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Abstract:
We discuss the circumstances under which improvements in productivity in a trading partner are beneficial to the home country and when they are detrimental. We show that there are gains from trade that a country can capture from a partly developed trading partner that strongly exceed the gains it can obtain by trading with either a fully developed trading partner or one that is relatively undeveloped. Once a trading partner passes this partly developed state, further improvements in the trading partner's productivity usually *decrease* the welfare of the home country so that we have an inherent conflict in the interests of the two countries.

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National Trade Conflicts Caused By Productivity Changes

by Ralph E. Gomory[1] and William J. Baumol[2]

In what circumstances does a country gain and when does it lose through improvements in productivity of its trading partner? Does the U.S. gain or lose when U.S. companies make major investments in plants in South East Asia? Does the U.S. gain or lose when it trains students in the latest technology and they return to their home countries? Does it matter whether that home country is relatively undeveloped or is a fully developed industrial country like Japan? This paper will deal with these and related subjects, and will make contributions in two areas, first, theory, much of it with policy implications, and second, the introduction of new and powerful analytic tools.

First, the *theory pertinent to policy* includes the conclusion that the gains from trade that a country can capture from a partly developed trading partner can substantially exceed the gains it can obtain by trading either with a fully developed country or one that is extremely underdeveloped. We also determine what characteristics of a nation's trading partner allow it to serve the interests of the home country most effectively. To be such an "ideal trading partner" the partner's economy must have low productivity in most industries so that it cannot compete effectively with the home country in those industries, even at its relatively low wage. But it should be very productive in a small proportion of industries so that in these industries it can produce more cheaply than the home country.

We will show that if the trading partner is below this state of development, for example if it is an almost entirely undeveloped country trading with a well-developed one, there is a natural symbiosis: improvements in the undeveloped trading partner usually benefit both countries. But once the ideal trading partner stage of development is reached, further improvements in the trading partner's productivity usually *decrease* the welfare of the home country. From that point on we no longer have symbiosis but, rather, an inherent conflict in the interests of the two countries. The nature of this conflict is fundamentally different from those that result from trade restrictions and protectionist policies, more commonly noted as sources of conflicting national interests.

The second contribution of this paper is to provide *analytic tools* that are new to the study

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of linear trade models.[3] These tools enable us to display as points of a graph each of the equilibria that can emerge from a linear trade model as a result of different values of the productivity parameters in the various industries of two trading countries. We can obtain these equilibria and this graph not only for two industry models but for models with n industries as well. We show that the set of these possible equilibria has robust attributes with the clear economic implications described above. And we find that models with large numbers of industries behave fundamentally differently from the more familiar two industry models.

These methods also enable us to quantify our qualitative conclusions. We can compute the gains and losses to the two countries when a country goes from underdeveloped to being the ideal trading partner, or from ideal trading partner to being fully developed. This shows, for example, that, for the home country, the difference between having an ideal trading partner and having a fully developed one is often as large as the difference between trading with a fully developed partner and being in a state of autarky.

Many of the comparative statics results of general equilibrium trade theory have been inherently local in character, providing partial derivatives of an endogenous variable with respect to an infinitesimal change in the value of one of the parameters. Our analysis is quite different in that it yields results for *all* possible parameter values. Consequently, for any arbitrary starting point our analysis yields comparative statics results for changes in the relevant parameters of any magnitude within the ranges consistent with the constraints of the model. These larger changes in parameter values can, of course, have effects upon the endogenous variables that can be very different from the results of local changes. The non-local character of our results explains why they also enable us to connect linear models with those having economies of scale such as those in Gomory [1994].

Roots in the Previous Literature: Our subject has long been discussed by specialists in international trade -- the circumstances under which improvements in productivity in a trading partner are beneficial to the home country. A significant part of the economic literature on this subject has been based on the analysis of Ricardian models whose trade equilibrium is shifted by improvements in productivity in one country or the other. In his noted inaugural lecture Professor Hicks [1953] sketched an intuitive Ricardian model of the effect of improved productivity in Country A on its own welfare and that of its trading partner Country B. He concluded, first, that *uniform* improvements in productivity in a trading partner benefitted both countries, and then distinguished two other cases. In the first the improvements in Country B are concentrated in its export industries, and he concluded that this improvement is beneficial to both Countries. In the second case the improvements are concentrated in Country B's import industries and he showed that although this is good for Country A, Country B is worse off.

