PRODUCTIVITY DIFFERENCES, WORLD-MARKET SHARES AND CONFLICTING NATIONAL INTERESTS IN LINEAR TRADE MODELS

Ralph E Gomory* and William J. Baumol**[a]

*President, Alfred P. Sloan Foundation

**Director, C.V. Starr Center for Applied Economics, New York University

We have shown elsewhere (Gomory 1994, Gomory and Baumol 1994) that where the world economy is characterized by scale economies the market mechanism will entail a vast number of equilibria that differ substantially in the welfare they provide each of the trading countries. The relative benefit of a particular trade equilibrium to a country is related directly to the share of world income it yields to that nation. When the share of that country is extremely low, a rise in its income share can benefit not only itself, but its trading partner's as well. Similarly, the country's share of world income can reach levels that are excessive, in the sense that a rise in its trading partner's share can be desirable for both. However, when income share lies in the intermediate range there is conflict in the interests of the trading countries, with a rise in the share of one country benefitting that country, but harming the other.

We will show here that much of this story carries over to situations where there

---

[a] We are deeply indebted to Gene Grossman, Paul Samuelson and Frank P. Stafford for their very helpful comments. We are also grateful to the Sloan Foundation and the C.V. Starr Center for their support of our work. For pertinent literature on the subject of this paper see Grossman and Rogoff 1995, and Ethier 1982.

are neither economies nor diseconomies of scale, to which it is convenient to refer as the linear case. Specifically, for the classical linear case this paper will demonstrate that

1. There is often inherent conflict in the interests of trading partners, in the sense that a result that is best or relatively good for one is often poor for the other.

2. On the assumption that, given the current state of technological knowledge, there is in any industry a maximal level of (labor) productivity attainable by a producer, one obtains a region of equilibria for families of linear models. There exists both an upper and a lower boundary for this region that can be approximated closely with the aid of linear programming.

3. The characteristic shape of the region is robust, and is the same as that generated by the many equilibria of a scale-economies model. That shape is what leads directly to the conclusion that there can be conflict in the interests of the trading partners of the sort already described.

4. Any perfectly specialized solution, in which any good is produced in only one country, will be the equilibrium of some suitably selected linear model. These equilibria can usefully be subdivided into those where a country has attained maximal productivity in each good it produces and those where it has not. In the latter case, increases in the productivity of the producing nation will benefit both countries. However, when one of the countries' equilibria entails maximal productivity then it can be harmed by increases in the productivity of the other country in an industry currently supplied exclusively by the former, because the improving country may then be able to take the industry away from the current producer.

5. The model also permits an analysis of the gains and losses to the welfare of a country from a ceteris paribus increase in the productivity level of a single industry in the other country in the model. Thus, the analysis helps us to see when it is (or is not) detrimental to the interest of one country to help another country to improve its technology and its productivity more generally.

6. It is possible to determine from the point of view of one of the countries (J) the attributes of the other country that will best serve J's interests. This set of attributes will be said to constitute J's "ideal trading partner."

The linear case is much more familiar to economists than that of scale economies in international trade. It goes back to Ricardo and earlier. Consequently, the extension of our scale economies results to the linear model may facilitate understanding of the regions of equilibria and of the implications that follow from them.

Yet the idea that there can be a region of equilibria for linear models may at first seem more than a bit odd. There are good grounds for the standard presumption that a linear model of international trade will yield only a single equilibrium rather than the thousands or millions of candidate equilibria that we have shown to characterize the scale economies case, and that together constitute the equilibrium region. This is correct, and we will, indeed, deal with models with the unique equilibria to be expected in the linear case. However, rather than studying just a single linear model we will, instead, examine entire families of such linear models. These will all be the same in every respect but one: they will differ in terms of productivity performance. That is, one model in a family will differ from another in that in the first model the average productivity of labor in Country J in the production of good I will be different from the corresponding productivity level in the second model. Each such model will then yield a different equilibrium, and we will focus on the set of equilibria corresponding to the entire family of models -- one equilibrium per model. This is in contrast with our scale-economies analysis, where all of the equilibria in the equilibrium region are possible outcomes from a single model.

I. The Model and the Graph

Our model is simply the standard classical model of international trade with Cobb-Douglas demand, and it is identical with that used in our scale-economies analysis, except that the production relationships are no longer characterized by economies of scale. Specifically, in this model it is assumed that there is a single input, labor, and that each of the two countries is endowed with its given quantity of labor. The market forces produce an equilibrium that determines the quantity of each good that will be produced, the amount of this good that will be produced in each country, the price of each good, the wage rate that prevails in each country, and the quantity of labor that will be used in each country. An equilibrium must satisfy three sets of requirements that among them yield the value of each of the preceding variables. These three requirements are 1) prices and outputs must be such that the quantity of each good demanded is equal to the quantity supplied; 2) because of competition, each good that is produced must yield zero economic profit, that is, no profit above the prevailing competitive level; 3) the demand for labor must equal the available supply (there must be full employment).

That, in essence, is all there is to the model. It is, incidentally, not difficult to show that in our model, if all three equilibrium conditions are satisfied, trade must be in balance.

II. The Scale Economies Analysis: Review

First, as a foundation for the linear analysis, it is necessary to review our scale-economies results, showing why, where scale economies are pervasive, we can expect the norm to be specialized equilibria, meaning that no good is produced in more than one country, and why we can expect these equilibria to be locally stable. For simplicity we deal with a situation in which only two countries are present, initially referring to the countries as "the U.K." and "France." We assume that the two countries are not extremely different in efficiency and that economies of scale are substantial. Then, if France takes a strong volume-of-production lead in the widget market, the U.K. will simply be unable to compete because its small scale of production will make its costs too high. The reverse will clearly be true if the U.K. happens to preponderate. Thus, it is plausible that either the U.K. or France alone will end up producing all the widgets because the larger the share of world production of widgets attained by a single country the lower its relative cost of widget production will be. Consequently, as has been recognized for some time, under ubiquitous scale economies there is a strong long-range tendency toward specialized production, and these equilibria will be stable returning back to the equilibrium from any small deviation (we call it "locally stable") because entry on a small scale by nonproducers is bound to fail.

Moreover, each and every possible specialized assignment of the task of production among different countries will be a locally-stable equilibrium in a world of universal scale economies. It does not matter whether the U.K. happens to be the only producer of widgets and France the only producer of schmidgets or vice versa. Whichever of these possibilities happens to occur, the country that happens to have specialized in one of the products, call it product X, will benefit from automatic protection against invasion of that field by the other country, because large-scale production of x will give its producer a cost advantage, and the other country will be unable to succeed if it tries to venture into the field on a small scale, with the high costs that this entails.

How many specialized equilibria are available for the market to choose among? There are as many as the many different ways in which the task of production of the world's various goods can be divided up among the different countries. Here we are dealing with the number of combinations of n products that can be assigned to one or the other of the two countries. Since each commodity can be assigned either to the U.K. or to France the number of such combinations is clearly 2^n. If we rule out the possibility that either country will be excluded totally from the production of anything, the number of combinations becomes 2^n-2. Thus, as the number of products in the model increases, the number of equilibria grows far faster than the number of products. In a model that includes ten commodities the number of equilibria already exceeds one thousand. It exceeds 100 million in a 27-product world.

Our research has shown that it is possible to organize the many equilibria graphically in a way that greatly clarifies their implications. Using the absolute and relative incomes of two different countries (which we continue to call "France" and "the U.K."), it is possible to devise a model describing the nature of the entire set of equilibria for a given number of industries. The main instrument used for the required analysis is a graph depicting each of the equilibrium points that would result from every possible variation in the choice of countries that will produce the world's different products. Each equilibrium in our two-country model is represented by two points corresponding to what we may think of as that assignment's payoffs to each of the countries. Every equilibrium is defined by the prices and output quantities determined by the supply-demand relationships that are at the heart of our model. This price and output information obviously permits computation of the total national income (GDP) of each country that any given equilibrium contributes. Once we have the national income of each country, we can obviously also calculate relative income, or that country's share of the total, in terms of a common currency equivalent, say in dollars.[b]

Using this information on absolute and relative income for each country, we construct a graph on which we can plot these two important characteristics of the many equilibria of our model. The U.K.'s relative national income, called Z_1, is represented on the x axis (thinking of the U.K. as "Country 1"). On the vertical (y) axis of our graph we

---

[b]. Our calculations were not actually carried out in terms of dollars or any other currency, but were expressed in terms of (Cobb-Douglas) utilities. However, for clarity of exposition it is convenient to describe the graph, with no loss of substance, in monetary terms, using a common currency for both countries.

measure social utility, taken as a function of the quantities of the various commodities consumed in each country, if we are given a utility function for each country. Or, instead, we can use some form of real (absolute) national income as an indicator of "social utility."

We now have a graph in which the relative income (income share) of two countries appears on the horizontal (x) axis, while what can be described as the absolute (real national) income of one of the countries appears on the vertical axis (y). To draw the graph for one of the countries, say, the U.K., consider an equilibrium, D, in which U.K. income is 40% of the total and is equivalent to $8 trillion. Figure 1 contains a dot, marked D, corresponding to these two numbers that characterize this illustrative equilibrium. This, of course, is only one of the huge number of possible equilibria. The other marker, labeled E, shows another possible equilibrium, one that gives the U.K. 60 percent of the total, and a national income indicated on the y axis as $10 trillion.

Figure 2 is the completed graph, depicting the data for Country 1 (the U.K.), calculated from a model involving 11 commodities. Each of the many dots shown in the diagram represents one of the more than 2000 equilibria in the way the previous diagram described.

The first surprising and interesting feature of the graph is the fact that these dots do not fall into a haphazard and disorderly pattern, such as that depicted in Figure 3. Rather, as in Figure 2, when the number of commodities is fairly large, they always constitute a gray (or black), half-moon shaped region that lies between two hill-shaped curves marked "U.K. Upper Income Frontier"and "U.K. Lower Income Frontier." The dots are spread fairly evenly throughout the region, accounting for its gray-shaded appearance. The upper and lower frontiers of the region of equilibria meet one another at the vertical axes, that is, at the left at the point where the U.K. share of world income is zero, and at the right where the U.K. share is 100%.

As we move from left to right on the graph the shaded region at first moves uphill until it reaches a point where it begins to descend as we continue to move to the right. This offers a lesson that can perhaps be described as "the penalty of excessive greed", for it tells us that, up to a point, as one would expect, a country's income, and hence its standard of living, will benefit if it succeeds in acquiring a greater share of world income. That is, when the U.K. share rises, say, to 40 percent of world income (point B in the graph) it can clearly attain a higher absolute income than when its share is only 30 percent (point A).

But once we get toward the right-hand end of the graph, matters reverse. When the U.K. share of world income rises from 80 up to 90 percent of the total, instead of becoming better off, its standard of living, that is, its absolute income is forced downward. The country will have grabbed off a greater percentage of a vastly impoverished world economy, so that not only France, but even the U.K. with its improving relative position, will suffer.

