CHAPTER V

EMPIRICAL TESTS OF SELECTED MODELS: SOME PARAMETER ESTIMATES

In this chapter a summary of the basic results of the regression tests will be presented. As indicated in Chapter III, the basic models tested are formulations suggested by Durand, Modigliani-Miller, Barges, Benishay, and Gordon. The emphasis throughout the chapter will be on statistical performance rather than economic interpretation of the parameter estimates. There is an obvious reason for such emphasis. Unless the parameters meet some of the stability criteria outlined in Chapter IV, any economic interpretation of average parameter estimates or pooled parameter estimates is likely to be spurious.

The sections that follow summarize the results obtained for each of the model types. The body of the text reports the pooled regression estimates obtained while the results from the ninety-six basic regressions themselves are reported in an appendix to this chapter. For each model there is a table indicating parameter significance (t-tests) for the basic regressions performed. There is also a table for each section reporting the (F) ratios for the analysis of covariance tests outlined in Chapter IV. Although it is impossible to report all the alternative regressions tried, some of the results of important alternatives are reported in those cases where it seemed probable such information would add to an understanding of the basic model performance.

5.1 The Durand Models

The Durand model attempted to assess the relative impact of per share net income, dividends, and net worth on equity share prices. Two model forms, logarithmic and linear, were tested to see if these alternative forms made any difference in this simple model.

Results of the pooled regression tests are summarized in Table V-1. Using the "ALL" data estimates for discussion purposes, we see that earnings and dividends are highly significant in both the logarithmic and linear specifications.[1] Per share net worth was insignificant in the logarithmic case but significantly negative in the linear case. Using approximate average values of $45 for price, $3.00 for (ni), and $1.80 for (dv), we find that the differentials at those values indicate that the impact of a change in earnings is less in the logarithmic specification than in the linear case (see end of Table V-I). In most of the cases tested, the impact of a change in dividends on price relative to the impact of a change in earnings was less than the value of four times found by Durand for bank stocks. However, such comparisons are almost meaningless since there was very little stability across groups or years for the pooled regressions.

The overall significance of the parameters for the basic regressions is indicated in Table V-2. As expected, net income and dividend parameters are almost always significantly different from zero, and the net worth parameter is usually not significant. The fact that the sign

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[1]The analysis of covariance tests which use the pooled regression estimates are described in Section 4.5. See Table IV-9 for a summary of the pooling procedures.

TABLE V-1

DURAND EQUATION: (1) $P = a \cdot (ni)^b \cdot (dv)^c \cdot (nw)^d$

(2) $P = a + b(ni) + c(dv) + d(nw)$

		ALL	Pooled by Groups Across All Years I	II	III	IV
	ln a	3.051	2.845	2.715	3.816	3.026
	b	.260* (.028)	.429* (.067)	.202* (.035)	.504* (.069)	.127 (.070)
(1)	c	.382* (.030)	.579* (.055)	.195* (.045)	.312* (.046)	.478* (.078)
	d	.052 (.030)	-.034 (.042)	.161* (.059)	-.188* (.055)	.115 (.063)
	R^2	.464	.818	.377	.540	.405
	F	257.3	328.7	50.9	78.9	50.6
	a	8.480	4.643	16.149	15.230	1.668
	b	7.331* (.775)	3.139* (.747)	3.755* (.643)	7.829* (1.410)	12.365* (2.083)
(2)	c	12.559* (1.405)	14.005* (1.157)	7.601* (1.379)	19.596* (2.112)	6.315 (3.950)
	d	-.310* (.065)	-.094 (.055)	-.089 (.061)	-.704* (.113)	-.121 (.181)
	R^2	.405	.792	.436	.642	.344
	F	202.3	278.7	64.5	120.0	39.4

TABLE V-1 (CONTINUED)

DURAND EQUATION: (1) $P = a \cdot (ni)^b \cdot (dv)^c \cdot (nw)^d$

(2) $P = a + b(ni) + c(dv) + d(nw)$

		Pooled by Years Across All Groups			
		1956	1957	1958	1959
	ln a	3.044	3.021	3.201	3.273
	b	.785*	.406*	.184*	.278*
		(.069)	(.055)	(.038)	(.069)
(1)	c	.281*	.542*	.442*	.290*
		(.049)	(.057)	(.051)	(.057)
	d	-.156*	-.097	.071	.038
		(.056)	(.056)	(.050)	(.055)
	R^2	.657	.603	.568	.435
	F	142.3	112.9	97.9	57.6
	a	4.910	1.211	8.322	9.189
	b	9.986*	6.496*	8.408*	9.718*
		(1.571)	(1.118)	(1.551)	(1.890)
(2)	c	10.813*	12.257*	12.232*	13.195*
		(2.807)	(2.015)	(3.008)	(2.888)
	d	-.596*	-.330*	-.121	-.365*
		(.149)	(.094)	(.133)	(.129)
	R^2	.420	.518	.443	.442
	F	54.3	80.3	59.6	59.4

Log: dP/d(ni) = \$3.90
dP/d(dv) = \$9.50

Linear: dP/d(ni) = \$7.30
dP/d(dv) = \$12.60

TABLE V-2

SIGNIFICANT PARAMETERS IN THE DURAND REGRESSIONS

Groups		I	II	III	IV	Totals
	b	1 1 1 1	1 1 1 1	1 1 1 1	1 1 0 0	14
Log	c	1 1 1 1	1 1 1 0	1 1 1 1	1 1 1 0	14
	d	0 1 0 0	0 0 0 0	0 1 0 1	0 0 0 0	3
	b	0 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	15
Linear	c	1 1 1 1	0 1 1 1	1 1 1 1	0 1 0 0	12
	d	0 1 0 0	1 1 0 0	1 1 1 1	0 0 0 0	7

1 = parameter significant at 5% level.
0 = parameter not significant
(The four indices in each cell represent the four years 1956-1959).

for the net worth variable was negative in twenty-five of the thirty-two regressions may be an indication of a slight size effect. Since the "size" effect could be a surrogate for a number of variables---- (1) desirable price trading range, (2) lack of firm leverage, (3) absolute firm size, (4) number of shares outstanding, etc.----it would not seem wise to attach too much significance to this variable.

Some may be more interested in the relative magnitudes of the net income and dividends parameters. As indicated in Table V-3, there is little over-all difference in the logarithmic model and a slight preference toward dividends in the arithmetic specification. Two factors must be remembered however. First, these results are sample sensitive. For example, Group I gives much heavier weighting to dividends than does

TABLE V-3

	Variable Receiving Higher Weighting		
	dv	ni	equal
Log	8	7	1
Linear	11	5	0

Group III. This means that a researcher testing only a few samples could be easily misled. Second, the results may be due to a statistical bias. It is conceivable that measured dividends are a close representation of their true investor expected value, whereas measured current earnings may contain a large error term.[2]

To see whether or not the regression estimates from the various samples could come from the same underlying population, a series of analysis of covariance tests was performed. These tests essentially compare the residuals from the sample regressions assumed to come from the same population with the residuals from a pooled regression made up of all the sample data under consideration.[3] The results of the tests on the Durand regressions are presented in Table V-4. The (F) ratios are significant in every instance. That is, in none of the situations tested can the sample regression parameters be regarded as coming from the same underlying population. These are consistent with Durand's findings, which should have served as a warning to subsequent

[2]For a discussion of this problem see Section 2.2.3.

