CHAPTER III

ELEMENTS FOR A POSITIVE THEORY OF EQUITY VALUATION: SOME EXISTING MODELS

3.1 A Note on Financial Assets

Financial assets are a class of goods that differ fundamentally from other forms of wealth in our society. This difference is immediately apparent when two factors are considered: (1) the degree of liquidity or mobility of exchange for financial assets is generally much greater than that for real assets; and (2) the end use, the purpose of purchase, of financial assets is usually quite different from that for other forms of wealth.

The degree of mobility is, of course, a reflection of the broadness of the market for particular financial assets and the ease and exactness with which financial assets can be described. It is quite possible to pick up a telephone in most parts of the world today and place an order to buy or sell a stock on the New York Stock Exchange. Moreover, a reasonable estimate of the current market price for a particular stock can be had by asking the broker or, in some cases, by merely examining one of the newspapers that carry the names and prices of many of the more actively traded financial goods. All this is quite in contrast to the market for other types of wealth. In buying or selling a home, a car, a refrigerator, there may be sizable lags between the time a sale is initiated and the time it is finally consummated. More important, secondary markets for physical assets are highly imperfect and in many cases virtually nonexistent.

The difference in end use between financial assets and other forms of wealth is a fundamental difference that poses questions not ordinarily considered in detail in the study of markets for physical goods. For example, to ask the question, "What is it that is really being bought when the buyer buys the bushel of barley?" is verging on metaphysics.[1] The same question would appear to have more empirical content when applied to financial assets, for a financial asset is seldom a desired good in its own right. Few people purchase stock certificates just to collect stock certificates. What is "really being bought" when a financial good is purchased is a store of wealth, a claim to other financial assets such as money (return of principal) or money flows (dividends, earnings, capital appreciation) or a claim to some physical assets (inventory, equipment). It should be possible, therefore, to explain prices of financial goods in terms of the prices and relative importance of the underlying assets.

3.2 Determinants of Security Share Prices

The price of a bond can be mostly explained by three of four factors. For example, in a stable economy the price of United States Government bonds, which always have a ready market and no risk of default, will be a function of four variables: (a) the principal on the bond; (b) the

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[1] I do not mean to imply that economists do not or should not engage in metaphysics. Anyone who has read some of the discussions on utility theory and the attempts to measure utiles would know this is just not the case. See, for example, Ward Edwards' review article "The Theory of Decision Making" in Albert H. Rubenstein and Chadwick J. Haberstroh, Some Theories of Organization, (Homewood, Ill.: Irwin, 1960).

coupon interest rate; (c) the length of maturity; and (d) the market

discount rate. To generalize to the case of any pure bond instrument

in a nonstationary economy, three additional factors would probably be

important: (e) expectations about interest rate changes; (f) the riskiness

of default; and (g) the breadth of the market for that particular bond.[2]

Unfortunately, the determinants of equity share prices are not so

easily discerned. Indeed, I will present evidence later which suggests

that research to date has been unable to uncover variables that con-

sistently explain even two-thirds of the variation in equity share prices.

Uncertainty about the variables behind an equity's value arises partially

because of the legal difference between debt and equity instruments.

Whereas the bondholder has a legal claim to a specified principal and

interest payment, the legal claim of the stockholder is to the rights of

ownership. What benefits accrue from these rights in the form of obtain-

able stocks or flows is a matter of some uncertainty.[3]

Present models of equity valuation are restricted to a few of the

stock and flow variables reported in the financial statements of the firm.

Table III-1 summarizes the types of financial variables used by some of

the more widely known equity valuation models. The actual independent

variables used are often combinations of these financial variables or some

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[2]See Lawrence Fisher, "Determinants of Risk Premiums on Corporate Bonds," Journal of Political Economy, 67 (June, 1959), pp. 217-237. There is an additional factor that may be relevant to some groups of investors, and that is the tax status of the bonds.

[3]While it is usually possible, of course, to sell the stock certificate itself for some capital gain or loss, the price at which it can be sold is ultimately determined by the values inherent in future ownership rights.

Table III-1

"A gross indication of the principal variables used in various models of equity valuation"

	Independent Variables										
		Stock Variables							Flow Variables		
	P	IN	CA	PE	TA	CL	LL	NW	S	NI	D
Ahlers	X									X	X
Barges							X	X	X		
Benishay	X						X			X	X
Beranek				X			X	X		X	X
Durand								X		X	X
Gilles										X	X
Gordon	X	X	X	X	X	X	X	X		X	X
Kolin							X	X	X		
Lerner-Carleton				X			X	X		X	X
Modigliani-Miller	X						X		X		
Mod.-Miller (1966)	X			X			X			X	X
Ortner	X										X
Peterson		X		X				X		X	X
Whitbeck-Kisor										X	X

P = market price of equity
IN = inventory
CA = current assets
PE = plant and equipment
TA = total assets
CL = current liabilities
LL = long-term liabilities
NW = book value of net worth
S = sales
NI = net income after taxes
D = dividends

statistical transformation of the variables, such as variance estimates.
These particular models were chosen for two reasons: (1) they are models
that have been subjected to some empirical testing; (2) some of the
models have been so well received in the professional literature that
the models are not only standard journal references, but are also be-
coming textbook examples.[4]

Note that the models all restrict themselves to two flow variables--
net income and dividends. These are certainly the most widely reported
variables in the flow class, but they are not the only such variables
available. A content analysis of company financial statistics appear-
ing in the news media would reveal that sales are almost always reported,
and quite often some measure of net cash flow (i.e., net income plus
depreciation charges) is mentioned. If the implicit models found in re-
ports by professional investment advisory services are examined, they
will be found to contain reference to sales, net cash flows, capital
expenditures, managerial ability, intraindustry and interindustry com-
petition, and more.[5] While these latter variables may dominate the
professional investor's implicit models of equity valuation, such variables
are virtually absent from the present economic models of equity valuation.

This chapter will concentrate on several financial models that have
been proposed to answer questions relating to explanatory variables for

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[4]See, for example, R. Lindsay and A. W. Sametz, Financial Management:
An Analytic Approach, (Homewood, Ill.: Irwin, 1963) or W. Beranek,
Analysis for Financial Decisions, (Homewood, Ill.: Irwin, 1963).

[5]Examples can be found in any issue of such publications as the
Value Line Survey, Forbes, Barron's.

the valuation of common stocks. These models might all be called "economic models" of equity valuation, in contrast to the "investor models" used by most professional stock investors. The distinction is important, for the economic models postulate equilibrium stock prices to be a function of firm financial variables, whereas investor models are more likely to incorporate current market dynamics and economic factors outside the firm. The focus in economic models is on explaining relative share price differences in cross-section models whereas investor models focus on predicting relative stock price adjustments over time. Parameters for the economic models are estimated from regression equations, using cross-section and time-series firm financial data. Such parameters as may exist for the investor models are usually estimated from graphical inspection or personal introspection.