The graph for France (Figure 4) is, of course, identical in its qualitative properties with that of the U.K. For purposes of comparison, the French graph can be superimposed in Figure 5 over that for the U.K., taken from Figure 2. If we plot the region of equilibria for the two countries we must, of course, read the region for Country

1 (the U.K.) from left to right, starting off where that country's share of world income is zero. For the same reason, the region for Country 2 (France) must be read from right to left, starting off from the point where Country 1 has 100 percent of world income, so that Country 2's corresponding share must be zero. The shape of Country 2's equilibrium region must then be a very rough mirror image of that of Country 1. The upper income frontiers of the two countries must each have a highest point. However, the peak of Country 2's frontier must always lie to the left of that of Country 1. This will prove very important in the subsequent discussion. Intuitively, the reason is clear. When Country 1 has its largest absolute income (the peak of its upper income frontier) this will give it a larger share of world income, Z_1, than it obtains when Country 2's income is at its peak.[c] Thus Country 1's peak must occur where Z_1 is relatively large, while Country 2's peak must occur where Z_1 is relatively small. This, however, is just another way of saying that Country 2's peak must lie to the left of that of Country 1. In a scale economies model it follows immediately, as stated in the introduction, that the equilibrium that is best for one of the countries entails a relative sacrifice for the other. This must be so, because, as shown in Figure 5, when the value of relative income, Z_1, is consistent with an equilibrium locating the U.K. at or very near to the peak of its upper utility frontier, the upper frontier of France at that same Z value

---

[c]To see this compare Country 1's peak (at Z_1) with Country 2's peak at (at Z'_1) . Country 1's income has gone down in going from Z_1 to Z'_1 and Country 2's has increased, so that Country 1's share has decreased.

must be below the French peak. The same reasoning shows that the value of Z that permits maximum utility for France means a loss of prospective utility for the U.K. This, then, is one manifestation of the possibility of conflict in the interests of the trading partners.

III. The Graph in the Linear Model

We turn now to our central focus in this article: the case of constant returns to scale, where the production function is assumed to take a particularly simple form. It is assumed that the quantity of good I produced in Country J is equal to the quantity of labor, $l_{i,j}$ used by the Ith industry in Country J multiplied by a constant, $e_{i,j}$, which is the productivity of Country J labor in the production of good I. We assume that each of these productivity numbers has some upper bound, for which we use the symbol $e_{i,j}^{max}$. The productivity of Country J labor in industry I cannot exceed this amount.

As in the scale economies case, the equilibrium point derived from such a model can be represented by two dots in the same graph. Since the equilibrium gives us the output of each good produced in each country as well as the price of each good, we can multiply the price by the quantity to determine the revenue yielded by each such good, and adding these numbers together for each of the goods produced in Country J, we obtain Y_j the <u>absolute income of Country J</u>, that is, the sum of the revenues obtained in that country from the sale of its products. We can also calculate Z_j, Country J's corresponding <u>share</u> of world income yielded by the equilibrium in question, by determining the absolute income of the other country in the same way. and then computing each country's share of the total. The graph then describes an equilibrium by plotting relative income, Z, for Country 1 on the horizontal axis and Y_j, the absolute income of Country J, on the vertical axis. That is, for Country 1 if its absolute income is 5 trillion dollars and it has 45 percent of world income, we plot the point whose height is 5 and that lies to the right of the zero point at the distance 45 percent of the length of the horizontal axis. We plot the point representing Country 2's position in this equilibrium in the corresponding way.

In what follows we will also at times use utility as the vertical axis. The results are the same whether national income or utility is used. Most of our actual computational experience and the rigorous development has been based on utility, but at times a description in terms of absolute national income makes the results more intuitively accessible.

Next, to obtain other equilibrium points, we produce a family of linear models by just varying the productivity numbers. That is, suppose we start off with any linear model given by the three equilibrium requirements described in Section I. Then change any or all of the productivity numbers, leaving everything else absolutely unchanged. The result will be a new linear model that we consider to be in the same family as the model with which we began. The two equilibria will be said, correspondingly, to be in the same family of equilibria.

A family of linear models will yield a set of equilibria, each represented a point in the graph for each country. Each such region of equilibria is completely specified by the sizes of the labor forces of the two countries, the set of demand parameters, and the number of traded commodities. For either of the countries the set of points representing all of the equilibria for a family of linear models forms a region. It can be shown that this region is bounded from above, and this upper boundary, the upper utility frontier (or upper income frontier) for the country in question, can be approximated from above [d] in the following manner. Letting $x_{i,j}$ represent Country J's share of world output of good i, and let x be the vector of the $x_{i,j}$ and e be the vector of the productivity parameters, $e_{i,j}$. Each equilibrium can be shown to be characterized by the values of x, Z and e, so we can represent an equilibrium as (x,Z,e). Then it should be plausible that an upper boundary of the equilibrium region can be found by determining the solution for each value of Z the solution of the linear program

$$\text{Maximize } U(x,Z,e^{max})$$

$$\text{subject to } \quad \Sigma_i(d_{i,1}Z_i + d_{i,2}Z_2)x_{i,1} = Z_1, \qquad \text{and } \quad 0 \leq x_{i,1} \leq 1.$$

In this maximization problem e^{max} is the vector whose components are the $e^{max}{}_{i,j}$. For every Z, one determines the equilibrium that yields the highest utility subject to the requirements that the x's are consistent with the definition of the country's relative national income, and that they can represent output share for the good and country in question by being numbers no lower than zero and no greater than unity.

From among such a family of models and their equilibria we can select a smaller set of equilibria that we call maximal-productivity equilibria. These represent the equilibria at which each country has developed skills sufficient to attain maximal productivity in each of the industries in which it is a producer.

This concept is important because, as will be discussed in the following section, the set of maximal productivity equilibria yields a subregion of equilibria very similar to that derived from a model with scale economies in production. Indeed, subject to a requirement that is described in the next section, there is a one-to one correspondence between selected equilibria in the maximal-productivity region of a family of linear models and the equilibria of an appropriately-selected model with scale economies. All the maximum productivity equilibria lie in a subregion that we call the maximal productivity subregion.

The maximal-productivity equilibrium subregion is bounded in the upward direction by an curve, the upper utility frontier of Country J. This is obtained from the linear program that was just described, so that the upper boundary of the subregion of maximal productivity equilibria is exactly the same as the boundary for the region of all of the equilibria for the family of linear models in question. In addition, the subregion of maximal-productivity equilibria is bounded from below by another curve, the lower utility frontier of Country J. This lower boundary is obtained by minimizing the utility in the preceding linear program for each value of Z. These two curves are always roughly hill shaped and they encompass a crescent-shaped region of equilibria. For Country 1, that region starts off at the bottom of the graph, that is, at the zero point. As we move to the right in the graph the region always begins to move uphill (Figure 2) until it finally reaches a peak, and then it begins to descend again and reaches a point on the right-hand axis of the graph, a point that is higher than zero and that represents the level of absolute income Country 1 would obtain in autarky, that is, if it produced all of its goods for itself, and imported nothing from Country 2.

The reasons for this shape are exactly the same as in the scale economies case. As we move to the right in the graph it means that Country 1's share of world income is increasing. Starting from the left hand end, initially Country 1 produces virtually none of the world's goods, but that means it can produce more goods in which it enjoys a substantial comparative advantage, thereby adding to the world's output capacity. Consequently, the world output pie must be increasing in size at the same time that Country 1's share is rising as well. With an ever-larger share of a growing pie, Country 1's income must clearly be rising. Thus the equilibrium region of Country 1 will continue to go upward -- up to a point. What finally stems the rise is the fact that when the country has moved too far to the right it will have taken some (or many) industries from Country 2 in which the latter is potentially the larger producer. By putting such industries into the wrong hands from the point of view of maximization of world output capacity, the size of the world's output pie must be reduced. Thus, despite the continued rise in its share as one moves still further to the right, the pie will ultimately shrink so much that even Country 1's income will be constricted by the move. Ultimately, as one moves as far to the right as one can in the graph, Country 1 will obtain from the other country none of the goods it consumes. The former will have attained a state of autarky and will obtain its autarky level of national income.

The similarity of the shapes of the graphs for the linear models with that for the scale-economies case extends also to the location of the peaks of the upper utility (income) frontiers. We again have the result that the peak of the frontier for the country whose relative income, Z, one reads from left to right, will always lie to the right of the peak for the other country. This again immediately leads to a result showing the conflict in the interests of the two countries: a value of Z that permits one country to maximize its utility will always force the other country to a utility level below the latter's maximum.

Only one last item must be noted again in our description of the portion of the linear story that corresponds directly to the scale-economies scenario: As before, when the number of goods included in the model expands, the number of equilibria increases far more than proportionately (it equals $2^n - 2$ in the case where production is perfectly specialized so that no good is produced in more than one county, and the number of goods is n). The number of dots in the graph therefore also explodes and gradually tends to fill in the entire region between the upper and lower income frontiers. All of the conclusions that have just been reported can be proven mathematically, but the intuitive description provided here should suggest the logic of the proofs.

IV. The Correspondence Theorems

The properties of the region of maximal productivity that were just described were originally derived from an analysis of the scale economies case. As already noted, this qualitative similarity derives from a correspondence between the equilibria and their regions for the scale-economies and the linear cases to which we now turn.

Given any particular equilibrium of a scale-economies model one can construct from it a directly corresponding equilibrium of a linear model, meaning that the two equilibria have the same income-share values, Z_j, as well as the same output share values, $x_{i,j}$, and for any good actually produced in Country J the average productivities of labor at the given output are the same in both models (with nonzero startup costs for any good not produced in Country J).[e]

One can prove

Correspondence Theorem 1. The linear model that yields an equilibrium corresponding directly to the given scale economies equilibrium can be found directly by taking the scale-economies production functions $f_{i,j}(l_{i,j})$, calculating from them the average productivity figures, $f_{i,j}(l_{i,j})/l_{i,j}$, for each good, I, produced in Country J, and then setting the corresponding productivity parameter, $e_{i,j}$, equal to that ratio, and by giving $e_{i,j}$ a value small enough to make Country j non-competitive in the goods not produced in Country j.

Next, suppose we start with a family of linear models and plot all of its equilibrium points. It is logical to ask whether there exists a scale-economies model that yields exactly those same equilibrium points and, if so, what is the character of that model. That is, given a family of linear models, can we describe the scale-economies model, if any exists, that yields the same equilibria? To answer this question, we focus exclusively on perfectly specialized equilibria, that is, on equilibria in which no commodity is produced in more than one of the countries. There are two reasons for doing so. First, as we know, in the presence of scale economies there is at least a strong

---

[e]The role of the non zero startup costs is, of course, to make entry into good-I production difficult for a non producer, thereby making the given equilibrium stable.

tendency for the (locally) stable equilibria to be perfectly specialized; any unspecialized assignment tends to be unstable since if one of two producer countries of good I increases its market share it will obtain a cost advantage over the other producer country which will enable it to expand even further and drive the other country from the industry. Hence, if we want to see whether the equilibria of a family of linear models coincides with the stable equilibria of a scale economies model it is convenient to deal exclusively with the specialized equilibria. Second, for analysis of the region of equilibria, rather than the individual equilibrium points, there is almost no loss of substance in confining the discussion to specialized equilibria. That is because in an model with a reasonably large number of traded goods (say, one with more than a dozen goods) there are specialized equilibrium points near any given point in the diagram. That is the meaning of assertion that for the scale economies case the region between the upper and lower income frontiers of a country tends to "fill up" rapidly with specialized equilibrium points as the number of goods in the model grows.

We then can prove

Correspondence Theorem 2. Any finite set of equilibria, having different specializations, deriving from a family of linear models will correspond to equilibria of a single economies of scale model if and only if for two linear models in the family in question, the corresponding equilibria, E* and E** satisfy the following condition: Let E** yield a higher share of world income in County 1 than E* does. Then the productivity of Country 1's labor at E** in every industry in which Country 1 is a producer will be no higher at E** than it is at E*, and the productivity of Country 2 labor in each industry in which Country 2 is a producer will be at least as high at E** as it is at E*. In other words, the family of linear models must be defined by changes in productivity levels for Country 1 that do not increase as one moves to the right in the graph, while the productivity levels for Country 2 do not decrease as one moves in the same direction (considering for each country only the industries in which it is a producer).

