for the price of used games and resale value ($t = 2$)

	price of used copies		resale value	
variable	estimate	s.e.	estimate	s.e.
price of new copy	0.737*	0.046	0.638*	0.077
dummy for handheld games	43.6	116.7	22.5	197.5
dummy for sequel games	-128.7	116.7	166.1	197.5
dummy for story-based games	8.25	71.3	72.9	120.5
dummy for multi-player games	51.3	69.2	200.6	117.1
critic rating	132.8*	48.8	210.3*	82.5
user rating	-8.27*	2.89	-8.22	4.89
constant	-156.1	406	-1491.4*	686.7
Adjusted R-squared	0.974		0.915	
# observations	41		41	

Notes: * p < 0.05

Table 2.2: Regressions for the price of used games, resale value, and inventory level ($t > 2$)

	price of used copies		resale value		inventory level	
variable	estimate	s.e.	estimate	s.e.	estimate	s.e.
lagged value	0.964*	0.003	0.930*	0.004	0.996*	0.003
dummy for handheld games	36.6*	6.61	51.6*	7.23	1813.4*	219.7
dummy for sequel games	-23.0*	10.0	-44.1*	11.4	442.7	348.6
dummy for story-based games	-11.6	7.44	-32.3*	8.49	609.1*	258.0
dummy for multi-player games	31.4*	7.59	34.9*	8.68	309.6	262.8
critic rating	9.21	4.89	14.6*	5.59	617.0*	168.9
user rating	0.898*	0.306	1.22*	0.349	-1.63	10.6
lagged inventory level	-7.29e-4*	7.89e-5	8.30e-4*	9.40e-5	-	-
constant	-8.79	42.9	-20.5	48.0	-5496.4*	1459.3
Adjusted R-squared	0.988		0.985		0.993	
# observations	1343		1343		1384	

Notes: * p < 0.05

Chapter 3

A Structural Analysis of the Japanese Video Game Market

3.1 Introduction

The objective of this chapter is to apply the new empirical structural framework proposed in Chapter 2 to the Japanese video game market, and examine the impact of the used video game market on the sales and revenues of new video game copies.

I will start by describing the detailed estimation setup, including the functional form assumptions, prior distributions on model parameters, and other parameters of the proposed algorithm. Then, I discuss the parameter estimates. Finally, I conduct two policy experiments.

3.2 Estimation setup

To apply my model described in Chapter 2, I need to assume functional forms for (i) single-period consumption values, $v_i^g(t,\tau)$, (ii) the one-time transaction cost for buying a used video game, $l_Y(Y_t^g)$, where Y_t^g is the inventory of used copies for game g at time t, and (iii) the competitive effect from other newly introduced games, $l_C(C_t^g)$, where C_t^g is the cumulative number of newly introduced games at time t since the introduction of game g.

For the single-period consumption value, $v_i^g(t,\tau)$, I will assume the following parsimonious functional form. In the release week ($t = 1$), I set $v_i^g(1,0) = \gamma^g \ \forall i$, where γ^g is the game-specific constant for game g. To capture the freshness-based depreciation for potential buyers, I allow $v_i^g(t,0)$ to decay as a function of t.[1] Specifically, I model the depreciation rate as:

$$v_i^g(t+1,0) = (1-\varphi_{it})v_i^g(t,0),$$

where

$$\varphi_{it} = \frac{\exp(\phi_i + \phi_t \ln(t))}{1+\exp(\phi_i + \phi_t \ln(t))}. \tag{3.1}$$

Recall that i indexes consumer type, and thus, the freshness-based depreciation rate, φ_{it}, is heterogeneous across consumer type i.

I capture the satiation-based depreciation for owners by modeling the depreciation rate as:

$$v_i^g(t+1,\tau+1) = (1-\kappa_\tau^g)v_i^g(t,\tau),$$

where

$$\kappa_\tau^g = \frac{\exp(X'_{g\tau}\delta)}{1+\exp(X'_{g\tau}\delta)}. \tag{3.2}$$

$X_{g\tau}$ includes observed product characteristics of game g and the duration of ownership (τ).[2] I include all the observed product characteristics in the data: handheld game dummy, sequel game dummy, story-based game dummy, multi-player game dummy, average critic rating, and average user rating.

Next, the one-time transaction cost that consumers incur when buying a used video game, $l_Y(Y_t^g)$, is specified as

$$l_Y(Y_t^g) = \lambda_0 + \lambda_1 \ln(1+Y_t^g).$$

I assume it depends on the inventory of used copies in order to capture the idea that search costs may change with the availability of used copies. In the empirical result, I expect λ_1 to be negative (i.e., as the availability increases, the one-time transaction cost decreases). Finally, the competitive effect from other newly introduced games, $l_C(C_t^g)$, is modeled as

$$l_C(C_t^g) = \pi_0 + \pi_1 \ln(C_t^g).$$

[1] I do not allow for the appreciation of consumption values, given that the length of my sample is at most one year.

[2] After trying several functional form, this parsimonious specification has so far fit the data best.

The unit of time period in my empirical application is a week. In estimating the dynamic model, I set the terminal period (T) to be 100. Also, I allow for two types of consumers who differ in freshness-based depreciation (i.e., ϕ_i). In the Bayesian estimation algorithm, I set $N = 1,000$ (the number of past pseudo-value functions used for the approximation of the expected future value) and $h^2 = 0.01$ (kernel bandwidth). Finally, I use the diffuse inverted gamma prior on σ_{ξ_1}, σ_{ξ_2}, and σ_{ξ_s}, and the flat priors for the rest of the parameters.

3.3 Parameter estimates

The estimation results are reported in Table 3.1. Most of the parameter estimates are statistically significant. The estimated discount factor is 0.89. Because the unit of periods in my application is week, my estimate is lower than the discount factor translated from a weekly interest rate ($\simeq 0.999$), typically assumed when a dynamic model does not have any exclusion restriction to help identify the discount factor. Nevertheless, my result falls into the range of consumers' discount factors found by previous studies in experimental/behavioral economics (see Frederick et al. 2002 for a survey on consumers' discount factors). Price-sensitivity (α) is positive and significantly different from zero. Recall that the price enters the utility function as a negative term in the utility function for buying decision. A negative sign of λ_1 indicates that costs for purchasing used video games diminish as the inventory level rises. This result is intuitive: as the availability of used copies increases, the search costs for consumers may decrease. The parameter for the competitive effect from other games (π_1) is positive, suggesting that increasing number of new game introduction may reduce the size of potential buyers who still consider purchasing the focal game.

Parameters for the depreciation rate of potential buyers' consumption values (freshness-based depreciation) include a type-specific intercept and a time effect. The estimated depreciation rate from the release week to the second week is about 5% for type-1 consumers and 47% for type-2 consumers. These are computed by substituting the parameter estimates for ϕ's (intercepts for type-1 and type-2 consumers under depreciation rates: potential buyers in Table 3.1) into Equation (3.1). After estimating the structural model, I am able to monetize

the declining consumption value for potential buyers from the release week to the second week. Based on game-specific intercepts and the price coefficient, I find that on average, potential buyers' consumption values decline by JPY 677 ($\simeq$ USD 7) for type-1 consumers and JPY 6,672 ($\simeq$ USD 66) for type-2 consumers from the release week to the second week. Given that the average price of new copies is JPY 6,389 (USD 64), the decline for type-2 consumers is large. However, a negative coefficient for the time effect suggests that the depreciation rate becomes much smaller over time.

For the depreciation rate of owners' consumption values (satiation-based depreciation), I include the following product characteristics in $X_{g\tau}$: an intercept, handheld game dummy, sequel game dummy, story-based game dummy, multi-player game dummy, critic rating and user rating. A positive coefficient of a variable implies that the variable will increase the satiation-based depreciation. My estimates suggest that on average, games released in handheld platforms (GameBoyAdvance, Nintendo DS, PlayStation Portable) and games with a higher user rating exhibit lower depreciation rates. On the other hand, sequel games, story-based games, multi-player games, and games with a higher critic rating exhibit higher depreciation rates. However, by examining the magnitude of the coefficient for each game characteristic, I find that the user rating plays the largest role in influencing the depreciation rates. For example, the depreciation rate at $\tau = 1$ (for the first week of ownership) for the game with the highest user rating in my sample is 0.71 while that for the game with the lowest user rating is 0.83. The depreciation rates for all other games fall within this range. Again, these are computed by substituting the parameter estimates for δ's (variables under depreciation rates: game owners in Table 3.1) into Equation (3.2). Finally, the coefficient for the duration of ownership suggests that the per-period depreciation rates become lower as consumers keep the game longer periods.

The parameters for pseudo-policy functions are reported in Table 3.2. While potential buyers' consumption values have a very little impact on both the price and the resale value of used games, owners' consumption values averaged across ownership durations have a positive impact. The difference in unobserved demand shocks between used and new games ($\xi_{2t} - \xi_{1t}$) has a negative impact, but it is small for the resale value of used games. The unobserved supply shock to used games has a positive impact on both the price and the resale value. The size of

potential buyers has a positive impact on the price of used games and a negative impact on the resale value of used games. The average size of owners across different durations of ownership has a negative impact on the price of used games and a positive impact on the resale value of used games. Finally, the inventory level has a negative, but small, impact on both the price and the resale value of used games.

3.4 Substitutability between new and used games

3.4.1 The cross-price elasticity of demand

One conventional way of examining the potential competition from used games is to examine the cross-price elasticity of demand/revenue for new goods with respect to the price of used goods (e.g. Ghose et al. 2006). Based on the estimates, I first compute the average cross-price elasticity of single-period revenues for new games for the first 15 weeks (starting from the second week). Figure 3.1 plots the result. It shows that the cross-price elasticity is low at the beginning and increases to 0.035 after 15 weeks. On average, the cross-price elasticity is quite low, which suggests that new and used video games in Japan may not be close substitutes. This result may be surprising because video games hardly depreciate physically, and thus used games should be almost perfect substitutes for new games. One reason is because of the high costs for purchasing used games. In particular, unlike new copies, used copies are available at a retailer only when other consumers sold their copies to the retailer. Thus, in the earlier periods consumers may think that the search costs are too high for purchasing used games and thus tend to buy new games. As a result, the cross-price elasticities are lower in the earlier periods. However, as the supply of used goods increases over time, the cross-price elasticity increases.

Note that the cross-price elasticity of single-period revenues for new games only accounts for the change in the revenue for new copies in the current period. However, a change in the price of used goods could have a dynamic effect. Some consumers may simply delay their purchase instead of switching to new copies when the price of used goods increases. Thus, there may be an increase in the demand for new and used copies in the subsequent periods.

After estimating the structural model, I am able to examine such a dynamic effect. In order to examine the dynamic effect, I evaluate the percentage change in the expected present discounted value (PDV) of the current and future revenues (up to the terminal period T) for new games in response to an increase in the price of used goods in each of the 14 weeks (week 2-15).[3] In computing the PDV of revenues, I assume that video game publishers' discount factor is 0.999 based on a weekly interest rate. I do not apply the estimated discount factor for consumers to video game publishers because (1) it is likely that publishers have different discount factors from consumers as forward-looking decisions critically affect their business, and (2) that the interest rate is used in firms' accounting and thus it is more reasonable to use the discount factor based on a weekly interest rate.

In Figure 3.1, I plot the percentage changes in the PDV of new-game revenues. As the figure shows, the dynamic effect is even smaller than the cross-price elasticity of the single-period revenue. It may appear puzzling that the dynamic effect is smaller than the static effect. To understand this, notice that the change in the price of used games today lowers the demand for used games today,[4] leading to a larger inventory level of used games tomorrow. Consumers who delay purchase today then may find it more attractive to buy a used copy since the search costs for finding a used copy is now lower compared to the baseline case. Therefore, an increase in the price of used games today could lower the sales of new games in the future, resulting in a smaller cross-price elasticity of the PDV of new-game revenues. However, the main results in this section show that regardless of whether I examine the static or PDV of new-game revenues, the substitutability between new and used games with respect to the price of used games is very low.

3.4.2 The inventory elasticity of demand

The comparison of the static and dynamic effects in the previous section points out the potential importance of examining the impact of the inventory level of used goods on the substitutability

[3]Notice that in my application, the price of new games is constant over time. Thus, the percentage change in the PDV of the current and future revenues is numerically equivalent to the PDV of the current and future sales.