This fruitful line of thought was taken up by Dornbush, Fischer, and Samuelson [1977] using a ground breaking approach to deal with an infinity of goods. Their conclusion was, like that of Hicks, that technological change spread uniformly among the products of the improving

[3] One of us, Gomory [1994], has used similar methods for the study of trade models with economies of scale.

country was good for both countries. They also pointed out, however, that the international transfer of technology from a high-wage country to a less advanced low-wage country can be harmful to the welfare of the transferring country. In an illuminating paper that builds on the ideas of both these articles, Krugman [1986] considered the subject of trade between a technologically advanced country and its less advanced trading partner using some new and realistic premises. He found an interesting asymmetry. Progress in the advanced country was always beneficial to both countries, but progress in the less advanced country, while always beneficial to it, could, depending on circumstances, be either harmful or beneficial to the more advanced country. He pointed out that these results can be interpreted in terms of the tendency of the advanced country to make export biased improvements and of the less advanced country to make improvements that were more import biased.

More recently Johnson, Hyman and Stafford [1993,1995] have analyzed the effect of the improvement in a single industry in one of the countries. They found, consistent with the earlier work, that if the industry starts from a very low level of productivity and improves to a competitive level, the initial effects benefit the improving country but harm the trading partner, but later, when the industry is entirely shifted to the improving country, further improvements (which can now be regarded as export biased) are beneficial to both. Whether this is a net gain or net loss for the other country depends on the balance of the two phases.

All this work indicates that productivity improvements in one country are always good for it, but that the effect on its trading partner depends on the balance between the damaging effect on importing industries and the beneficial effect on exporting industries.

Approach of this Paper: In this paper we discuss a different but related issue. Instead of looking at the effect of changes in productivity around an existing equilibrium, we investigate how the overall characteristics of a country affect the interests of its trading partner. Is Country 1 better off if Country 2, the Country with which it trades, has a high income and its productivity levels in many of its industries are high? Or is Country 1 better off if Country 2 is relatively poor and in many of its industries has low productivity?

To answer these questions we consider the full range of productivity parameters that do not exceed some natural or current technological limit. Specifically, for each industry i in each Country j we consider *all* levels of productivities $e_{i,j} \leq e^{max}_{i,j}$. We then ask which values of these productivity parameters yield the best results for one country or the other, or possibly simultaneously for both. Using integer and linear programming methods we give sharp answers to these questions.

Limits on productivity play an important role in our analysis as they do in real economic activity. We know that plants will shift to a low labor cost country when it is no longer possible to increase productivity in the home country to compensate for the cheapness of labor abroad. Or, at the analytical level, we know that in the last phase of the Johnson, Hyman, Stafford analysis, limits on productivity will determine the overall balance of beneficial and detrimental effects. Our methods allow us to deal systematically with the effects of limits on productivity.

Relative national income plays a large role in our analysis. We find that there is a range of relative incomes in which productivity parameter changes that increase utility in one country usually increase utility in the other as well. In this range *even import biased improvement in the* *trading partner is usually good for the home country.* But there is another range of relative national incomes where the utilities of the two countries move in opposite directions as productivity parameters change. Here there is inherent conflict in the interests of the countries.

We distinguish in our work between models with only a few industries and those with many. The number of industries matters. The results we have just described are valid for models with a larger number of industries, usually six or more. These models are different from small models, such as the familiar England-Portugal wine-textile example, discussed below. There, the outcome at which Portugal specializes in wine and attains its maximal productivity, and England specializes in textiles and attains its maximal productivity, is best for both countries. It remains the best even when we consider, as we do, all models with lower productivities, i.e., $e_{i,j} \leq e^{max}_{i,j}$. However, we will see that this single best outcome is a property of some small models, with two, three, or perhaps four industries, that does not carry over to large ones.