[3]See Section 4.5.

researchers in the equity markets.

TABLE V-4

COVARIANCE TESTS ON DURAND REGRESSIONS

Test	F Ratios Log	F Ratios Linear
ALL	245.4	105.7
I	47.3	45.3
II	56.5	46.5
III	25.3	28.5
IV	20.1	8.0
1956	20.8	15.9
1957	34.3	14.2
1958	30.0	14.7
1959	32.8	22.1

All ratios significant at 5% level.

5.2 Variations in the Simple Linear Model

There is a great temptation to believe, after examining the results of tests on models such as Durand's, that the results are not as bad as they seem statistically. The temptation is to believe that an addition of one or two more essential variables to the model will substantially improve results and reduce existing statistical problems. Each researcher, of course, has his own candidate for these "magic variables." This author must conclude, after examining the attempts of a score or more other researchers and trying several variations of each model tested himself, that the probability is very high researchers will search for the magic variables and the probability is very low that such variables will be found.

In this section, four simple variations of the linear Durand model are discussed. The variations include (1) adjustment for stock splits, (2) adjustment for price level changes, (3) the substitution of smoothed earnings for actual earnings, (4) the addition of industry dummy variables to the model. Discussion of the effects of these variations will be somewhat impressionistic since cost considerations of computer time did not permit the making of the covariance tests or other tests on the residuals.[4] Nevertheless, perhaps the reported statistics are sufficient to enable the reader to conclude with me that the variations are interesting but would not ameliorate the existing problems.

5.2.1 Adjustment for Stock Splits

The first variation tried was to adjust all per share data for any stock splits that occurred during the years 1956-1959. The year 1959 was taken as the base year. If there were no strong firm effects, there should be little difference in parameter estimates using adjusted and unadjusted shares. Prices are presumably corrected for any size effect by the net worth variable. On the other hand, least squares estimating procedures are particularly sensitive to extreme values. To the extent that stock splits are more likely to occur for high priced stocks, there is going

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[4]A complete set of tests for one model specification required about twenty minutes of 7070-94 computer time at a charge of $80. This does not include the time required to read and write data files from user tapes to disc files.

to be some realignment in extreme points when adjusted data are used. If high priced shares are more likely to depart from the sample price/earnings or price/dividends norms than ordinary shares, then the realignment of upper extreme values may cause some changes in parameter values.

Parameter estimates for the adjusted per share regressions are found in Table V-5. For Group I there seems to be little difference between these results and the original regression results.[5] In Group II there was one switch (1956) in the relative weightings of dividends-earnings. Group III produced the only systematic changes noted. The adjusted data seemed to provide some increase in over-all explanatory power (adjusted R^2), a depressing of earnings weights, and some increase in dividends weights for this group. Still, the year to year variability in weights was not reduced but increased. In Group IV two switches in the relative weightings of dividends-earnings occurred. One of the reasons for this may be that high priced shares tend to have high earnings but relatively low dividends so that adjustment for stock splits reduces the impact of the price-earnings dimensions much more absolutely than it does the dividend dimension. Since there may be some benefit in reducing the relative importance of these high priced shares and since there is no reduction in degrees of freedom, the adjusted per share data were used for the other variations described in this section.

5.2.2 Adjustment for Market Level Effects

The second variation attempted was to try to adjust the data for changes in the general price levels of financial assets. An almost

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[5]The original regression results are found in Appendix Tables 1-4 at the end of Chapter V.

TABLE V-5

ADJUSTED FOR STOCK SPLITS

		1956	1957	1958	1959
	a	2.22	1.20	4.27	2.50
	b	2.40*	2.57*	2.64	6.73*
I	c	12.82*	14.00*	16.48*	11.47*
	d	-.04	-.13*	-.04	-.09
	R^2	.89	.91	.86	.85
	a	12.27	8.25	14.05	17.19
	b	5.71*	2.60*	4.11*	3.58*
II	c	6.62*	8.20*	10.15*	10.08*
	d	-.24	-.16*	-.02	-.05
	R^2	.63	.59	.63	.49
	a	2.45	4.16	6.83	.75
	b	7.89*	5.93*	9.41*	20.25*
III	c	19.50*	18.49*	22.15*	16.49*
	d	-.49*	-.47*	-.43*	-1.12*
	R^2	.78	.79	.78	.78
	a	13.43	6.41	10.64	-1.58
	b	2.93	4.30	6.04	22.61*
IV	c	16.32*	16.03*	13.84	-4.00
	d	-.29	-.37	-.07	-.16
	R^2	.41	.40	.24	.35

* = significant at 5%; all R^2 significant at 5%.

universal procedure in the testing of equity valuation models is to test the same sample for two or more years and compare parameter estimates across years. Seldom, if ever, are data adjusted for changing interest rate levels or other possible indexes of changing financial asset price levels.

There are several possible explanations for this indifference. The most obvious possibility is that researchers do not consider fluctuations in money price indexes an important variable in the models they are considering. A more likely possibility is that some sort of significant relationship exists, but the dynamics of the situation are so complex the relationship cannot be discerned by static equilibrium analysis. In fact, there seems to be considerable uncertainty about the extent and timing of the impact of monetary changes on the equity markets. In a popular study among market professionals, Beryl Sprinkel argues that there is a positive relationship between stock prices and a smoothed rate of growth in the money supply.[6] But the money supply change index leads the stock series by a highly variable two to eighteen months. Another popular theory attempts to relate stock price trends to Federal Reserve discount rates and prime commercial loan rates, but here again the lead time of these rates has a large variance.[7] The level of various rates during the period of interest is indicated in Table V-6. Several such indexes

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[6] See Beryl W. Sprinkel, Money and Stock Prices, (Homewood, Ill.: R.D. Irwin, 1964).

[7] See E. Gould, "Three Steps and a Stumble," Barrons (Dec. 27, 1965), p. 3.

TABLE V-6

SELECTED INTEREST RATES, 1955-1960

	1955	1956	1957	1958	1959	1960
U. S. Treasury, 3-5 Years	2.5	3.1	3.6	2.9	4.3	4.0
Aaa Corporate Bonds	3.1	3.4	3.9	3.8	4.4	4.4
S&P, Dividends/Price	3.5	3.5	4.3	3.0	2.9	3.2
S&P, Earnings/Price	7.8	7.1	8.2	5.0	5.5	5.5
S&P, $\frac{\text{Dividends + Capital Gains}}{\text{Price}}$	26.6	6.8	-12.5	30.4	11.5	-1.7

Herbert E. Dougall, Capital Markets and Institutions, (Englewood Cliffs, N. J.: Prentice Hall, 1965), p. 150, and Standard and Poor's Trade and Securities Statistics, 30 (1964), pp. 123-124.

were tried to see if they had any impact on the parameter levels. Again, the objective was to increase apparent parameter stability across years for each sample group. The best results seemed to be obtained when sample prices, earnings, and dividends were separately adjusted by their corresponding Standard and Poor industrial average value relative to the 1959 values.[8] That is, S&P values were made into indexes with 1959 equal to 1.00 and then the data were adjusted by dividing by the appropriate index number.

