

Internet Appendix to "Efficient Recapitalization"*

This Internet Appendix serves as a companion to the paper "Efficient Recapitalization." It reports results not reported in the main text due to space constraints. We present results in the order they appear in the main text.

I. Continuous Asset Distribution

Our benchmark model assumes a binary payoff structure for assets in place. We can generalize our model to a distribution $F(a|\varepsilon)$ over $[0, \infty)$. As before, we assume that ε parameterizes the quality of assets in place. We now discuss how our main results change in this more general setup.

It is clear that Theorem 1 continues to hold. It is relatively easy to check that Proposition 5 also carries over to a general distribution of asset values. The issue of optimal interventions under asymmetric information is more delicate. Consider a generalization of the program described in Theorem 2. The government offers cash m in exchange for preferred stock with face value $(1 + h)m$ and a portfolio of $(1 - \eta)/\eta$ warrants with strike price $\max(a - D, 0)$. The key difference is that the strike price of the warrants is conditional on the realized asset payoff. The schedule of payoffs to the old shareholders is now

$$f(y^e) = \min(y^e, \max(a - D, 0)) + \eta \max(y^e - \max(a - D, 0), 0)$$

and total shareholder payoff is

$$y^e = \max(a - D + i \cdot (v - rx + (r - 1)m) - hm, 0),$$

where r is the break-even rate conditional on ε, v , and m (in the binary case, it was simply $r = 1/p_\varepsilon$). The NIP constraint is $h > 0$ and for simplicity we take the limit $h \rightarrow 0$. The first key point is that $E[y^e]$ is increasing in i if and only if $v + (r - 1)m > rx$. This pins down the investment constraint conditional on participation. For any $\eta > 0$ it also pins down the lower participation schedule $L(\varepsilon, v; \Gamma) = 0$. By not participating, these types get $\max(a - D, 0)$. It is easy to check that they prefer to participate (and invest) if and only if $v + (r - 1)m > rx$.

Consider now the upper participation schedule $U(\varepsilon, v; \Gamma) = 0$. These types invest alone and a fortiori with the help of the government. For these types, $y^e > \max(a - D, 0)$ so $f(y^e) = \max(a - D, 0) + \eta \max(y^e - \max(a - D, 0), 0)$. As we decrease η towards zero, the schedule $f(y^e)$ becomes $\max(a - D, 0)$, which is the payoff without investment. It is strictly lower than the outside option of any type that would invest alone. In other words, in the (ε, v) space, the schedule $U(\varepsilon, v; \Gamma) = 0$ converges to the schedule $L(\varepsilon, v; \emptyset) = 0$ and opportunistic participation disappears. By the same argument, the rents

*Citation format: Philippon, Thomas, and Philipp Schnabl, 2012, Internet Appendix to "Efficient Recapitalization," *Journal of Finance* [vol #], [pages], <http://www.afa.jof.org/supplements.asp>. Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

of all participating types also disappear since their payoff $f(y^e)$ converges to their outside option $\max(a - D, 0)$. We therefore obtain our last result.

PROPOSITION IA.1: Theorem 1 and Proposition 5 hold for any asset distribution. Theorem 2 holds when the strike price of the warrants is set ex-post to $\max(a - D, 0)$.

Proof. Consider the equity payoffs for a bank in the program

$$y^e = \max\left(a - D + i \cdot \left(v - \frac{x - m}{p_\varepsilon} - m\right) - hm, 0\right).$$

In the good state, as soon as $y^e > A - D$, the warrants are in the money and the number of shares jumps to $1 + \frac{1-\eta}{\eta} = \frac{1}{\eta}$. So the old shareholders get only a fraction η of the value beyond $A - D$. The payoff function for old shareholders is therefore

$$f(y^e) = \min(y^e, A - D) + \eta \max(y^e - A - D, 0).$$

Old shareholders are full residual claimants up to the face value of old assets $A - D$ and η residual claimants beyond. Now let us think about their decisions at time 1. The NIP-constraint is simply $h > 0$. The value for old shareholders conditional on participation and investment is

$$E_1[f(y^e) | \varepsilon, v, \Gamma, i = 1] = p_\varepsilon \left(A - D + \eta \left(v - \frac{x - m}{p_\varepsilon} - (1 + h)m \right) \right).$$