I. Bounded Productivity Families, Equilibria, and the Basic Graph

Families of Linear Models In our linear model, the quantities $q_{i,j}$ produced of each good i in Country j are determined by linear production functions $e_{i,j}l_{i,j}$. Each of the two countries participating in trade has a given utility function of Cobb-Douglas form[4] with demand parameters $d_{i,j}$. We fix the labor-force sizes L_j of the two countries as well as n, the number of industries. A single model is then completely specified by the vector of productivity coefficients $\epsilon=\{e_{i,j}\}$. However, instead of dealing with just one model we will discuss the equilibrium outcomes of the *family of models* obtained by considering *all values of the productivity coefficients* ϵ *subject only to a maximal productivity condition* $e_{i,j} \le e^{max}_{i,j}$ and holding everything else constant. We will refer to these as bounded productivity families or *BP families.* Working with BP families enables us to analyze the effect of different productivity levels on the welfare of the two countries.

Each equilibrium of a BP family is represented by a pair of points in a graph of utility versus relative national income, described below. The regions of that graph that result from plotting *all* the equilibrium pairs for a BP family of linear models have definite and characteristic shapes for large models. These robust shapes lead to the economic implications of this article.

The Basic Graph: For any given vector of productivity parameters $\epsilon= \{e_{i,j}\}$of a BP family, there is a stable equilibrium giving a national income Y_j and a utility U_j for each country. From the Y_j we can compute *relative* national income $Z_j=Y_j/(Y_1+Y_2)$.

In our basic graph (Figure 1.1) Country 1 relative income Z_1 is measured horizontally from 0 to 1. Country 2 relative income $Z_2=1-Z_1$ is measured back from the right end of the same axis. Country 1 utility U_1 is measured vertically on the *right* vertical axis. Country 2 utility is measured vertically on the *left* vertical axis. On both vertical axes the scale is chosen so that unity represents each country's utility in autarky using its maximal productivities $e^{max}_{i,j}$.

The equilibrium from any $\epsilon=\{e_{i,j}\}$ of a BP family gives us a point $r_1(\epsilon)=(Z_1,U_1)$ and a point $r_2(\epsilon)=(Z_2,U_2)$ in our diagram. Figure 1.1 represents 14 equilibria. The large dots represent the $r_1(\epsilon)$, their utility is measured vertically on the right vertical axis. We also have the 14 corresponding $r_2(\epsilon)$ as small dots. Each $r_2(\epsilon)$ has the same relative national income Z_1 as its corresponding $r_1(\epsilon)$ but its height represents U_2 and is measured on the left vertical axis.

Stable Equilibria: Next we describe the equilibrium conditions that yield these equilibria. For this we need some notation. Z_j just defined, is Country j' s (relative) national income (Country j's *share*). We normalize analogously all our pecuniary expressions, so p_i the price of good i, and w_j, the wage in Country j, are also divided by total income $Y_1.+ Y_2$. Country j's *consumption* of good i is denoted by $y_{i,j}$ and its *production* of good i by $q_{i,j}$. Country j's *production share* or *market share* of world output of good i is represented by $x_{i,j}=q_{i,j} / (q_{i,i}+q_{i,2})$, so that the vector $x=(x_{i,j})$ describes the pattern of production.

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[4]Using the Cobb-Douglas demand function enables us to carry out explicit computations and therefore provide *quantitative* results about the effects we describe. However, as we will see in Section 6, many of our *qualitative* results do not require this assumption.

We can now describe our equilibrium conditions, noting that, henceforth, the term "equilibrium" will mean *stable* equilibrium. First, (relative) national income of Country j must equal the total revenue from domestic and foreign sales of that country's products. Since with a Cobb-Douglas utility function Country 1's expenditure on good i will be $d_{i,j}Z_j$, this condition[5] is:

(1.1) $$\sum_i x_{i,j}(d_{i,1}Z_1 + d_{i,2}Z_2) = Z_j.$$

Second, we have a zero-profit condition. World expenditure on Country j's output of good i all goes into the wages of the labor $l_{i,j}$ employed in that industry, so:

(1.2) $$w_j\, l_{i,j} = x_{i,j}(d_{i,1}Z_1 + d_{i,2}Z_2).$$

Third, is the full-employment requirement for each country. This is expressed as the condition that the wage rate times the country's total labor force equals national income:

(1.3) $$w_j\, L_j = Z_j.$$

Fourth, we have the requirement that, for each good, quantity supplied equals quantity demanded, or equivalently, that the value of the output of good i at the equilibrium price equals the amount consumers are willing to spend on it