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[8]For example, earnings were adjusted by the following earnings index (EI).

$$(EI)_t = \frac{S\&P(ni)_t}{S\&P(ni)_{1959}}$$

Then per share net income for every firm for year (t) was adjusted by dividing by the index $(EI)_t$.

The results of these index number adjustments are reported in Table V-7. By comparing these results with those in Table V-5, the reader can

TABLE V-7

ADJUSTED BY STANDARD AND POOR INDEXES

		1956	1957	1958	1959
	a	2.86	1.80	4.68	2.50
	b	3.09*	3.83*	2.41	6.73*
I	c	15.47*	20.40*	16.98*	11.47*
	d	-.06	-.20*	-.04	-.09
	R^2	.89	.91	.86	.85
	a	15.80	12.41	15.36	17.19
	b	7.36*	3.88*	3.75*	3.58*
II	c	7.99*	11.96*	10.46*	10.08*
	d	-.31	-.24*	-.02	-.05
	R^2	.63	.59	.63	.49
	a	3.16	6.26	7.48	.75
	b	10.16*	8.85*	8.60*	20.25*
III	c	23.53*	26.95*	22.82*	16.49*
	d	-.63*	-.71*	-.47*	-1.12*
	R^2	.78	.79	.78	.78
	a	17.29	9.65	11.64	-1.58
	b	3.77	6.42	5.52	22.61*
IV	c	19.69*	23.36*	14.26	-4.00
	d	-.37	-.56	-.08	-.16
	R^2	.41	.40	.24	.35

* = Significant at 5%; all R^2 significant at 5%. Data also adjusted for stock splits.

see that the variability of the estimates was not reduced, but increased in several cases. Nor is the direction of parameter change always what might be predicted from the index adjustments made from Table IV-2. The facts seem to be that in these samples, parameter variability created by "industry" effects or "firm" effects swamps any variability due to market level changes. This must admittedly be considered a rather gross attempt to measure such market level changes, but in view of the complex dynamics of the situation a more sophisticated attempt does not seem warranted until models of equity valuation are available to handle the more serious underlying problems found in current models.

5.2.3 Effects of Smoothing Earnings

The third variation tried was to study the effects of smoothing the earnings variable. For a variety of reasons current earnings are probably only a rough approximation of investor expected earnings. If current earnings contain a random or noise component superimposed on an underlying trend that is the "true" measure of investor expected earnings, then some sort of averaging will add emphasis to this underlying trend and presumably yield results superior to the use of unsmoothed earnings.

Several averaging processes were tried and most of them gave consistent results. Table V-8 reports the results of smoothing earnings using an exponential smoothing process described by Kolin.[9] The weights for the earnings for the past five years are given by:

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[9]See Marshall Kolin, "The Relative Price of Corporate Equity with Particular Attention to Investor Valuation of Retained Earnings and Dividends," Ph.D. Thesis, University of Chicago (1965).

TABLE V-8

ADJUSTED BY SMOOTHING EARNINGS

		1956	1957	1958	1959
	a	2.27	1.36	4.35	1.97
	b	2.84	2.88*	3.62	7.55*
I	c	12.41*	14.02*	15.59*	11.19*
	d	-.05	-.15*	-.08	-.10
	R^2	.88	.91	.86	.83
	a	11.67	8.00	11.98	16.82
	b	10.29*	4.53*	10.31*	6.71*
II	c	3.16	6.33*	3.76	6.78*
	d	-.40	-.24*	-.25*	-.13
	R^2	.62	.61	.72	.50
	a	3.13	3.90	8.35	6.74
	b	9.49*	8.44*	11.57*	20.10*
III	c	20.23*	17.37*	20.60*	17.85*
	d	-.68*	-.66*	-.69*	-1.24*
	R^2	.78	.80	.77	.74
	a	7.06	6.53	7.29	.05
	b	9.63*	5.76	11.46	20.90*
IV	c	10.21	15.10*	8.85	-.19
	d	-.39	-.45	-.12	-.13
	R^2	.44	.40	.25	.28

* = Significant at 5%; all R^2 significant at 5%. Data also adjusted for stock splits.

$$(5.1) \quad W(t) = H.B(1-B)^t \qquad t = 0, ---, -4$$

$$B = .40$$

$$H = 1/(B. \sum_{0}^{t-4} (1-B)^t)$$

Kolin ran several tests using different values for (B) ranging from the ordinary arithmetic average (B=0) to the other extreme of having all weight on the current value (B=1) and concluded that empirically (B=.40) seemed to give the best results.

The results in Table V-8 are rather interesting. In almost every case the earnings coefficient increased for the smoothed earnings relative to the unsmoothed earnings coefficients reported in Table V-5. The increase was small for Groups I and III but rather striking for Groups II and IV. Note, however, that the adjusted R^2's were not materially changed from one test to the other. Nor was the stability of the parameter estimates increased by the smoothing process.

If we are to believe this evidence, there must be serious doubts about the so-called proofs that the marginal worth of a dollar of dividends is not worth substantially more than the marginal worth of a dollar of earnings.[10] These proofs use models with current earnings as "straw-man" targets and then generate the proof by substituting smoothed "normalized" earnings for current earnings. As Table V-8 indicates, the smoothing process does indeed increase the weighting given to earnings sometimes, but the causality of the process is highly suspect. What the

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[10]See Kolin, op. cit. and I. Friend and M. Puckett, "Dividends and Stock Prices," American Economic Review, 54 (September, 1964), pp. 656-682.

smoothing process does is (1) reduce the average value of the earnings variable,[11] (2) reduce the variance of the earnings variable, and (3) substantially increase the simple correlation between dividends and the smoothed earnings. Since there appears to be no substantial increase in the overall explanatory power of the model (adjusted R^2) and no substantial reduction in the variability of parameter estimates, it seems doubtful that smoothed earnings models should be considered to reflect the "true" relationship between earnings and dividends, while the unsmoothed models represent an "impure" situation. Indeed, we might even blaspheme and ask the question, "Are smoothed earnings an informational surrogate for expected dividends?" As we indicated in Section 2.2.3, the answer to this question or the question of whether stockholders value a dollar of dividends or earnings more probably depends on considerably more sophisticated tests than have been made to date.

5.2.4 Adjustment for Industry Effects

The fourth variation in the simple linear model was to add dummy "industry" variables to each sample to see if a redistribution of the constant term might at least increase the stability of that parameter. Three zero-one dummy variables were added to the basic equation permitting four "industry" categories to be assigned for each sample. Firm classification was as indicated by Table IV-8 in Section 4.4.

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[11] Over most of the five year intervals in the 1950's average earnings for typical samples would be growing. The smoothing processes used do not adjust for this trend and there is no separate growth variable in the model. This downward bias in the adjusted earnings variable may tend to increase the associated parameter estimate.