The lower schedule (efficient participation) is therefore

$$L(\varepsilon, v; \Gamma) = \eta(p_\varepsilon v - x + m(1 - (1 + h)p_\varepsilon)).$$

For any $\eta > 0$, we can see that the lower schedule is equivalent to that of an equity injection with $\frac{\alpha}{1-\alpha} = \frac{m(1+h)}{A-D}$, and that of an asset purchase with $m = qZ$ and $q = \frac{1}{1-h}$. If we take $h \rightarrow 0$ we get the lower bound of a simple cash injection program, with an investment set simply equal to $I(\bar{a}, m)$. In general, we have an investment set \mathcal{I} . The upper schedule (opportunistic participation) is:

$$U(\bar{a}, \varepsilon, v; \Gamma) = \eta m(1 - (1 + h)p_\varepsilon) - (1 - \eta)(p_\varepsilon v - x).$$

When $\eta \rightarrow 0$, the upper bound schedule $\{U = 0\}$ converges to the schedule $\{p_\varepsilon v - x = 0\}$. In this limit, there is no opportunistic participation and

$$\lim_{\eta \rightarrow 0} \Omega(\Gamma) = \mathcal{I} \setminus I(\bar{a}, 0) = \Omega^{\min}.$$

Finally, the expected payments to the old shareholders converge to $p_\varepsilon(A - D)$. So the government receives expected value $p_\varepsilon v - x + m$ by paying m at time 1. The total cost therefore converges to

$$\lim_{\eta \rightarrow 0} \Psi_1(\bar{a}, \Gamma) = - \int \int_{\Omega^{\min}} (p_\varepsilon v - x) dF(\varepsilon, v) = \Psi_1^{\min}.$$

Q.E.D.

The key point of the implementation is that banks whose assets perform well ex-post are rewarded by a lower dilution of their equity. Technically, by adjusting the strike price based on the realized asset value a , the government can provide the same incentives as in the model with binary asset payoffs. While the security design is not standard, it is perfectly consistent with the assumptions we have made regarding information and contracts. If assets trade, the government only needs to use ex-post market prices. Even if assets do not trade, the government can implement the optimal intervention by buying a small random sample of assets, observing the ex-post performance, and setting the strike price accordingly.

II. Calibration

A. Assets and Senior Debt

Suppose that the assets with face value A are financed by debt D and equity E , so that $A = D + E$. We can normalize by the face value A . So without loss of generality, we assume payoffs $a \in [0, 1]$ and legacy leverage

$$b \equiv \frac{D}{A}.$$

We assume that a follows a beta distribution. The beta distribution is well suited to model recovery rates on assets with a fixed upper-bound payoff (such as loans that pay at most face value).[†] The recovery a takes on values between zero and one, allowing for partial to full recovery. The pdf is

$$(a, \epsilon) = \epsilon a^{\epsilon-1},$$

and the cdf is

$$F(b; \epsilon) = \int_0^b \epsilon a^{\epsilon-1} da = b^\epsilon.$$

The probability of solvency is

$$\Pr[a > b] = \int_b^1 \epsilon a^{\epsilon-1} da = 1 - b^\epsilon.$$

[†]The beta distribution has two shape parameters, ϵ and χ , that can be used to calibrate the mean and variance to match an empirical recovery distribution. Its pdf is given by

$$\frac{\Gamma(\epsilon + \chi)}{\Gamma(\epsilon)\Gamma(\chi)} a^{\epsilon-1} (1-a)^{\chi-1}, \quad 0 \leq a \leq 1.$$

The mean is given by

$$E[a] = \frac{\epsilon}{\epsilon + \chi}.$$

If we are only interested in matching the mean, and we want a uniparametric distribution, we can set $\chi = 1$. The pdf then simplifies to

$$\frac{\Gamma(\epsilon + 1)}{\Gamma(\epsilon)\Gamma(1)} a^{\epsilon-1} = \epsilon a^{\epsilon-1},$$

since $\Gamma(1) = 1$ and $\Gamma(\epsilon + 1) = \Gamma(\epsilon)\epsilon$.