[4]The own-price elasticity of used-game sales averaged across weeks 2-15 is 2.21.

between new and used goods. Previous studies on the substitutability between new and used goods do not incorporate the excess supply information about used goods, thus no prior research has investigated the importance of the inventory level of used goods. After incorporating the excess supply information and estimating its impact on the utility function for used game purchase (λ_1), I am able to quantify its impact on the substitutability between new and used games.

To quantify the economic significance of λ_1, I first examine the inventory elasticity of used-game sales, i.e., the percentage change in used-game sales due to one percent change in the inventory level, from week 3 to 15 (note that the inventory level is zero for the first two weeks by construction). I find that the average inventory elasticity of used-game sales is 0.132. Compared to the own-price elasticity of used-game sales (2.21), the inventory elasticity of used-game sales turns out to be smaller. Next, I examine the inventory elasticity of new-game revenues (both static and PDV). This analysis provides insights on how an increase in the availability of used games induces consumers to switch from new games to used games. In Figure 3.2, I plot the percentage changes in both static and PDV of new-game revenues from week 3 to 15. The figure shows that both elasticities are increasing over time in absolute value, but remain small. The results further support the results in the previous section that new and used games in Japan are not close substitutes.

3.4.3 The price elasticity of used-game supply

An important factor that determines the inventory level of used games is the resale value. While a higher resale value will lower retailers' profit-margin, it will induce more consumers to sell their games and increase the inventory level of used games. Consequently, the sales of used games may increase. Therefore, it is of retailers' interest to examine the price elasticity of used-game supply with respect to the resale value. Figure 3.3 shows the results. The average price elasticity of used-game supply is initially 1.58 and declines over time. This number is comparable but smaller than the average own-price elasticity of used-game demand, which is 2.21.

3.5 Elimination of used markets

Video game publishers often claim that the existence of used game markets lowers the sales of new games. The claim is often based on the conjecture that if there were no used game market, most of the consumers who would have purchased a used copy would instead buy a new copy. However, the results in the previous section suggest that the extent to which consumers switch from new to used games may be small. Moreover, the role of used game markets for consumers is not to simply offer a cheaper alternative. They also serve as outlets to sell games that consumers no longer want to keep. If the used game market shuts down, it is possible that consumers' willingness to pay for new games becomes lower because the expected future value from buying a new game could become lower due to the lack of selling opportunities. The reduced-form analysis on the elasticity commonly used to examine the substitutability between new and used games cannot take this important factor into account (e.g., Ghose et al. 2006). After estimating the dynamic structural model, I am able to quantify the overall impact for new-game revenues due to the existence of the used game market.

In order to examine the impact of eliminating used video game markets, I simulate my demand model by shutting down the used video game market. I achieve this objective by setting the parameters that capture the transaction costs of buying and selling used games (λ_0 and μ) to be extremely large so that the used game market shuts down in effect. I set $\lambda_0 = \mu = 1000$ in the simulation. In the first experiment, I simply take the observed price of new games as given. In general, the optimal price of new games in the absence of the used game market could be very different from the flat-pricing strategy I observe in the data. However, some marketing managers could be conservative in pricing strategy and may not want to change the prices drastically. Thus, it is still of video game publishers' interest to examine how the elimination of the used game market changes the total revenue for new games given the current flat-pricing scheme. Alternatively, one can think of the results presented here as the lower bound (i.e., in general, video game publishers could do better by optimally setting the price).

Based on the simulation, I compute statistics for the percentage change in the total revenue for new games following the elimination of the used game market. Here, the total revenue is

the summation of the revenues in T periods. The first row of Table 3.3 shows the statistics. Averaging across games, the new-game revenue declines by 4%, and there are only two games that exhibit an increase in the total revenues. This finding suggests that the future selling opportunity plays an important role in determining consumers' purchasing decisions. To further investigate this conjecture, I plot the average percentage change in new-game demand across games for the first 30 weeks in Figure 3.4. It shows that the demand for new games actually declines right after the game release. Then, the percentage changes in demand increase and become positive after 22 weeks. The initial decline is due to the lowered future value from purchasing: when the resale value is still high, consumers' purchasing decision is critically influenced by the future selling opportunity. The increasing trend of the percentage change in demand reflects the fact that after the resale value of used games drops, it is no longer important for consumers to have the future selling opportunity as they probably keep the game even in the presence of the used game market. Thus, the elimination of the used game market will be profitable for video game publishers in the later periods.

Recall that the results above are based on the observed flat-pricing strategy. In reality, video game publishers may adjust the price optimally after the used video game market shuts down. In the second experiment, I compute the optimal price for each video game to quantify the maximum possible benefits from the elimination of the used video game market.[5] I assume that the marginal cost of producing a new copy is JPY 1,000 ($\simeq$ USD 10). The optimal price is computed by solving for the optimal prices from the terminal period. I assume that the consumer expectation about the future price of new copies is consistent with the optimal pricing strategy by video game publishers. Similar to the first experiment, I compute the statistics on the percentage change in the total revenue for new games after eliminating the used game market. The second row of Table 3.3 shows the statistics. The results suggest that on average, the revenues for a new game increase by 57% as a result of eliminating the used video game market, which is a huge increase. However, this increase represents the upper-bound of the benefits from the elimination of the used video game market. The optimal prices computed

[5] Another interesting experiment could be to derive the optimal flat-pricing strategy. I plan to compute this in the future research.

here are on average 35-40% lower than the currently observed prices. It is possible that the complex structure of the distribution system in Japan (many middlemen between manufacturers and retailers) makes it difficult for manufacturers to significantly lower retail prices. Another issue is that from this experiment, we cannot conclude if the increase in revenues is purely due to the elimination of the used video game market. It may be the case that the currently observed price is not optimally set and too high. Thus, even in the presence of the used video game market, the optimal prices may be lower than the current prices.[6] To investigate this issue, I plan to develop a supply-side model where used video game retailers compete with video game publishers, and derive the optimal prices in the presence of the used video game market.

3.6 Is the flat-pricing strategy optimal?

As mentioned repeatedly in this paper, one interesting fact in the Japanese video game industry is that video game publishers typically keep the price of new copies constant over time. This is in contrast to the price-skimming strategy employed in the US video game industry or in typical durable goods markets. Although investigating why they employ a flat-pricing strategy is beyond the scope of this paper, it is interesting to investigate whether the observed flat-pricing strategy is optimal.

To investigate the optimality of the flat-pricing strategy, I run a counterfactual experiment in which video game publishers marginally reduce the price of new games over time, fixing the price and resale value of used copies at observed values. More specifically, I reduce the price of new copies by 0.1% in each period, simulate the model, and compute the change in total revenue for new copies. I choose the reduction to be very small so as to derive a marginal change in the total revenue.

Table 3.4 reports the summary statistics on the percentage change in the total revenue for a new game. First, I find that the "marginal price-skimming" strategy is on average more beneficial than the flat-pricing strategy: it increases the total revenue by 0.5%. Given that the

[6] In the absence of the used video game market, Nair (2007) finds that during his sample period, the prices set by video game publishers in the U.S. were higher than the optimal prices that he computed using his parameter estimates.

median revenues of the games in my data set is about 65 million US dollars, this amounts to an increase in total revenues by 0.33 million US dollars.

However, of the 41 games in my sample, there are several games that exhibit a decline in total revenues after switching to the marginal price-skimming. In future research, it will be interesting to find out if this is actually the case for some video games in the Japanese video game industry by developing an equilibrium model and examining the optimal pricing strategy for video game publishers and retailers.

3.7 Conclusion

This chapter applies the empirical framework proposed in Chapter 2 to the Japanese video game market described in Chapter 1, and studies consumers' preferences and the impact of the used video game market on new-game sales and revenues in Japan.

I find evidence that consumers are forward-looking in the Japanese video game market, and that the substitutability between new and used video games is very low. Using the parameter estimates, I quantify the impact of eliminating the used video game market on new-game sales and revenues. I find that the elimination of the used video game market reduces the demand for new copies in the earlier part of video games' lifecycle. Also, I compute the optimal price response by video game publishers and show that the elimination of the used video game market could increase the revenues for a new game. These two results can serve as the lower- and upper-bound of the costs and benefits from eliminating the used video game market. In the future, digital products such as video games could be distributed electronically, which would essentially eliminate the used video game market. Industry experts argue that the shift from hard copies to electronic copies will lower the price of digital products as distribution costs fall. However, the results of this research suggest that producers also need to account for how consumers react to the elimination of future selling opportunities when setting the prices.

Like any other research, the empirical analysis presented here also has limitations. First, I do not control for other marketing variables such as advertising. It is possible that high sales in the release week could be due to a high amount of pre-release advertising. Without controlling

for advertising, it is possible that my estimate for the freshness-based depreciation could be biased upwards. I have obtained weekly TV advertising data and plan to control for its impact on demand. Second, I do not allow for the impact of newly introduced games on consumer selling decisions. It is possible that when a new game is released, some consumers may sell their own games to finance the new game. I plan to incorporate this effect into the model in future research.

One interesting observation in my data set is the flat-pricing strategy commonly used by video game publishers. It is of our interest to examine whether this observation is a result of optimal response by video game publishers to the active used video game market. On the one hand, the theoretical studies suggest that the existence of used goods markets could work as a device for producers to commit to a high price over time (Liang 1999), thereby allowing producers to overcome the Coase conjecture. On the other hand, Cho and Rust (2010) investigate the flat rental pricing strategy used by car rental companies, and conclude that actual rental markets are not fully competitive and firms may be behaving suboptimally. Although I provide a preliminary analysis in this chapter, I plan to further investigate this issue by incorporating the supply-side model into the empirical framework presented in Chapter 2.

Table 3.1: Demand estimates

	mean	s.d.
preference parameters		
discount factor (β)	0.885	0.006
price sensitivity (α)	4.03E-04	2.71E-06
costs for buying used goods		
intercept (λ_0)	3.890	0.049
inventory level (logged) (λ_1)	-0.112	0.008
costs for selling used goods (μ)	6.117	0.072
no-purchase option		
intercept (π_0)	0.069	0.025
competitive effect from other games (π_1)	0.236	0.066
depreciation rates		
potential buyers (φ)		
intercept for type-1 consumer	-2.998	0.057
intercept for type-2 consumer	-0.126	0.046
time since release	-14.710	0.014
proportion of type-1 consumers	1.192	0.064
game owners (κ)		
intercept	0.123	0.009
handheld	0.520	0.037
sequel	1.299	0.011
story-based	0.224	0.015
multi-player	0.262	0.035
critic rating	0.037	0.001
user rating	-0.022	0.001
duration of ownership (logged)	-0.007	7.92E-05
parameters for error terms		
η	0.071	0.027
s.d. of measurement error for Q_1^d	68.2	2.94
s.d. of measurement error for Q_2^d	125.0	3.26
s.d. of measurement error for Q^s	101.0	5.07
s.d. (ξ_1)	0.362	0.020
s.d. (ξ_2)	1.67	0.14
s.d. (ξ_s)	0.553	0.035

- 41 game specific intercepts are not reported here.