(1.4) $$p_i(q_{i,1} + q_{i,2}) = d_{i,1}Z_1 + d_{i,2}Z_2 \quad or \quad p_i q_{i,j} = w_j\, l_{i,j}$$

where the second form of (1.4) follows directly from the first by multiplying through by $x_{i,j}$ and using (1.2). Finally, we have the stability conditions that make entry by non-producers unprofitable. These require producers not to have higher unit costs than non-producers For example, if Country 1 is the producer in industry i and Country 2 is a non-producer, we must have $e_{i,1}/w_1 \geq e_{i,2}/w_2$. More generally:

(1.5) $$\begin{array}{llll} if\ x_{i,1}>0 & and\ \ x_{i,2}=0 & then & e_{i,1}/w_1 \geq e_{i,2}/w_2 \\ if\ x_{i,2}>0 & and\ \ x_{i,1}=0 & then & e_{i,1}/w_1 \leq e_{i,2}/w_2 \\ if\ x_{i,2}>0 & and\ \ x_{i,1}>0 & then & e_{i,1}/w_1 = e_{i,2}/w_2. \end{array}$$

The conditions (1.5) are, of course, a form of the familiar comparative-advantage criterion.

It is easily shown that at equilibria satisfying the preceding conditions trade must also be in balance and that the exchange rate is w_1/w_2.

II. Preliminaries: Utility and the Classical Assignment

We adopt the notation (x,Z_1,ϵ) for the equilibrium determined by the productivity parameters $\epsilon=\{e_{i,j}\}$ and having market shares $x=\{x_{i,j}\}$, and relative national income Z_1 for Country 1. Generally a large ϵ in (x,Z_1,ϵ) means high productivity and high utility, and therefore yields points high up in the diagram. When the $e_{i,1}$ are large relative to the $e_{i,2}$, Country 1 is the

[5] There are of course two equations, one for each j value. However, since $Z_1+Z_2=1$ and $x_{i,1}+x_{i,2}=1$ the two equations are dependent. If one is satisfied the other is too.

producer in most industries so Country 1's income share is large. The equilibrium (x,Z_1,ϵ) will have a large Z_1, and the equilibrium point $p_1(\epsilon)$ will be near the right edge in the diagram. Similarly, a large $e_{i,2}$ relative to the $e_{i,1}$ yields points near the left edges.

Cobb-Douglas Utility: Cobb-Douglas Utility $U_j=\Pi_i\, y_{i,1}^{\,d_{i,j}}$ is a function of $y_{i,j}$, the consumption of good i in Country j. At any equilibrium (x,Z_1,ϵ) the consumption $y_{i,j}$ can be found by multiplying world output of good i, which is $q_{i,1}(x,Z_1,\epsilon)+q_{i,2}(x,Z_1,\epsilon)$, by the fraction $F_{i,j}$ that County j obtains, so the (log) utility is:

$$\textbf{(2.1)} \qquad \ln U_j(x,Z_1,\varepsilon)= u_j(x,Z_1,\varepsilon)=\sum_i d_{i,j}\ln\, y_{i,j}=\sum_i d_{i,j}\ln\, F_{i,j}\,\{q_{i,1}(x_{i,1},Z_1,\epsilon)+q_{i,2}(x_{i,2},Z_2,\epsilon)\}.$$

With Cobb-Douglas utility, the fraction Country j obtains is $F_{i,j} = d_{i,1}Z_1/(d_{i,1}Z_1+d_{i,2}Z_2)$ and the quantities produced in each country can be expressed in terms of x, Z_1, and ϵ by:

$$\textbf{(2.2)} \qquad q_{i,j}(x_{i,j},Z_1,\epsilon)=e_{i,j}l_{i,j}=e_{i,j}\frac{x_{i,j}(d_{i,1}Z_1+d_{i,2}Z_2)}{w_j}=e_{i,j}\frac{x_{i,j}(d_{i,1}Z_1+d_{i,2}Z_2)L_j}{Z_j}.$$

Therefore (2.2) and (2.1) give us the utility value for any equilibrium (x,Z_1,ϵ), in terms of x, Z_1, and ϵ. In fact they assign a utility value for any triple (x,Z_1,ϵ) whether or not it is an equilibrium.