Equity value without the new project is

$$E[\max(a - b, 0)] = \int_b^1 (a - b) \epsilon a^{\epsilon-1} da = \frac{\epsilon}{\epsilon + 1} (1 - b^{\epsilon+1}) - b(1 - b^\epsilon).$$

The market value of existing debt without taking into account new investment is

$$\bar{v}(b) = \int_0^b a \epsilon a^{\epsilon-1} da + b(1 - F(b; \epsilon)) = \frac{\epsilon}{1 + \epsilon} b^{\epsilon+1} + b(1 - b^\epsilon) = b \left(1 - \frac{b^\epsilon}{1 + \epsilon} \right).$$

The discount is

$$\bar{p}^b = 1 - \frac{b^\epsilon}{1 + \epsilon}$$

In December 2008, the 5 year CDS spreads for the largest banks implied a discount of about 0.88 (see Table I in the main text).

Assuming that senior debt is assets minus deposits, we thus have

$$0.88 = 1 - \frac{0.5^\epsilon}{1 + \epsilon}.$$

B. New Project and Junior Debt

New investment opportunities with fixed payoff v (per unit of asset) require an investment of x (per unit of asset). Let j be the face value of the junior debt

$$E[\min(\max(a - b + v, 0), j)] = x.$$

Writing out the integral leads to

$$\int_{b-v}^{b-v+j} (a + v - b) \epsilon a^{\epsilon-1} da + j(1 - F(b + j - v; \epsilon)) = x$$

$$\epsilon \frac{(b - v + j)^{\epsilon+1} - (b - v)^{\epsilon+1}}{\epsilon + 1} + (v - b) [F(b + j - v) - F(b - v)] + j(1 - F(b + j - v; \epsilon)) = x.$$

Since this is a nonlinear equation it has to be solved numerically to yield $j = J(\epsilon, b, v, x)$.

C. Investment Region

C.1. Laissez Faire

We first characterize L^o , the lower schedule for investment. For a bank to invest without participating in a government program, the cutoff point (ϵ, v) such that a bank is indifferent between investing and not is:

$$E[\max(a + v - b - j, 0)] = E[\max(a - b, 0)].$$

This holds when $v = j$, so we have

$$L^o = \{(\epsilon, v) \mid v = J(\epsilon, b, v, x)\}.$$

For a firm with additional cash m to be indifferent between investing and not, we need

$$E[\max(a + v - b - j(m), 0)] = E[\max(a - b + m, 0)].$$

This holds if and only if $v = J(\epsilon, b, v, x - m) + m$.

C.2. Equity Injection

- Government injects m and receives share α of equity.
- Conditional on participation and investment, we have $j_m = J(\epsilon, b, v, x - m)$, and the old shareholders get

$$\begin{aligned} (1 - \alpha)E[\max(a + v - b - j_m, 0)] &= (1 - \alpha) \int_{b+j-v}^1 (a + v - b - j_m) \epsilon a^{\epsilon-1} da \\ &= (1 - \alpha) \left(\frac{\epsilon}{1 + \epsilon} (1 - (b + j_m - v)^{\epsilon+1}) - (b + j_m - v) (1 - (b + j_m - v)^{\epsilon}) \right) \end{aligned}$$

- Lower schedule for equity. Firms that are indifferent between participating and investing and not participating without investing:

$$\begin{aligned} (1 - \alpha)E[\max(a + v - b - j_m, 0)] &= E[\max(a - b, 0)] \\ &= \int_b^1 (a - b) \epsilon a^{\epsilon-1} da \\ &= \frac{\epsilon}{\epsilon + 1} (1 - b^{\epsilon+1}) - b(1 - b^{\epsilon}). \end{aligned}$$

- Upper schedule for equity. Firms that are indifferent between participating and investing and not participating and investing:

$$\begin{aligned} (1 - \alpha)E[\max(a + v - b - j_m, 0)] &= E[\max(a + v - b - j_0, 0)] \\ &= \frac{\epsilon}{1 + \epsilon} (1 - (b + j_0 - v)^{\epsilon+1}) - (b + j_0 - v) (1 - (b + j_0 - v)^{\epsilon}). \end{aligned}$$

C.3. Preferred Equity and Warrants

- m is a junior loan (preferred equity), h is interest, η is the share of equity income left to existing shareholders above strike price S .