Table 3.2: Estimates for pseudo-policy functions

pseudo-pricing policy function parameters			
price of used goods			
	ω_{10}	8.472	0.025
	ω_{11}	0.005	0.005
	ω_{12}	0.015	0.047
	ω_{13}	-0.178	0.094
	ω_{14}	0.016	0.066
	ω_{15}	3.50E-09	1.25E-24
	ω_{16}	-1.98E-07	4.00E-07
	ω_{17}	-1.06E-06	5.55E-07
	s.d. (v^p)	0.205	0.019
resale value of used goods			
	ω_{20}	8.236	0.013
	ω_{21}	-0.012	0.004
	ω_{22}	0.081	0.012
	ω_{23}	-0.035	0.027
	ω_{24}	0.168	0.143
	ω_{25}	-1.29E-08	2.79E-11
	ω_{26}	1.21E-06	2.69E-07
	ω_{27}	-2.78E-06	1.73E-07
	s.d. (v^r)	0.276	0.004

Table 3.3: The percentage change in the total revenue of new games due to the elimination of used games

	Average	S.D.	Max	Min
% change in total revenues under observed prices	-3.85%	2.33%	4.62%	-7.61%
% change in total revenues under optimal prices	57.36%	40.74%	131.80%	4.04%

Table 3.4: The percentage change in the total revenue of new games from switching to the marginal price-skimming strategy

	Average	S.D.	Min	Max
% change in total revenues	0.54%	1.15%	-3.99%	3.29%

Figure 3.1: Cross-price elasticity of new-game revenues

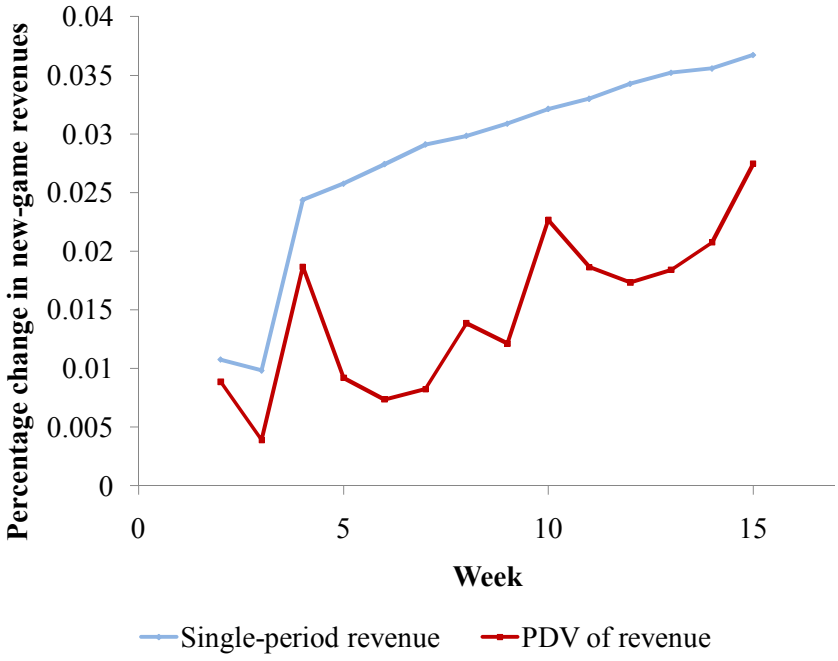


Figure 3.2: Inventory elasticity

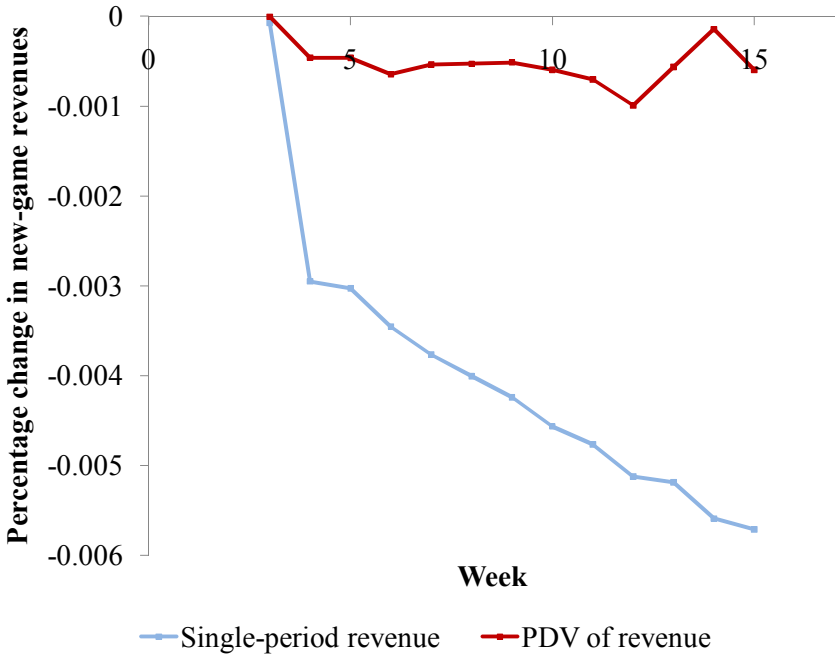


Figure 3.3: Elasticity of used-game supply

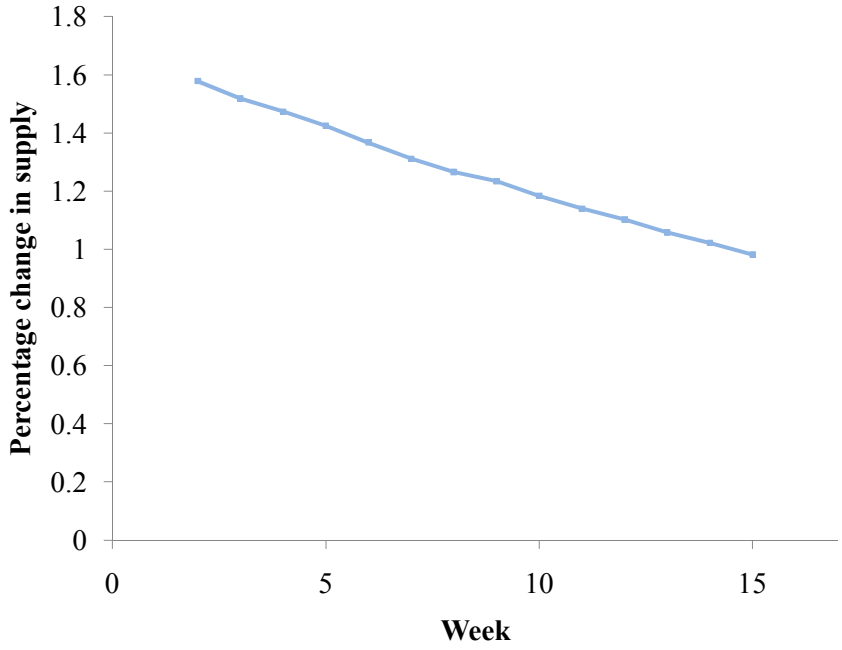
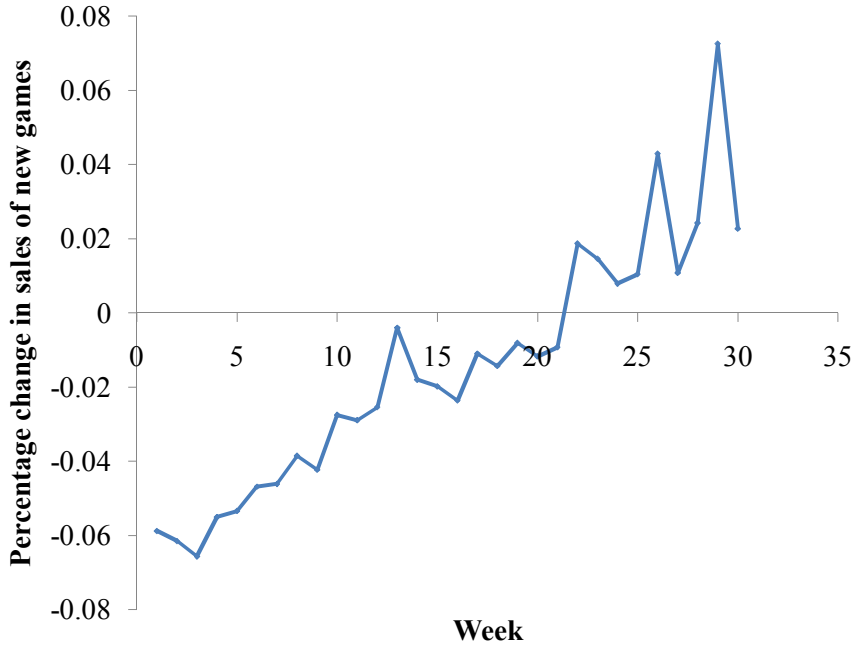


Figure 3.4: Percentage changes in new-game demand due to the elimination of used games



Appendix A

Monte Carlo exercises

This section of the appendix presents two Monte Carlo exercises for the proposed non-stationary Bayesian estimation of discrete choice dynamic programming models. In the first exercise, I simulate the data assuming that the prices are exogenous and there is no unobserved demand shock. I then apply the proposed Bayesian MCMC algorithm to recover the parameter values. In the second exercise, I use a known pricing equation and assume that the prices are correlated with unobserved demand shocks. I then combine the proposed Bayesian MCMC algorithm with the pseudo-policy function approach to control for the price endogeneity problem.

A.1 The estimation algorithm without price endogeneity

A.1.1 The model

Consider a simple dynamic consumer model of durable good purchase where consumers have uncertainty about future prices. The model is a simplified version of Nair (2007) with a finite-horizon. Let $t = 1$ be the release period of a product, and let $t = T$ be the terminal period. I will use the video game market to illustrate the model. Let g denote a video game ($g = 1, \ldots, G$). I assume that consumers' purchase decisions on G games are independent.

In each period, consumers decide whether or not to purchase video game g. Let $j = 0, 1$ denote no purchase ($j = 0$) and purchase ($j = 1$). If consumers decide to purchase today, then they receive the present discounted value of the game and exit the market. If consumers decide not to purchase today, then they will receive nothing today and face the same decision problem tomorrow. The utility function for purchase at time t is given by

$$U_{gi1t}(p_{gt}; \theta) = \gamma_0 + \gamma_1 CR_g - \alpha p_{gt} + \epsilon_{gi1t}$$

where $\theta = (\gamma_0, \gamma_1, \alpha)$, CR_g is the average critic rating for video game g; p_{gt} is the price of game g at time t; ϵ_{gi1t} is the idiosyncratic error for purchase ($j = 1$) and assumed to follow *i.i.d.* extreme value distribution. I assume zero mean utility for no purchase ($j = 0$) for all t. In this representation, $\gamma_0 + \gamma_1 CR_g$ is the present discounted value of game g.[1] To simplify the notation, below I will drop the g subscript from all the variables.

Consumers' objective is to choose an optimal stopping time that maximizes the present discounted value of future utilities:

$$\max_t E_{\epsilon,p}\left[\sum_{\tau=1}^{t-1} \beta^{\tau-1}\epsilon_{i0\tau} + \beta^{t-1}U_{i1}(p_t; \theta)\middle| p_1\right].$$

In this model, there is only one state variable, p_t. Assume for now that the transition probability for p_t is given by

$$p_{t+1}|p_t \sim N(\rho_p p_t, \sigma_p^2) \quad \forall t = 1, \ldots, T-1.$$

Let me denote this density by $f(p_{t+1}|p_t)$. I assume that consumers know this distribution and use it to form the expectation about future prices.

The Bellman equation for $t < T$ can be written as

$$V_t(p_t; \theta) = E_\epsilon \max_{j\in\{0,1\}} \{V_{jt}(p_t; \theta) + \epsilon_{ijt}\}$$

where

$$V_{1t}(p_t; \theta) = \bar{U}_{1t}(p_t; \theta),$$

$$V_{0t}(p_t; \theta) = \beta E_p[V_{t+1}(p_{t+1}; \theta)|p_t],$$

[1]Note that in my proposed framework, I allow for freshness-based depreciation of the single-period consumption values. To keep the illustration simple, in this exercise I assume that the present discounted value of the game does not change over time. This is the same approach by Nair (2007).

where $\bar{U}_{1t}(p_t;\theta) = U_{i1t}(p_t;\theta) - \epsilon_{i1t}$ and β is the discount factor. The expected future value is given by

$$E_p[V_{t+1}(p_{t+1};\theta)|p_t] = \int V_{t+1}(p_{t+1};\theta) f(p_{t+1}|p_t) dp_{t+1}.$$

At $t = T$, we have

$$V_T(p_T;\theta) = E_\epsilon \max_{j\in\{0,1\}} \{V_{jT}(p_T;\theta) + \epsilon_{ijT}\}$$

where

$$V_{1T}(p_T;\theta) = \bar{U}_{1T}(p_T;\theta),$$
$$V_{0T}(p_T;\theta) = 0.$$

Let $s_t(p_t;\theta)$ be the share in period t when the price is p_t. Given the extreme value error assumption, it can be obtained by

$$s_t(p_t;\theta) = \frac{\exp(V_{1t}(p_t;\theta))}{\sum_{j'=0}^{1} \exp(V_{j't}(p_t;\theta))}.$$

Let M_t be the size of potential buyers at time t. The observed quantity demanded, $Q_t^o(p_t;\theta)$ is then assumed to be

$$Q_t^o(p_t;\theta) = M_t s_t(p_t;\theta) + e_t,$$

where e_t is a measurement error and assumed to be normally distributed with mean zero and variance σ_e^2.

