Practical Agnostic Active Learning

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* introductory slide credit
Active learning

Labels are often much more expensive than inputs:
- documents, images, audio, video,
- drug compounds, ...

Can interaction help us learn more effectively?

Learn an accurate classifier requesting as few labels as possible.
Active learning

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Why should we hope for success?

Threshold functions on the real line. Target is a threshold.

![Threshold function graph]

Supervised: Need $\approx 1/\epsilon$ labeled points. With high probability, any consistent threshold has $\leq \epsilon$ error.
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\[ w^- \quad w^+ \]

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Nonseparable data? Other hypothesis classes?
Typical heuristics for active learning

Start with a pool of unlabeled data
Pick a few points at random and get their labels
Repeat
   Fit a classifier to the labels seen so far
   Query the unlabeled point that is closest to the boundary (or most uncertain, . . .)
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Biased sampling: the labeled points are not representative of the underlying distribution!
Sampling bias

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Example:

![Bar chart example]

45% 5% 5% 45%
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Example:

Even with infinitely many labels, converges to a classifier with 5% error instead of the best achievable, 2.5%. Not consistent!

“Missed cluster effect” (Schütze et al, 2006)
Setting:

- Hypothesis class $H$. For $h \in H$,

$$\text{err}(h) = \Pr_{(x,y)}[h(x) \neq y]$$

- Minimum error rate $\nu = \min_{h \in H} \text{err}(h)$

- Given $\epsilon$, find $h \in H$ with $\text{err}(h) \leq \nu + \epsilon$

Desiderata:

- General $H$
- Consistent: always converge
- Agnostic: deal with arbitrary noise, $\nu \geq 0$
- Efficient: statistically and computationally
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Is this achievable?
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Importance Weighted Active Learning

\[ S_0 = \emptyset \]

For \( t = 1, 2, \ldots, n \)

1. Receive unlabeled example \( x_t \) and set \( S_t = S_{t-1} \).
2. Choose a probability of labeling \( p_t \).
3. Flip a coin \( Q_t \) with \( \mathbb{E}[Q_t] = p_t \). If \( Q_t = 1 \), request \( y_t \) and add \((x_t, y_t, \frac{1}{p_t})\) to \( S_t \).
4. Let \( h_{t+1} = \text{LEARN}(S_t) \).

Empirical importance-weighted error

\[
\text{err}_n(h) = \frac{1}{n} \sum_{t=1}^{n} \frac{Q_t}{p_t} 1[h(x_t) \neq y_t]
\]

Minimizer \( \text{LEARN}(S_t) = \arg \min_{h \in H} \text{err}_t(h) \)

**Consistency:** The algorithm is consistent as long as \( p_t \) are bounded away from 0.
How should $p_t$ be chosen?

Let $\Delta_t = \text{increase in empirical importance-weighted error rate if learner is forced to change its prediction on } x_t$.

Set $p_t = 1$ if $\Delta_t \leq O\left(\sqrt{\frac{\log t}{t}}\right)$; otherwise, $p_t = O\left(\frac{\log t}{\Delta_t^2 t}\right)$.

- $h_t = \arg\min\{err_{t-1}(h) : h \in H\}$
- $h'_t = \arg\min\{err_{t-1}(h) : h \in H \text{ and } h(x_t) \neq h_t(x_t)\}$
- $\Delta_t = err_{t-1}(h'_t) - err_{t-1}(h_t)$
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How can we compute $\Delta_t$ in constant time?
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How can we compute $\Delta_t$ in constant time?

$\triangleright$ Find the smallest $i_t$ such that $h_{t+1} = h'_t$ after $(x_t, h'_t(x_t), i_t)$ update.

$\triangleright$ We have $(t - 1) \cdot \text{err}_{t-1}(h'_t) \leq (t - 1) \cdot \text{err}_{t-1}(h_t) + i_t$. Thus $\Delta_t \leq i_t/(t - 1)$. 
Importance weight aware SGD updates [Karampatziakis and Langford]. Solve for $i_t$ directly. E.g., for logistic,\

$$i_t = \frac{2w_t^T x_t}{\eta_t \text{sign}(w_t^T x_t) x_t^T x_t}$$

- **astrophysics**
  - invariant
  - implicit
  - gradient multiplication
  - passive

- **rcv1**
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Guarantees

IWAL achieves error similar to that of supervised learning on $n$ points:

Accuracy Theorem: For all $n \geq 1$,

$$\text{err}(h_n) \leq \text{err}(h^*) + \sqrt{\frac{C \log n}{n - 1}}$$

with high probability.
Guarantees

IWAL achieves error similar to that of supervised learning on $n$ points:

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with high probability.

**Label Efficiency Theorem:** With high probability, the expected number of labels queried after $n$ iterations is at most

$$O(\theta \text{err}(h^*) n) + O \left( \theta \sqrt{n \log n} \right)$$

where $\theta$ is the disagreement coefficient.
The crucial ratio: Disagreement Coefficient (Hanneke-2007)

\( r \)-ball around a minimum-error hypothesis \( h^* \):

\[
B(h^*, r) = \{ h \in H : \Pr[h(x) \neq h^*(x)] \leq r \}
\]

Disagreement region of \( B(h^*, r) \):

\[
\text{DIS}(B(h^*, r)) = \{ x \in X | \exists h, h' \in B(h^*, r) : h(x) \neq h'(x) \}
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The disagreement coefficient measures the rate of

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\theta = \sup_{r > 0} \frac{\Pr[\text{DIS}(B(h^*, r))]}{r}
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Example:
The crucial ratio: Disagreement Coefficient (Hanneke-2007)

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Example:

- Thresholds in \( \mathbb{R} \), any data distribution. \( \theta = 2 \).
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1. Always consistent.
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2. Efficient.
   2.1 Label efficient, unlabeled data efficient, computationally efficient.
3. Compatible.
   3.1 With online algorithms
   3.2 With any optimization-style classification algorithms
   3.3 With any Loss function
   3.4 With supervised learning
   3.5 With switching learning algorithms (!)
4. Collected labeled set is reusable with a different algorithm or hypothesis class.
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Applications

- News article categorizer
- Image classification
- Hate-speech detection / comments sentiment
- NLP sentiment classification (satire, newsiness, gravitas)
Active learning in Vowpal Wabbit

Simulating active learning: \((\text{knob } c > 0)\)
\[
\text{vw} --\text{active\_simulation} --\text{active\_mellowness} \ c
\]

Deploying active learning:
\[
\text{vw} --\text{active\_learning} --\text{active\_mellowness} \ c --\text{daemon}
\]

- \(\text{vw}\) interacts with an \text{active\_interactor} \((\text{ai})\)
- receives labeled and unlabeled training examples from \text{ai} over network
- for each unlabeled data point, \(\text{vw}\) sends back a query decision (and an importance weight if label is requested)
- \text{ai} sends labeled importance-weighted examples as requested
- \(\text{vw}\) trains using labeled importance-weighted examples
Active learning in Vowpal Wabbit

active_interactor.cc (in git repository) demonstrates how to implement this protocol.
Needle in a haystack problem: Rare classes

Learning interval functions $h_{a,b}(x) = 1[a \leq x \leq b]$, for \(0 \leq a \leq b \leq 1\).

**Supervised learning**: need $O(1/\epsilon)$ labeled data.

**Active learning**: need $O(1/W + \log 1/\epsilon)$ labels, where $W$ is the width of the target interval. No improvement over passive learning.
Needle in a haystack problem: Rare classes

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...but given *any* example of the rare class, the label complexity drops to $O(\log 1/\epsilon)$.

Dasgupta 2005; Attenberg & Provost 2010: Search and insertion of labeled rare class examples helps.
How can editors feed observed mistakes into an active learning algorithm?
predicted label: Entertainment
How can editors feed observed mistakes into an active learning algorithm?
Is this fixable?

Beygelzimer-Hsu-Langford-Zhang (NIPS-16):

Define a **Search** oracle:

The active learner interactively restricts the searchable space guiding Search where it’s most effective.
**Oracle Search**

**Require:** Working set of candidate models $V$

**Ensure:** Labeled example $(x, y)$ s.t. $h(x) \neq y$ for all $h \in V$

(systematic mistake), or $\perp$ if there is no such example.
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How can a counterexample to a version space be used?
Search Oracle

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---

How can a counterexample to a version space be used?

Nested sequence of model classes of increasing complexity:

$$H_1 \subseteq H_2 \subseteq \ldots \subseteq H_k^* \ldots$$

Advance to more complex classes as simple are proved inadequate.
Search + Label

Search + Label can provide exponentially large problem-dependent improvements over Label alone, with a general agnostic algorithm.

Union of intervals example:

• $\tilde{O}(k^* + \log(1/\epsilon))$ Search queries
• $\tilde{O}((\text{poly log}(1/\epsilon) + \log k^*)(1 + \frac{\nu^2}{\epsilon^2}))$ Label queries
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How can we make it practical?

- Drawbacks as with any version space approach
- Can we reformulate as a reduction to supervised learning?
Interactive Learning

Interactive settings:

- Active learning
- Contextual bandit learning
- Reinforcement learning

Bias is a pervasive issue:

- The learner creates the data it learns from / is evaluated on
- State of the world depends on the learner’s decisions
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How can we use supervised learning technology in these new interactive settings?
For $t = 1 \ldots T$:

1. Observe $x_t$
2. Predict label $\hat{y}_t \in [1, \ldots, K]$
3. Pay and observe $1[\hat{y}_t \neq y_t]$ (ad not clicked)

No stochastic assumptions on the input sequence.

Compete with multiclass linear predictors $\{W \in \mathbb{R}^{K \times d}\}$, where

$$W(x) = \arg \max_{k \in [K]} (W \cdot x)_k$$
Mistake Bounds

Banditron [Kakade-Shalev-Shwartz-Tewari, ICML-08]
SOBA [Beygelzimer-Orabona-Zhang, ICML-17]

<table>
<thead>
<tr>
<th></th>
<th>Perceptron</th>
<th>Banditron</th>
<th>SOBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L + \sqrt{T}$</td>
<td>$L + T^{2/3}$</td>
<td>$L + \sqrt{T}$</td>
<td></td>
</tr>
</tbody>
</table>

where $L$ is the competitor’s total hinge loss.

Per-round hinge loss of $W$

$$l_t(W) = \max_{r \neq y_t} [1 - (W x_t)_y + (W x_t)_r] + \geq 1[y_t \neq \hat{y}_t]$$

Resolves a COLT open problem (Abernethy and Rakhlin’09)
The Multiclass Perceptron

A linear multiclass predictor is defined by a matrix $W \in \mathbb{R}^{k \times d}$. For $t = 1 \ldots T$:

- Receive $x_t \in \mathbb{R}^d$
- Predict $\hat{y}_t = \arg \max_r (W^t x_t)_r$
- Receive $y_t$
- Update $W^{t+1} = W^t + U^t$ where
  
  $U_t = 1[\hat{y}_t \neq y_t] (e_{y_t} - e_{\hat{y}_t}) \otimes x_t$
Bandit Setting

- If $\hat{y}_t \neq y_t$, we are blind to the value of $y_t$
- Solution: Randomization!

SOBA:

- A second order perceptron with a novel unbiased estimator for the perceptron update and the second order update.
- Passive-aggressive update (sometimes updating when there is no mistake but the margin is small)
The End