The results, as indicated in Table V-9, were somewhat disappointing. The estimated parameters for the dividends and earnings variables remained about as they were for the simple linear case adjusted for stock splits. This is as one would predict since there should be little correlation among these variables and the dummy variables. The variability of the constant term was not reduced, however, as one might predict. Nor were the estimated parameters reasonably stable for the dummy variables. It appears that the "industry" effects found exist not only as level effects associated with the constant term, but also as a part of each of the other basic parameters. Fortunately, sample size limitations on degrees of freedom and other statistical problems prevented an expansion of the dummy variable sets to the other basic variables. Such procedures cannot provide answers to most of the central issues in the equity valuation area even if they were successful in reducing parameter variability.

5.3 The Modigliani-Miller, Barges Models

The Modigliani-Miller (M-M) and Barges models, in the specification tested, hypothesize a relationship between measured rate of return on common equity and firm capital structure. The essential difference in the M-M and Barges specifications is that Barges has substituted book net worth in the independent variable for the market value of the equity.[12] In the crude form of these equations, where risk measures and growth variables have been omitted, one should not expect high R^2 values or

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[12]Note that in tests on both models the book value of long-term debt was used. This point is discussed in Section 3.4.

TABLE V-9

ADJUSTED BY INDUSTRY INDICES

		1956	1957	1958	1959
	a	-2.96	-2.88	-3.43	-4.52
	b	2.83*	3.26*	3.66*	6.71*
I	c	12.95*	13.57*	16.07*	12.05*
	d	-.07	-.15*	-.07	-.08
	R^2	.92	.93	.90	.89
	a	6.89	3.52	10.47	13.86
	b	6.12*	3.07*	3.54*	3.43*
II	c	6.67*	8.46*	11.32*	10.58*
	d	-.21	-.17	.00	-.02
	R^2	.69	.65	.63	.48
	a	1.64	1.77	-6.30	-12.17
	b	8.43*	7.66*	15.45*	22.22*
III	c	18.28*	15.65*	15.02*	13.70*
	d	-.75*	-.60*	-.58*	-1.28*
	R^2	.86	.85	.87	.84
	a	4.11	-.80	-7.17	-17.06
	b	1.88	4.52	7.27*	21.60*
IV	c	18.74*	16.23*	14.07	-1.20
	d	-.29	-.37	.08	-.16
	R^2	.42	.38	.25	.36

* = Significant at 5%; all R^2 significant at 5%. Data also adjusted for stock splits.

TABLE V-9 (CONTINUED)

DUMMY PARAMETER ESTIMATES

		1956	1957	1958	1959
	e	9.32*	7.95*	13.71*	15.16*
I	f	1.16	.37	1.49	.17
	g	6.78*	4.74	9.69*	8.91
	e	-3.53	1.70	1.35	-.31
II	f	12.51*	10.44*	6.81	5.70
	g	6.07	3.52	4.11	4.96
	e	16.07*	14.56*	27.56*	30.75*
III	f	-3.81	-3.00	.50	4.51
	g	17.50*	6.41	14.02	19.18
	e	3.89	4.76	6.80	6.98
IV	f	11.30	5.39	12.48	10.19
	g	19.13	13.51	32.63	35.39

* = Significant at 5% level.

completely stable parameters. The basic regression results bear out these expectations.[13]

The pooled regression estimates reported in Table V-10 tend to support the original findings of M-M and Barges. The values of the constant (a) are positive and of a reasonable magnitude. The values of the parameter (b) are positive and highly significant. For the ALL regression, the Barges (b) is less than one-half the M-M (b). While this relationship is not consistent across all the pooled samples tested, one gets the definite impression that the Barges (b) is less than the M-M (b). Note the relatively greater slope variability across pooled groups compared to slope variability across years. This suggests that sample effects or "risk class" may be a more important omitted variable than indexes of interest rate levels.

Table V-11 summarizes the t-statistics for the sixteen basic M-M and sixteen basic Barges regressions. The results are extremely sample sensitive. That is, the M-M slope coefficient was significant in every case in Groups I and III and in no case in Group II; in Group IV (b) was significant the first two years, but not the last two years, 1958-1959. Tests of other samples suggest that this behavior is fairly typical. The coefficient is significant for some samples and not for others, for some years and not others for the same sample.

Modigliani and Miller would probably argue that such behavior arises because the samples have not been properly constrained for risk class. So long as "risk class" remains a crucial variable in the M-M theory and so

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[13]The basic results for tests of the M-M, Barges regressions are reported in Appendix Tables 5-8.

TABLE V-10

MODIGLIANI-MILLER EQUATION: (NI)/S = a + b(LL)/S
BARGES EQUATION: (NI)/S = a + b(LL)/(NW)

		Pooled by Groups Across All Years			
	ALL	I	II	III	IV
a	.077	.081	.085	.061	.073
b	.036* (.003)	.056* (.005)	.024 (.013)	.032* (.003)	.034* (.008)
R^2	.138*	.337*	.010	.426*	.071*
F	142.5	112.2	3.4	148.4	17.7
a	.083	.084	.093	.059	.083
b	.015* (.003)	.055* (.008)	-.009 (.015)	.038* (.004)	.002 (.005)
R^2	.021*	.172*	.000	.261*	.000
F	20.4	46.3	0.4	71.1	0.2

* = Significant at the 5% level.

MODIGLIANI-MILLER EQUATION: (NI)/S = a + b(LL)/S
BARGES EQUATION: (NI)/S = a + b(LL)/(NW)

	Pooled by Years Across All Groups			
	1956	1957	1958	1959
a	.088	.100	.057	.067
b	.036* (.005)	.033* (.005)	.031* (.006)	.024* (.006)
R^2	.162*	.167*	.097*	.063*
F	43.7	45.4	24.8	15.8
a	.093	.105	.061	.071
b	.019* (.006)	.026* (.008)	.012* (.005)	.007 (.005)
R^2	.036*	.043*	.020*	.003
F	9.2	10.9	5.5	1.6

* = Significant at the 5% level.

TABLE V-11

SIGNIFICANT PARAMETERS IN THE M-M, BARGES REGRESSIONS

(b)	I	II	III	IV	Totals
M-M	1 1 1 1	0 0 0 0	1 1 1 1	1 1 0 0	10
Barges	1 1 1 1	0 0 0 0	1 1 1 1	0 0 0 0	8

1 = Parameter significant at 5% level.
0 = Parameter not significant.
(The four indices in each cell represent the four years 1956-1959).

long as the concept remains undefined and not an explicitly measured variable, their argument remains irrefutable. Of course, such a position makes any attempt to test the theory meaningless. Occasionally, M-M have argued that "risk class" is similar to industry classification. Here the evidence of the tests on the M-M hypothesis seems rather strong. There is little, if any, indication that the significance or non-significance of the parameter (b) is directly related to the purity of the product industry class of the sample. The results from Group IV, which contains firms from fifty industries, suggest the absence of such a relationship. Group III is made up of three sharply defined product industries and even Group I contains significantly different industry categories. Other samples tested produced similar results, so it seems improbable that the fluctuating significance of the slope parameter is due to product industry sample impurities. The question of what does cause such fluctuations or what alternative definitions of risk class might be more appropriate must be left to those who are strong advocates of an M-M type world.