The intuitive reason underlying the theorem is not difficult to grasp. In a scale economies model the productivity of labor must, by definition, rise as the amount of labor devoted to an industry increases. Now, as we move to the right in the graph, Country 2's share of world income and of the world's industries decreases. With a fixed labor force and full employment, this means that there must be a rise in employment in the average industry of the smaller number of industries remaining to that Country. Indeed, it is easy to prove that in our model employment in each of those industries must rise. Hence, if this move is to be consistent with scale economies, productivity in such an industry must grow higher. Similarly, with a move to the right in the graph Country 1 must spread its fixed labor force over a larger number of industries than before, reducing the quantity of labor per industry, so that if there are economies of scale, average labor productivity must fall. Consequently, only if the condition of Theorem 2 is satisfied by the family of linear models is it possible for there to exist a single scale economies model that can yield the same set of equilibria.

More than that; once the condition of the theorem is satisfied it is not difficult to construct a single scale-economies model that does the trick -- that yields the equilibria of the family of linear models. For this purpose one simply constructs the desired scale economies model in the following two steps: i) recognizing that all of the relationships in the family of linear models, except for the production function are identical (e.g., all the linear models have the same demand functions) we adopt that common set of relationships as part of the constructed scale-economies model as well; 2) to complete the scale economies model we now only need to determine its production function, call it $f_{i,j}(l_{i,j})$. Then at any equilibrium point, simply set the average product of labor of Country J in good I, $f_{i,j}(l_{i,j})/l_{i,j}$, equal to the corresponding and fixed average productivity figure, $e_{i,j}$, in the linear model that yields the equilibrium in question. It should then be plausible that the resulting production relationship is what we are looking for. It has scale economies and it clearly is compatible in the pertinent features with our family of linear models.

The importance of the preceding results for our purposes is this. These mean that the unfamiliar features that have emerged from the growing literature on scale economies in international-trade analysis are not phenomena peculiar to that case alone. Rather, what follows from this discussion is that those same features arise also in the more-familiar linear case, whose well known attributes may facilitate absorption of the new observations.

V. The Ideal Trading Partner From the Viewpoint of a Given Country's Welfare

The magic of comparative advantage is its offering of physical gains from trade to both parties who participate in the exchange process. Proper assignment of the productive task can enhance the outputs of all commodities without any expansion in the use of inputs. But though both parties can gain, the magnitude of the benefit to either of the parties depends on the number and identity of the industries that devolve upon the other, and the productive techniques it employs in those industries. The question to be considered here is what assignment of industries and what levels of productivity of Country 2 in its assigned industries will be consistent with maximization of the benefits of trade to Country 1? That is, we will examine what assignment of outputs and what levels of productivity for Country 2 will make it into the ideal trading partner for Country 1, as evaluated in terms of the latter's selfish interests.

Let us first consider the model in which Country 1 has all its $e_{i,j}=e^{max}_{i,j}$. In terms of our graphs, the ideal trading partner for Country 1 entails production arrangements for Country 2 that bring the former as close as possible to the summit point on its absolute income (upper utility) frontier. This is shown by the dot marked A_1 in Figure 6. Several things become apparent from this graph. First, because the peak of the Country 1 frontier always lies to the right of that for Country 2, we may expect that the ideal trading partner for Country 1 will be one that produces a fairly limited share of the world's commodities. In the model underlying Figure 6, one entailing 22 industries, six of those products are supplied by Country 2 when it is an ideal trading partner, with the remaining 16 produced by Country 1. This is obviously a very plausible result, asserting simply that the ideal trading partner for Country 1 will be an economy that has succeeded in capturing a very modest share of the world's industries.

To deal with the case where Country 1 does not have $e_{i,j}=e^{max}_{i,j}$, it is only necessary to start a new analysis by setting $e^{max}_{i,j}=e_{i,j}$ and considering the new region that results and performing the same analysis.

The result attains additional support from analysis of the case where the two countries have the same maximal productivities. In that case it is possible to carry out rigorous calculations of general validity. Such a calculation shows that where the countries are identical a country's ideal trading partner will always turn out just 24 percent of the world's commodities.

Second, the fact that the peak of Country 1's upper income frontier lies to the right of that of Country 2 also means that the position of ideal trading partner for Country 1 entails a sacrifice that can be substantial. Because then Country 2's equilibrium point (point A_2 in Figure 6) is well to the right of Country 2's own peak it will lie in the descending region of the equilibrium region. This clearly means that Country 2 must give up some, and possibly a good deal, of its maximum potential income in order to become an ideal trading partner.

A third observation emerges from the analysis of the preceding section. As will be described later, when a country is the sole producer of some good, a ceteris paribus increase in its productivity in that commodity will always benefit itself and the country with which it trades. Consequently, to serve as ideal trading partner Country 2 must always attain maximal productivity in each of the (few) industries in which it is a (sole) producer. This is again very plausible. If Japan imports all of its oil, it obviously serves the interests of that country if the petroleum-supplying nations produce oil as efficiently and cheaply as possible.

Finally, we will see that the story can be very different for Country 2's productivity in a commodity it is not initially producing. We will see later that a rise in such a productivity figure in, say, industry I can enable that country to compete away some or all of the product I sales of Country 1, thereby reducing the latter's real income. Indeed, if Country 2 (as well as Country 1) attain maximal efficiency in all products the result can be a material sacrifice for Country 1 relative to its position if Country 2 had remained an ideal trading partner. The equilibrium with ubiquitous maximal productivity is shown by the square markers in Figure 6 (points B_1 and B_2). These points, for obvious reasons, lie between the peak points of the upper income frontiers of the two countries, and clearly constitute a compromise in their interests, with neither serving as the ideal trading partner of the other.

To summarize, for a country to constitute an ideal trading partner it must accept a position with three attributes: i) it must be the producer of a modest share of the traded commodities, ii) it must be a maximally efficient producer of just those goods that it does supply, and iii) it must be an inefficient producer of all the remaining commodities, so that it constitutes no competitive threat in those industries.

One can well think of a number of agricultural countries serving as nearly ideal trading partners for some of the leading industrialized economies. This example and our analysis both suggest that the position of ideal trading partner is hardly ideal for the economy that assumes this role.

This also has the following important consequence: any departure whatsoever from the ideal trading partner production parameters by Country 2 has a detrimental effect on Country 1. If Country 2's parameters become worse, this hurts Country 1. If

Country 2's parameters become better, that too hurts Country 1.

VI. Evolving Productivity and Intertemporal Performance Patterns

Productivity performance does not just grow fortuitously. It clearly is affected by a variety of economic influences. For example, technology is improved by experience. There is, of course, a considerable literature on the phenomenon of learning by doing. Such a learning process means that a nation actually engaged in the production of a particular commodity is apt to learn more rapidly than others that are not doing so, other things being equal, how to improve both the product and the production process. Thus, the productivity performance of country J in commodity I is likely to be stimulated by J's production of that good. New knowledge gained in this way accumulates, every piece of information acquired in this way thereby adding to the economy's inventory of knowledge. Productive knowledge, however, has a way of growing obsolete. Thus, there are also influences that tend to deplete the stock of knowledge as time passes. Such observations can readily be built into an economic model and can be analyzed systematically to determine the resulting intertemporal trajectory of productivity and income performance of an economy.

The analysis can easily encompass built-in influences on productivity of both varieties: those that serve to enhance productivity and those that cause it to decline. One need only provide a relationship among these variables that describes the mechanism that produces adaptation in their values over time. For example, one can investigate the time path of productive efficiency of the trading countries in the simple case where there is no erosion of the stock of knowledge through obsolescence and productivity grows by experience in any industry in an economy, with the growth rate an increasing function of the shortfall of the current productivity level below the maximum productivity level. This last premise implies that, other things being equal (and only up to some limit), the further behind an economy is in its productivity the faster that productivity will tend to grow. Our model's assumption is a formalization of the hypothesis that moderate backwardness is an advantage for growth. It rests on the role of technology transfer and the notion that a moderate technological laggard can learn a good deal from the world's technology leaders, but that as the laggards gradually catch up with the leaders the scope for further acquisition of technology narrows, thereby reducing growth stimulation from this source.

One can then prove the intuitively plausible

Theorem 3. Convergence toward maximal productivity. In a model of learning by doing technology transfer and the absence of erosion of the stock of knowledge all trajectories of the productivity levels of industries in which Country J is a producer will converge toward the maximal productivity levels, $e_{i,j}^{max}$. That is to say all trajectories will approach points within the region of maximal productivity.

The same sort of approach readily enables us to extend the dynamic analysis to deal both with obsolescence and with different lags in the speed with which technology improves and with which its applicability and usefulness deteriorates with age (obsolescence). Not surprisingly, obsolescence per se has two consequences. First, it slows the rate of productivity growth of an industry and, second, it can drive the economy toward an equilibrium point in which the values of the productivity parameters systematically fall short of their maximal levels. Moreover, given the model, it is possible to calculate the magnitude of the shortfall.

Those who have worked with models containing lags will also not be surprised that such lags can easily result in fluctuations in which productivity grows in fits and starts and even falls backward periodically. The reason is that a period of rapid growth of productivity and the accompanying accumulation of knowledge in a moderately-laggard country will be followed by an increase in the absolute amount of knowledge in that country that later grows obsolete. With a given percent of the stock of knowledge growing obsolete every year, the larger the amount of knowledge there is at any particular date, the more there is to grow obsolete at a later date. Thus, rapid growth will be followed by a period of slowdown of accumulation and a possible decline in the amount of useful knowledge available in that country. In that event, there will be a growing shortfall between the current productivity level and its maximum value. Here, there comes into play the premise that the further behind, within limits, that a country is, the greater is its scope for rapid productivity growth. That is, the decline in the country's productivity in the preceding stage will serve to stimulate current growth. This sequence in which enhanced growth sows the seeds of slowdown that in turn restimulates growth is the foundation of the cyclical pattern the model can engender.

It is clear that the intertemporal patterns of growth that emerge from the linear model are, as is to be expected, highly dependent on the structure of the growth process, the identity of its major determinants, its lag structure, etc. The discussion has shown, however, that the analysis is not inconsistent with a mechanism that generates systematic growth of productivity and that drives the economy toward the region of maximal-productivity equilibria upon which the discussion has focussed.

The analysis can also be helpful in the theory of economic growth, an area that has recently elicited a good deal of renewed attention. That, however, is beyond the scope of our discussion here.

VII. Consequences of Improvements in One Country's Productivity for a Trading Partner

Among non-economists who view unrestricted freedom of trade with some suspicion a particular concern has been improvement in the productivity of foreign countries -- of increases in "the competitiveness" of other nations. Does the rise in productivity in a number of Japanese industries really threaten the welfare of Western Europe and the U.S.? Is the welfare of Japan, in turn, threatened by progress in productivity in Taiwan, South Korea and Singapore? Economic historians have debated similar issues, such as the effect of the growth of productivity in nineteenth century U.S. and Germany upon well being in Great Britain. Though more-careful economic analysis has suggested that the matter is not open and shut, a superficial view of the standard theory leaves the impression that it claims a rise in productivity in one country must always be beneficial to all. By increasing world output and lowering prices the benefits are, on this view, generally transmitted throughout the globe, with trade barriers serving as a primary impediment to universal distribution of the gains.

The suspicion by economists is indeed aroused by the claimed employment effects of a country's declining (relative) competitiveness, for the number of jobs provided by an economy does not depend on the success of a few particular industries; and many believe that macroeconomic policies rather than success in exporting are the prime determinants of the number of jobs offered by an economy. The analysis reported here uses a full-employment assumption to avoid the issue of loss of jobs through decline in competitiveness. It shows, nevertheless, that while in some circumstances growth in the productivity in one economy can, indeed, benefit another, it can, however, in other cases harm the latter and reduce its real income, perhaps substantially. More specifically, we will describe circumstances under which foreign productivity progress is beneficial to the country in which no such advance has occurred, and the circumstances in which the foreign productivity growth will be detrimental to it..