With a preferred equity and warrants policy (m, h, η, S) , the lower schedule is

$$E[\min(\max(a+v-b-j_m-(1+h)m, 0), S)+\eta \max(\max(a+v-b-j_m-(1+h)m, 0)-S, 0)] = E[\max(a-l, 0)].$$

The upper schedule would be given by setting the LHS of the above equation equal to the same expression for investing without government help derived before.

D. Comparisons

We choose the parameters of the model to match some data:

- Precommitted senior debt is 50% of assets ($A = 1, D = 0.5$).
- x is 30% of assets ($x = 0.3$).
- The distribution of ϵ is set to match the distribution of five-year CDS spreads.
- The distribution of v is set to match the distribution of market-to-book. Before the crisis (2005), median bank market to book equity was 2.1 (for instance, 1.67 for BoA and 3 for Wells Fargo). If equity is 10% of assets, this give a median market-to-book for assets of $1 + 0.1 * (2.1 - 1) = 1.11$. Neglecting credit risk, this implies $\bar{v} - x = 0.11$. If v is uniform from x to V , this means $\frac{V-x}{2} = 0.11$ so $V = 0.52$

The results of the calibration are reported in the main text. Investment is relative to first best. Costs are in percent over the minimum cost.

III. Proof of Proposition 7

A. Time-0 Programs

Full Transfer: $v < \Delta$.

For simplicity, we suppress the macro state \bar{a} in all expressions. The expected values of deposits at time 1 and time 0 are

$$\begin{aligned} E_1 [y^\Delta (m)] &= p(\epsilon) \Delta + (1 - p(\epsilon)) m \text{ if } (\epsilon, v) \in T \setminus I(m) \\ &= p(\epsilon) \Delta + (1 - p(\epsilon)) v \text{ if } (\epsilon, v) \in I(m) \\ E_0 [y^\Delta (m)] &= \bar{p} \Delta + (1 - \bar{p}) m + \int \int_{I(m)} (1 - p) (v - m) dF(\epsilon, v). \end{aligned}$$

The expected cost of deposit insurance at time 0 is

$$\begin{aligned} \Psi_0^F (m) &= \Delta - E_0 [y^\Delta (m)] \\ &= (1 - \bar{p}) (\Delta - m) - \int \int_{I(m)} (1 - p(\epsilon)) (v - m) dF(\epsilon, v). \end{aligned}$$

The change in the expected cost of deposit insurance as a result of the cash injection m is

$$\begin{aligned}\Lambda_0^F(m) &= \Psi_0^F(m) - \Psi_0^F(0) \\ &= -(1 - \bar{p})m + m \int_{I(m) \setminus I(0)} \int (1 - p) dF(\varepsilon, v) - \int_{I(m) \setminus I(0)} \int (1 - p(\varepsilon)) v dF(\varepsilon, v).\end{aligned}$$

The net cost of government intervention is

$$\bar{\Psi}_0(m) + \Lambda_0^F(m) = - \int_{I(m) \setminus I(0)} \int v dF(\varepsilon, v).$$

Note that this term is negative because the benefits of incremental investments accrue to the government.

Partial Transfer: $\Delta < v$.