The economic models described in this chapter by no means exhaust the list of proposed models in this class. The five models selected for testing were chosen because they are all widely known and referenced, and because they represent a cross-section of the types of issues that are under study in the general area of equity valuation. The five models are fomulations suggested by Durand, Modigliani-Miller, Barges, Benishay, and Gordon. The Durand model was an early attempt to ascertain the relative importance of net income, dividends, and book net worth in the valuation of equity shares. Although this particular model has become a convenient straw man for subsequent researchers, Durand's careful statistical analysis set a standard that should have been more closely followed. The Modigliani-Miller and Barges models are attempts to measure the impact of capital financing decisions on the value of the

corporation. The emphasis here is on external financing of the debt-equity type rather than the internal financing of the retained earnings-dividends type implicit in Durand's model. Haskel Benishay attempts to explicitly incorporate internal and external financing factors as well as risk factors in his model for examining the determinants of the differences in rates of return on corporate equities. The final model selected is the explicit model of equity share price determination proposed by Myron Gordon. This model, constructed by extending earlier models on which Gordon had worked, is probably the most elaborate attempt to find explanatory variables to describe the equity valuation process available in the literature to date.

3.3 The Durand Model[6]

Durand undertook his study on "Bank Stock Prices and the Bank Capital Problem" in 1952 as part of the National Bureau of Economic Research financial research program. The purpose of the study was to measure the relative importance of some basic variables that might affect the market price of bank stocks. Durand was concerned that some stocks were selling for less than book value; for he believed that, in the long run, the ability of a company to raise capital through a stock flotation depends on whether or not a stock is selling for more than its book value. While a bank could certainly sell stock even if present book value were greater than present market value, it appeared at the time of the study that such equity financing could only be done at rather prohibitive costs.
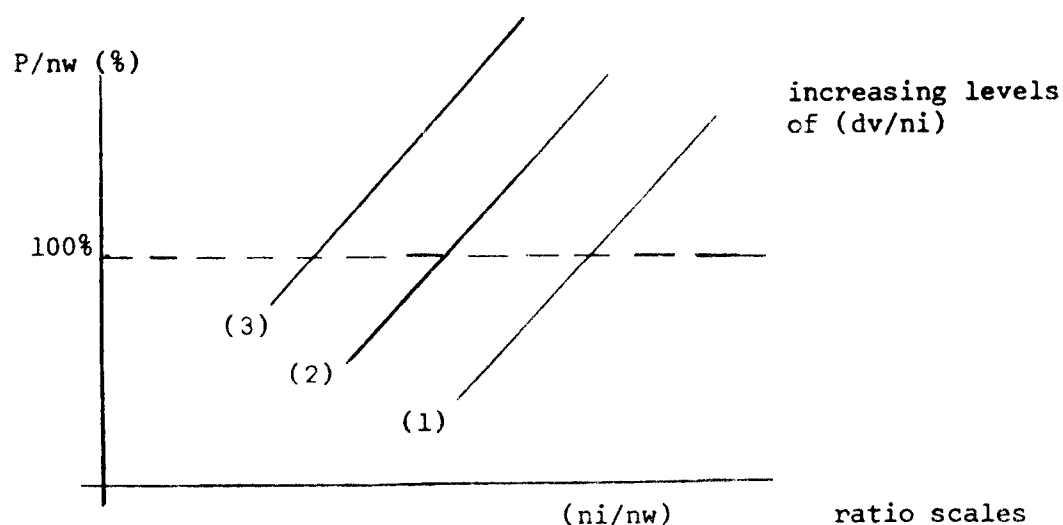
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[6]See David Durand, Bank Stock Prices and the Bank Capital Problem, (New York: National Bureau of Economic Research, occasional paper 54, 1957) and "Bank Stock Prices and the Analysis of Covariance, "Econometrica 23 (January, 1955), pp. 30-45.

At least, this was one of the inferences Durand made from his study

of the drastic decline in bank capital ratios.

Figure III-1 indicates the type of relationship Durand was in-

terested in investigating.[7] The question that concerned him was, "Given

a ratio of market price to book net worth, what level of the bank's rate

of return (net income/book net worth) would be necessary to maintain a

(P/nw) ratio of at least 100%?" At least one additional factor was

thought to influence this relationship, and that was the dividend payout

rate. An increase in this rate, ceteris paribus, was assumed to decrease

the rate of return necessary to maintain (P/nw) at a given level. This

basic relationship can be written as:[8]

$$(3.1) \quad \frac{P}{nw} = a \quad . \quad [ni/nw]^e \quad \cdot \quad [dv/ni]^f$$

Figure III-1

P/nw (%)

increasing levels
of (dv/ni)

100%

(3)

(2)

(1)

(ni/nw)   ratio scales

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[7]See Durand, Bank Stock Prices and the Bank Capital Problem, p. 30.

[8]A summary of most of the symbols used in this chapter can be found in Table III-2.

Durand decided to test (3.1) in a slightly different form. The actual test equation is indicated by (3.3).

$$(3.2) \quad P = a \cdot [ni/nw]^e \cdot [dv/ni]^f \cdot [nw]$$
$$= a \cdot [ni]^{e-f} \cdot [dv]^f \cdot [nw]^{1-e}$$
$$(3.3) \quad = a \cdot [ni]^b \cdot [dv]^c \cdot [nw]^d$$

Since the coefficients of [nw] are now unconstrained, one test of the reasonableness of equation (3.1) is to see whether or not the parameter sum (b+c+d) is approximately equal to one.

Durand examined several other variables to see whether additional factors besides the [dv/ni] ratio should be incorporated into the basic model. These factors included: (a) total capital, as a measure of size of bank; (b) ratio of assets to capital; (c) ratio of risk assets to capital; (d) ratio of current dividend rate to average past dividend rate; (e) average annual rate of increase in earnings as measured by the slope coefficient of the regression of earnings on time for each bank; and (f) stability of earnings as measured by the standard deviation of earnings about the trend line in (e). None of these variables performed well enough to warrant being added to the basic regression.