A.1.2 Results

The model parameters includes γ_0 (intercept for utility function common across games), γ_1 (effect of critic rating), α (price-sensitivity), β (discount factor), ρ_p (effect of lagged price on current price), σ_p (standard deviation for the price process), and σ_e (standard deviation for the measurement errors). In this Monte Carlo exercise, I will estimate $(\gamma_0, \gamma_1, \alpha)$ and fix the rest of the parameters as follows: $\beta = 0.98$, $\rho_p = 0.98$, $\sigma_p = 0.2$, and $\sigma_e = 5.0$. Also, I set $p_1 = 5.0$ (the price at $t = 1$) for all games. The average critic rating for each game, CR_g, is simulated from a standard normal distribution. I assume a flat prior on $(\gamma_0, \gamma_1, \alpha)$.

I simulated 10 game markets for 20 periods. In the simulation, I discretize the state space for price by using 2000 grid points. The terminal period is set to 100 (i.e., $T = 100$). The true parameter values for $(\gamma_0, \gamma_1, \alpha)$ are set as follows: $\gamma_0 = 2.0$, $\gamma_1 = 1.0$, $\alpha = 0.5$. In the estimation, I set the total number of the MCMC iterations to be 30,000. Also, I tried two versions of N (the number of past pseudo-value functions used for the approximation of the expected future value): (1) $N = 100$ for all MCMC iterations, (2) $N = 100$ for MCMC iterations $\leq 15,000$ and $N = 1000$ for MCMC iterations $> 15,000$.

I computed the posterior distribution based on $25,001-30,000$th iterations. The estimation results are shown in Table A.1. True parameter values are well recovered. Figure A.1 plots the MCMC draws for each parameter. It can be seen that the draws converge after 10,000 MCMC iterations.

A.2 The estimation algorithm with price endogeneity

A.2.1 The model

Now, I discuss how to combine the finite horizon IJC with the pseudo-policy function approach proposed by Ching (2000;2010). I will extend the model presented in the previous section by including unobserved demand shocks which are correlated with prices.

Inclusion of unobserved demand shocks

Before explaining how I combine these two approaches, it will be useful to discuss how we estimate the model with unobserved demand shocks in a Bayesian framework.

I modified the example in the previous section by including an unobserved demand shock that is *i.i.d.* across time and game, ξ_{gt}:

$$U_{gi1t}(p_{gt}, \xi_{gt}; \theta) = \gamma_0 + \gamma_1 CR_g - \alpha p_{gt} + \xi_{gt} + \epsilon_{gi1t}.$$

Let me assume that $\xi_{gt} \sim N(0, \sigma_\xi^2)$. In a classical likelihood-based approach, we need to

integrate ξ out to form the simulated likelihood function. In a Bayesian approach, we augment ξ by treating each ξ_{gt} as a parameter. More precisely, for each game g and $t \leq T^o$ (T^o is the number of periods for which we observe the data from the introduction of products), we draw ξ_{gt} using the Metropolis-Hastings algorithm. I use the diffuse inverted gamma distribution as a prior for σ_ξ^2.

Pricing equations

I maintain the assumption that the price process consumers use to form the expectation about future prices is given by

$$p_{gt+1}|p_{gt} \sim N(\rho_p p_{gt}, \sigma_p^2) \;\; \forall t = 1, \ldots, T-1. \tag{A.1}$$

However, I assume that firms use the following pricing equation to determine the prices:

$$\ln p_{gt} = \omega_0 + \omega_1 CR_g + \omega_2 \xi_{gt} + \omega_3 M_{gt} + \nu_{gt}, \tag{A.2}$$

where M_{gt} is the size of potential buyers for game g at time t; ν_{gt} is a cost shock and assumed to be distributed normally with mean zero and variance σ_ν^2. Note that prices depend on the state variables, CR_g and M_{gt} (observed) and ξ_{gt} (unobserved). Also, in this setup, M_{gt} plays a role of instrumental variables as it is correlated with p_{gt} but uncorrelated with ξ_{gt}.

Similar to my proposed framework in Chapter 2, I assume that the consumer expectation process and firms' pricing process could be different. In the data simulation, I generate the prices using Equation (A.2). When consumers form the expectation about future prices conditional on today's price, they use Equation (A.1).

A.2.2 Estimation procedure with pseudo-policy function approach

Here, I provide a step-by-step procedure for estimating the model described above. In the description below, it is useful to partition the parameter vector into four parts:

- $\{\xi_{gt} \; \forall g, t \leq T^o\}$: augmented parameters.

- σ_ξ^2: population variance of ξ_{gt}.
- $\theta_d = (\gamma_0, \gamma_1, \alpha)$: other demand-side parameters.
- $\theta_s = (\omega_0, \omega_1, \omega_2, \omega_3)$: supply-side parameters (i.e., parameters for pseudo-pricing policy functions).

Other parameters, $(\beta, \rho, \sigma_p^2, \sigma_e^2, \sigma_\nu^2)$, are fixed and not estimated.

Let $\mathbf{Q^o} = \{Q_{gt}^o \ \forall g, t \leq T^o\}$ and $\mathbf{p^o} = \{p_{gt}^o \ \forall g, t \leq T^o\}$ be the observed sales and price vector. Also, let $\mathbf{D^o} = \{D_{gt}^o = (CR_g, M_{gt}) \ \forall g, t \leq T^o\}$ be the rest of observed data. Let $L_d(\mathbf{Q^o}|\mathbf{p^o}, \mathbf{D^o}; \sigma_\xi^2, \{\xi_{gt}\}, \theta_d)$ and $L_s(\mathbf{p^o}|\mathbf{D^o}; \{\xi_{gt}\}, \theta_s)$ be the pseudo-likelihood for the demand-side model and the pricing equation, respectively ("pseudo" in the sense that they will be based on pseudo-value functions).

The procedure involves (i) drawing σ_ξ^2 conditional on $\{\xi_{gt}\}$, (ii) drawing individual ξ_{gt} conditional on σ_ξ^2 and (θ_d, θ_s) using the Metropolis-Hastings algorithm (M-H), (iii) drawing θ_d conditional on σ_ξ^2 and $\{\xi_{gt}\}$ using the M-H, and (iv) drawing θ_s conditional on $\{\xi_{gt}\}$ using the M-H. I use the diffuse inverted gamma distribution as a prior for σ_ξ^2. I assume a flat prior on the rest of the parameters.

In the Metropolis-Hastings algorithm below, I denote the accepted parameter vectors in iteration r as ξ_{gt}^r, θ_d^r, and θ_s^r, and denote the candidate parameter vectors in iteration r as ξ_{gt}^{*r}, θ_d^{*r}, and θ_s^{*r}. Note that since we directly draw σ_ξ from its posterior distribution computed based on the previously accepted values of $\{\xi_{gt}\}$, there is no candidate value of σ_ξ.

Procedure

1. In iteration r, we start with a history of past outcomes

$$H^r = \{(\sigma_\xi^2)^l, \theta_d^{*l}, \{\tilde{p}_{gt}^l, \tilde{V}_{gt}^l(\tilde{p}_{gt}^l; (\sigma_\xi^2)^l, \theta_d^{*l}) \ \forall g, t = 2, \ldots, T\}_{l=r-N}^{r-1},$$

where $(\sigma_\xi)^l$ is the draw of σ_ξ in iteration l, θ_d^{*l} is the candidate demand-side parameter vector in iteration l, $\tilde{p}_{gt}^l$ is a random draw of price for game g at time t in iteration l, $\tilde{V}_{gt}^l$ is the pseudo-value function for game g at time t. N is the number of pseudo-value functions used to approximate the expected future values. Note that the pseudo-value

functions depend on σ_ξ^2 and θ_d, but not on $\{\xi_{gt}\}$, as I integrate out ξ_{gt} when computing the pseudo-value functions.

2. Draw $(\sigma_\xi^2)^r$ conditional on $\{\xi_{gt},\ \forall g,t \leq T^o\}$. This can be drawn directly from the posterior distribution.

3. For each g and $t \leq T^o$, draw ξ_{gt}^r by the M-H.

   (a) Draw a candidate value, ξ_{gt}^{*r}, from $N(0,(\sigma_\xi^2)^r)$, the prior on ξ_{gt}.

   (b) The acceptance probability, λ, is given by

$$\lambda = \min\left\{\frac{L_d(Q_{gt}^o|p_{gt}^o,D_{gt}^o;(\sigma_\xi^2)^r,\xi_{gt}^{*r},\theta_d^{r-1})L_s(p_{gt}^o|D_{gt}^o;\xi_{gt}^{*r},\theta_s^{r-1})}{L_d(Q_{gt}^o|p_{gt}^o,D_{gt}^o;(\sigma_\xi^2)^r,\xi_{gt}^{r-1},\theta_d^{r-1})L_s(p_{gt}^o|D_{gt}^o;\xi_{gt}^{r-1},\theta_s^{r-1})},1\right\}.$$

   (c) The computation of $L_d(.|.)$ requires the approximation of the expected future values. For example, for $L_d(.|p_{gt}^o,D_{gt}^o;(\sigma_\xi^2)^r,\xi_{gt}^{*r},\theta_d^{r-1})$, we approximate the expected future value for game g at time $t<T$ as follows:

$$\hat{E}_{p'}^r[V_{gt+1}(p';(\sigma_\xi^2)^r,\theta_d^{r-1})|p_{gt}^o] = \sum_{l=r-N}^{r-1}\tilde{V}_{gt+1}^l(\tilde{p}_{gt+1}^l;(\sigma_\xi^2)^l,\theta_d^{*l})\frac{K_h((\sigma_\xi^2)^r-(\sigma_\xi^2)^l)K_h(\theta_d^{r-1}-\theta_d^{*l})f(\tilde{p}_{gt+1}^l|p_{gt}^o)}{\sum_{k=r-N}^{r-1}K_h((\sigma_\xi^2)^r-(\sigma_\xi^2)^k)K_h(\theta_d^{r-1}-\theta_d^{*k})f(\tilde{p}_{gt+1}^k|p_{gt}^o)},$$

   where $\hat{E}_{p'}^r$ is the approximated expected future value ($\hat{\cdot}$ indicates that this is an approximation and the r superscript indicates that this approximation is based on the set of pseudo-value functions in iteration r; K_h is a Gaussian kernel with bandwidth h; $f(p'|p)$ is the transition density based on consumers expectation process in Equation (A.1).

   Note that this is the step in which $\{\xi_{gt}\}$ links the likelihood of the demand-side model to the likelihood of the pricing equation, and controls for the price endogeneity problem.

4. Draw θ_d^r by the M-H.

   (a) Draw a candidate value, θ_d^{*r}, from $N(\theta_d^{r-1},\sigma_{\theta_d}^2)$ (random-walk M-H).

(b) The acceptance probability, λ, is given by

$$\lambda = \min\left\{\frac{L_d(\mathbf{Q^o}|\mathbf{p^o},\mathbf{D^o};(\sigma_\xi^2)^r,\{\xi_{gt}^r\},\theta_d^{*r})}{L_d(\mathbf{Q^o}|\mathbf{p^o},\mathbf{D^o};(\sigma_\xi^2)^r,\{\xi_{gt}^r\},\theta_d^{r-1})},1\right\}.$$

(c) The computation of $L_d(.|.)$ requires the approximation of the expected future values, which is similar to Equation (A.3).

5. Draw θ_s^r by the M-H.

(a) Draw a candidate value, θ_s^{*r}, from $N(\theta_s^{r-1},\sigma_{\theta_s}^2)$ (random-walk M-H).

(b) The acceptance probability, λ, is given by

$$\lambda = \min\left\{\frac{L_s(\mathbf{p^o}|\mathbf{D^o};\{\theta_{gt}^r\},\theta_s^{*r})}{L_s(\mathbf{p^o}|\mathbf{D^o};\{\theta_{gt}^r\},\theta_s^{r-1})},1\right\}.$$

(c) The computation of $L_s(.|.)$ does not require the approximated expected future values.

6. Compute and store one pseudo-value function at each of $t = 2,\ldots,T$.

(a) Draw $\tilde{p}_g^r = (\tilde{p}_{g2}^r,\ldots,\tilde{p}_{gT}^r)$, each from, say, $U[\underline{p}_g^o,\overline{p}_g^o]$, where $\underline{p}_g^o$ and $\overline{p}_g^o$ are the largest and smallest values of observed prices for game g, respectively.