Underlying the analysis is a concrete definition of the competitiveness of Country J in a particular industry, I. In our one-input model, what matters is the wage rate, w_j in country J relative to that in the other economy, and the relative productivity of J's labor in industry I. The notions that follow can be extended in an obvious way to the more-realistic case where production makes use of a variety of inputs. The crucial competitiveness figure, call it the competitiveness fraction, is the wage rate divided by the average productivity number , $w_j/e_{i,j}$, for this fraction gives us the per-unit-of-output cost of production of product I in country J. The numerator of the fraction is the cost of an hour of labor in Country J, while the denominator is the quantity of good I that this hour of labor produces in that country. The fraction, cost divided by output, is the cost per unit of good I produced in J.

Then using the obvious definition of competitiveness as the ability to supply a good at a lower cost than a rival can, the following terms will facilitate the discussion. We will say

Country 1 is uncompetitive in the production of good I if Country1's commodity I competitiveness fraction is higher than that of Country 2. In other words, this means that Country 1 cannot supply I as cheaply as Country 2. Similarly, we will say that Country 1 is competitive in good I if it has the lower competitive fraction. Finally, we will say that both countries are marginally competitive in I if the fractions for the two countries are equal. Then, we have the following result:

Theorem 4. Competitiveness and Ceteris-Paribus productivity Growth in One Country. Other things being equal, if the average productivity of Country 1 labor in industry I increases, all other parameter values remaining the same, then (i) if Country 1 was and remains uncompetitive in I the rise in its productivity will have no effect on real income in either country; (ii) if Country 1's productivity rises sufficiently to make it marginally competitive with Country 2 then a further rise in that productivity will raise wages in Country 1 and reduce them in Country 2, permitting the two countries to remain marginally competitive in I despite 1's continuing productivity advance. In that case, so long as marginal competitiveness prevails, any further rise in Country 1's I-productivity will raise the real income of Country 1, and reduce the real income of Country 2; Finally, (iii) if the rise in Country 1's productivity makes Country 2 uncompetitive in I, so that the latter is driven out of commodity I production altogether, any further rise in country 1's productivity will raise the absolute real income of both countries, and cause no further change in the relative incomes of the two countries.[f]

---

[f]A very similar result was derived earlier by Johnson and Stafford [1993] in a very illuminating note that does not seem

The results are most easily summarized by dividing the pertinent range of the changing productivity parameter into three zones: zone 1 in which Country 1 is uncompetitive in I, zone 2 in which the two countries are marginally competitive, and zone 3 in which Country 2 is uncompetitive in I. Theorem 4 asserts that in zone 1 a rise in Country 1's I competitiveness has no effect on the income of either country. In zone 3, where Country 1 is the sole producer of I, a rise in J's I productivity benefits both countries proportionately. However, in the central zone 2, where the two countries are and remain marginally competitive, a rise in Country 1's I productivity benefits 1 at 2's expense. The higher the productivity level in that zone, the greater is the real income of Country 1 and the smaller is the real income of Country 2.

Note that this result is not offset by the possibility that the rise in Country 1's I productivity will have ancillary consequences upon its other industries. This rise in productivity will raise country 1 wages and will therefore handicap that country's competitiveness in other industries. Moreover, that productivity gain will enhance 1's <u>comparative</u> advantage in I, thereby tautologically reducing the comparative advantage of its other industries. Hence, while the productivity rise will enable Country 1 to expand its exports of I it may reduce its exports of some other products or perhaps even eliminate them altogether. Nevertheless, the theorem continues to be valid. Country 1 is always unharmed

---

yet to have received the attention it merits. See also Hymans and Stafford [1995], particularly for a very clear geometric discussion of extension of the analysis to the case of diminishing returns.

and generally gains from a rise in the average productivity of labor in some industry, and while in some circumstances the result can also be beneficial to Country 2, in other circumstances the effect on that country's real income will be unambiguously detrimental. More sophisticated trade theorists such as Viner would undoubtedly not have been shocked at this result that offers some legitimate grounds for the fears of nonspecialists about international trade rivalry, but the result does diverge from the conclusions suggested by more naive discussions of trade theory.

Why does the theorem hold? The explanation is slightly different for the cases of the three zones, with that for the intermediate zone of universal marginal competitiveness perhaps the most difficult. The argument underlying the result for zone 1, in contrast, is trivial. In that zone Country 1 starts off and remains a non-producer of I because it is uncompetitive in that good. Consequently, a rise in its competitiveness in that item has no effect on anything, given the absence of any activity in that industry by Country 1. At the other extreme, in zone 3, Country 1 is the only producer of I. The growth in output per worker-hour in Country 1 then makes the good steadily cheaper for both countries and so gives them both more of the good for a given expenditure. Something similar holds for Country 1 in the intermediate zone 2, where both countries produce good I.

But why does Country 2 always lose out in that zone when the good-I productivity of Country 1 rises? The answer is not that Country receives less of good I. On the contrary, since that Country produces some or all of the good I that it consumes, with the good I productivity of labor in that country constant, an hour of labor will earn just enough to purchase just as much of good I as before, despite the growth in the other country's I productivity. Curiously, Country 2 loses out because it gets less to consume of some goods other than I. There are two cases in which this is clearly true. **Case I.** goods other than I of which Country 1 continues to be the sole producer despite its rising productivity in good I. For with the resulting increase in Country 1 wages imports of those items by Country 2 will grow ever more expensive. The rising price does not damage the welfare of Country 1 because its increased wage provides additional purchasing power that offsets the price rise. Country 2, however, obviously obtains no such rising wage offset, so the increased prices of the goods it buys from Country 1 reduce the real national income of Country 2 but not that of Country 1. **Case II.** goods that are transferred to Country 2 from Country 1 because rising wages in Country 1 have made it uncompetitive in those commodities. Paradoxically, Country 2 also loses out in getting less to consume of these goods in which it has become competitive. The reason is not far to seek. Consider a good I* that formerly was produced exclusively by Country 1 at a price of $5. Suppose Country 2 could have produced that item for $6 per unit. Rising wages in Country 1 resulting from increased I Productivity now raise the cost of production of I* in that country to $6.25, so that Country 1 becomes uncompetitive in production of good I*. The cost to Country 2 of a unit of good I* therefore rises from our illustrative $5 to $6, without any offsetting rise in Country 2 wages like that enjoyed by Country 1. In the case of Country 1 the wage offset is even more effective than it would have been if it had continued to be the producer of good I* because the transfer of I* production to Country 2 limits the price rise to $6, rather than letting it go all the way to $6.25, as it would have to if Country 1 were to have continued as I* producer.

In sum, we see how rising I productivity in Country 1 benefits it both by a greater abundance of good I, and by a rise in wages that at least keeps pace with the resulting rise in prices of other goods. Country 2 in this case loses out because rising Country 1 prices raise the cost of 2's imports, with no offsetting rise in 2's purchasing power, despite any industries that may as a consequence be transferred to it from Country 1.

Despite the apparent definitiveness of the discussion of this section its results must be interpreted with caution. First, the increase in I-productivity in Country 1 may span several zones. It may, for instance, bring Country 2 from being marginally competitive in good I (zone 2) to a state of uncompetitiveness in I (zone 3). Then Country 2 will lose out from the rise in Country 1's productivity when they were in zone 2, but the further rise in Country 1's I-productivity when zone 3 is attained will benefit Country 2. The net gain or loss to Country 2 from the entire change, therefore cannot be determined in general terms. An interesting special case of this can occur when the two countries share a large industry. If a smaller industry improves in Country 1, so that it becomes more competitive than Country 2 the net result can be that the wage rates remain the same, the small industry shifts to Country 1, and their is a realignment of shares in the large industry. In this case Zone 2 has zero length and this kind of change is always beneficial to both countries. Second, the discussion as reported takes no account of the premise of the analysis that at any time there is a ceiling, $e_{i,j}^{max}$, upon each productivity parameter.

VIII. Concluding Comment

Our discussion has focussed upon the applicability of the new analyses of the role of scale economies in international trade theory to cases from which scale economies are absent. We have shown that the new theory is also applicable to the more-familiar linear case, implying that any insights it offers are valid not only in the less-familiar terrain. In the process some other substantial implications were derived. Most notable for application were the results describing the wide range of circumstances in which conflict in the interests of two trading partners can arise. The graphics of the region of equilibria, which enabled us to arrive at this result, served more generally as a basis for an evaluation of the possibilities of mutual gain and the possibilities of conflict in the trading relations of the two countries, in an analysis now extended to the case of constant returns to scale. We were also able to study the effects of improvement in productivity of one country on the prosperity of other countries with which it trades. It was shown that these effects are more complex and heterogeneous than might have been suspected, but that they fall into clear patterns that the analysis is able to describe concretely.

Appendix A

The Model: Equilibria Under Constant Returns and Under Scale Economies

This appendix describes our model and derives its equilibria. In what follows the production functions are written as $f_{i,j}$ and the quantities of labor are $l_{i,j}$. The price of the ith good is p_i.

We define national income by

$$Y_j = \sum_i p_{i,j} f_{i,j}(l_{i,j}) . \qquad (A.1)$$

Our equilibria are defined by three requirements. The first is the condition that supply equals demand for each good. The demand is derived from the assumption of Cobb-Douglas utility. So we have as our first equilibrium requirement

$$p'(f_{i,1}(l_{i,1}) + f_{i,2}(l_{i,2})) = d_{i,1} Y_1 + d_{i,2} Y_2 . \qquad (A.2)$$

require zero profit in each industry, so if the wage in Country j is w'_j.

$$p' f_{i,j}(l_{i,j}) = w' l_{i,j} . \qquad (A.3)$$

The third and last requirement of equilibrium is full employment in each economy, so that

$$\sum_i l_{i,j} = L_j . \qquad (A.4)$$

where L_j is the size of the labor force of the j th country.

We can rewrite (A.1)-(A.4) using the market share variables $x_{i,j}$ and the normalized variables Z_j, P, and w. We define the market share of Country j in the ith industry by $x_{i,j}$ by

$$x_{i,j}=\frac{f_{i,j}(l_{i,j})}{f_{i,j}(l_{i,1})+f_{i,j}(l_{i,2})}. \qquad \text{(A.5)}$$

We define the normalized or relative national income Z_j by

$$Z_j=\frac{Y_J}{Y_1+Y_2} \qquad \text{(A.6)}$$

and, similarly, p and w by

$$p=\frac{p'}{Y_1+Y_2} \quad and \quad w=\frac{w'}{Y_1+Y_2}. \qquad \text{(A.7)}$$

In terms of these variables (A.1)-(A.4) become

$$Z_j=\sum_i pf_{i,j}(l_{i,j}) \qquad \text{(A.8)}$$

$$p(f_{i,j}(l_{i,1}))=x_{i,j}(d_{i,1}Z_1+d_{i,2}Z_2) \qquad \text{(A.9)}$$

$$pf_{i,j}(l_{i,j})=wl_{i,j} \qquad \text{(A.10)}$$

$$\sum_i l_{i,j}=L_j \qquad \text{(A.11)}$$

We can rearrange and combine (A.8)-(A.11) to get the equivalent set of equations:

$$Z_j = \sum_i x_{i,j}(d_{i,1}Z_1 + d_{i,2}Z_2) \qquad \text{(A.12)}$$

$$w_j l_{i,j} = \sum_i x_{i,j}(d_{i,1}Z_1 + d_{i,2}Z_2) \qquad \text{(A.13)}$$

$$w_j L_j = Z_j \qquad \text{(A.14)}$$

$$p f_{i,j}(l_{i,j}) = w l_{i,j} \qquad \text{(A.15)}$$

(A.12)-(A.15) will be are our basic equilibrium equations.