Linearized Utility: It simplifies both theory and computation to use the linearized utility function introduced in Gomory [1994]. It is easily shown that this linearized utility $Lj(x,Z_1,\epsilon)$ is equal to the utility $u_j(x,Z_1,\epsilon)$ at all equilibria and on all specialized production shares x. Its great advantage is that it is linear in x. We use linearized utility in all our calculations, so while we discuss utility, we always compute with the much simpler linearized utility.

World Utility: In addition to country utility, we will often refer to *world utility*, the total utility of all goods produced. This is most useful when both countries have the same utility functions but we will use it to some extent when they are different as well. We define world utility as $U_w=\Pi_i\,(q_{i,1}+q_{i,2})^{d_{i,1}}$. Using $d_{i,1}$ means we are measuring the utility of total world output using Country 1's preferences. The expression for world utility in terms of (x,Z_1,ϵ) is obtained immediately from (2.1)and (2.2) by putting j=1 and all the fractions $F_{i,j}=1$.

The Classical Assignment: If we choose any value of Z_1 we can determine the wages $w_1=Z_1/L_1$ and $w_1=Z_2/L_2$ at Z_1 and therefore, for each industry, we can find which country would be the cheaper producer if both were at their maximal productivities. As in Gomory [1994] we define the *classical assignment* at Z_1, $x^C(Z_1)$, to be the assignment of industries that gives each country the industries in which it would be the cheaper producer *at maximal productivity.* This specialized assignment is usually not an equilibrium. However, there is one important exception

Consider the equilibrium that results when $\epsilon=\epsilon^{max}=\{e^{max}_{i,j}\}$, i.e. when each country *is* at its maximal productivity in every industry. We call this equilibrium the *classical equilibrium.* The level of relative income Z_1 at this equilibrium we call the *classical level* Z_C. Since (1.5) tells us that at any equilibrium only the cheaper producer produces in each industry, at the classical equilibrium, each country produces in the industries in which it is the cheaper producer using its full productivity potential. Therefore, at Z_C the classical assignment produces the classical equilibrium.

However, the classical assignment is never an equilibrium except at $Z_1=Z_C$. For example, for relative income Z_1 sufficiently small, Country 1's wage is extremely low. Therefore the classical assignment would give Country 1 the entire production of almost all industries. But the large resulting fraction of world revenue Country 1 would obtain would not match the small relative income Z_1, and (1.1) would not hold. Generalizing this thought gives us a theorem we will use repeatedly.

Theorem 2.1 - Classical Assignment Theorem: *The classical assignment gives Country 1 a revenue greater than Z_1 for $Z_1<Z_C$ and a revenue less than Z_1 for $Z_1>Z_C$. The classical assignment gives Country 2 a revenue greater than Z_2 for $Z_1>Z_C$, and a revenue less than Z_2 for $Z_1<Z_C$.* Given the background we have just discussed, this theorem is very intuitive. A proof is in Gomory and Baumol [1998].

III. Basic Structure of the Region

The regions we plot are composed of all the equilibrium points of a BP family. For a given BP family we can plot (1) R_1, the region of all points $r_1(\epsilon)$ representing Country 1 utility, (2) R_2 the region of all points $r_2(\epsilon)$ representing Country 2 utility and (3) the world utility region R_w, composed of the points representing world utility. The general regional structure is the same for all three regions and for all families. Each region always consists of an upper boundary curve $C_j(Z_1)$ and *all* the points below it. Figure 3.2 shows the regions for a small model. It is the boundary curve $C_j(Z_1)$ that completely determines the shape of each region, and it is from that shape that the economic consequences flow.

That there is such a boundary curve is certainly plausible If at an equilibrium (x,Z_1,ϵ) we can vary all the productivity parameters up and down around ϵ we should get equilibria all around the $r_1(\epsilon)$ point in the diagram corresponding to (x,Z_1,ϵ), so that point would be an interior point of the region R_1. But if at (x,Z_1,ϵ) most of the productivity parameters are at their maximal values so that the parameter variation upward is restricted, the point could be a boundary point. Since the larger $e_{i,j}$ give higher points in the diagram, a completely filled out region of equilibria, topped by a boundary curve involving equilibria with many $e_{i,j}=e^{max}_{i,j}$, seems intuitively plausible. It is proven in Gomory and Baumol [1998]