(b) Start with $t = T$. Compute $\tilde{V}_{gT}^r(\tilde{p}_{gT}^r;(\sigma_\xi^2)^r,\theta_d^{*r})$ and store it.[2]

(c) Move to $t = T-1$. First, approximate the expected future value at $(\tilde{p}_{gT-1}^r;(\sigma_\xi^2)^r,\theta_d^{*r})$ by the weighted average of pseudo-value functions at $t = T$. That is,

$$\hat{E}_{p'}^r[V_{gT}(p';(\sigma_\xi^2)^r,\theta_d^{*r})|\tilde{p}_{gT-1}^r] = \sum_{l=r-N}^{r-1}\tilde{V}_{gT}^l(\tilde{p}_{gT}^l;(\sigma_\xi^2)^l,\theta_d^{*l})\frac{K_h((\sigma_\xi^2)^r-(\sigma_\xi^2)^l)K_h(\theta_d^{*r}-\theta_d^{*l})f(\tilde{p}_{gT}^l|\tilde{p}_{gT-1}^r)}{\sum_{k=r-N}^{r-1}K_h((\sigma_\xi^2)^r-(\sigma_\xi^2)^k)K_h(\theta_d^{*r}-\theta_d^{*k})f(\tilde{p}_{gT}^k|\tilde{p}_{gT-1}^r)}$$

(d) Given $\hat{E}_{p'}^r[V_{gT}(p';(\sigma_\xi^2)^r,\theta_d^{*r})|\tilde{p}_{gT-1}^r]$, compute the pseudo-value function at $(\tilde{p}_{gT-1}^r;(\sigma_\xi^2)^r,\theta_d^{*r})$ (i.e., $\tilde{V}_{gT-1}^r(\tilde{p}_{gT-1}^r;(\sigma_\xi^2)^r,\theta_d^{*r})$) and store it.

(e) Repeat (c) and (d) by backward induction until $t = 2$.

7. Go to iteration $r+1$.

[2]Note that $\tilde{V}_{gT}^r(\cdot)$ does not depend on past pseudo-value functions. Thus, $\tilde{V}_{gT}^r(\cdot) = V_{gT}$ for all r.

A.2.3 Results

The model parameters includes γ_0 (intercept for utility function common across games), γ_1 (effect of critic rating), α (price-sensitivity), σ_ξ^2 (variance of ξ), $\omega = (\omega_0, \omega_1, \omega_2, \omega_3)$ (parameters for pseudo-pricing policy functions), β (discount factor), ρ_p (effect of lagged price on current price), σ_p (standard deviation for the price process), σ_e (standard deviation for the measurement errors), and σ_ν (standard deviation for the cost shocks). In this Monte Carlo exercise, I will estimate $(\gamma_0, \gamma_1, \alpha, \sigma_\xi^2, \omega)$ and fix the rest of the parameters as follows: $\beta = 0.98$, $\rho_p = 0.98$, $\sigma_p = 0.2$, $\sigma_e = 5.0$, and $\sigma_\nu = 0.2$. Also, the average critic rating for each game, CR_g, is simulated from a standard normal distribution.

I simulated 30 game markets for 30 periods. In the simulation, I discretize the state space for price by using 1000 grid points. The terminal period is set to 100 (i.e., $T = 100$). The true parameter values for $(\gamma_0, \gamma_1, \alpha, \sigma_\xi^2, \omega)$ are set as follows: $\gamma_0 = 2.0$, $\gamma_1 = 1.0$, $\alpha = 0.5$, $\sigma_\xi = 0.2$, $\omega_0 = 0.5$, $\omega_1 = 0.2$, $\omega_2 = 0.3$, $\omega_3 = 0.001$. In the estimation, I set the total number of the MCMC iterations to be 70,000. I fix $N = 100$.

I computed the posterior distribution based on $65,001 - 70,000$th iterations. The estimation results are shown in Table A.2. I estimated three versions of the model. In Model 1, I treat prices as exogenous and estimate the model without the pricing equation. In Model 2, I treat prices as endogenous, but fix the parameters for the pricing equation at the true values. Finally, in Model 3, I treat prices as endogenous and estimate the parameters for the pricing equation as well.

The posterior means under Model 1 in Table A.2 show some biases, in particular for γ_0 (1.67) and α (0.44). These make sense because (i) we expect α to be biased downward if we do not control for the price endogeneity, and (ii) since α is biased downward, γ_0 will be biased downward to balance out. However, it is possible that these biases come from the poor approximation of the expected future values in the proposed algorithm. Thus, in Model 2, I estimate the four demand-side parameters by using the joint likelihood of the demand-side model and the pricing equation, but fixing the parameters for the pricing equation at the true values. The results are under Model 2 in Table A.2. The posterior means are reasonably close

to the true values this time, suggesting that the biases found in Model 1 are not generated by the algorithm. In Model 3, I estimate all the parameters. The results are under Model 3 in Table A.2, and true parameter values are recovered reasonably well.

Figure A.2 plots the MCMC draws for each parameter under Model 1, and Figures A.3 and A.4 plot the MCMC draws for demand-side and pricing equation parameters under Model 3, respectively. The draws for all the parameters converge to the posterior distribution after about 10,000 MCMC iterations.

Table A.1: Estimation results: exogenous prices

		N=100		N=100 -> 1000	
parameter	True value	mean	s.d.	mean	s.d.
γ_0	2.0	2.081	0.033	1.985	0.040
γ_1	1.0	0.980	0.009	0.975	0.008
α	0.5	0.508	0.007	0.494	0.009

Notes

Sample size: 10 games for 20 periods.

Fixed parameters: $\beta = 0.98$, $\rho_p = 0.98$, $\sigma_p = 0.2$, $\sigma_e = 5.0$.

Tuning parameters: $N = 100$ or $N = 1000$, $h = 0.01$ (bandwidth).

Table A.2: Estimation results: endogenous prices

		Model 1		Model 2		Model 3	
parameter	True value	mean	s.d.	mean	s.d.	mean	s.d.
γ_0	2.0	1.667	0.027	2.122	0.032	2.150	0.031
γ_1	1.0	0.956	0.010	1.050	0.011	1.058	0.011
α	0.5	0.439	0.008	0.519	0.009	0.523	0.009
σ_ξ	0.2	0.202	0.009	0.206	0.009	0.205	0.010
ω_0	0.5			0.5		0.490	0.018
ω_1	0.2			0.2		0.204	0.007
ω_2	0.3			0.3		0.327	0.051
ω_3	0.001			0.001		1.02E-03	2.94E-05

Notes

Sample size: 30 games for 30 periods.

Fixed parameters: $\beta = 0.98$, $\rho_p = 0.98$, $\sigma_p = 0.2$, $\sigma_e = 5.0$, $\sigma_\nu = 0.2$.

Tuning parameters: $N = 100$, $h = 0.01$ (bandwidth).

Model 1: Treat prices as exogenous.

Model 2: Treat prices as endogenous, but fix ω at true values.

Model 3: Treat prices as endogenous.

Figure A.1: MCMC plots: exogenous prices

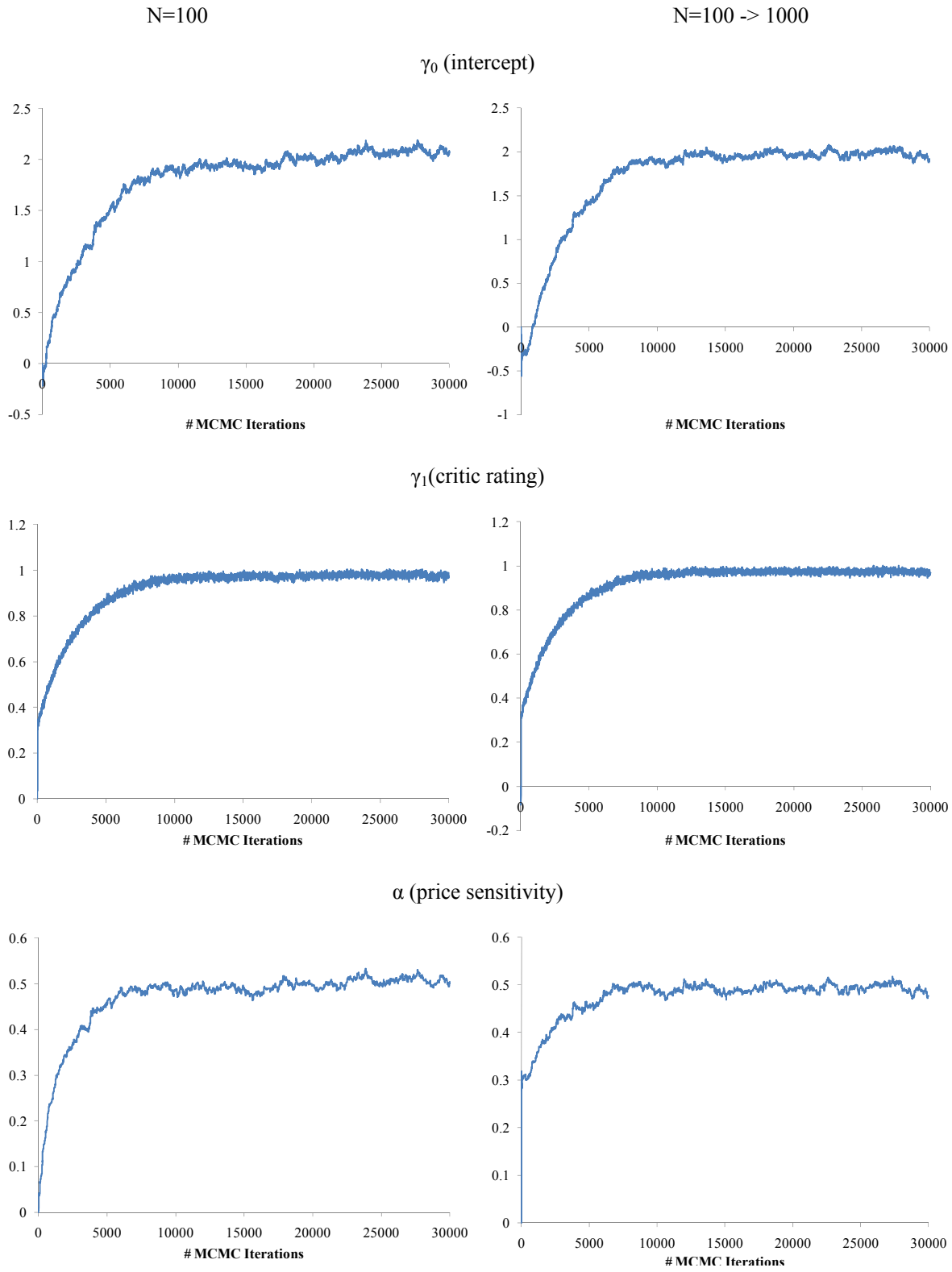


Figure A.2: MCMC plots: Model 1 (without controlling for the price endogeneity)