Estimated parameters for the M-M and Barges tests are of the same order of magnitude as those authors' original tests on industrial firms. For the M-M equation, we can derive a value for [dP/d(ni)] which will turn out to be the reciprocal of the constant term. For the ALL pooled regression, the M-M [dP/d(ni)] turns out to be 13.0. This is somewhat higher than the estimate of 7.3 from the ALL regression for the linear Durand model, but such results are not particularly surprising since there is no dividend term in the M-M equation. As Table V-12 suggests, the implied magnitude for the cost of debt capital seems lower than is reasonable. These low values may partially result from a downward bias in (b). Both (a) and (b) could be biased in the same direction if there exist errors in the variables, but it is not clear which parameter would have proportionately greater downward bias.

TABLE V-12

IMPLIED COST OF DEBT CAPITAL FOR M-M REGRESSIONS

Group	1956	1957	1958	1959
I	.008	-.002	-.043	-.032
II	.082	.127	.066	.022
III	.010	.005	-.006	.008
IV	.000	.021	-.005	.051

Assumed tax rate of 50%.

The results of the analysis of covariance tests are reported in Table V-13. The F ratios are all significant at the 5% level indicating again that the estimated parameters for the combinations tested did not come from the same underlying populations. The higher F ratios for tests I-IV relative to the ratios for tests 1956-1959 do not confirm our previous impression that there is greater variability across groups than from year to year for the pooled data.

TABLE V-13

COVARIANCE TESTS ON M-M, BARGES REGRESSIONS

Test	F Ratios M-M	F Ratios Barges
ALL	361.5	418.6
I	49.1	59.1
II	125.9	131.1
III	53.9	72.5
IV	39.6	38.4
1956	29.8	37.5
1957	35.3	47.8
1958	39.6	46.7
1959	25.0	26.1

All ratios significant at 5% level.

In the discussion of the Modigliani-Miller model in Chapter III, we noted that Weston and others had suggested that the absence of a growth term could seriously bias the results in favor of the M-M proposition. We went further and speculated that Barges' results might be due to the absent growth terms. Several variations of the M-M model with growth terms added were tested to study the impact of growth on this model.

Growth terms tested included both average rates of growth in earnings and rates of growth in equity value. Since there is some evidence that rates of growth in equity value are more closely related to changes in expected corporate income than alternative growth measures, the results from using this growth variable are reported in Table V-14.[14] The growth variable used was simply:

$$g = \sum_{T-4}^{T} \left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)/5$$

The results in Table V-14, where the M-M model with growth term is compared to the original model, are a distinct surprise. The estimated coefficients for the leverage variable are higher, not lower as expected. The parameters for the growth variable are exceedingly unstable and have the wrong sign when significant. These results cannot be entirely attributed to the particular form of the growth variable. Equally unstable results occurred when other growth variables were tried. There is some evidence that the results are due to a small number of atypical firms in each sample.[15] Indeed, the low simple R^2 between the dependent variable and each of the independent variables, and the low adjusted R^2 for the regressions as a whole raise the possibility that the entire set of Modigliani-Miller tests is near spurious results due to incomplete model specification.

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[14]Kolin reports results of tests of both types of growth variables. See Kolin, op. cit., p. 6 and Chapter IV.

[15]An atypical firm in this context is just one that has an extreme value in one of the variable dimensions greater than three standard deviations from the mean.

TABLE V-14

M-M MODEL WITH GROWTH TERM ADDED

	1956		1957		1958		1959	
	B	N	B	N	B	N	B	N
a	.10	.07	.10	.09	.07	.04	.07	.07
b	<u>.05</u>	<u>.11</u>	<u>.05</u>	<u>.08</u>	<u>.06</u>	<u>.10</u>	<u>.05</u>	<u>.05</u>
c		<u>.49</u>		<u>.36</u>		<u>.27</u>		.02
R^2	<u>.26</u>	<u>.62</u>	<u>.36</u>	<u>.53</u>	<u>.27</u>	<u>.60</u>	<u>.40</u>	<u>.39</u>
a	.10	.12	.13	.13	.06	.04	.07	.07
b	.01	.04	.00	<u>.04</u>	-.01	.02	.03	.03
c		-.02		-.08		<u>.15</u>		.01
R^2	.00	.00	.00	.09	.00	.07	.00	.00
a	.07	.10	.08	.11	.05	.05	.06	.07
b	<u>.03</u>	<u>.05</u>	<u>.04</u>	<u>.04</u>	<u>.03</u>	<u>.04</u>	<u>.02</u>	.00
c		-.03		.03		.12		-<u>.20</u>
R^2	<u>.33</u>	.14	<u>.62</u>	.18	<u>.45</u>	.12	.19	<u>.60</u>
a	.08	.15	.10	.14	.06	.05	.06	.06
b	<u>.04</u>	.02	<u>.04</u>	.02	.03	<u>.05</u>	.01	.01
c		-.31		-.16		.06		-.01
R^2	.12	.04	.09	.00	.03	.06	.00	.00

B = Basic M-M model --- see Appendix Tables 5-8.
N = Model with growth term added.
Underlined parameters are significant at the 5% level.

5.4 The Benishay Model

The Benishay model is an attempt to find some of the determinants of relative returns on equity share prices. This model incorporates explicit risk variables. All the variables are weighted time series averages of firm financial data. We would therefore expect the parameters of this model to exhibit greater stability than the parameters of the previous models discussed.

The results of the Benishay regressions are somewhat puzzling. An examination of the pooled regression estimates in Table V-15 shows that some of the parameter values there differ in sign and magnitude from the results Benishay himself obtained. For example, on the cross-group pooled samples (1956-1959), which should be analogous to Benishay's cross-section samples, the following differences can be noted between the estimates from these firms pooled by years across all groups and Benishay's estimates as reported in Table III-6: (1) the earnings growth coefficient (b) is larger and almost significant; (2) the equity growth coefficient (c) is larger absolutely in the present tests; (3) the pay-out ratio does not seem to be significant in the present tests; (4) the debt-equity ratio is highly significant in the present tests, whereas this was not the case in Benishay's reported results.

These differences in results do not seem to be so striking when the Benishay estimates are compared to the basic regression estimates reported in the Appendix.[16] Part of the answer to why these apparent differences exist may be that both in Benishay's own tests and in the sixteen basic

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[16] See Appendix Tables 9-12.