These equations have a very definite structure. If an arbitrary $x=\{x_{i,j}\}$ is chosen a unique $Z=(Z_1,Z_2)$ is determined by the need to satisfy (A-12). With Z known $w=\{w_1,w_2)$ is determined by (A.14). With x, Z, and w known all the $l_{i,j}$ are uniquely determined by (A.13). Consequently, for *any* choice of x, all the equations except (A.15) can be satisfied. What remains is to choose x so that (A.15) is also satisfied.

To analyze the effect of (A.15) we need to consider two cases:

Case 1: In industry I there are two producers, i.e., neither $x_{i,1}$ nor $x_{i,2}$ is 0. Then neither $f_{i,j}(l_{i,j})$ nor $l_{i,j}$ is 0 and either equation (A.15) with j=1 or equation (A.15) with j=2 already uniquely determine i. The task of obtaining an equilibrium point then comes down to choosing x so that the two values of i obtained from j=1 and j=2 are the same.

We remark parenthetically that the stability of the resulting equilibrium point is not addressed in this analysis. A very rough rule, to which there are exceptions, is that the equilibrium will tend to be stable if the production functions all have diseconomies of scale, and will tend to be unstable if they all have economies of scale.

Case 2: One of the $x_{i,j}$, say $x_{i,1}$ is 0. This gives us zero output in Country 2, while (A.13) gives us $l_{i,2}=0$. The j=2 equation of (A.15) is satisfied for any i so choosing i to satisfy j=1 alone will give an equilibrium point.

Clearly similar reasoning applies for the case $x_{i,1}=0$.

However, although we can obtain an equilibrium point in this way, it could be quite unstable if the non-producer could enter with a very small output quantity and make a positive profit. For this reason we will use a further stability condition which we will add to our basic equilibrium equations.

If $x_{i,j}=0$, we will require that the former non-producer, j, will not make a positive profit upon entering. This means:

$$\lim_{l_{i,j}\to 0} p_i f_{i,j}(l_{i,j}) \le w_j l_{i,j} \text{ , or equivalently} \qquad \text{(A.16)}$$
$$p_i f'_{i,j}(0) \le w_j .$$

Consequently if there are two producers we will use (A.15), but if Country 1 is the sole producer we will use instead

$$p_i f_{i,1}(l_{i,1}) = w l_{i,1} \qquad \text{(A.17)}$$
$$p_i f_{i,2} \le w_2 ,$$

and if Country 2 is the sole producer we will require

$$p_i f_{i,2}(l_{i,2}) = w_2 l_{i,2} \qquad \text{(A.18)}$$
$$p_i f_{i,1} \leq w_1.$$

There are two important special cases:

I. Linear production functions:

In this case we have $f_{i,j}(l_{i,j}) = e_{i,j} l_{i,j}$ so that (A.16), (A.17), and (A.18) can be simplified

If $x_{i,1} \geq 0$ and $x_{i,2} = 0$

$?_{i,1} = w_1, \quad p_i e_{i,2} \leq w_2.$ *or equivalently* $e_{i,1}/e_{i,2} \geq w_1/w$(A.19)

If $x_{i,1} = 0$ and $x_{i,2} \geq 0$

$e_{i,1} \leq w_1, \quad p_i e_{i,2} = w_2.$ *or equivalently* $e_{i,1}/e_{i,2} \leq w_1/w$(A.20)

If both $x_{i,1}$ and $x_{i,2}$ are positive

$_{,1} = w_1, \quad p_i e_{i,2} = w_2.\ w_2.$ *or equivalently* $e_{i,1}/e_{i,2} = w_1/$(A.21)

Clearly these are the familiar criteria of comparative advantage. If County 1 is the sole producer in industry i, then $e_{i,1}/e_{i,2} \geq w_1/w_2$. If Country 2 is the sole producer then $e_{i,1}/e_{i,2} \leq w_1/w_2$ and if both produce we have $e_{i,1}/e_{i,2} = w_1/w_2$.

The calculation that yields the actual equilibrium x can be sketched as follows:(1)set all $x_{i,j}=0$ (2) choose the $x_{i,1}$ with the largest $e_{i,1}/e_{i,2}$ ratio, increase this $x_{i,1}$ until (a) it reaches 1 or (b) equation (A.12) is satisfied. If case (a) obtains chose the next variable in order of $e_{i,1}/e_{i,2}$ and repeat until case (b) occurs. Then the equilibrium x is the one whose $x_{i,1}$ and $x_{i,2}$ values are those that arise when case (b) occurs and $w_1/w_2=e_{k,1}/e_{k,2}$.

II. Production Function with Economies of Scale and Start-Up Costs.

We use "economies of scale" to mean that $f_{i,j}(l_{i,j})/l_{i,j}$ increases with $l_{i,j}$. We express start up costs as the requirement $f'_{i,j}(0)=0$.

In this case *all* integer $x=\{x_{i,j}\}$, i.e., $x_{i,1}=0$ or 1, or equivalently all specialized production patterns, are equilibria. For example if $x_{i,1}=1$ and $x_{i,2}=0$, then the second condition in (A.17) is automatically satisfied since $f'_{i,2}(0)=0$, and the first condition in (A.17) determines p_i.

Appendix B

In what follows it is useful to have a normal form for equilibria belonging to the same equivalence class. We will take as our normal form, $e_{i,j}=0$ for every case where Country j is a non-producer of good i.

Lemma B.1 There are equilibria for every Z, $0<Z_1<1$.

Actually, as the proof indicates, there are many equilibria for each such Z, but one will suffice for the moment.

In this proof we will need to refer to the notation and equations of Appendix A which contains the equilibrium equations that we need.

Proof: Choose any x satisfying the income share definitions (A.12) of Appendix A for the given Z. if $x_{i,1}=1$ and $x_{i,2}=0$, choose any $e_{i,1}>0$ and $e_{i,2}=0$. Similarly if $x_{i,2}=1$ and $x_{i,1}=0$, choose $e_{i,2}>0$ and $e_{i,1}=0$. If both variables are positive for a some industry i, choose $e_{i,1}=w_1$ and $e_{i,2}=w_2$. These choices clearly satisfy (A.19) (A.20) and (A.21) of Appendix A.

Lemma B.2 If (x,Z,ϵ) is an equilibrium, so is $(x,Z,\lambda\epsilon)$ for all λ such that $0<\lambda<1$.[8]

Proof: Clearly the equilibrium conditions (A.12)-(A.15) and (A.19)-(A.21) of Appendix A are satisfied.

Utility: The (Cobb-Douglas) utility U_1 of an equilibrium is given by

$$u_1(x,Z,\epsilon)=\ln U_1(x,Z)=\sum_i d_{i,1}\ln y_{i,1}$$
$$=\sum_i d_{i,1}\ln F_{i,1}(Z)\{q_{i,1}(x_{i,1},Z,\epsilon)+q_{i,2}(x_{i,2},Z,\epsilon)\}. \quad \text{(A.22)}$$

where x and Z are the equilibrium x and Z and

$$Z,\epsilon_{i,j})=e_{i,j}l_{i,j} \quad with \quad l_{i,j}w_j=(d_{i,1}Z_1+d_{i,2}Z_2)x_{i,j} \quad ana$$
$$and \quad F_{i,j}(Z)=\frac{d_{i,j}Z_j}{d_{i,1}Z_1+d_{i,2}Z_2} \quad \text{(A.23)}$$

Note that the definitions given in (B.2) enables us to calculate a utility value from a given triple (x,Z,ϵ) whether or not (x,Z,ϵ) is an equilibrium.

---

[9] Is also an equilibrium for all $\lambda>1$ such that $\lambda e_{i,j}\leq e^{max}_{i,j}$.

If we fix x and Z at x' and Z', the utility given by (2.1) and (2.2) decreases with decreasing $e_{i,j}$. Therefore if we plot the equilibria $(x,Z,\lambda\epsilon)$ as points $v(\lambda\epsilon)$ in the (Z,U) plane, the points $v(\lambda\epsilon)$ move steadily *down* the vertical line given by $Z=Z'$ as λ decreases. This decrease can continue down to the axis, passing through all the points on the line $Z=Z'$ that are below the starting point. This gives us :

Lemma B.3 If (x', Z', ϵ) is an equilibrium with utility U_1, all the points below (Z', U_1) on the vertical line $Z=Z'$ are also equilibria.

Next we have:

Theorem B.1 There is a curve $U_1=B^*_1(Z)$ such that every point of the (Z,U_1) diagram under $U_1=B^*_1(Z)$ is an equilibrium point.

Proof: Clearly, for any Z' the utility is bounded because the $e_{i,j}$ and the $x_{i,j}$ are. There are points corresponding to equilibria on the vertical line $Z=Z'$ by Lemma B.1. Define $B^*_1(Z')$ to be the supremum of those points. Suppose there is a point v' on Z' below B^*_1 that is *not* an equilibrium point. Since $B^*_1(Z')$ is the supremum there are equilibria on $Z=Z'$ between it and v'. But then Lemma B.3 asserts that v' is also an equilibrium. This contradiction shows that for each Z every point on the line from $B^*_1(Z)$ down to the Z axis corresponds to an equilibrium. This ends the proof.

Linearized utility: The analysis becomes much easier if we substitute for the utility function a closely related linearized function. Just as in [1] and [2] we define the linearized utility by:

$$\sum_i \{x_{i,1} d_{i,1} \ln F_{i,1}(Z)\, q_{i,1}(1, Z_1, \epsilon) + x_{i,2} d_{i,1} \ln F_{i,1}(Z)\, q \qquad (A.24)$$

where the $q_{i,j}$ and $F_{i,j}$ have the meanings assigned in (B.2). Like utility, the linearized utility is well defined for any triple (x,Z,ϵ) whether or not it is an equilibrium.

Theorem B.2: The utility $u_i(x,Z,\epsilon)$ and the linearized utility $Lu_i(x,Z,\epsilon)$ are equal at every equilibrium point.

Proof: We will compare the ith terms in (2.3) and (2.1).

Case 1: $x_{i,1}=1$, $x_{i,2}=0$. The ith term is $d_{i,1}\ln F_{i,1}q_{i,1}(1,Z,\epsilon)$ in both expressions.

Case 2: $x_{i,1}=0$, $x_{i,2}=1$. The ith term is $d_{i,1}\ln F_{i,1}q_{i,2}(1,Z,\epsilon)$ in both expressions.

Case 3: $1> x_{i,1}>0, 1> x_{i,2}>0$. Then $e_{i,1}/w_1=e_{i,2}/w_2$ so:

$$_{i,1}\frac{(d_{i,1}Z_1+d_{i,2}Z_2)x_{i,1}}{w_1} \quad and \quad q_{i,2}(x,Z,\epsilon)=e_{i,2}l_{i,2}=e$$

$$(x,Z,\epsilon)+q_{i,2}(x,Z,\epsilon)=\frac{e_{i,1}}{w_1}(d_{i,1}Z_1+d_{i,2}Z_2)=q_{i,1}(1,Z \quad (A.25)$$

$$(x,Z,\epsilon)+q_{i,2}(x,Z,\epsilon)=\frac{e_{i,2}}{w_2}(d_{i,1}Z_1+d_{i,2}Z_2)=q_{i,2}(1,Z$$

so both terms $= d_{i,1}\ln F_{i,1}q_{i,1}(1,Z,\epsilon)= d_{i,1}\ln F_{i,1}q_{i,2}(1,Z,\epsilon)$.