The expected values of deposits at time 1 and time 0 are

$$\begin{aligned}E_1 [y^\Delta(m) | \varepsilon, v] &= p(\varepsilon) \Delta + (1 - p(\varepsilon)) \max(\Delta, m) \text{ if } (p, v) \in T \setminus I(m) \\ &= \Delta \text{ if } (\varepsilon, v) \in I(m) \\ E_0 [y^\Delta(m)] &= \Delta - \int_{T \setminus I(m)} \int (1 - p(\varepsilon)) (\Delta - \max(\Delta, m)) dF(\varepsilon, v).\end{aligned}$$

The expected cost of deposit insurance is

$$\Psi_0^F(m) = \int_{T \setminus I(m)} \int (1 - p(\varepsilon)) (\Delta - \max(\Delta, m)) dF(\varepsilon, v).$$

The change in the expected cost of deposit insurance is

$$\Lambda_0^F(m) = \int_{T \setminus I(m)} \int (1 - p(\varepsilon)) (\Delta - \max(\Delta, m)) p(\varepsilon) - \int_{T \setminus I(0)} \int (1 - p(\varepsilon)) (\Delta) dF(\varepsilon, v).$$

Note that this expression is negative. The government's cost is $\Lambda_0^F(m) + \bar{\Psi}_0(m)$. The results apply to all programs because all programs have the same cost function at time 0.

B. Time-1 Programs

Full Transfer: $v < \Delta$.

The expected values of deposits at time 1 and time 0 given an asset purchase program (q, Z) are

$$\begin{aligned} E_1 [y^\Delta(q, Z)] &= p(\varepsilon) \Delta \text{ if } (\varepsilon, v) \in T \setminus (I(0) \cup \Omega_1(q, Z)) \\ &= p(\varepsilon) \Delta + (1 - p(\varepsilon)) v \text{ if } (\varepsilon, v) \in I(0) \cup \Omega_1(q, Z) \\ E_0 [y^\Delta(q, Z)] &= \bar{p} \Delta + \int_{I(0) \cup \Omega_1(q, Z)} \int (1 - p(\varepsilon)) v dF(\varepsilon, v) \end{aligned}$$

The expected cost of deposit insurance is

$$\Psi_0^F(q, Z) = (1 - \bar{p}) \Delta - \int_{I(0) \cup \Omega_1(q, Z)} \int (1 - p(\varepsilon)) v dF(\varepsilon, v)$$

The change in the cost of deposit insurance due to the injection is

$$\Lambda_0^F(q, Z) = - \int_{\Omega_1^q(q, Z) / I(0)} \int (1 - p(\varepsilon)) v dF(\varepsilon, v)$$

The expected cost to the government is $\Psi_1^a(q, Z) + \Lambda_0^F(q, Z)$.

Partial Transfer: $\Delta < v$.

The expected values of deposits at time 1 and time 0 are

$$\begin{aligned} E_1 [y^\Delta(Z, q)] &= p \Delta \text{ if } (\varepsilon, v) \in T \setminus (I(0) \cup \Omega_1(q, Z)) \\ &= \Delta \text{ if } (\varepsilon, v) \in I(0) \cup \Omega_1(q, Z) \\ E_0 [y^\Delta(Z, q)] &= \Delta - \int_{T \setminus (I(0) \cup \Omega_1(q, Z))} \int (1 - p(\varepsilon)) \Delta dF(\varepsilon, v). \end{aligned}$$

The expected cost of government insurance is

$$\Psi_0^F(Z, q) = \int_{T \setminus (I(0) \cup \Omega_1(q, Z))} \int (1 - p(\varepsilon)) \Delta dF(\varepsilon, v).$$

The change in the expected cost of deposit insurance is

$$\Lambda_0^F(Z, q) = - \int_{\Omega_1(q, Z) / I(0)} \int (1 - p(\varepsilon)) \Delta dF(\varepsilon, v).$$

The cost to the government is $\Psi_1^a(q, Z) + \Lambda_0^F(q, Z)$. The results also apply to debt guarantees at time 1 because asset purchases and debt guarantees have the same cost function at time 1.

C. Cash Against Equity at Time 1

Note that we can compute the expected cost of time-1 cash against equity similarly to the time-1

asset purchase program. The only difference is the participation region for cash against equity $\Omega^e(m, \alpha)$ and the participation region for asset purchases $\Omega_1^g(q, Z)$. It turns out that the change in the expected cost of deposit insurance $\Lambda_0^F(m)$ is equivalent under both programs because both in the full and in the partial transfer case the difference in the participation region cancels out when computing the difference in the expected cost of deposit insurance. It follows that the relative ranking of programs is unchanged.