If there is such a boundary curve $C_1(Z_1)$ then the point on it at Z_1 represents an equilibrium that maximizes utility for that particular Z_1. This suggests that the points of the boundary curve can be found by solving the following programming problem: *maximize utility subject to the constraint of being an equilibrium, i.e. satisfying the equilibrium conditions (1.1)-(1.5).* This approach can in fact be carried out. The result is the remarkably simple:

Theorem 3.1 - Basic Regional Structure Theorem: *R_1 consists of a boundary curve $C_1(Z_1)$ and all the points on or below it. The utility of each point of the boundary curve $C_1(Z_1)$ for $Z_1 \leq Z_C$ is obtained by solving the linear programming problem in integer x:*

$$\textbf{(3.1)} \qquad \begin{array}{c} C_1(Z_1) = Max_x\; u_1(x, Z_1, e^{max}) \\ \sum_i x_{i,1}(d_{i,1}Z_1 + d_{i,2}Z_2) \le Z_1, \qquad x_{i,1} + x_{i,2} = 1 \end{array}$$

There are similar statements for $Z_1 > Z_C$ and for $C_2(Z_1)$ and $C_w(Z_1)$.

The inequality in (3.1), which is derived from the equilibrium condition (1.1), asserts that x must give Country 1a revenue equal or less than the specified income Z_1. The $x_{i,1}+x_{i,2}=1$ constraint on nonnegative integer variables $x_{i,j}$ simply means that each $x_{i,j}$ can only be either 0 or 1. So the problem (3.1) is simply to find the *specialized* assignment x of industries to countries that, (1) maximizes utility at Z_1 when both countries are fully developed ($e_{i,j}=e^{max}i,j$, all i), (2) does not assign a quantity of revenue to Country 1 that exceeds Z_1. How the equilibrium conditions (1.1)-(1.5) lead to this surprisingly simple conclusion is shown in Gomory and Baumol [1998].

Constructing the Equilibrium x: Although the x determined from (3.1) does give the maximizing utility, the Basic Regional Structure Theorem does not assert that x *is* the equilibrium x. In fact x is usually not an equilibrium because: (1) it will usually violate the stability conditions, (2) it may satisfy the inequality in (3.1) as an inequality while equilibrium condition (1.1) requires equality. However x can be converted into the maximizing equilibrium of the BP family by a simple process that deals with both these problems.

The Classical Assignment Theorem tells us that the Country 1 revenue from the industries in which it is the cheaper producer exceeds Z_1. Therefore there are always industries in which Country 1 is the cheaper producer and, in which, with the assignment x, it is not producing. This violates equilibrium condition (1.5). To cure this we lower the productivity of Country 1 in each such industy until its unit cost rises to the unit cost of Country 2. This means decreasing $e_{i,1}$ from $e^{max}_{i,1}$ to $e_{i,1}=(w_1/w_2)e^{max}_{i,2}$ in those industries. Doing this in all the industries that x^C assigns to Country 1, but the assignment x does not, makes x stable.

To deal with strict inequality in (3.1) we move parts of industries from Country 2 to Country 1 thus increasing Country 1 revenue. This means increasing the $x_{i,1}$ from zero in either one or several of the industries that are in $x^C(Z_1)$ but not in x. We do this until we have equality in (3.1). This new x can easily be shown to be an equilibrium. The industry shifting is between producers of equal unit cost, and therefore does not affect utility. The new equilibrium x has the same utility as the original non-equilibrium x and therefore has the maximizing utility value.

Next we will use these methods to find the boundary equilibria of a specific small model. This exercise has a double purpose: to show just how our methods work in finding such equilibria, and to demonstrate that the results obtained from the two good model so frequently used in the literature are in an important sense atypical. They suggest a commonality in the interests of two trading countries that, we will show, vanishes rapidly as the number of traded goods increases.

The Ricardo Example and the Outcome Best for Both Countries: Country 1 (England) is assumed to excel in the first industry (textiles) with $e^{max}_{1,1}=1$ while its wine production is characterized by $e^{max}_{1,2}=0.55$. The other country (Portugal) excels in the other industry (wine), with $e^{max}_{2,2}= 1$ and $e^{max}_{2,1}=0.45$. The demand parameters for textiles and for wine are the same in both countries, $d_{i,1}$=.55 for textiles, $d_{i,2}$=.45 for wine, and the countries are of the same size.