An Upper Bound for the Equilibrium Points Using Linear Programming: We will adopt the notation ϵ_1 for the vector $e_{i,j}=e_{i,j}^{max}$. Consider the linear programming problem

$$\ln B_1(Z) = Max_x\ Lu_1(x, Z, \epsilon_1)$$
$$ubject\quad to\quad \sum_i (d_{i,1}Z_1 + d_{i,2}Z_2)x_{i,1} = Z_1 \quad and \quad 0 \leq x_{i,1} \leq 1 \qquad (A.26)$$

The solution x(Z) to this problem for each Z gives us a curve $U_1 = B_1(Z)$ in the Z,U diagram. We assert that $B_1(Z)$ is always above the region of equilibria. More precisely.

Theorem B.3: $B^*_1(Z) \leq B_1(Z)$

Proof: Let (x', Z', α) be an equilibrium point. For that Z consider (2.5). Since x' corresponds to an equilibrium point, it satisfies (A.12) in Appendix A, which is the equation in (2.5). Therefore x'was among the values of x considered in the maximization calculation and does not yield a larger result than x(Z). Therefore, $Lu_1(x', Z', \epsilon_1) \leq Lu_1(x(Z'), Z', \epsilon_1)$. Since we certainly have $Lu_1(x, Z, \alpha) \leq Lu_1(x, Z, \epsilon_1)$, it follows that $Lu_1(x', Z', \alpha) \leq Lu_1(x'(Z'), Z', \epsilon_1) = B_1(Z')$.

In addition, $B_1(Z)$ approximates $B^*_1(Z)$, the curve beneath which every point is an equilibrium point j that is, the two curves can not be very far apart. Specifically:

Theorem B.4: $1 \leq B_1(Z)/\ B_1^*(Z) \leq (\max\ (R_1/(w_1/w_2),\ R_2/(w_2/w_1))^D$ where $w_1 = Z_1/L_1$ and $w_2 = Z_2/L_2$., $D = \max_i d_{i,1}$, and $R_1 = \max_i e^{max}_{i,1}/e^{max}_{i,2}$, $R_2 \max_i e_{i,2}/e_{i,1}$. That is, R_j is the maximum relative productivity advantage Country j enjoys in any good.

Proof: Consider the equilibrium point obtained from the maximizing x(Z) by constructing $\alpha(x,Z)$ as follows: if $x_{i,1} = 1$, $e_{i,1} = e^{max}_{i,1}\ e_{i,2} = 0$; if $x_{i,2} = 1$, $e_{i,2} = e^{max}_{i,2} e_{i,1} = 0$. For the one non-integer term $e_{i,1} = \min(e^{max}_{i,1}, (w_1/w_2)e^{max}_{i,2})$, $e_{i,2} = \min(e^{max}_{i,2}, (w_2/w_1)e^{max}_{i,1})$. We can directly verify that this satisfies all the conditions for an equilibrium point.

If $x_{i,1} = 1$ and $x_{i,2} = 0$ the ith term of the log utility of $(x(Z), Z, \alpha(x, Z))$ is

$$d_{i,1} lnF_{i,1} q_{i,1}(1, Z, e^{\max}_{i,1})$$

which is exactly the same as the ith term of the utility of (x(Z),Z,ϵ_1). Equality also holds when $x_{i,2}=1$ and $x_{i,1}=0$. The linearized utilities of (x(Z),Z,α(x, Z)) and (x(Z),Z,ϵ_1) only differ on the *one* term where both $x_{i,1}$ and $x_{i,2}$ are positive. For that term we have

$$for \quad (x(Z), Z, \epsilon_1) \qquad d_{i,1}\{x_{i,1} lnF_{i,1} e_{i,1} l_{i,1} + x_{i,2} lnF_{i,1} e_{i,2} l_{i,2}\}$$

$$, Z, \alpha) \qquad d_{i,1}\{x_{i,1} lnF_{i,1} \min(e_{i,1}, (w_1/w_2) e_{i,2}) l_{i,1} + x_{i,2} lnF_{i,1} \min(e_{i,2}, (w_2/w$$

The difference between the two log utilities is

$$\frac{{}^{\max}_{i,1}}{w_1} \geq \frac{e^{\max}_{i,2}}{w_2} \qquad d_{i,1}\{x_{i,1} \ln \frac{e^{\max}_{i,1}}{(w_1/w_2)\, e^{\max}_{i,2}} + x_{i,2} \ln \frac{e^{\max}_{i,2}}{e^{\max}_{i,2}}\} = d_{i,1}\{\ln \frac{e^{\max}_{i,1}}{(w_1/w_2)\, e^{\text{ma}}}$$

$$\frac{{}^{\text{ax}}_{i,1}}{w_1} \leq \frac{e^{\max}_{i,2}}{w_2} \qquad d_{i,1}\{x_{i,1} \ln \frac{e^{\max}_{i,1}}{e^{\max}_{i,1}} + x_{i,2} \ln \frac{e^{\max}_{i,2}}{(w_2/w_1)\, e^{\max}_{i,1}}\} = d_{i,1}\{x_{i,2} \ln \frac{e^{\max}_{i,}}{(w_2/w_1)\, e}$$

so the difference in linearized utility between (x(Z),Z,ϵ_1) and (x(Z),Z,α(x, Z)) is less than the larger of these two amounts. If we allow for the fact that this amount is in the difference in the log utilities and the theorem refers to the utilities, we see that the amount appearing in the theorem is larger than either of these differences. Since B^*_1 lies above (x(Z),Z,α(x, Z)) and below (x(Z),Z,ϵ_1) this proves the theorem.

Each region of equilibria is completely specified by its country labor-force sizes L_j its demand parameters $d_{i,j}$, the $e^{\max}_{i,j}$ and the number of industries n. We now consider the regions resulting from a sequence of models with increasing numbers of industries n. We will use D^n for max $d^n_{i,j}$, R^n_1 and R^n_2 for the R_1 and the R_2 of the nth problem. We have immediately.

Theorem B.5 (Convergence Theorem): For any Z, $B^n_1{}^*(Z) \to B^n_1(Z)$ as $n \to \infty$ provided that $D^n \to 0$, and ϵ_1 and R^n_1 and R^n_2 are both bounded as $n \to \infty$.

Proof: For any fixed Z, w_1 and w_2 are fixed. Therefore, $(\min(R^n_1(w_1/w_2), R^n_2\,(w_2/w_1))$ is bounded. Since $D^n \to 0$, $(\min(R^n_1(w_1/w_2), R^n_2\,(w_2/w_1))^{Dn} \to 1$ and the result follows from Theorem B.4.

Therefore, for problems with large numbers of industries the space of all possible points corresponding to equilibria becomes almost identical with the space under the curve $B_1(Z)$.[h] Note that in Theorem B.5 the requirement $D^n \to 0$ simply means that as the number of goods in the world economy becomes very large, the share of total income consumers devote to any one of these goods becomes very small.

---

[h]A more sophisticated analysis is possible and shows by numerous examples that the convergence of B_1 to B_1 * is far more rapid than these crude estimates would indicate.

References

[1] **Baumol, William J. and Gomory, Ralph E.,** April 1994, "On Efficiency and Comparative Advantage in Trade Equilibria Under Scale Economies," C. V. Starr Economic Research Report RR #94-13, New York University.

[2] **Ethier, W. J.,** 1982, Decreasing cost in international trade and Frank Graham's argument for protection, Econometrica, 50, 1243-1268.

[3] **Gomory, Ralph E.,** April 1994, "A Ricardo Model with Economies of Scale," Journal of Economic Theory, (62),394-419.

[4] **Gomory, Ralph E. and Baumol, William J.,** October 1994, "Shares of World Output, Economies of Scale, and Regions Filled with Equilibria," C. V. Starr Economic Research Report RR #94-29, New York University.

[5] **Grossman, Gene M. and Helpman, Elhanan,** November 1994, "Technology and Trade," National Bureau of Economic Research, Inc., Working Paper No. 4926.

[6] **Grossman, Gene M. and Rogoff, Kenneth,** Editors, 1995 (forthcoming), "Technology and Trade", Handbook of International Economics, Vol. 3, North Holland Press.

[7] **Hymans, Saul H. and Stafford, Frank P.,** 1995, "Divergence, Convergence, and the Gains from Trade", Review of International Economics, 3 (1), p. 118-123.

[8] **Johnson, George E. and Stafford, Frank P.,** May 1993, "International Competition and Real Wages," American Economic Association Papers and Proceedings, Vol. 83 No. 2, 127-130.

[9] **Johnson, George E. and Stafford, Frank P.,** January 1995, "The Hicks Hypotheses, Globalization and the Distribution of Real Wages", Econometric Society

Meeting, (unpublished).

Figure 1.

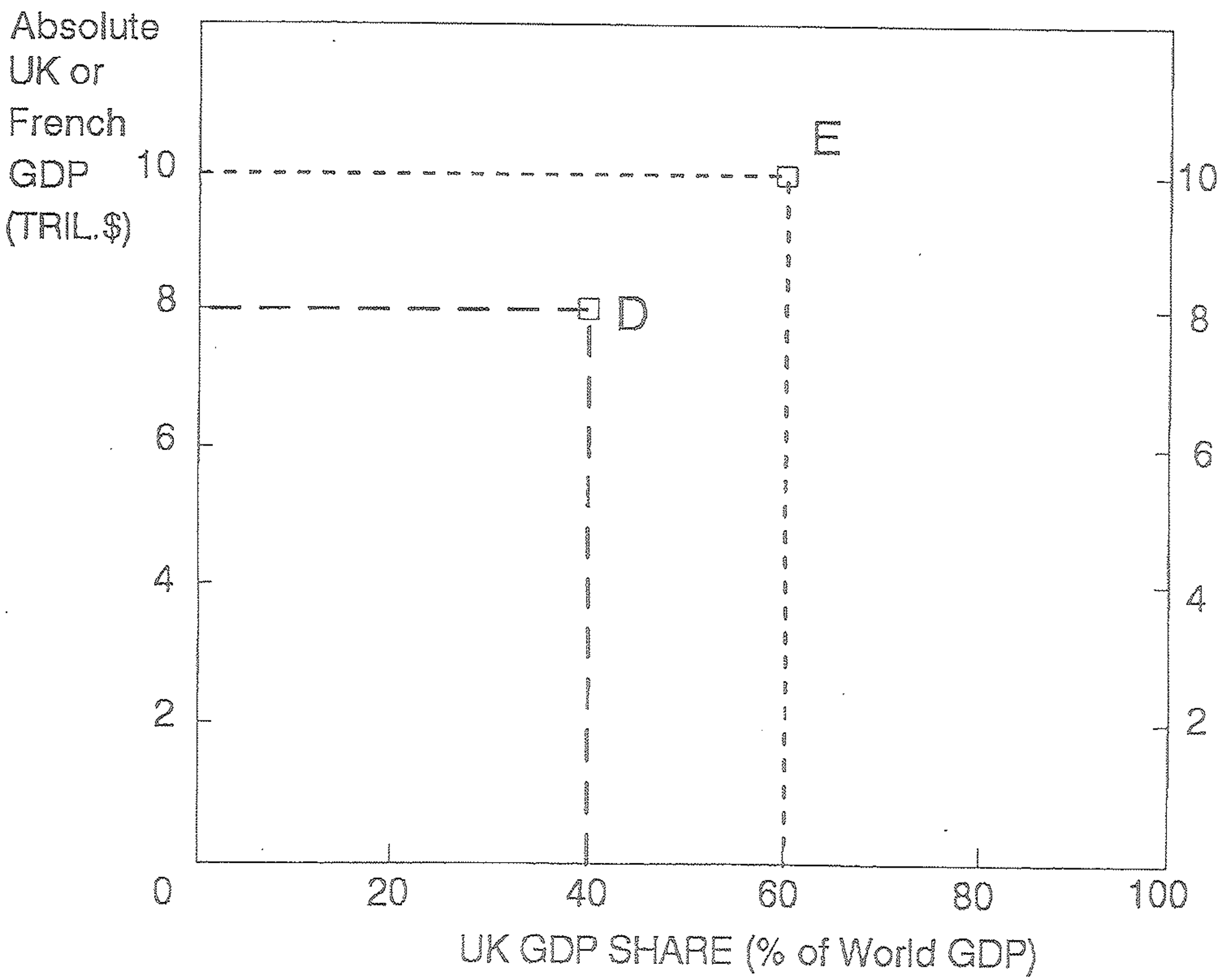

Absolute
UK or
French
GDP
(TRIL.$)
10
8
6
4
2
0
20
40
60
80
100
E
D
UK GDP SHARE (% of World GDP)

Figure 2.