Although Durand attempted to control for size effects, this is very difficult to successfully do in equity valuation models. There are a number of ways spurious relationships can be introduced into such models.[9]

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[9] I am indebted to Professor Kalman J. Cohen (Carnegie Institute of Technology) for pointing out the potential seriousness of some of these biases. It is very difficult to remove such biases, and about the only one who has seriously considered the problem is Marshall Kolin in his dissertation, "The relative Price of Corporate Equity with Particular Attention to Investor Valuation of Retained Earnings and Dividends," University of Chicago, 1965.
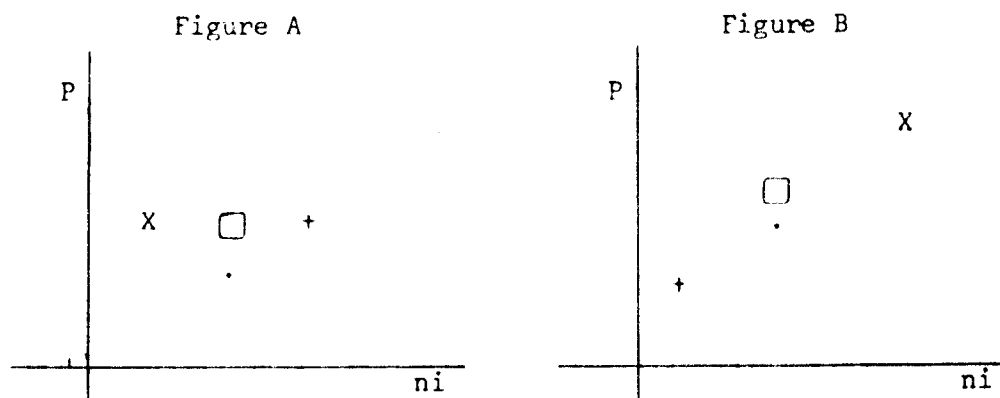
Table III-2

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                        SUMMARY OF SYMBOLS


      P = price of a stock at the end of period t

     DV = dividends for the period t

     dv = dividends per share

     NI = net income after taxes

     ni = NI per share

     NW = book value of the net worth of the corporation

     nw = net worth per share

      S = market value of common stock equity = (P·N)

     LL = long-term debt for the firm, including any

          preferred stock, at book value

     DT = total balance sheet liabilities for the firm

     IY = inventory at the end of period t

     NP = net plant and equipment (gross plant and equip-

          ment less depreciation at stated book values)

     CH = cash at the end of period t

     CL = current liabilities

     AR = accounts receivable

     TA = total assets at book value
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For example, suppose at some point in time there were no relationships between price per share and net income per share for a group of stocks. (Figure A) Suppose then that 25% of the stocks were selected at random and subjected to a 2- for- 1 stock split, and that another 25% of the

stocks selected at random were subjected to a reverse 1- for-2 stock split. Then a positive correlation would be introduced if the original price/earnings ratios continued to hold. (Figure B)