TABLE V-15

BENISHAY EQUATION:

$$Y = a \cdot e^{(bx_1 + cx_2 + hx_7)} \cdot (x_3)^d \cdot (x_4)^e \cdot (x_5)^f \cdot (x_6)^g$$

	ALL	Pooled by Groups Across All Years I	II	III	IV
ln a	-2.107	-1.300	-1.879	-2.445	-2.064
b	.684*	-1.484*	.459	-.867	2.642*
	(.167)	(.263)	(.247)	(.562)	(.340)
c	-2.117*	.421	-3.081*	-1.335*	-3.498*
	(.212)	(.318)	(.351)	(.512)	(.481)
d	.014	-.266*	.025*	-.000	.041
	(.008)	(.047)	(.009)	(.020)	(.073)
e	.040	-.049	.205*	.068	.055
	(.021)	(.026)	(.043)	(.049)	(.047)
f	.034	.146*	-.160*	.086	.007
	(.030)	(.034)	(.053)	(.068)	(.070)
g	-.108*	-.084*	-.087*	-.083*	-.112*
	(.008)	(.011)	(.020)	(.017)	(.017)
h	.285*	.323*	.139	.288*	.066
	(.027)	(.047)	(.081)	(.060)	(.067)
R^2	.491	.691	.421	.582	.473
F	123.0	71.0	26.7	40.6	29.1

Y = a weighted rate of return

x_1 = a growth in earnings factor

x_2 = a growth in equity value factor

x_3 = pay-out ratio

x_4 = stability of income measure

x_5 = stability of equity value measure

x_6 = size, as measured by equity value

x_7 = a debt-equity ratio

TABLE V-15 (CONTINUED)

BENISHAY EQUATION:

$$Y = a \cdot e^{(bx_1 + cx_2 + hx_7)} \cdot (x_3)^d \cdot (x_4)^e \cdot (x_5)^f \cdot (x_6)^g$$

	Pooled by Years Across All Groups			
	1956	1957	1958	1959
ln a	-1.848	-1.819	-2.183	-2.246
b	-.149	.288	1.167*	1.650*
	(.268)	(.309)	(.373)	(.336)
c	-1.193*	-2.287*	-2.819*	-3.324*
	(.362)	(.414)	(.442)	(.409)
d	-.014	.008	.012	.007
	(.019)	(.018)	(.014)	(.013)
e	-.055	.075	.152*	-.011
	(.041)	(.044)	(.046)	(.036)
f	.055	-.053	-.039	.100
	(.046)	(.056)	(.063)	(.064)
g	-.111*	-.109*	-.103*	-.090*
	(.013)	(.014)	(.015)	(.015)
h	.272*	.237*	.279*	.245*
	(.050)	(.045)	(.052)	(.068)
R^2	.597	.568	.499	.481
F	47.8	42.5	32.4	30.3

Y = a weighted rate of return

x_1 = a growth in earnings factor

x_2 = a growth in equity value factor

x_3 = pay-out ratio

x_4 = stability of income measure

x_5 = stability of equity value measure

x_6 = size, as measured by equity value

x_7 = a debt-equity ratio

regressions in this study many of the parameters were not significant. In fact, in the current test series only fourty-one of a possible 112 coefficients were significant.

TABLE V-16

SIGNIFICANT PARAMETERS IN THE BENISHAY REGRESSIONS

	I	II	III	IV	Totals
b	1 1 0 0	0 0 0 0	0 0 0 0	0 0 1 1	4
c	0 0 0 0	1 1 1 1	0 1 0 0	0 1 1 1	8
d	1 1 1 1	0 0 0 0	0 0 0 0	0 1 0 1	6
e	0 0 0 0	0 0 1 0	0 1 0 0	0 0 1 0	3
f	0 0 0 1	0 0 0 0	0 0 0 0	0 0 0 0	1
g	1 1 1 1	1 0 0 0	1 1 0 0	1 1 1 1	11
h	1 1 1 1	0 0 0 0	1 1 1 0	1 0 0 0	8

1 = Parameter significant at 5% level.
0 = Parameter not significant.
(The four indices in each cell represent the four years 1956-1959).

Table V-16 indicates the significant coefficients were concentrated on four independent variables. The parameter of the first of these four variables, the growth in equity value (x_2), was negative as would be predicted. The second parameter, the pay-out ratio (x_3), was negative in the case of Group I. In Group IV, however, the two significant cases were of opposite signs. Size, as measured by equity value (x_6), proved to be the most significant variable in the Benishay set. In both Benishay's original results and these later regressions, this variable had an estimated coefficient of about (-.09). As we pointed out in Section 3.6, this is consistent either with the notion that large, well-known corporations tend to sell at a slightly higher price-earnings ratio than smaller, less well-known companies or with the notion that the results are a spurious effect of the ratio error bias. The fourth parameter, significant for Groups I and III, is the debt-equity ratio (x_7) coefficient. This parameter is positive when significant which makes it consistent with the Modigliani-Miller (b) coefficient. Moreover, the order of magnitude for [$dY/d(x_7)$] in the Benishay regression is about (.045) which is close to the (.05) estimate for the significant values of (b) in the M-M regression. Note that the growth in earnings parameter was almost never significant even though it was highly significant in several of the pooled regressions where more observations were available.

Two things must be noted at this point. First, all of the independent Benishay variables contain either measures of net income or equity value or both. Since these are the same measures that define the dependent variable, there may be very substantial biases in this regression model. Second, the risk measures [(x_4), (x_5)] were usually not significant. In an adaptive multiple regression mode, these variables were often dropped from the "in" set of covariances.[17]

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[17]The adaptive multiple regression procedure is a heuristic search procedure developed by Efroymson to determine a minimal set of predictors which (a) contains all forced variables and (b) exhausts the capacity of the potential predictors for significantly improving the criterion. (F tests) The algorithm does not guarantee to find a minimal set since the adjustments are made by adding or dropping one variable at a time in the sequence specified by the investigator. Nevertheless, the procedure is widely used to investigate potential variable sets. See M. A. Efroymson, "Multiple Regression Analysis," in Ralston and Wilf (eds.), <u>Mathematical Methods for Digital Computers</u>, (New York: John Wiley, 1960).

In terms of the covariance tests, as indicated by Table V-17, the Benishay parameters do relatively well. The (F) ratios are somewhat lower than corresponding ratios for other models tested. However, the ratios are still significant so that the sample regression parameters cannot be regarded as coming from the same underlying populations for the situations tested.

TABLE V-17

COVARIANCE TESTS ON BENISHAY REGRESSIONS

Test	F Ratios
ALL	81.6
I	7.7
II	11.7
III	5.4
IV	12.7
1956	7.6
1957	9.9
1958	11.4
1959	19.1

All ratios significant at 5% level.

5.5 The Gordon Model

The Gordon model represents one of the most ambitious attempts to date to develop a single equation model for the explicit determination of equity share prices. Like the Benishay model, the Gordon model contains variables to measure growth and risk. Unlike the Benishay model, the Gordon model contains independent variables that are not statistical variations of the dependent variable.

The overall performance of the Gordon model is quite impressive. As indicated by the estimated coefficients for the pooled regressions (Table V-18), the signs and magnitudes of the coefficients are generally consistent with the results of Gordon's own tests.[18] In the case of the food industry sample, where two-thirds of the firms are the same as firms in Gordon's own sample, the results are particularly close. The major difference is in the size of the earnings instability measure; for in the more recent tests, this measure was smaller and less significant. But in both groups of tests, parameter estimates for this variable were extremely unstable.

Table V-19 summarizes the significance of the various parameters for the basic regressions. As was true of the other models, the significance of the coefficients varied considerably from sample to sample. Four of the six parameters were significant at least half the time. The dividends coefficient was significant and positive in every regression. The dividends growth coefficient almost matched this performance, being significant and positive in fourteen regressions. The third and fourth ranked coefficients--the leverage index and firm size index--were significant in Group I, but the results were mixed for the other groups.

It is interesting to compare the effects of a dividend change on the Gordon model with the effects of a similar change on the Durand models.

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[18]See Table III-7.