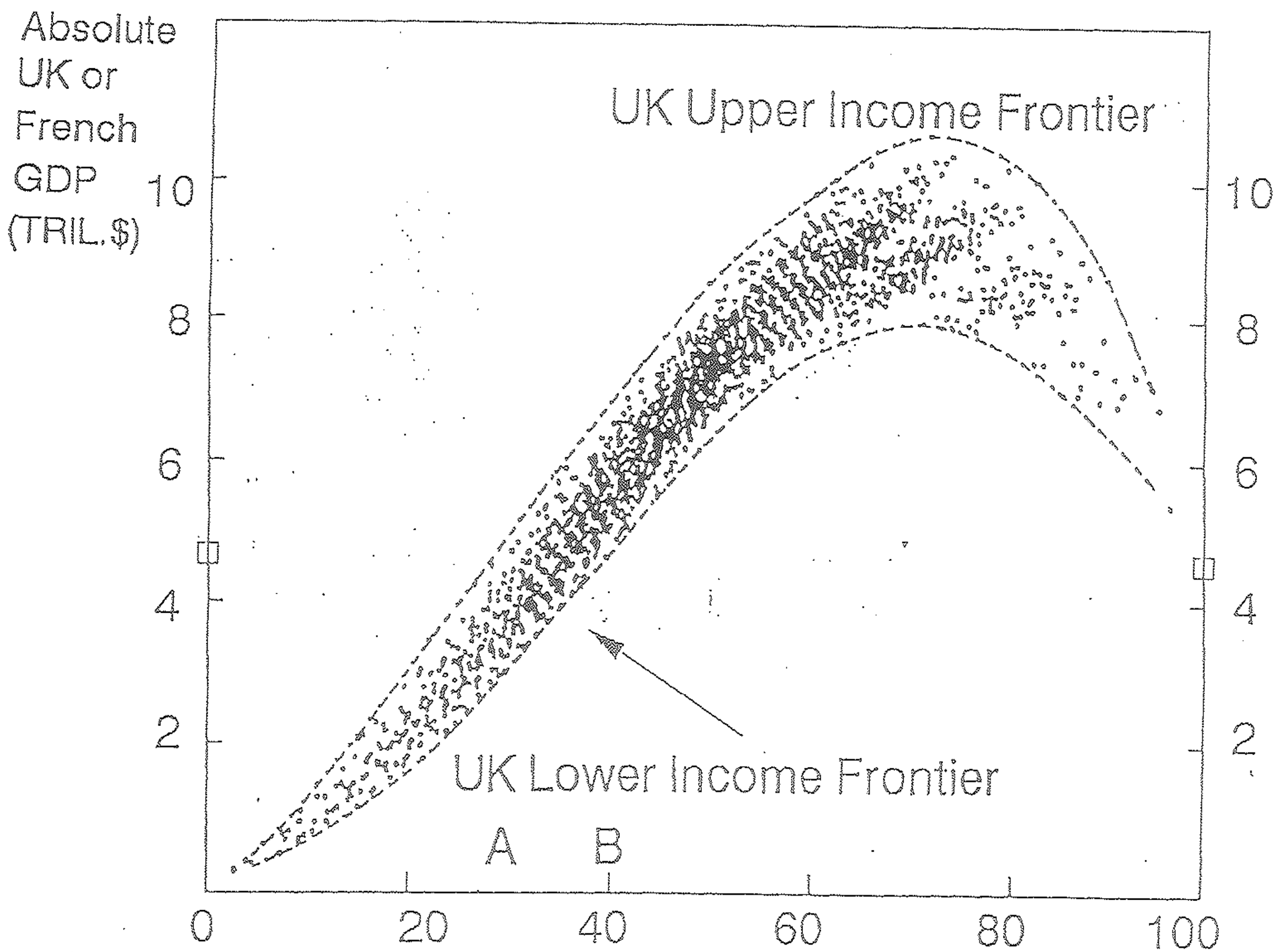

Absolute UK or French GDP (TRIL.$)
10
8
6
4
2
UK Upper Income Frontier
UK Lower Income Frontier
A
B
0
20
40
60
80
100

Figure 3.

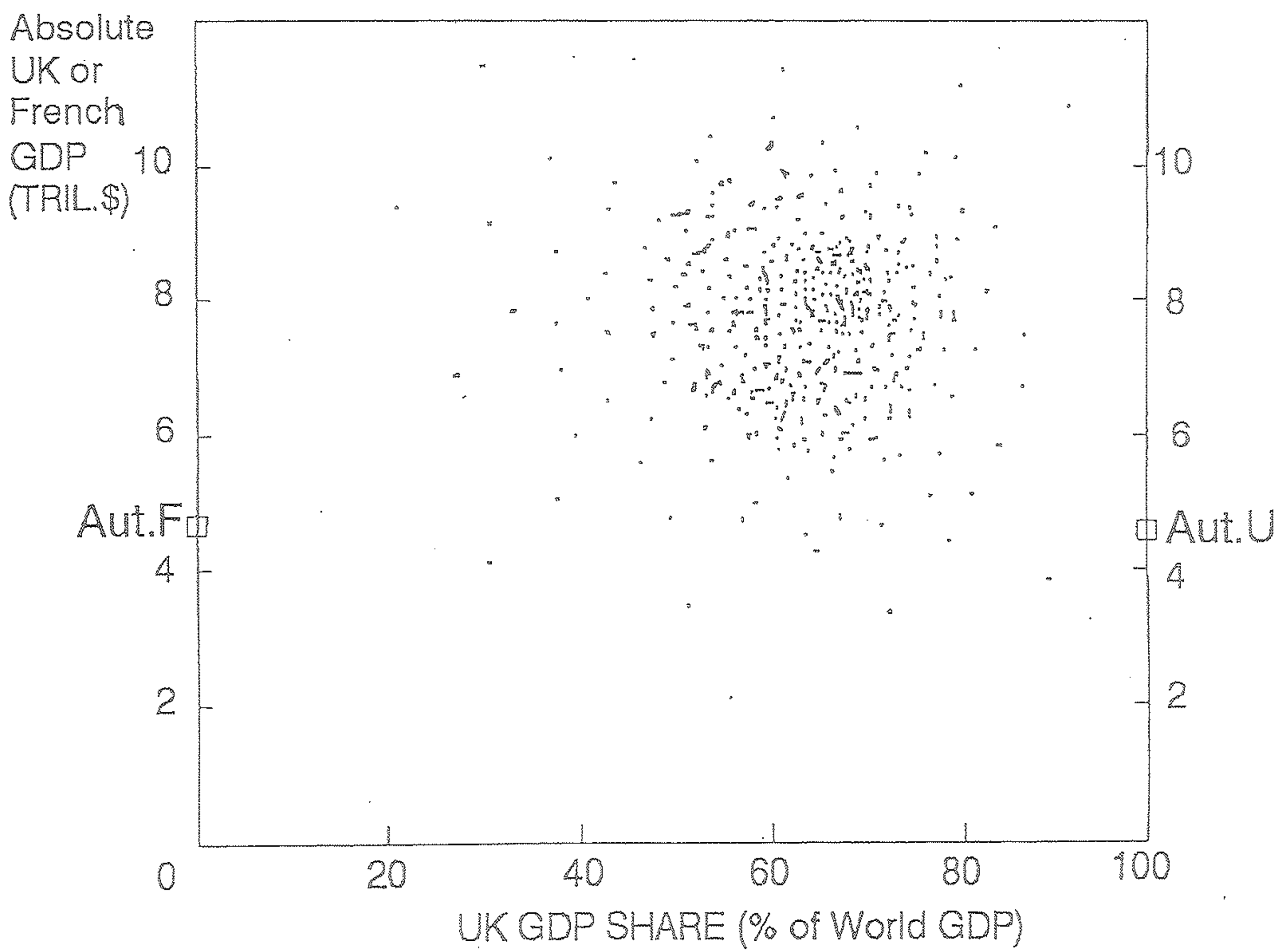
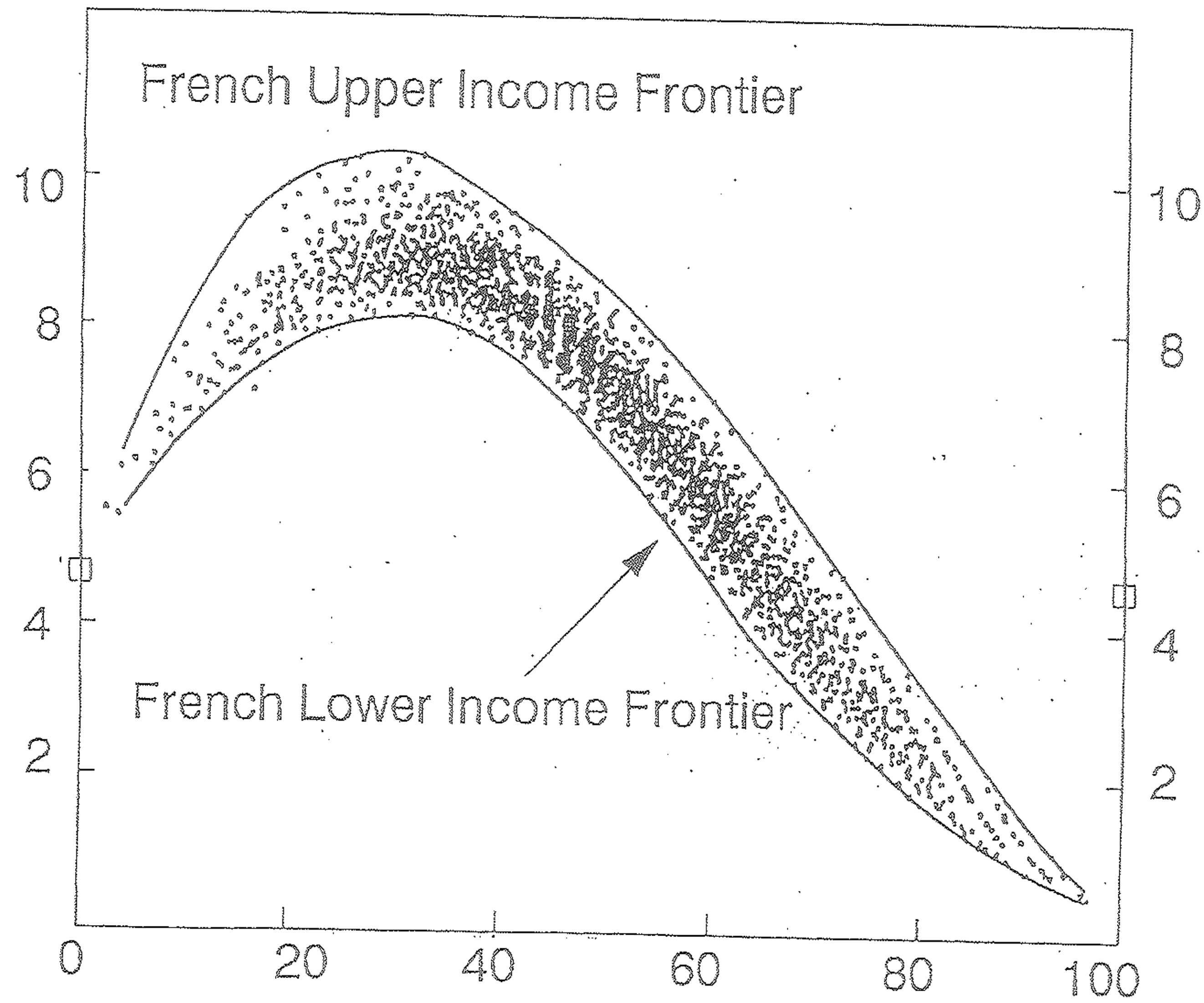

Absolute
UK or
French
GDP
(TRIL.$)
10
8
6
Aut.F
4
2
0
20
40
60
80
100
Aut.U
UK GDP SHARE (% of World GDP)

Figure 4.