Figure A

Figure B

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P|                              P|
 |                               |
 |                               |        X
 |                               |
 |                               |       []
 |    X   []   +                 |       .
 |        .                      |   +
 |                               |
 |_____ni      |_____ni
```

Problems due to stock splits will disappear if aggregate value models are used. Aggregate values have their own biases however. Mergers can create spurious relationships as effectively as stock splits. Differences in the ages of the firms and differences in market opportunities arising from concentration in different industries also make for disparate firm sizes.

The data Durand used in testing the model were for the years 1946-1953 and for six groups of banks: (1) 17 New York City banks; (2) 25 large banks outside New York City; (3) 17 northeastern banks; (4) 17 midwestern banks; (5) 17 southeastern banks; (6) 24 western banks. It is not clear why these particular groupings were made for the 117 banks for which data could be obtained. Durand felt that the scope of activities and size of asset portfolios for the New York City banks were different enough from most banks outside the city to warrant making this group a separate sample. But it was apparently convenience in data collection that led

to the other groupings, for Durand did not postulate any relationship about the impact of regional differences in the capital markets within the United States.

Representative results from the 48 basic regressions are indicated in Table III-3. These are not to be construed as average values; for, as we

TABLE III-3

DURAND PARAMETER ESTIMATES FOR 1950

	a	b	c	d	b+c+d
Group (1)	.53	.17	.23	.57	.97
(2)	.81	.06	.74	.33	1.13
(3)	.89	.34	.54	.22	1.10
(4)	1.19	.15	.78	.09	1.02
(5)	.93	.17	.64	.26	1.07
(6)	.56	.32	.24	.49	1.05

shall see, the concept of average or typical parameter estimates is almost meaningless. In fact, the variability in parameter estimates from year to year within a sample group is as great as the indicated variability among samples. Durand documented this variability by performing a series of covariance analysis tests on the data. He concluded that the samples could not be regarded as coming from the same underlying population either on a cross-section (pooling of groups) or time-series (pooling of years) basis.[10] Nor could additional variables be found that would reduce these sample heterogeneities.

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[10] See Durand, "Bank Stock Prices and the Analysis of Covariance," pp. 34-37.

In equation form (3.3), the Durand model is a direct test of the question of the relative importance of earnings and dividends. In this particular equation form, the marginal change in price for marginal changes in earnings or dividends is given by:

$$(3.4) \quad \frac{dP}{d(ni)} = b \cdot P/ni$$

$$\frac{dP}{d(dv)} = c \cdot P/dv$$

For the bank samples, the dividend payout ratio averaged about 50%; and for the majority of samples, the dividend parameter (c) was roughly twice the earnings parameter (b). This implies that a change in dividends has four times the impact on price that an equivalent change in earnings would have. Since others have examined this question using a linear hypothesis, the Durand model will be tested in both its logarithmic and linear forms. In summary form, the Durand models to be tested are:

Durand Equation

$$(3.5) \quad P = a(x_1)^b \cdot (x_2)^c \cdot (x_3)^d$$

$$(3.6) \quad P = a + b(x_1) + c(x_2) + d(x_3)$$

(x_1) = net income per share

(x_2) = dividends per share

(x_3) = book net worth per share

3.4 The Modigliani-Miller Model[11]

Professors Modigliani and Miller did not set out explicitly to develop

an explanatory model of equity valuation. Instead, they were interested in

examining the theory of the impact of leverage on the cost of capital to a

firm in a world where yields are uncertain and where capital can be acquired

either through debt or equity financing. Nevertheless, Proposition II in

their now famous set of conjectures is an implicit model of equity valua-

tion. The proposition is stated in the form:[12]

(3.7) $i = a + (a-r)(1-t)(LL'/S)$

where i = expected yield on a share of stock, (LL') is the

market value of (LL), (t) is the corporate tax rate, and

(r) is the cost of debt capital.

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[11]See Franco Modigliani and Merton Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review,48 (1958), pp. 261-297 and also their "reply," American Economic Review,49 (1959), pp. 655-669 and their "Corporate Income Taxes and the Cost of Capital: A Correction," American Economic Review,53 (1963), pp. 433-443.

Their most recent article, "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954-1957," American Economic Review,56 (1966), pp. 333-391, appeared after the tests for this dissertation had been completed. At first pause this recent paper seems to be a significant improvement over previous empirical work since a growth term is explicitly included and expected earnings yields are now estimated via a two-stage regression process incorporating dividends. There are already several working papers in preparation on the West coast suggesting among other things that (a) M-M do not entirely understand the regulatory process in the electric utility industry with regard to the accounting base on which permissible rates are determined, (b) empirical tests of their model with the same firms for the years 1955, 1961-63 give substantially different (and not significant) parameters, (c) M-M have badly mixed up earnings and dividends in the two-stage process so that almost the same results can be gotten by using dividends where they say earnings and earnings where they say dividends. Unfortunately these comments must remain hearsay criticism until the working papers are completed.

[12]Modigliani and Miller, "Corporate Income Taxes and the Cost of Capital," p. 439.

The parameter (a) may be interpreted as the capitalization rate for a
pure equity stream in the "risk class" in which the particular stock is
a member. Since the expected yield on a share of stock (i) is defined by
M-M as expected net income relative to market value [E(ni)/P], the following
equity valuation equation can be derived:

$$(3.8) \quad P = \frac{E(ni) - (a-r) \cdot (1-t) \cdot (11')}{a}$$

where (11') is the per-share market value of (LL).

This indicates an inverse relationship between price and long-term debt,
whereas traditional theories of the impact of financial structure seem to
suggest there is either no relationship between price and long-term debt
or a slightly positive relationship.[13]

Proposition II was tested by the following specification:

Modigliani-Miller Equation

$$(3.9) \quad Y = a + b(x_1)$$

$$(Y) = \frac{\text{after tax net income}}{\text{market price of equity}}$$

$$(x_1) = \frac{\text{market value debt}}{\text{equity ratio}}$$

The original tests of Propositions I and II by M-M relied on sample data
provided by F. B. Allen (utility sample) and Robert Smith (oil company

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[13]There is some uncertainty as to exactly what position the tradi-
tionalists hold. It seems likely there is no single traditionalist theory,
but a straw man erected by M-M. See Modigliani and Miller, "The Cost of
Capital," pp. 276-281.

sample).[14] The utility sample contained data for 43 large electric

utilities for 1947-1948. The sample points used were average figures from

these two years. The Smith study contained data for 42 oil companies for

the year 1953. Earnings data for 1952 were collected to provide an average

earnings figure, but this average produced results little different from