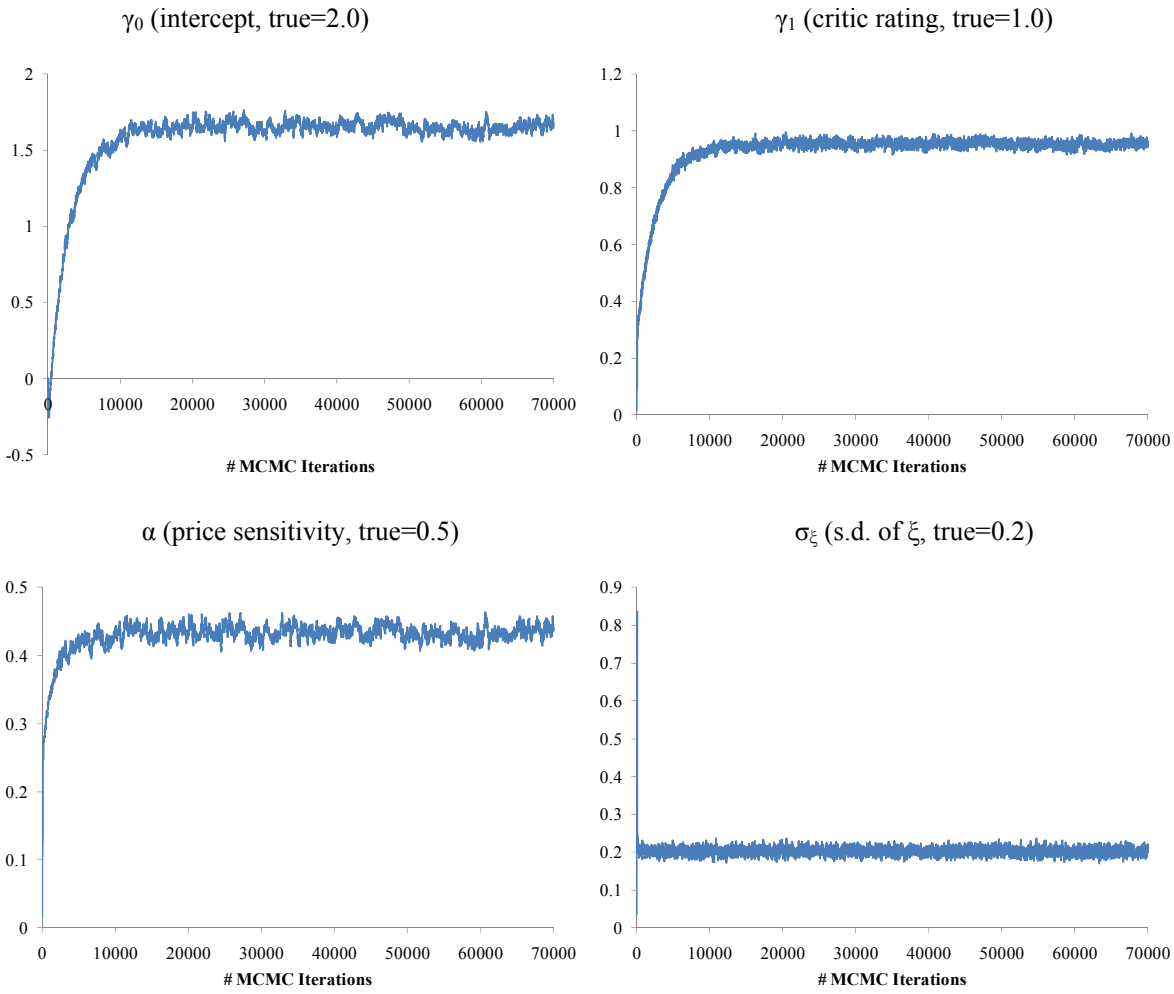


Figure A.3: MCMC plots: Model 3, demand-side parameters

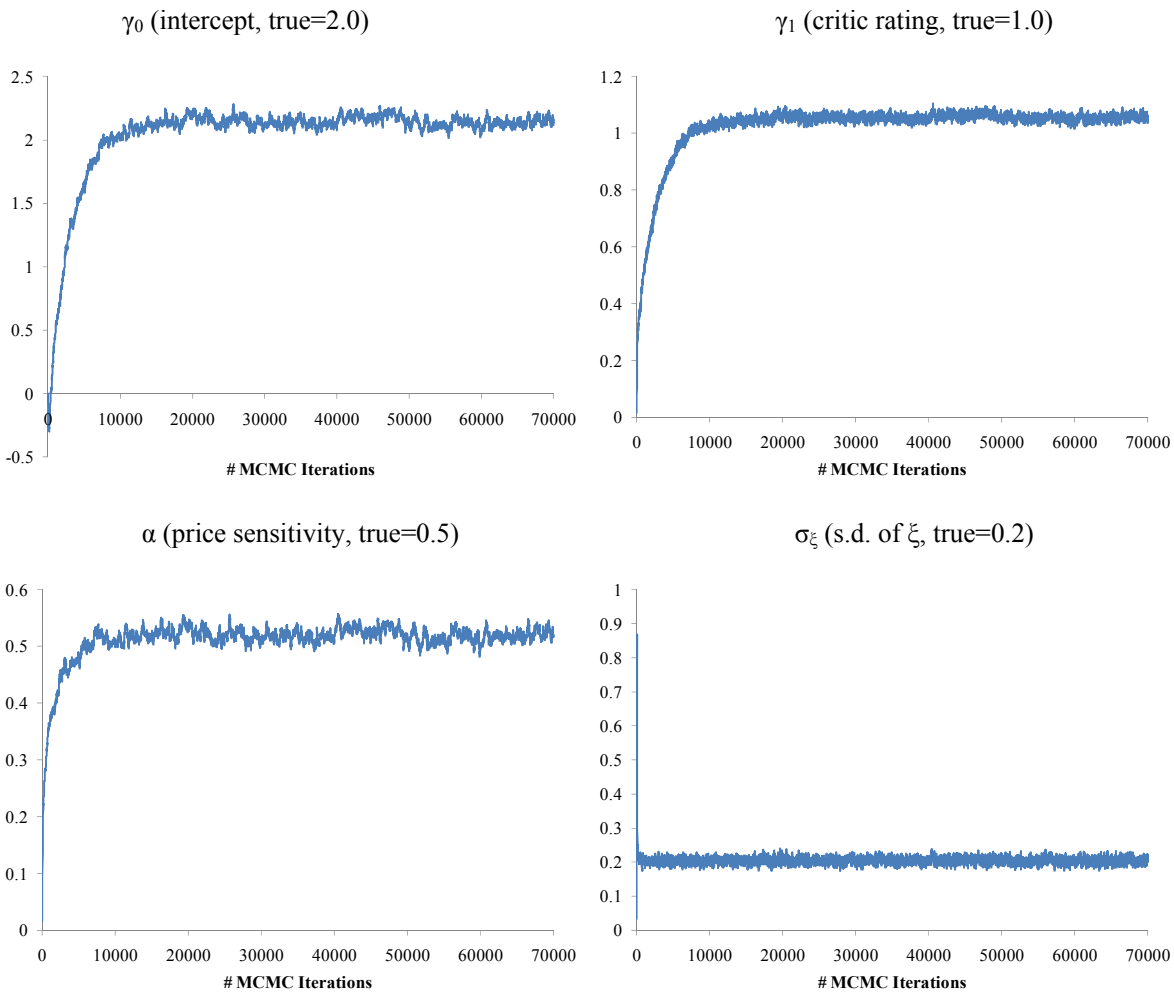
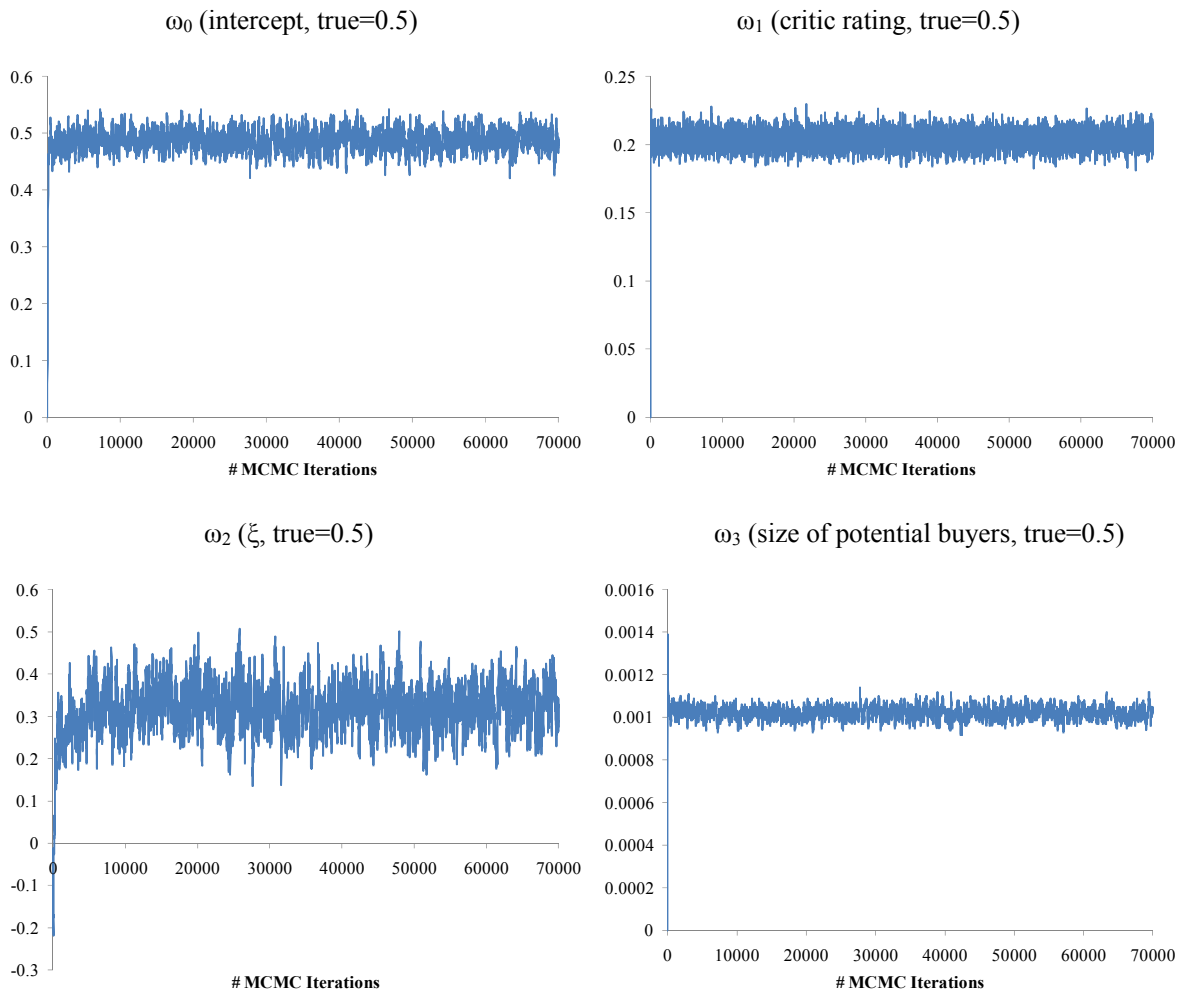


Figure A.4: MCMC plots: Model 3, pricing equation parameters



Appendix B

The likelihood function

Assuming that the prediction errors, ν_t^p and ν_t^r, in Equations (2.6) and (2.7) are normally distributed, I obtain the conditional likelihood of observing (p_{2t}^g, r_t^g),

$$f_s(p_{2t}^g, r_t^g | \{M_{it}^{d,g}, v_i^g(t,0), \{M_{it}^{s,g}(\tau), v_i^g(t,\tau)\}_{\tau=1}^{t-1}\}_{i=1}^{I}, \xi_{1t}^g, \xi_{2t}^g, \xi_{st}^g, Y_t^g; \theta_s)$$

where θ_s is the parameter vector of pseudo-policy functions. Note that (i) $v_i^g(t,\tau)$ depends on product characteristics, X_g; (ii) $M_{it}^{d,g}$ (size of potential buyers) and $M_{it}^{s,g}(\tau)$ (size of owners) are a function of X_g, p_1^g, $\{p_{2m}^g, r_m^g, Y_m^g\}_{m=2}^{t-1}$, $\{C_m^g\}_{m=1}^{t-1}$, $\{\xi_{1m}^g\}_{m=1}^{t-1}$, $\{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^{t-1}$,[1] $M_1^{d,g}$ (initial size of potential buyers), and $\{N_m^g\}_{m=2}^t$ (potential buyers who entered at time m). Thus, I can rewrite f_s as

$$f_s(p_{2t}^g, r_t^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, Z_t^g; \theta_s).$$

where $Z_t^g = \{X_g, p_1^g, \{p_{2m}^g, r_m^g, Y_m^g\}_{m=2}^{t-1}, \{C_m^g\}_{m=1}^{t-1}, M_1^{d,g}, \{N_m^g\}_{m=2}^t\}$ is a vector of observed variables.

Assume further that the measurement errors, ε_{1t}, ε_{2t}, ε_{st}, in Equations (2.2) and (2.3) are normally distributed. Then, the conditional likelihood of observing $(Q_{1t}^g, Q_{2t}^g, Q_{st}^g)$ is written as

$$f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{M_{it}^{d,g}, v_i^g(t,0), \{M_{it}^{s,g}(\tau), v_i^g(t,\tau)\}_{\tau=1}^{t-1}\}_{i=1}^{I}, \xi_{1t}^g, \xi_{2t}^g, \xi_{st}^g, p_1^g, p_{2t}^g, r_t^g, C_t^g, Y_t^g; \theta_d),$$

where θ_d is the vector of demand-side parameters. Similar to f_s, f_d can be rewritten as

$$f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, p_{2t}^g, r_t^g, C_t^g, Y_t^g, Z_t^g; \theta_d).$$

[1] ξ_{2m}^g and ξ_{sm}^g start with $m = 2$ as at time $m = 1$, there is no used goods market.