French Upper Income Frontier
French Lower Income Frontier
10
8
6
4
2
0
20
40
60
80
100

Figure 5.

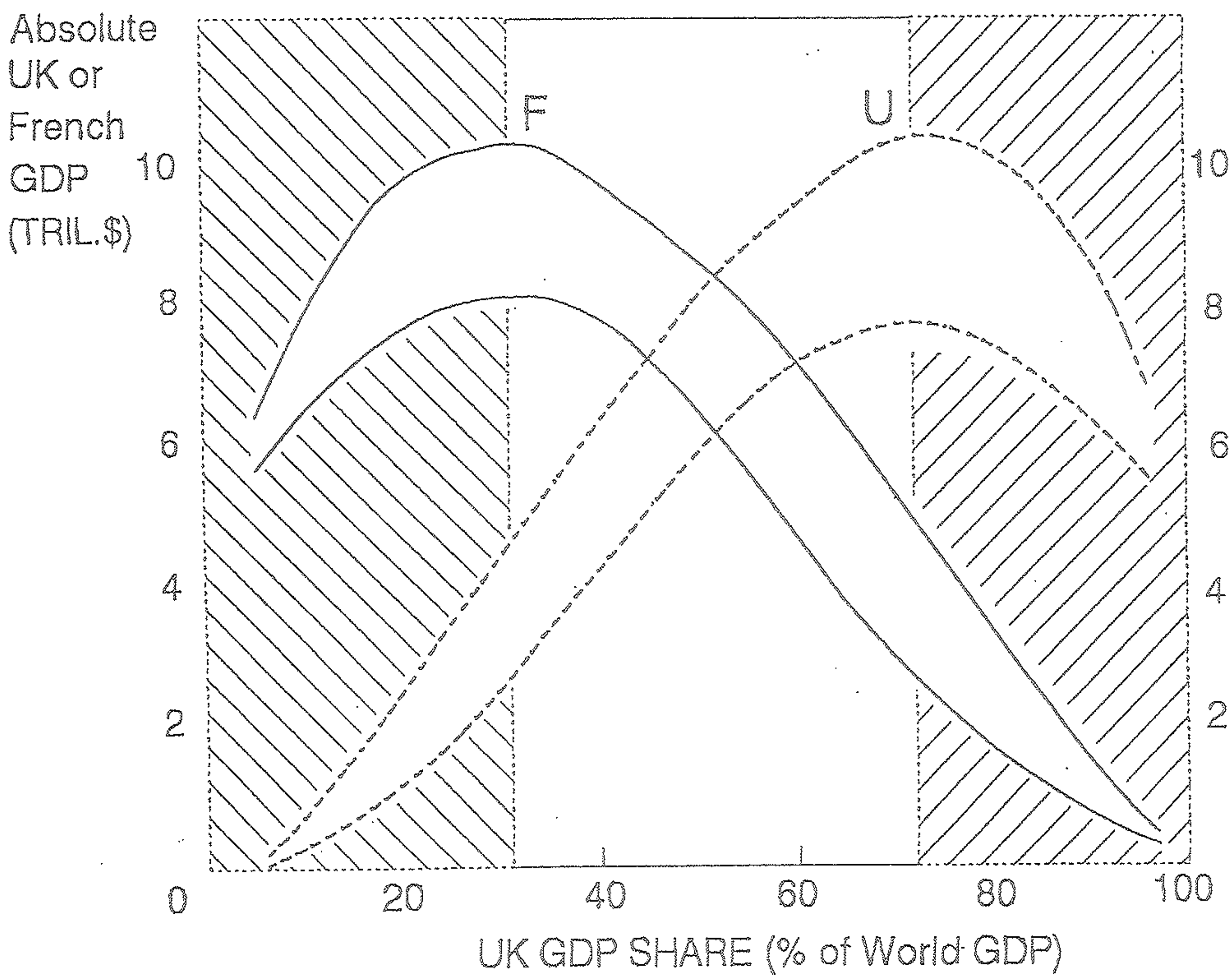
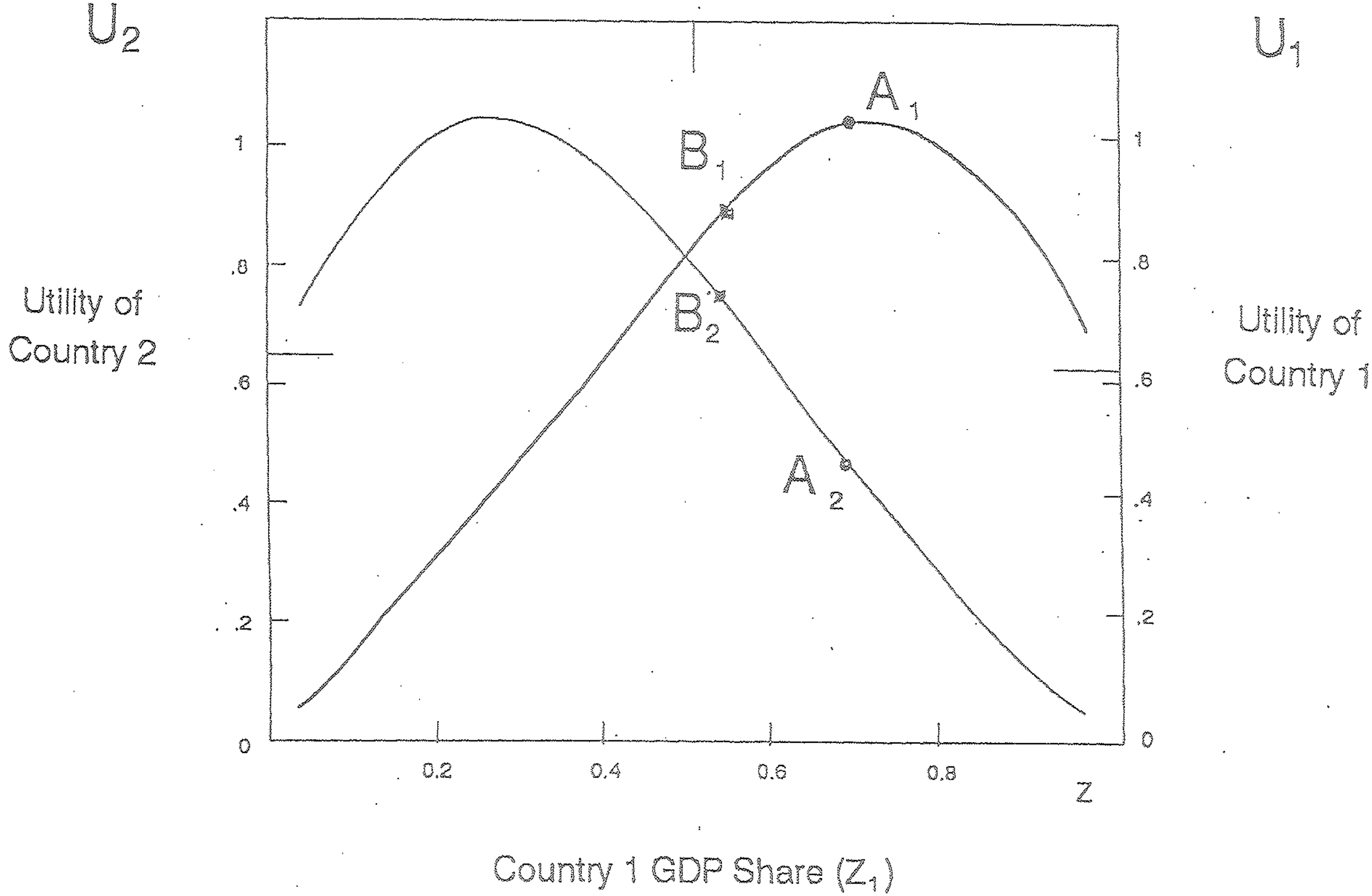

Absolute
UK or
French
GDP
(TRIL.$)
10
8
6
4
2
0
F
U
20
40
60
80
100
UK GDP SHARE (% of World GDP)

Figure 6.


U2
U1
A1
B1
B2
A2
Utility of Country 2
Utility of Country 1
1
.8
.6
.4
.2
0
0.2
0.4
0.6
0.8
Z
Country 1 GDP Share (Z1)

C.V. STARR CENTER RESEARCH REPORTS RECENTLY PUBLISHED:

9501 *A Moving-Knife Solution to the Four-Person Envy-Free Cake-Division Problem*
Steven J. Brams, Alan D. Taylor, and William S. Zwicker, January 1995

9502 *Indeterminacy and Sector-Specific Externalities*
Jess Benhabib and Roger E.A. Farmer, January 1995

9503 *The Analysis of Foreign Exchange Data Using Waveform Dictionaries*
James B. Ramsey and Zhifeng Zhang, January 1995

9504 *If NonLinear Models Cannot Forecast, What Use Are They?*
James B. Ramsey, February 1995

9505 *Strategic Behavior in Two-Sided Foreign Exchange Auctions*
Linda Goldberg and Rafael Tenorio, February 1995

9506 *Fixed versus Flexible Exchange Rates: Which Provides More Fiscal Discipline?*
Aaron Tornell and Andres Velasco, February 1995

9507 *What Remains of Purchasing Power Parity*
Kenneth Rogoff, March 1995

9508 *On Growth and Indeterminacy: Some Theory and Evidence*
Jess Benhabib and Jordi Gali, March 1995

9509 *Product Diversity, Endogenous Markups, and Development Traps*
Jordi Gali, March 1995

9510 *How Should Voting on Related Propositions be Conducted?*
Steven J. Brams, D. Marc Kilgour, and William Zwicker, May 1995

9511 *Modeling Free Choice in Games*
Steven J. Brams, May 1995

9512 *Real Business Cycles with Involuntary Unemployment*
Jordi Gali,, May 1995

9513 *The Law of One Price Over 700 Years*
Kenneth A. Froot, Michael Kim, and Kenneth Rogoff, May 1995

9514 *Investment, Pass-Through and Exchange-Rates: A Cross Country Comparison*
Jose Campa and Linda Goldberg, May 1995

C.V. STARR CENTER RESEARCH REPORTS RECENTLY PUBLISHED:

9515 *Inside the Black Box: The Credit Channel of Monetary Policy Transmission*
Ben Bernanke and Mark Gertler, June 1995

9516 *OMB's Circular A-76 and Its Implications For Municipal Contracting*
Jonas Prager and Swati Desai, July 1995

9517 *Regions of Linear Trade-Model Equilibria and the Conflicting Interests of Trading Partners*
Ralph E. Gomory and William J. Baumol, July 1995

9518 *N-Product Natural Monopoly As "Natural Cartel" -- On Scale Economies Under Capital Rationing*
William J. Baumol and Ralph E. Gomory, July 1995

9519 *Productivity Differences, World-Market Shares and Conflicting National Interests in Linear Trade Models*
Ralph E. Gomory and William J. Baumol, July 1995