those using 1953 data alone. The debt figures for the oil companies con-

tained some book value estimates instead of market value estimates. This

is not unreasonable in view of the almost insuperable difficulties in

obtaining meaningful market value estimates for most corporate debt. The

parameter estimates obtained by M-M were:[15]

TABLE III-4

M-M PARAMETER ESTIMATES

	a	b	
Electric utilities	6.6	.017	(.004)
Oil companies	8.9	.051	(.012)

As Table III-4 indicates, in both samples (b) was positive and significant

and both parameters were of reasonable orders of magnitude.

There are serious statistical methodological questions that arise

when trying to test Proposition II. First is the problem of possible

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[14]See F. B. Allen, "Does Going into Debt Lower the Cost of Capital,"
Analysts Journal,10 (August, 1954), pp. 57-61 and R. Smith, "Cost of
Capital in the Oil Industry" (hectograph) (Pittsburgh: Carnegie Institute
of Technology, 1955). Data for both samples are best obtained by writing
to Professor Miller (University of Chicago).

[15]See Modigliani and Miller, "The Cost of Capital," pp. 284-286.

biases in the measures of debt, equity, and net income used. It seems
likely that the measures of debt utilized are likely to have relatively
small errors. In the cases where the market value of the debt was used,
there is no reason to believe this is not an accurate estimate of both
the aggregate debt or small bundles of it. If the book value of the debt
is used, there will be systematic errors if the level of interest rates
has changed. This will be important in cross-section samples only to the
extent different companies have different debt maturities or have floated
debt at different past dates.

The estimation of the aggregate value of the firm's equity presents
a more serious challenge. In a world of perfect capital markets and
atomistic firms, it is reasonable to estimate the aggregate value of the
equity by market price times the number of shares outstanding. The only
question concerns the appropriateness of using a price high-low average
as the estimate of market value. If, however, there is a positive growth
trend, and if earnings estimates are also averaged, there will be a
positive bias imparted to the slope parameter.

Another question that arises in testing the M-M proposition is
whether a simple average of recent earnings figures is a satisfactory
surrogate for expected net income. In particular, what is the impact
of growth on the model variables? In their original paper, Modigliani
and Miller neglect growth by assuming that all potential investments
earn a rate of return equal to the cost of capital; actually, if one
assumes a homogeneous risk class can be selected, then serious bias will
not occur so long as growth (defined as opportunities to invest at a
rate of return greater than the current cost of capital) is constant for

all firms in the sample. There is no evidence to suggest this is a reasonable first approximation, however.

Weston has argued that the absence of a growth term strongly biases the results in favor of the M-M proposition.[16] A test of the hypothesis (3.10) by him indicated that (b) was zero and (d) was negative and

$$(3.10) \quad \frac{NI}{S} = a+b \left[\frac{LL'}{S+LL'} \right] + c(TA) + d(G)$$

where (G) is the ten-year compound growth rate in

earnings per share

significant. Unfortunately, it is not obvious how one relates (3.10) to the M-M theory of (3.7). Weston does have corroborating evidence that is more useful. He found that a significant inverse relationship existed when $\left[\frac{LL'}{S+LL'} \right]$ was regressed on (G). That is, firms with higher growth rates tend to have lower market debt/equity ratios. Suppose it is also true that equity share prices are directly related to expected earnings. Reported earnings will probably be lower than expected earnings for firms with higher than average growth rates. Given these assumptions and Weston's findings, we can infer that the absence of an explicit growth measure could impart a serious bias in favor of the M-M proposition.

There are other omitted variables that could have an impact on Proposition II. The most obvious omission is the absence of any measure of risk class, which M-M cite as a critical variable. There is also some question as to whether investors use book value debt/equity ratios or market value ratios when examining the financial structure of the firm.

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[16]See J. F. Weston, "A Test of Cost of Capital Propositions," Southern Economics Journal, 30 (1963), pp. 105-112.

Finally, it is not clear that the dividends can be ignored in defining
expected investor returns, even under the equilibrium conditions M-M
specify.

3.5 The Barges Elaboration[17]

The award-winning Ford Foundation doctoral dissertation by Alexander
Barges had as its thesis an analysis of why the empirical tests of
Modigliani-Miller were biased in favor of the M-M proposition and what
could be done about the bias. Briefly, Barges' argument is one of con-
tamination.[18] Suppose we have three firms that are identical in the
financial variables reported except that they are in different risk
classes. If these firms happen to get picked for the same sample, they
would show up in the Figure III-2 in the manner suggested. Expected net

FIGURE III-2
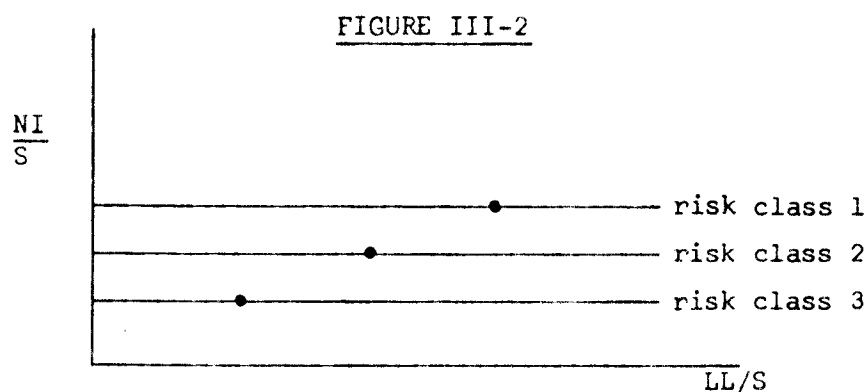
income, the market value of debt, and actual capital invested (NW) are
identical for all three firms. But the firms are in different risk

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[17]See Alexander Barges, The Effect of Capital Structure on the Cost
of Capital, (Englewood Cliffs, N.J.: Prentice-Hall, 1963).

[18]Ibid., pp. 20-34.

classes so the market value of the equity will differ from firm to firm. That is, high-risk firms would have a higher debt/equity ratio than low-risk firms; so that, even if there were no real relation, a contaminated sample could favor the M-M hypothesis. One way to get rid of this bias is to substitute (LL)/(NW) for (LL)/(S), in which case the three points would be vertically distributed on the same graph.

But is Barges correct in asserting this possible bias exists? If we extend his partial parameter variation analysis to include variation of the other two variables in the equation (i.e., (LL) and (NI), which could also be the variables that differ for different risk classes), it would appear that one can readily get all the points specified in Figure III-3. In such a case it is hard to assert before tests are made that this bias really does exist.

FIGURE III-3
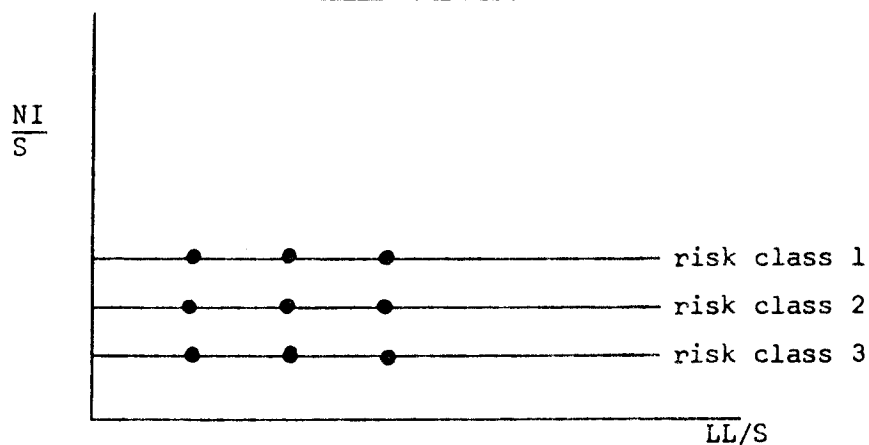
This is not to imply that Barges' tests are without merit. The basic test form that we shall consider is given by:

Barges Equation

(3.11)   Y   = $a+b(x_2)$

(Y) = $\dfrac{\text{after tax net income}}{\text{market price of equity}}$

(x_2) = book value debt/equity ratio

In terms of the M-M formulation for Proposition II, Barges' equation would be:

(3.12)   i = $a+(a-r) \cdot (1-t) \cdot (LL/NW)$

This implies an equity valuation equation of the form:

(3.13)   P = $\dfrac{E(ni)}{a+(a-r) \cdot (1-t) \cdot (LL/NW)}$

Equation (3.13) can be compared to the M-M valuation equation (3.8).
In the M-M equation a negative linear relationship between price and debt
is suggested. In equation (3.13) $\partial P/\partial(LL)$ is still negative, but the
relation is non-linear suggesting that larger and larger increments of
debt reduce price by smaller and smaller amounts. Of course, at some
point in both models, it would no longer be a reasonable first approxi-
mation to consider (a,r) stable parameters.

Although Barges' specification cannot, at least on an a priori basis,
reduce biases caused by having different risk classes in the same sample,
this specification can, under certain circumstances, partly reduce the