The joint likelihood of observing $(Q^g_{1t}, Q^g_{2t}, Q^g_{st}, p^g_{2t}, r^g_t)$ is the product of f_s and f_d:

$$
\begin{aligned}
&l(Q^g_{1t}, Q^g_{2t}, Q^g_{st}, p^g_{2t}, r^g_t | \{\xi^g_{1m}\}^t_{m=1}, \{\xi^g_{2m}, \xi^g_{sm}\}^t_{m=2}, C^g_t, Y^g_t, Z^g_t; \theta_d, \theta_s) = \\
&\quad f_d(Q^g_{1t}, Q^g_{2t}, Q^g_{st} | \{\xi^g_{1m}\}^t_{m=1}, \{\xi^g_{2m}, \xi^g_{sm}\}^t_{m=2}, p^g_{2t}, r^g_t, C^g_t, Y^g_t, Z^g_t; \theta_d) \times \\
&\quad f_s(p^g_{2t}, r^g_t | \{\xi^g_{1m}\}^t_{m=1}, \{\xi^g_{2m}, \xi^g_{sm}\}^t_{m=2}, Y^g_t, Z^g_t; \theta_s).
\end{aligned}
$$

The likelihood of observing $\mathbf{D} = \{\{Q^g_{1t}\}^{T^g}_{t=1}, \{Q^g_{2t}, Q^g_{st}, p^g_{2t}, r^g_t\}^{T^g}_{t=2}\}^G_{g=1}$ is

$$
\begin{aligned}
&L(\mathbf{D} | \xi, C, Y, Z; \theta_d, \theta_s) = \\
&\quad \prod_{g=1}^{G} \left[f_d(Q^g_{11} | \xi^g_{11}, C^g_1, Z^g_1; \theta_d) \prod_{t=2}^{T^g} l(Q^g_{1t}, Q^g_{2t}, Q^g_{st}, p^g_{2t}, r^g_t | \{\xi^g_{1m}\}^t_{m=1}, \{\xi^g_{2m}, \xi^g_{sm}\}^t_{m=2}, C^g_t, Y^g_t, Z^g_t; \theta_d, \theta_s) \right]
\end{aligned}
$$

where G is the total number of games, T^g is the length of observations for game g, $C = \{\{C^g_t\}^{T^g}_{t=1}\}^G_{g=1}$, $Y = \{\{Y^g_t\}^{T^g}_{t=1}\}^G_{g=1}$, and $Z = \{\{Z^g_t\}^{T^g}_{t=1}\}^G_{g=1}$.

Note that $\{\xi^g_{1t}\}^{T^g}_{t=1}, \{\xi^g_{2t}, \xi^g_{st}\}^{T^g}_{t=2}$ are unobserved to the econometricians. In the proposed Bayesian framework, these variables are augmented from the corresponding distributions to form the likelihood $L(\mathbf{D}|\cdot)$.

Appendix C

The estimation procedure for the proposed model

This appendix describes the details of the estimation algorithm described in section 2.3.1.

Let θ_d and θ_s be the vectors of demand-side parameters and pseudo-policy function parameters, respectively. In the context of the present model, the output of the new algorithm in iteration m is

$$H^m = \{\theta_d^l, \theta_s^l, \theta_d^{*l}, \{\tilde{V}_{it}^l(p_1, \tilde{p}_{2t}^l, \tilde{r}_t^l, \tilde{Y}_t^l, \tilde{C}_t^l, \tilde{\xi}_d^l, t; \theta_d^{*l})\ \forall i\}_{t=2}^T, \{\{\tilde{W}_{it}^l(\tilde{r}_t^l, \tilde{Y}_t^l, \tilde{\xi}_s^l, t, \tau; \theta_d^{*l})\ \forall i\}_{\tau=1}^{t-1}\}_{t=2}^T\}_{l=m-N}^{m-1},$$

where $\tilde{V}_{it}^l$ and $\tilde{W}_{it}^l$ are type-i consumers' pseudo-value functions for purchasing and selling decisions at time t in iteration l, respectively; N is the number of past pseudo-value functions used for approximating the expected future value functions; θ_d^l and θ_s^l are the accepted parameter vectors of the demand-side model and the pseudo-policy functions in iteration l, respectively; θ_d^{*l} is the candidate parameter vector for the demand-side model in iteration l; $(\tilde{p}_{2t}^l, \tilde{r}_t^l, \tilde{Y}_t^l, \tilde{C}_t^l)$ are a draw of state vector at time t in iteration l; $(\tilde{\xi}_d^l, \tilde{\xi}_s^l)$ are draws of unobserved shocks for buying and selling decisions. I assume $(\tilde{\xi}_d^l, \tilde{\xi}_s^l)$ are *i.i.d.* across time, and thus(1) I can use the same draws for all periods, and (2) the unobserved shocks are integrated out by the simple average of the past pseudo-value functions.

The pseudo-value functions for selling decision at time t in iteration m are defined as follows:

$$\begin{aligned}&\tilde{W}_{it}^{m}(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) \\ &\quad = E_e \max\{\tilde{W}_{i0t}^{m}(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) + e_{i0t}, \tilde{W}_{i1t}^{m}(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) + e_{i1t}\},\end{aligned}$$

where $\tilde{W}_{ikt}^{m}$'s are type-i consumer's pseudo alternative-specific value functions in iteration m, which are given by

$$\begin{aligned}&\tilde{W}_{ikt}^{m}(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) \\ &\quad = \begin{cases} \alpha \tilde{r}_t^m - \mu + \tilde{\xi}_s^m & \text{selling,} \\ v_i(t, \tau) + \beta \hat{E}^m[W_{it+1}(r', Y', \xi_s', t+1, \tau+1; \theta_d^{*m}) | (\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau)] & \text{keeping.} \end{cases}\end{aligned}$$

The *pseudo*-expected future value function for selling decision, $\hat{E}^m[W_{it+1}(.; \theta_d^{*m})|.]$, is defined as the weighted average of the past pseudo-value functions for selling decision in period $t+1$. It is constructed as follows:

$$\begin{aligned}&\hat{E}^m[W_{it+1}(r', Y', \xi_s', t+1, \tau+1; \theta_d^{*m}) | (\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau)] \qquad \text{(C.1)} \\ &\quad = \sum_{l=m-N}^{m-1} \tilde{W}_{it+1}^{s}(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, \tau+1; \theta_d^{*l}) \frac{K_h(\theta_d^{*m} - \theta_d^{*l}) f(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l | \tilde{r}_t^m, \tilde{Y}_t^m)}{\sum_{q=m-N}^{m-1} K_h(\theta_d^{*m} - \theta_d^{*q}) f(\tilde{r}_{t+1}^q, \tilde{Y}_{t+1}^q | \tilde{r}_t^m, \tilde{Y}_t^m)},\end{aligned}$$

where $K_h(.)$ is a Gaussian kernel with bandwidth h, and $f(.|.)$ is the transition density estimated in the first step. Note that the kernel captures the idea that one assigns higher weights to the past pseudo-value functions which are evaluated at parameter vectors that are closer to θ_d^{*m}. Also, this weighted average integrates out both ξ_s' and r'. In particular, ξ_s' is integrated out by the simple average since ξ_s^l's are drawn from its distribution. In contrast, r' is integrated out by the weighted average, where weights are given by the transition probability.

The pseudo-value functions for the purchasing decision at time t in iteration m are defined as follows:

$$\begin{aligned}\tilde{V}_{it}(p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m}) \;=\; & E_{\zeta,\epsilon} \max\{\tilde{V}_{i0t}^{m}(p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m}) + \zeta_{i0t} + \epsilon_{i0t}, \\ & \tilde{V}_{i1t}^{m}(p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m}) + \zeta_{i1t} + \epsilon_{i1t}, \\ & \tilde{V}_{i2t}^{m}(p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m}) + \zeta_{i1t} + \epsilon_{i2t}\},\end{aligned}$$

where $\tilde{V}_{ijt}^m$'s are type-i consumer's pseudo alternative-specific value functions in iteration m, which are given by

$$\tilde{V}_{ijt}^m(p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m})$$

$$= \begin{cases} v_i(t,0) - \alpha p_1 + \tilde{\xi}_1^m + \beta \hat{E}^m[W_{it+1}(r', Y', \xi_s', t+1, 1; \theta_d^{*m}) | (\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, 0)] & \text{new copy,} \\ v_i(t,0) - \alpha \tilde{p}_{2t}^m + \tilde{\xi}_2^m - l_Y(\tilde{Y}_t^m) + \beta \hat{E}^m[W_{it+1}(r', Y', \xi_s', t+1, 1; \theta_d^{*m}) | (\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, 0)] & \text{used copy,} \\ l_C(\tilde{C}_t^m) + \beta \hat{E}^m[V_{it+1}(p_1, p_2', r', Y', C', \xi_d', t+1; \theta_d^{*m}) | (p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t)] & \text{no purchase.} \end{cases}$$

The *pseudo*-expected future value function for the purchasing decision, $\hat{E}^m[V_{it+1}(,;\theta_d^{*m})|.]$, is defined as the weighted average of the past pseudo-value functions for the purchasing decision in period $t+1$, and is constructed as follows:

$$\begin{aligned} &\hat{E}^m[V_{it+1}(p_1, p_2', r', Y', C', \xi_d', t+1; \theta_d^{*m}) | (p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t)] \qquad \text{(C.2)} \\ &= \sum_{l=m-N}^{m-1} \tilde{V}_{it+1}(p_1, \tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l, \tilde{\xi}_d^l, t+1; \theta_d^{*l}) \\ &\times \frac{K_h(\theta_d^{*m} - \theta_d^{*l}) f(\tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l | \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m)}{\sum_{q=m-N}^{m-1} K_h(\theta_d^{*m} - \theta_d^{*q}) f(\tilde{p}_{2t+1}^q, \tilde{r}_{t+1}^q, \tilde{Y}_{t+1}^q, \tilde{C}_{t+1}^q | \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m)}. \end{aligned}$$

Note again that this weighted average integrates out p_2', r', Y', C' and ξ_d'.

Each MCMC iteration in the proposed algorithm consists of five blocks:

1. Draw $\sigma_\xi^m = (\sigma_{\xi_1}, \sigma_{\xi_2}, \sigma_{\xi_s})$ directly from their posterior distributions conditional on $\xi_t^{m-1} = (\xi_{1t}^{m-1}, \xi_{2t}^{m-1}, \xi_{st}^{m-1})$ for all observed t and g.

2. Draw ξ_t^m for all observed t and g conditional on the data, σ_ξ^m, θ_d^{m-1} and θ_s^{m-1}. In the Metropolis-Hastings algorithm, the joint-likelihood of the demand-side model and the pseudo-policy functions will be used to compute the acceptance probability.

3. Draw θ_d^m conditional on the data, $\{\xi_t^m\}$ and θ_s^{m-1} using the random-walk Metropolis-Hastings algorithm. In the Metropolis-Hastings algorithm, the joint-likelihood will be used.

4. Draw θ_s^m conditional on the data, $\{\xi_t^m\}$ and θ_d^m using the random-walk Metropolis-Hastings algorithm. In the Metropolis-Hastings algorithm, only the likelihood of the pseudo-policy functions will be used since θ_s^m does not enter the demand-side model.

5. Compute the pseudo-value functions for purchasing and selling decision problems. Starting from the terminal period, I sequentially compute the pseudo-value functions backwards at only one randomly drawn state point in each period. I store them and update H^m to H^{m+1}.

In deriving the posterior distribution of parameters, I use an inverted gamma prior on σ_ξ, and a flat prior on θ_d and θ_s. Also, note that the likelihood used in the IJC algorithm is called *pseudo*-likelihood as it is a function of *pseudo* alternative-specific value functions. Below, I provide a step-by-step procedure for the five blocks described above.

1. Suppose that we are at iteration m. We start with

$$H^m = \{\theta_d^l, \theta_s^l, \theta_d^{*l}, \{\tilde{V}_{it}^l(p_1, \tilde{p}_{2t}^l, \tilde{r}_t^l, \tilde{Y}_t^l, \tilde{C}_t^l, \tilde{\xi}_d^l, t; \theta_d^{*l})\ \forall i\}_{t=2}^T, \{\{\tilde{W}_{it}^l(\tilde{r}_t^l, \tilde{Y}_t^l, \tilde{\xi}_s^l, t, \tau; \theta_d^{*l})\ \forall i\}_{\tau=1}^{t-1}\}_{t=2}^T\}_{l=m-N}^{m-1},$$

   where N is the number of past iterations used for the expected future value approximation.