In Figure 3.1 Country 1's equilibrium region R_1 is everything on or under the solid line marked C_1. Country 2's equilibrium region R_2 is on or under its boundary, the dashed line C_2. R_w, the region of world utility measured in Country 1 units, is on or under the solid line C_w.

For very small Z_1 England has a very low wage, so the classical assignment assigns both industries to it. But, since both textiles and wine have a relative income share of .45 or more, the *integer* x can assign neither one to England without violating the constraint in (3.1). So the maximizing x assigns no industry to England. Both textiles and wine therefore are industries that would be cheaper in England, but x does not assign them to England. So stability requires England's productivity to be reduced to $e_{i,1}=(w_1/w_2)e^{max}_{i,2}$ for both industries. Then England, with low wages and low productivity, competes at equal unit cost with high wage high productivity Portugal in both textiles and wine, getting a small share of the market for each.

As Z_1 increases, England's wage and productivities rise together to keep it equal with Portugal in both industries. The first change occurs at Z_1=.3548. There England's wine productivity has risen to its maximal value of .55. To the right of Z_1=.3548 its higher wages make it no longer competitive in wine. The classical assignment $x^{C}(Z_1)$ now assigns only textiles to England and assigns wine to Portugal. Portugal takes the entire wine market. England takes a larger share of the textile market to make up for its loss in wine and continues increasing its textile productivity. The next possible change is at Z_1=.45 where for the first time it is possible for x to assign an entire industry (wine) to England. But England is not competitive in wine, so solving (3.1) tells us that the maximizing x at Z_1=.45 does not change. So we go on as before with England taking an ever larger slice of the textile market. Finally, at Z_1=.55, which is the classical level Z_C, it is possible for the first time to assign the entire textile industry to England. This assignment turns out to be the maximizing x and to coincide with the classical assignment. In textiles England now has the productivity $e^{max}_{1,1}=1$ rather than the reduced productivity (w_1/w_2) $e^{max}_{1,2}$= (.55/.45).55=.672. Consequently, the boundary curve jumps up discontinuously. This is because England is capable of improving its textile productivity even after capturing the entire world market from Portugal with less than its maximal productivity. England now has all of textiles production and Portugal all of wine. Both are fully developed. It is the classical point.

The classical point is so high that the best outcome for each country is attained there, as Figure 3.1 shows. So in this two-product case the classical specialized outcome is the best possible result for *both* countries. But this result is far from typical.

Larger Examples and an Emerging Shape: Figure 3.2 is the region for a 4-industry model. In it Country 1 is more productive in two industries and Country 2 is more productive in two industries. In contrast with the two industry model, in this 4 industry model *the equilibria best for the individual countries are not the same* - they are point P_1 for Country 1 and point P_2 for Country 2. Figure 3.3 depicts a 6-industry model. Again different widely separated equilibria are best for the two countries.

These larger models show that an increase in the number of traded commodities can entirely change some important implications of the model. We have seen that two industries are not enough. It takes more than a pair of goods to bring out the conflicts that, as we will see, always play a significant role in larger models. This can be contrasted with, for example, the early indifference map analysis of consumer purchase behavior, where it was shown that two or three good models reveal a great deal that single good analysis cannot show, but that larger numbers of goods offer no additional major insights.

In the larger models we can also see that the boundary curves are becoming smoother and a definite shape is emerging. We will capture that underlying shape by a simple approximation.

The Linear Programming Model: Consider a modification of (3.2) in which we replace the inequality by equality and, importantly, allow continuous rather than only integer x. This gives us instead of (3.1):

$$\textbf{(3.2)} \qquad \begin{aligned} &\ln B_1(Z_1)=Max_x\ u_1(x,Z_1,\epsilon^{max}) \\ &subject\ to\ \ \sum_i \{d_{i,1}Z_1+d_{i,2}Z_2\}\ x_{i,1} = Z_1 \quad and \quad x_{i,1}+x_{i,2}=1. \end{aligned}$$

We will show below that the curve $B_1(Z_1)$ that emerges from (3.2) lies above and very close to the exact regional boundary $C_1(Z_1)$, and that it gets closer and closer as n, the number of industries, increases. First we will show that $B_1(Z_1)$ is very easily obtained.