TABLE V-18

GORDON EQUATION:

$$P = a \cdot (x_1)^b \cdot (x_2)^c \cdot (x_3)^d \cdot (x_4)^e \cdot (x_5)^f \cdot (x_6)^g$$

		Pooled by Groups Across All Years			
	ALL	I	II	III	IV
ln a	2.380	2.487	2.291	2.639	2.591
b	.750*	.814*	.564*	.661*	.822*
	(.025)	(.032)	(.048)	(.061)	(.057)
c	6.331*	7.830*	9.559*	7.834*	4.715*
	(.630)	(1.196)	(1.396)	(1.544)	(1.303)
d	.734*	-4.651*	2.475	-.440	.278
	(.345)	(2.021)	(1.534)	(1.602)	(.436)
e	-.241*	-.901*	-.075	0.917*	-.124*
	(.040)	(.128)	(.165)	(.162)	(.055)
f	.003	.502*	.291	.403	-.188
	(.075)	(.102)	(.222)	(.258)	(.146)
g	.123*	.105*	.109*	.118*	.090*
	(.010)	(.017)	(.031)	(.026)	(.022)
R^2	.642	.853	.518	.631	.585
F	265.8	213.0	45.2	57.8	52.5

x_1 = dividends per share

x_2 = dividend growth rate

x_3 = earnings instability index

x_4 = leverage index

x_5 = operating asset liquidity index

x_6 = firm size index

TABLE V-18 (CONTINUED)

GORDON EQUATION:

$$P = a \cdot (x_1)^b \cdot (x_2)^c \cdot (x_3)^d \cdot (x_4)^e \cdot (x_5)^f \cdot (x_6)^g$$

	Pooled by Years Across All Groups			
	1956	1957	1958	1959
ln a	2.237	2.004	2.585	2.634
b	.803*	.892*	.782*	.740*
	(.045)	(.052)	(.039)	(.046)
c	7.825*	7.254*	6.393	7.289*
	(1.121)	(1.156)	(1.023)	(1.285)
d	.580	.141	.749	1.306
	(.654)	(.628)	(.506)	(.711)
e	-.166	-.123	-.311*	-.184*
	(.102)	(.104)	(.085)	(.050)
f	-.034	-.089	-.017	.063
	(.134)	(.143)	(.121)	(.141)
g	.122*	.122*	.104*	.094*
	(.019)	(.020)	(.017)	(.020)
R^2	.702	.703	.753	.649
F	87.6	88.0	113.5	69.1

x_1 = dividends per share

x_2 = dividend growth rate

x_3 = earnings instability index

x_4 = leverage index

x_5 = operating asset liquidity index

x_6 = firm size index

TABLE V-19

SIGNIFICANT PARAMETERS IN THE GORDON REGRESSIONS

	I	II	III	IV	Totals
b	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	16
c	1 1 1 1	1 1 1 1	0 1 1 1	1 1 0 1	14
d	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0
e	1 1 1 1	0 1 0 0	0 1 1 0	1 0 0 1	9
f	1 1 1 1	1 0 0 0	0 0 0 0	0 0 0 0	5
g	1 1 1 1	0 0 0 0	1 1 1 0	1 0 0 1	9

1 = Parameter significant at 5% level.
0 = Parameter not significant.
(The four indices in each cell represent the four years 1956-1959.)

The change in price for a corresponding change in dividends for the Gordon model would be approximately equal to:

$$(5.2) \quad \frac{dP}{d(dv)} = [b-c(dv)/(nw + \bar{y} - dv)] \cdot \frac{P}{dv}$$

If we use representative parameters for the Gordon model, we find that [dP/d(dv)] would be about ten dollars.[19] This compares rather closely to the values of $9.50 for the logarithmic Durand and $12.60 for the linear Durand given the same variable values. Note that the Gordon value depends upon the growth rate parameter (c). A zero value for (c) would essentially reduce the Gordon [dP/d(dv)] to the logarithmic Durand rate

[19] The values used were: (dv) = $1.80, (ni) = ($\bar{y}$) = $3.00, (nw) = $30.00, (P) = $45.00, (b) = .80, (c) = 7.0.

of change; as (c), the growth time horizon, increases the impact of a change in dividends on price is considerably reduced.

The firm size index (g) varies from about .09 to .12 for the pooled regression estimates. This is consistent with the Benishay estimates even though quite different size measures were used in the two samples.[20] The leverage index (e) is negative as Gordon predicted and for Groups I and III almost the predicted value of -1.00. The reader is cautioned that the leverage index variable (x_4) must not be confused with the ordinary debt-equity ratio (h´). It is true that, ceteris paribus, price declines with an increase in the leverage index (x_4). But it is not true that $[dP/d(h')] < 0$. In fact, Gordon's simulations at average parameter levels suggest that price increases as the debt-equity ratio increases. The reason for the apparent incongruity is that the debt-equity ratio (h´) is implicitly a part of the dividends variable and the growth rate variable. The variable (x_4) measures the "risk" impact of leverage but it does not measure the impact of changes in the "return" dimension. Gordon's findings are not a direct contradiction of the M-M hypothesis but a contradiction by elaboration. That is, both Gordon and M-M agree that the risk attributes of leverage are such that increased risk through increased leverage would depress price. But Gordon goes further to argue that the capital markets are not the perfect markets[21]

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[20] The simple correlation between price, the Benishay size variable, and $[TA-CL]/10^6$, the Gordon size variable, is approximately .80.

[21] The phrase "perfect markets" is an unfortunate association that has become linked in the literature to M-M capital markets. It seems to put other authors on the defensive. Had some other phrase such as "simplistic markets" or "instant equilibrium markets" or "mechanical markets" been used, the M-M theory would probably have been dealt with quite differently by other researchers.

described by M-M and that there are "return" effects for stockholders associated with leverage. Furthermore, the return effects seem to dominate the risk effects so that the over-all impact of an increase in the debt-equity ratio is to increase price.[22]

The most interesting finding from the several variations of the Gordon model that were tried is the sensitivity of this model to the definition of dividends used. In describing the data that were actually used for the model variables, Gordon makes the following brief comment about dividends.[23]

> When income falls sharply or the firm feels a strong temporary need for cash, the dividend may be cut sharply as a temporary expedient. To deal with these situations 2% of the book value per share was used whenever the dividend was below this figure.

Limited testing suggests that this is not an insignificant alteration in the Gordon model. Table V-20 indicates what happened to the dividend coefficient (b) and the adjusted R^2 when the samples were tested using Gordon's dividend measure and a slightly different dividend measure. In every year for every sample the Gordon dividend coefficient was higher than the corresponding coefficient when the alternative dividend definition was used. The coefficient differences and differences in explained

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[22]The development of the theory to arrive at these conclusions can be found in Gordon, op. cit., pp. 100-113 and 189-193. A more recent simplified discussion using this same approach can be found in Eugene M. Lerner and Willard T. Carleton, A Theory of Financial Management, (New York: Harcourt, Brace, and World Co., 1966), pp. 179-198.

[23]See Gordon, op. cit., p. 157.