2. Draw $\sigma_\xi^m = (\sigma_{\xi_1}, \sigma_{\xi_2}, \sigma_{\xi_s})$ directly from their posterior distributions (inverted gamma) conditional on $\xi_t^{g,m-1} = (\xi_{1t}^{g,m-1}, \xi_{2t}^{g,m-1}, \xi_{st}^{g,m-1})$ for all observed t and g.

3. For each observed t and g, draw $\xi_t^{g,m}$ from its posterior distribution conditional on $\sigma_{\xi_1}^m$, θ_d^{m-1}, θ_s^{m-1}, $\{\xi_q^{g,m}\}_{q=1}^{t-1}$, and $\{\xi_q^{g,m-1}\}_{q=t+1}^{T^g}$. I will draw $\xi_{1t}^{g,m}$, $\xi_{2t}^{g,m}$, and $\xi_{st}^{g,m}$ separately. Below, I will describe how to draw $\xi_{1t}^{g,m}$, but the procedure can be applied for drawing $\xi_{2t}^{g,m}$ and $\xi_{st}^{g,m}$.

   (a) Draw $\xi_{1t}^{g,*m}$ (candidate parameter value) from $N(0, (\sigma_{\xi_1}^m)^2)$.

   (b) We compute the pseudo-joint likelihood at $\xi_{1t}^{g,*m}$ conditional on $\{\xi_q^{g,m}\}_{q=1}^{t-1}$, $\xi_{2t}^{g,m-1}$, $\xi_{st}^{g,m-1}$, $\{\xi_q^{g,m-1}\}_{q=t+1}^{T^g}$, θ_d^{m-1} and θ_s^{m-1}. Note that conditional on $\sigma_{\xi_1}^m$, the pseudo-joint likelihood prior to time t does not depend on $\xi_{1t}^{g,*m}$. Thus, we only need to compute the pseudo-joint likelihood at time t and later. To compute the pseudo-joint likelihood, we need to obtain the pseudo-alternative specific value functions for both purchasing and selling decisions at time t and later: $\tilde{V}_{ijt}^m(\cdot, t; \theta_d^{m-1})$ and $\{\tilde{W}_{ikt}^m(\cdot, t, \tau; \theta_d^{m-1})\}_{\tau=1}^{t-1}$. To obtain $\tilde{V}_{ijt}^m(\cdot, t; \theta_d^{m-1})$, we need to calculate both

$\hat{E}^m V_{it+1}(\cdot, t+1; \theta_d^{m-1})$ (pseudo-expected future value when consumers choose no purchase option) and $\hat{E}^m W_{it+1}(\cdot, t+1, 1; \theta_d^{m-1})$ (pseudo-expected future value when consumers choose to buy new or used game, thus the third argument of W_{it+1}, which is the duration of ownership, is set to 1), using Equations (C.2) and (C.1), respectively, as shown below:

i. For $\hat{E}^m V_{it+1}(\cdot, t+1; \theta_d^{m-1})$, we take the weighted average of $\{\tilde{V}^l_{it+1}(p_1, \tilde{p}^l_{2t+1}, \tilde{r}^l_{t+1}, \tilde{Y}^l_{t+1}, \tilde{C}^l_{t+1}, \tilde{\xi}^l_d, t+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (C.2).

ii. For $\hat{E}^m W_i(\cdot, t+1, 1; \theta_d^{m-1})$, we take the weighted average of $\{\tilde{W}^l_{it+1}(\tilde{r}^l_{t+1}, \tilde{Y}^l_{t+1}, \tilde{\xi}^l_s, t+1, 1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (C.1). Note that since potential buyers at time t will have owned the game for one period when they reach $t+1$, the set of past pseudo-value functions used here only include those evaluated at $\tau = 1$.

To obtain $\{\tilde{W}^m_{ikt}(\cdot, t, \tau; \theta_d^{m-1})\}_{\tau=1}^{t-1}$, we need to calculate $\{\hat{E}^m W_{it+1}(\cdot, t+1, \tau+1; \theta_d^{m-1})\}_{\tau=1}^{t-1}$ by the weighted average of the past pseudo-value functions $\{\tilde{W}^l_{it+1}(\tilde{r}^l_{t+1}, \tilde{Y}^l_{t+1}, \tilde{\xi}^l_s, t+1, \tau+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (C.1).[1]

(c) Similarly, we compute the pseudo-joint likelihood at $\xi_{1t}^{g,m-1}$ conditional on $\{\xi_k^{g,m}\}_{k=1}^{t-1}$, $\xi_{2t}^{g,m-1}$, $\xi_{st}^{g,m-1}$, $\{\xi_k^{g,m-1}\}_{k=t+1}^{T^g}$, θ_d^{m-1} and θ_s^{m-1}.[2]

(d) Based on the pseudo-joint likelihoods at $\xi_{1t}^{g,*m}$ and $\xi_{2t}^{g,m-1}$, we compute the acceptance probability for $\xi_{1t}^{g,*m}$ and decide whether to accept (i.e., set $\xi_{1t}^{g,m} = \xi_{1t}^{g,*m}$) or reject (i.e., set $\xi_{1t}^{g,m} = \xi_{1t}^{g,m-1}$).

(e) Using a similar procedure, draw $\xi_{2t}^{g,m}$ and $\xi_{st}^{g,m}$. One difference in drawing $\xi_{st}^{g,m}$ is that conditional on $\sigma_{\xi_s}^m$, $\xi_{st}^{g,*m}$ does not influence the likelihood function for purchasing decisions.

[1] Conditional on σ_ξ^m, pseudo alternative-specific value functions do not depend on $\xi_{1t}^{g,*m}$. This is also true for $\xi_{2t}^{g,*m}$ and $\xi_{st}^{g,*m}$. Thus, pseudo alternative-specific value functions can be pre-computed right after step 2.

[2] In a standard Metropolis-Hastings algorithm, this step is not necessary as this value has been computed in the previous iteration. However, the IJC algorithm updates the set of past pseudo-value functions in each iteration. Thus, the pseudo-likelihood at $\xi_{1t}^{g,m-1}$ in iteration $m-1$ will be different from that at $\xi_{1t}^{g,m-1}$ in iteration m.

4. Use the Metropolis-Hastings algorithm to draw θ_d^m conditional on $\{\xi_t^m\}$ and θ_s^{m-1}.

   (a) Draw θ_d^{*m} (candidate parameter vector).

   (b) We compute the pseudo-joint likelihood at θ_d^{*m} conditional on $\{\xi_t^m\}$ and θ_s^{m-1} based on the pseudo-alternative specific value functions for both purchasing and selling decisions at θ_d^{*m}: $\tilde{V}_{ijt}^m(\cdot, t; \theta_d^{*m})$ and $\{\tilde{W}_{ikt}^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$ for all observed t and g. To obtain $\tilde{V}_{ijt}^m(\cdot, t; \theta_d^{*m})$, we need to calculate both $\hat{E}^m V_{it+1}(\cdot, t+1; \theta_d^{*m})$ and $\hat{E}^m W_{it+1}(\cdot, t+1, 1; \theta_d^{*m})$, which are computed as the weighted average of past-pseudo value functions evaluated at time $t+1$:

      i. For $\hat{E}^m V_{it+1}(\cdot, t; \theta_d^{*m})$, we take the weighted average of $\{\tilde{V}_{it+1}^l(p_1, \tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l, \tilde{\xi}_d^l, t+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (C.2).

      ii. For $\hat{E}^m W_{it+1}(\cdot, t, 1; \theta_d^{*m})$, we take the weighted average of $\{\tilde{W}_{it+1}^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, 1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (C.1). Again, note that since potential buyers at time t will have owned the game for one period when they reach $t+1$, the set of past pseudo-value functions used here are all evaluated at $\tau = 1$.

   To obtain $\{\tilde{W}_{ikt}^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$, we only need to calculate $\{\hat{E}^m W_{it+1}(\cdot, t+1, \tau+1; \theta_d^{*m})\}_{\tau=1}^{t-1}$ by the weighted average of the past pseudo-value functions $\{\tilde{W}_{it+1}^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, \tau+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation (C.1).

   (c) Similarly, we compute the pseudo-joint likelihood at θ_d^{m-1} conditional on $\{\xi_t^m\}$ and θ_s^{m-1}.

   (d) Based on the pseudo-joint likelihoods at θ_d^{*m} and θ_d^{m-1}, we compute the acceptance probability for θ_d^{*m} and decide whether to accept (i.e., set $\theta_d^m = \theta_d^{*m}$) or reject (i.e., set $\theta_d^m = \theta_d^{m-1}$).

5. Use the Metropolis-Hastings algorithm to draw θ_s^m conditional on $\{\xi_t^m\}$ and θ_d^m.

   (a) Draw θ_s^{*m} (candidate parameter vector).

   (b) We compute the pseudo-likelihood for pseudo-policy functions at θ_s^{*m} conditional on $\{\xi_t^m\}$ and θ_d^m. Note that the pseudo-alternative specific value functions do not

depend on θ_s^{*m}, but are required to compute the pseudo-likelihood at θ_s^{*m} since they influence the evolution of equilibrium state variables. However, they have already been computed in step 4(b) (if θ_d^{*m} has been accepted) or 4(c) (if θ_d^{*m} has been rejected), there is no need to re-compute them here to form the pseudo-likelihood for pseudo-policy functions.

(c) To form the acceptance probability of θ_s^{*m}, we need the pseudo-likelihood for pseudo-policy functions at θ_s^{m-1} conditional on $\{\xi_t^m\}$ and θ_d^m. Note that this value has been computed in step 4 and needs not be re-computed here.

(d) Based on the pseudo-likelihood for pseudo-policy functions at θ_s^{*m} and θ_s^{m-1}, we compute the acceptance probability for θ_s^{*m} and decide whether to accept (i.e., set $\theta_s^m = \theta_s^{*m}$) or reject (i.e., set $\theta_s^m = \theta_s^{m-1}$).

6. Compute the pseudo-value functions for purchasing and selling decision problems.

(a) For each $t = 2, \ldots, T$, make a draw of used-game price ($\tilde{p}_{2t}^m$), resale value ($\tilde{r}_t^m$), inventory level ($\tilde{Y}_t^m$), and cumulative number of newly introduced games ($\tilde{C}_t^m$) from uniform distributions with appropriate upper- and lower-bound (e.g., upper- and lower-bound of observed values).

(b) Make a draw of $\tilde{\xi}_1^m$, $\tilde{\xi}_2^m$, and $\tilde{\xi}_s^m$ from the corresponding distribution based on $\sigma_{\xi_1}^m$, $\sigma_{\xi_2}^m$, and $\sigma_{\xi_s}^m$.

(c) Start from the terminal period T.

i. Compute the value functions $\tilde{V}_{iT}^m(p_1, \tilde{p}_{2T}^m, \tilde{r}_T^m, \tilde{Y}_T^m, \tilde{C}_T^m, \tilde{\xi}_d^m, T; \theta_d^{*m})$ and $\{\tilde{W}_{iT}^m(\tilde{r}_T^m, \tilde{Y}_T^m, \tilde{\xi}_s^m, T, \tau; \theta_d^{*m})\}_{\tau=1}^{T-1}$ for all i. Note that at time T, there is no need to compute the pseudo-expected future value. Thus, the value functions computed at time T are not pseudo-value functions.

ii. Store $\tilde{V}_{iT}^m(\cdot, T; \theta_d^{*m})$ and $\{\tilde{W}_{iT}^m(\cdot, T, \tau; \theta_d^{*m})\}_{\tau=1}^{T-1}$.

(d) For $t = T-1, \ldots, 2$, compute the pseudo-value function $\tilde{V}_{it}^m(p_1, \tilde{p}_{2T}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m})$ and $\{\tilde{W}_{it}^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$ for all i backwards.

i. To compute $\tilde{V}_{it}^{m}(\cdot, t; \theta_d^{*m})$, we need to calculate $\hat{E}^m V_{it+1}(\cdot, t+1; \theta_d^{*m})$ and $\hat{E}^m W_{it+1}(\cdot, t+1, 1; \theta_d^{*m})$ based on Equations (C.2) and (C.1), respectively.

ii. To compute $\{\tilde{W}_{it}^{m}(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$, we need to calculate $\{\hat{E}^m W_{it+1}(\cdot, t+1, \tau+1; \theta_d^{*m})\}_{\tau=1}^{t-1}$ based on Equation (C.1).

iii. Store $\tilde{V}_{it}^{m}(\cdot, t; \theta_d^{*m})$ and $\{\tilde{W}_{it}^{m}(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$.

7. Go to iteration $m+1$.

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