biases created by the absence of growth measures. If industries are
selected where the market value of an average firm can be approximated
by its book value, growth firms will be found to sell at a premium to
their book value and deteriorating firms at a discount. By substituting

(NW) for (S) in the independent variable, Barges tends to condense the horizontal dispersion created by the implied growth variables. However, the vertical dispersion remains, since the dependent variable, expected rate of return, has not been adjusted for growth. Although this substitution of (NW) for (S) could reduce the growth bias, the adjustment process is so crude it is not likely to eliminate it entirely.

Barges conducted tests on three samples: (1) a group of 61 Class I railroads; (2) a sample of 63 department stores; (3) a group of 34 cement producers. Data are for the year 1956. Companies with no market value for their common stock and companies with negative earnings were excluded from the tests. It would appear that two of the samples, railroads and department stores, had average firm book values greater than average firm market values. The samples are therefore probably not representative of American industrial firms in 1956.

The parameter estimates obtained by Barges are indicated in Table III-5.[19] Unfortunately, Barges did not provide a direct confrontation of the M-M test equation except for a subsample of 28 large department stores. In that subsample, the M-M (b) was significant and the Barges (b) was not. It is interesting to note that, in this subsample, Barges cites examples of growth firms whose points shift rightward when (LL/NW) is substituted for (LL/S) and nongrowth firms whose points shift leftward when the substitution is made.[20] Even so, Barges does not seem to realize that his results are probably as much due to the growth adjustment index properties of (NW) as to any corrections in risk-class biases.

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[19]Ibid., pp. 40-76.

[20]Ibid., p. 71.

TABLE III-5

BARGES PARAMETER ESTIMATES

	a	b
Railroads	11.4	.019
Department Stores	10.1	.050
Cement Companies	8.0	-.010
28 Large Department Stores	10.3	.007
(LL/S) Test of Stores	8.0	.025*

*parameter significant at 5%
 other parameters not significant

3.6 The Benishay Model[21]

Haskel Benishay attempted to examine empirically the determinants of the differences in rates of return on corporate equities. The rate of return for a common stock equity is hypothesized to be a function of seven variables: (1) the trend in earnings; (2) the trend in the market price of the common stock; (3) the payout ratio; (4) the expected stability of the equity value; (5) the size of the firm, represented by the market value of the equity; (6) the debt/equity ratio. The test equation used is of the form:

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[21]See Haskel Benishay, "Variability in Earning-Price Ratios of Corporate Equities," American Economic Review, 51 (1961), pp. 81-94.

Benishay Equation

$$(3.14) \quad Y = a \cdot e^{[b(x_1) + c(x_2) + h(x_7)]} \cdot (x_3)^d \cdot (x_4)^e \cdot (x_5)^f \cdot (x_6)^g$$

(Y) = a weighted rate of return of net income to market equity value

(x_1) = a growth in earnings factor

(x_2) = a growth in equity value factor

(x_3) = pay-out ratio

(x_4) = stability of income measure

(x_5) = stability of equity value measure

(x_6) = size, as measured by equity value

(x_7) = a debt/equity ratio

The variables were defined in the following manner:

$(Y) = NI*/\overline{S}$

$$NI* = [NI(t) + \sum_{t-9}^{t-1} NI(t)/9]/2$$

$$\overline{S} = N[P_H + P_L]/2$$

$$= N \cdot \overline{P}$$

(x_1) = a growth in earnings factor

= $b_1/\overline{NI}$ where b_1 is the coefficient from the regression

$NI(t) = a_1 + b_1(t)$ $t = t-8, \ldots, t$ and

$$\overline{NI} = \sum_{t-8}^{t} NI(t)/9$$

(x_2) = growth in equity value

= b_2/P^* where b_2 is the coefficient from the regression

$$\overline{P}(t) = a_2 + b_2(t) \qquad t = t-8, \ldots, t \text{ and}$$

$$P^* = \sum_{t-8}^{t} \overline{P}(t)/9$$

(x_3) = pay-out ratio

$$= [\sum_{t-2}^{t} DV(t)/NI(t)] \cdot [100/3]$$

(x_4) = stability of income measure

= $\overline{NI}/\sigma_{x1}$

(x_5) = stability of equity value

= P^*/σ_{x2}

(x_6) = a measure of size

= $\overline{S}$

(x_7) = a debt/equity ratio

= $DT/\overline{S}$

To derive the implied equity valuation model, it will be assumed that the debt/equity ratio changes slowly with time so that (x_7) will be more or less independent of $(\overline{P})$. If we let (W) stand for the right-hand side of (3.14) excluding (x_6), we get an approximate valuation formula of the form:

$$(3.15) \qquad \overline{P} = (ni^*/W)^{\frac{1}{1+g}}$$

This is only an approximation, since the current value of $(\overline{P})$ is contained in the (P^*) measure of variables (x_2) and (x_5) as well as the debt/equity ratio.

It seems clear that there must be serious doubts about the independence of the Benishay variables. Every variable contains data points used in at least one other variable. More important, the use of the same data as dependent and independent variables can seriously bias the results. Suppose, for example, variables $(x_1 \, . \, . \, . \, x_7)$ are irrelevant for determining cross-section estimates of investor rate of return. To simplify matters, suppose that all coefficients but (g) turn out to be zero and we end up testing an equation whose true form can be reduced to:

$$(3.16) \quad Y = a(x_6)^g \quad or \quad \frac{NI^*}{S} = a(\overline{S})^g$$

Suppose also that there is no real relation between expected stockholder returns and firm size as measured by the market value of the equity, but that the measure of equity value $(\overline{S})$ is subject to substantial measurement error. That is, $(\overline{S})$ is only an estimated value of firm size determined from an average of the year price high-low stock values. These high-low values are random deviation extremes from the actual expected value. Depending on whether the deviations of $(\overline{S})$ are plus or minus, the observed points will drift northwest or southeast to their true values (see Figure III-4). This will tend to create a slight spurious correlation. The implications for (3.16) are that estimates of (g) would indicate a slight negative value instead of the correct value, zero. Of course, the situation is not quite so simple as (3.16) implies, since $(\overline{P})$ is embedded in several other variables. Nevertheless, actual test results raise serious doubts about the structure of the Benishay model.

The same sort of analysis could be extended to any of the other independent variables of the Benishay model since every variable contains

FIGURE III-4

$\dfrac{NI*}{\overline{S}}$

$\overline{S}$
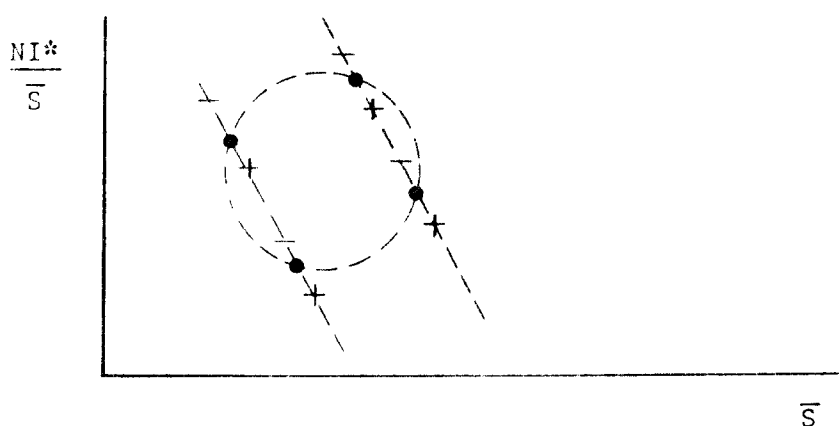
. = true values

+ = positive measurement error in $\overline{S}$

- = negative measurement error in $\overline{S}$

dotted lines used to clarify diagram

some measure of net income or market price. An examination of Benishay's results suggests, however, that none of the variables except size and perhaps payout ratio were statistically significant. A measurement error in the net income variables would produce the same type of bias in the payout ratio coefficient as suggested for the size variable coefficient.

The data that Benishay used were financial statistics for 56 industrial firms. Cross-section regressions were run for the years 1954-1957. Representative results are indicated in Table III-6.[22] The most consistently significant variable is firm size (x_6) with a slightly negative parameter. This is consistent with the notion that large, well-known firms command a premium in the market place. Unfortunately, the results are also

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[22]Ibid., p. 89.

TABLE III-6