TABLE V-20

GORDON DIVIDEND PARAMETERS UNDER ALTERNATIVE DEFINITIONS

			1956	1957	1958	1959
I	(1)	b	.837	.974	.832	.756
		R^2	.90	.91	.90	.87
	(2)	b	.813	.965	.823	.698
		R^2	.88	.91	.89	.84
II	(1)	b	.690	.838	.670	.563
		R^2	.75	.71	.79	.50
	(2)	b	.289	.373	.154	.164
		R^2	.56	.51	.40	.36
III	(1)	b	.724	.656	.702	.787
		R^2	.62	.66	.73	.65
	(2)	b	.166	.241	.290	.279
		R^2	.40	.57	.59	.48
IV	(1)	b	.856	.938	.826	.858
		R^2	.67	.61	.62	.56
	(2)	b	.355	.900	.758	.543
		R^2	.40	.60	.58	.37

(1) Dividends are the greater of actual reported dividends per share or 2% of per share net worth.

(2) Dividends are the greater of actual reported dividends per share or one cent per share.

variation are quite dramatic in several instances. And yet the number of firms that actually had to be adjusted was quite small--usually less than five firms. For example, in the Group IV-1956, sample dividends were altered for only one firm and yet (b) more than doubled.

The explanation for this dramatic adjustment would appear to be two-fold: (1) the logarithmic relationship becomes increasingly non-linear below (dv = $1.00) relative to logarithms for dividends greater than one dollar; (2) the variables other than dividends for firms paying very low dividends do not appear to change nearly as much from an average sample value as the change implied by the low dividends from the average dividends value.

Suppose we have a sample of size 50 with a firm that pays no dividends but has a 2% (nw) value of $1.00. Then the substitution of a one cent dividend (log. .01 = -4.6) for the 2% dividend would reduce the mean by (-4.6/50), .092, a significant amount when the average value of the logarithms of the dividends is only about .60. More important, however, is the fact that the one change would increase the standard deviation for the dividends variable from a value of about .60 to more than .90. The impact of the different dividends definitions is readily apparent in Table V-21 where the means and standard deviations for dividends under both definitions are presented.

Fortunately, the dividends variable is the only variable in the Gordon model that can reasonably be expected to assume values near zero. The reasonableness of the dividends surrogate Gordon uses to dispose of this problem will be further examined in Chapter VI. What this variation in the Gordon model illustrates is the sensitivity of least squares

TABLE V-21

MEANS AND STANDARD DEVIATIONS FOR LOG. DIVIDENDS

			1956	1957	1958	1959
I	(1)	M	.558	.554	.548	.477
		S	.612	.556	.600	.630
	(2)	M	.547	.551	.544	.463
		S	.621	.557	.601	.665
II	(1)	M	.646	.610	.429	.403
		S	.494	.453	.511	.513
	(2)	M	.584	.547	.257	.147
		S	.808	.762	1.035	1.227
III	(1)	M	.611	.622	.570	.433
		S	.505	.479	.539	.525
	(2)	M	.451	.449	.377	.264
		S	1.123	1.120	1.163	1.115
IV	(1)	M	.680	.673	.626	.579
		S	.572	.541	.558	.521
	(2)	M	.604	.665	.605	.406
		S	.898	.559	.582	1.096

M = Mean of logarithm of the dividends.
S = Standard deviation of log. dividends for sample.

(1): See (I), Table V-20; (2): See (2), Table V-20.

parameter estimates to extreme values of any variable. Extreme values (say greater than four standard deviations) seem to occur with greater than expected frequency for most of the independent or dependent variables considered in models of equity valuation. If such values are not exorcised from the data, these extremes will substantially influence parameter estimates.

What about the stability of the parameter estimates for the basic Gordon regressions? Is it legitimate to view the pooled estimates in Table V-19 as being derived from underlying homogeneous populations? Again the answer is no. Assumptions about the equivalence of population parameters must be rejected whether one considers the sample set as a whole or sub-sets of cross-section or time-series data.[24] There is one thing interesting in Table V-22. The (F) ratios for the pooled-groups tests are relatively low and are consistently lower than the (F) ratios for the pooled-years tests. This is generally the opposite of the situation found in the (F) tests for the other models and may be an indication that Gordon has succeeded in measuring some of the sample differences that plague all these models.

In view of the relatively good performance of the Gordon model, can this model be considered the prototype for successful models of equity valuation? It does not seem likely for several reasons. First, the variation in parameter values is still too great to have much utility to the individual investor or firm. Second, some of the variables Gordon

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[24]This means that there is some question about the appropriateness of averaging time-series parameter estimates as Gordon does. Ibid., pp. 163-173.

TABLE V-22

COVARIANCE TESTS ON GORDON REGRESSIONS

Test	F Ratios
ALL	86.7
I	19.3
II	35.0
III	8.1
IV	7.5
1956	13.1
1957	11.1
1958	7.1
1959	6.9

All ratios significant at 5% level.

went to great length to rationalize--earnings instability, firm leverage, asset liquidity, firm size--do not appear to be generally significant variables. Third, it is not obvious in what direction one might logically expand the variable set. Finally, the empirical prerequisites for the model are quite restrictive. The model is meant to be applied to firms that have a financial history of a decade or more and that have positive earnings and dividends. Despite these limitations, the Gordon monograph contains many stimulative sections that will undoubtedly be of great use in the development of future models, and the statistical performance is certainly good enough to warrant using this model as the "straw man" for future test comparisons of other models.

5.6 Some Concluding Comments

The models surveyed cannot be described as resounding successes. Indeed, considering the very large number of man hours and dollar resources that have been invested in the search for equity valuation models, one is tempted to conclude that he is viewing a major disaster area. What can we unequivocally conclude from the test results? If the sample data are representative of a somewhat larger population of significant American companies with positive earnings and dividends, we can conclude that:

> For a selected population of large American companies with positive earnings, there is a positive relation between equity share price and earnings or dividends.

Nothing more. This seems to be rather overdoing it in corroborating the obvious, for the amateur investor has held the same perception for decades.

If one does not mind equivocating a little, there are some useful inferences that may be drawn from these tests.

(1) There is a positive relation between equity share price and earnings or dividends for companies that have positive earnings or dividends.

(2) A reasonable range for a price-earnings multiplier might be 10-15. A reasonable range for a price-dividends multiplier might be 20-30. This may be just another way of stating that returns in the equity markets are not completely independent of returns in the other capital markets.

(3) The following variable types seemed to be useful in helping to explain equity share prices: (a) net income, (b) dividends, (c) growth index, (d) size index. Other variable types tested were not significant.

(4) There seems to be a "sample effect" or "industry effect" that was not explained as part of any of the models. This is true to a lesser extent for the "time effect."

(5) The most complex of the models tested, the Gordon model, had relatively more stable parameters than the simpler Durand model. Since the explanatory power of both models was about the same, it seems appropriate to use the Gordon model as a test alternative in future empirical studies.

One question remains. How much further can one expect to go with the Gordon type model or any other analogous model type? One cannot prove the search procedure being employed is fruitless any more than one could have proved to the alchemists two hundred years ago that their search for gold was doomed to failure. And yet one is left with the impression that the large number of attempts to find an equity valuation model through the regression equation add-a-variable, drop-a-variable search procedure has been and will be doomed to failure so long as the researchers rely primarily on historical firm financial variables and single equation model specifications.