BENISHAY PARAMETER ESTIMATES

	1954	1955	1956	1957
a	n	n	n	n
b	-.000	-.000	.000	.001
c	-.005	-.004	-.007	-.008*
d	-.543*	-.242	-.342	-.380*
e	-.104	-.099	-.141	-.079
f	.171	.102	.160	.131
g	-.111*	-.090*	-.084*	-.079*
h	-.000	-.001	-.002	-.002*

n - not available

* - significant at 5%

consistent with the notion that the parameter estimates for (g) are

spurious. The other parameters that were occasionally significant--the

equity growth measure (x_2), the payout ratio (x_3), the debt/equity ratio

(x_7)--also contained variations of the price and net income measures that

made up the dependent variable.

3.7 The Gordon Model[23]

Perhaps the most elaborate attempt to find explanatory variables to

describe the equity valuation process is the model proposed by Myron Gordon.

[23]See Myron J. Gordon, The Investment, Financing and Valuation of
the Corporation, (Homewood, Ill.: Irwin, 1962).

The model is constructed by extending earlier models on which Gordon had worked. Unlike the models of M-M and Benishay, this is a model which is explicitly designed to answer the question, "What are the variables that might explain the values of common stock equities?" There are six variables that are suggested as possible contributants to an equity valuation process: (1) the dividends of the firm; (2) the expected growth rate in dividends; (3) a measure of earnings instability; (4) a measure of the firm's leverage; (5) an index of the firm's operating asset liquidity; and (6) a measure of firm size. These variables then determine the price of the equity share in an equation of the form:

Gordon Equation

(3.17) $P = a \cdot (x_1)^b \cdot (x_2)^c \cdot (x_3)^d \cdot (x_4)^e \cdot (x_5)^f \cdot (x_6)^g$

(x_1) = dividends per share

(x_2) = dividend growth rate

(x_3) = earnings instability index

(x_4) = leverage index

(x_5) = operating asset liquidity index

(x_6) = firm size index

The six variables are defined in the following manner:

(x_1) = dividends per share

= (dv) or 2% (nw), whichever is greater

(x_2) = dividend growth rate

$\qquad$ = $[1 + b'i']$

$\qquad b'i' \quad = \left(\dfrac{\overline{y}(t)-dv}{\overline{y}(t)}\right) \cdot \left(\dfrac{\overline{y}(t)}{nw}\right)$

$\qquad \overline{y}(t) \quad = [.3NI(t) + .7\tilde{y}(t-1)(1+G[t])]/N$

$\qquad \tilde{y}(t-1) \quad = .3NI(t-1) + .7\tilde{y}(t-2)$

$\qquad G(t) \quad = .3[\tilde{y}(t) - \tilde{y}(t-1)]/\tilde{y}(t-1) + .7G(t-1)$

$\qquad \tilde{y}(1950) = NI(1950); \ G(1950) = .03$

(x_3) = earnings instability index

$\qquad$ = $(1 + \hat{\sigma}/NW)$

$\qquad \hat{y}(t) \qquad = \overline{y}(t-1)]1 + G(t-1)]$

$\qquad \Delta(t) \qquad = [|\hat{y}(t) - NI(t)|]/NW$

$\qquad \left(\dfrac{\sigma}{NW}\right)_t \quad = .3\Delta(t) + .7\left(\dfrac{\sigma}{NW}\right)_{t-1}$

$\qquad L(t) \qquad = CL + LL - CH - AR - \tilde{IY}(t)$

$\qquad \hat{IY}(t) \qquad = \dfrac{.4IY}{NW} + .6\hat{IY}(t-1)$

$\qquad \overline{IY}(t) \qquad = \hat{IY}(t) \cdot NW$

$\qquad \tilde{IY}(t) \qquad = IY - \overline{IY}(t)$

$\qquad \left(\dfrac{\hat{\sigma}}{NW}\right) \qquad = \left(\dfrac{\sigma}{NW}\right)\left(\dfrac{NW}{NW + L(t)}\right)$

$\qquad \left(\dfrac{\sigma}{NW}\right)_{1951} = \dfrac{NI(1951) - NI(1950)}{NW(1950)}$

$\qquad \overline{IY}(1950) = IY(1950)$

(x_4) = leverage index

$$= 1 + h' - rh'/k$$

$$h' = L(t)/NW(t)$$

$$k = \frac{(1-b')NI}{P} \cdot (1+h')^{-.3} + (b'i')$$

$$r = .045$$

(x_5) = operating asset liquidity index

$$= \frac{7\overline{IY} + 3NP}{5\overline{IY} + 5NP}$$

(x_6) = firm size index

$$= [TA-CL]/1,000,000$$

Although Gordon's model may appear more complex than the other models discussed, this model is really constructed in the same ad hoc fashion as the simpler models. Each variable is introspectively considered as a possible candidate and then some variation of the variable is added on to the multiplicative framework. This "add a variable" introspective approach taken by most researchers looking at equity valuation models contrasts with the systematic attempts to develop an integrated theory by researchers working on portfolio decision models.

For some of the variables, such as the operating asset liquidity index and the firm size index, only God and Gordon will ever know from whence the definitions come. For the other variables, the progenitors are more fully discussed. That is, we can trace the definition of these variables from their initial intuitive appeal, through a study of suitable

logical definitions of the variable, to the final simplified definition
for the empirical test of the model.

Consider, for example, the risk measure associated with income. It
seemed intuitively appealing to have a risk deflator associated with the
variability of income. After considering several candidates, Gordon
selected $(1+U)$ with $U = \sigma(1+wv)/NW$. The variable (σ) is the standard
deviation of income for a period where income has an expected value $(\bar{y})$;
(w) is a weighting factor between o and 1; (v) is equal to $(NW/\bar{y})$. Since
(w) could not be determined and (v) could not be measured independently
of the cost of capital, these factors are simply dropped. There is no
indication how the final empirical definition of $(1+\hat{\sigma}/NW)$ was derived
from the given assumptions.

The same type of procedure was followed in developing the dividend
growth rate and leverage variables. That is, a simplified model frame-
work was used to specify an initial relationship between price and the
variable under consideration. Then, after the variable had been logically
defined under a restrictive set of ceteris paribus conditions, a simpli-
fied definition of the variable was appended to the over-all model and
ignored while the next variable type was considered.

Gordon tested his model on two samples, one sample of 48 firms from
the food industry and another sample of 48 firms from the machinery in-
dustry. The food industry sample had identifiable subgroups from corn
refining and food processing, milling and baking, sugar and confectionery,
dairy, soap and vegetable oil, and tobacco industries. The machinery
sample subgroups were machinery, machine tools, and railroad equipment

manufacturers.[24] Separate regressions were run for each of the two samples for each of the years 1954-1958.

TABLE III-7

GORDON PARAMETER ESTIMATES

	Food Sample		Machinery Sample	
	1957	1958	1957	1958
ln a	2.541	2.632	2.330	2.975
b	.787*	.708*	.875*	.832*
	(.044)	(.051)	(.059)	(.056)
c	10.442*	10.038*	3.758*	2.176
	(1.535)	(1.685)	(1.638)	(2.135)
d	-8.675*	-4.000	-4.317*	-.788
	(1.988)	(2.251)	(1.992)	(1.995)
e	-.834*	-.852*	-.118	.428
	(.171)	(.222)	(.295)	(.379)
f	.283*	.245	.052	.666
	(.131)	(.154)	(.331)	(.334)
g	.085*	.113*	.131*	.064
	(.026)	(.033)	(.046)	(.046)
R^2	.941	.911	.882	.879

*Significant at 5% level.

Representative parameter estimates are presented in Table III-7.[25] In general, the Gordon results were impressive. The coefficient of determination was greater than .85 in every one of the ten regressions.

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[24] Ibid., pp. 247-248.

[25] Ibid., p. 169.

The regression coefficients seemed reasonably stable--at least in comparison to other models. Parameters for the dividend variable (x_1), the growth variable (x_2), and the size variable (x_6) were highly significant. The leverage variable (x_4) was significant about half the time. The risk measures--the earnings instability index (x_3) and the operating asset liquidity index (x_5)--were not generally significant.

Several factors concerning the Gordon model should be noted with care. First, although this is a "dividends" model, net income is an integral part of variables (x_2) and (x_3). Second, the dividend growth rate variable (x_2) is a special type of growth variable. This variable is essentially an expected retained earnings figure divided by net worth. The implications are that future dividend changes are a function of the magnitude of retained earnings and the expected marginal return on those earnings. Such assumptions are highly intriguing, but not substantiated by empirical evidence. Of course, neither are most traditional assumptions which link growth directly with historical rates of change in a variable.

The earnings instability index is not an earnings instability index, but an earnings yield instability measure essentially equal to the absolute value of expected net income (firm true value) less measured net income divided by net worth. This measure is not completely corrected for growth, so that under a constant growth situation the measure would be consistently positive, though small. Under such circumstances, the dividend growth rate variable (x_2) and variable (x_3) might tend to move together; or it is even possible that variable (x_2) is a surrogate for a payout index and (x_3) is the real growth variable. Finally, the leverage

index (x_4) has a parameter whose sign is opposite to that predicted by the Modigliani-Miller theory. Gordon predicted this would be the case from a theoretical analysis much too long to reproduce here.[26] Briefly, the differences between Gordon and M-M seem to derive from fundamentally different assumptions about the capital markets. Whereas M-M assumed perfect financial markets, Gordon assumed imperfect markets where "home-made" leverage is not equivalent to firm-generated leverage.

In a follow-up study of the Gordon work, David Peterson presented additional evidence on the power of the Gordon model compared to available alternatives.[27] Peterson's model is essentially the Gordon model with the leverage index and operating asset liquidity index dropped. In addition, the smoothing of the other variables has been considerably simplified. The index of earnings stability, for example, is now equal to:[28]

$$\text{Peterson stability index} = \sum_{t}^{t-6} \frac{|NI(t) - NI(t-1)|}{7 \cdot NW(t)}$$

Peterson tested his model on a sample of 92 large firms from a cross-section of Fortune's 500 largest industrials. Cross-section regressions were run for each of the years 1954-1960. Almost all of the Peterson parameters were significant at 5%.[29] The dividend coefficient averaged about (.78),

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[26] Ibid., pp. 100-112, 167-168.

[27] See David E. Peterson, "Corporate Investment Decisions and Financial Planning" (Urbana, Ill.: University of Illinois, Ph.D., 1963).

[28] Ibid., p. 88. Peterson's net income figures have been adjusted slightly by adding back one-half the interest charges on long-term debt.

[29] Ibid., pp. 89-93.

the growth coefficient (7.3), the risk parameter (-10.9), and the size

index parameter (.10). Note that the risk parameter is highly significant

and negative. This would seem to rule out the possibility of this variable

being a growth surrogate in this instance. As a practical consideration,

for mean values the product of $(x_2)^c$. $(x_3)^d$ in this model is (1.12),

suggesting that the growth variable and the stability index practically

may nullify one another if the inverse correlation between the two is high

enough.

Both Gordon and Peterson averaged their time-distributed parameters

to obtain benchmark estimates. Although both discussed the stability of

the parameters over time, neither presented confirming evidence that such

stability existed. Neither made corrections for changing price levels in

the capital markets. Even though their analysis of the parameter estimates

may be open to question, the Gordon model is certainly the most complex

attempt to date to develop and test a model of equity valuation.

3.8   Some Concluding Comments on Model Similarities

The models in this chapter differ in the number and type of firm

financial variables that are incorporated in each equation. These differ-

ences reflect the different questions the authors are trying to answer and

also the different attitudes of the authors as to the proper relation

between theory and an empirical specification of that theory. And yet,

there is a certain sameness that pervades all these models.

For example, net income appears as a dominant variable in all the

models. It is an explicit variable in all but the Gordon model, where

it is incorporated into the growth term and the stability index. So

firmly entrenched are historical net income measures in both academic

and professional models of equity share prices, it would probably be considered heresy to suggest that the equity valuation process could be accomplished without reference to reported net income figures. Yet it must be possible, for in a number of European countries reported net income figures are virtually meaningless as an indication of the earning power of corporations. In Canada a somewhat different situation prevails. There the stocks of many mining companies are actively traded even though these companies may have little or no income for years at a time. The point is, investor expectations need not be a function of historical or projected net income figures. The plethora of such data for American corporations may be leading investigators away from the true determinants of investor expectations with regard to corporate earning power.

Dividends are incorporated into four of the models and could, with no harm to the theory, be specified in the M-M, Barges models. Growth is an explicit variable only in the Benishay, Gordon models. Therefore, if growth is a relevant variable in the determination of equity share prices, there should be some differences in parameter estimates from samples of high-growth firms and samples of low-growth firms. In particular, if it is true that the growth that is of interest to investors is growth in net income per share, as many financial analysts suggest, the net income variable would probably receive a higher weighting in the growth sample than in the nongrowth sample.

Finally, it is rather surprising to note that at least one of the capital structure variables (LL, NW, TA) appears in every equation. As we shall see in the following chapters, the capital structure variables do not turn out to be particularly significant in any of the equations.

Why then are these variables almost always included in models of equity valuation? There may be at least two reasons. First is the notion that there is some degree of risk associated with the leverage of the firm. This riskiness, and hence the yield on a security, is supposed to vary continuously with the degree of leverage. Second is the possibility that capital structure variables are entering as surrogates for the investment activity of the firm. In this case, the variables could be indexes indicating risk diversification with size or indexes of structural stock changes made to meet changing economic conditions.

In this chapter we have examined several empirical specifications of models of equity valuation. From the simple Durand formulation to the complex Gordon model to the Peterson abridgment, we have traced a circular path. For Peterson uses essentially the same variable types that Durand did a decade earlier. Indeed, even in 1965, though the smoothing processes change, the variable types do not.[30]

In the next two chapters, the actual results from a large series of tests of some of these models are presented. We shall see that, from a statistical point of view, the models leave a great deal to be desired, whether one examines the models' performance over several time periods or the performance across several different samples.

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[30]See, for example, the model by Friend and Puckett which uses as variable types prices, dividends, and retained earnings. I. Friend and M. Puckett, "Dividends and Stock Prices," American Economic Review, 54 (September, 1964), pp. 656-682.