Calculating $B_1(Z_1)$: (3.2) is the problem of assigning industries or parts of industries to Country 1 in such a way that Country 1 maximizes its utility for a given national income Z_1. With its continuous variables x and our use of linearized utility (3.2) is a standard linear programming problem. In the maximizing solution entire industries are assigned to each country except for (at most) one which is shared between the two countries. The first result about the $B_1(Z_1)$ calculated in this way is:

Theorem 3.2 - Approximate Boundary Theorem: *$B_1(Z_1)$ lies above all the equilibria of the linear BP family.* Proof: Every equilibrium (x, Z_1,ϵ) must satisfy (1.1) which is the same as the equality in (3.2). Therefore, every equilibrium x is considered in the maximization with ϵ^{max} instead of ϵ, a change that only increases its utility.

However, $B_1(Z_1)$ is not only above the equilibria, it is close to them. We show this by producing for any Z_1 an equilibrium near $B_1(Z_1)$.

Converting the x to a Nearby Equilibrium: We convert the optimizing x to an equilibrium using the same basic approach as before. If in the ith industry the assignment x matches the classical assignment, we leave the $e_{i,j}$ at their maximal productivity values. If the assignment is the opposite of the classical assignment so that the cheaper potential producer does not produce, we lower the productivity of this non-producer to stabilize production in that industry. These changes in the productivity of non-producers have no effect on utility. Finally in the one shared industry we lower the productivity of the cheaper producer to match the cost of the other producing country. This step reduces utility because it decreases the productivity of a producer, but it also stabilizes this last industry and gives us an equilibrium.

In going from x to equilibrium only the last step, which involved only one industry, affected utility. If this change in utility is small relative to total utility, the resulting equilibrium will be near the original boundary point. We can expect this to happen as the number of industries increases and demand for any one industry's products becomes a small part of national income. In Gomory and Baumol [1998] we give a precise statement of the very wide range of conditions under which this convergence of the approximate boundary to the region of equilibria occurs as the number of industries increases.

The Boundary Shapes: We have used (3.2) to calculate more than 100 examples ranging in size from 5 industries to over 40. The calculation is simple and rapid. We always see the same characteristic shapes, the ones displayed in Figures 3.5 and 4.2 for different 22 industry models.

The Country 1 boundary $B_1(Z_1)$ always starts from a zero utility level at $Z_1=0$, rises steadily to a point that is always well to the right of the classical level Z_C, and then declines to the unit level which represents Country 1's utility in autarky. $B_2(Z_1)$ has a similar (reversed) shape. The theorem that describes that shape is:

Theorem 3.7 - Country Peak Theorem: *$B_1(Z_1)$ increases monotonically from zero at $Z_1=0$ to a peak value at $Z_{p1} \geq Z_C$ and is monotone decreasing thereafter to the Country 1 autarky value at $Z_1 = 1$. There is a similar statement for Country 2.* One important conseqence of the theorem is that the Country 1 peak can never be to the left of the Country 2 peak. Proof Outline: The zero value at $Z_1=0$ and the autarky value at $Z_1=1$ are clear from the fact that at $Z_1=0$ Country 1 has zero percent of the total income of the two countries and at $Z_1=1$ it is the sole producer. It is quite difficult to show the monotone increase to the peak to the right of Z_C and the decrease thereafter. We do this in Gomory and Baumol [1998] by proving the quasi-concavity of $B_1(Z_1)$.

However, the theorem can be made plausible: Although the approximate *world* boundary $B_W(Z_1)$ is *not* always quasi-concave, it always shows up in our computations as either the dome shape of Figure 3.6 or something like the very rough dome shape of Figure 4.2. This dome shape is economically plausible. We would expect a larger world output when both countries are significant producers than when one does almost everything. Now the shapes of the world and country boundaries are somewhat linked. At any Z_1 Country 1 has a share Z_1 of world income. It is plausible that its utility will be roughly equal to a Z_1 share of world utility. If we accept this, it follows that multiplying the height of the dome shaped world boundary by Z_1 should give us a curve close to the Country 1 boundary. Such a curve will rise with increasing Z_1 while the dome height is increasing, and it will turn down somewhere to the right of the world peak as it gets a larger fraction Z_1 of a dome height that is now descending to the autarky level. This resembles the behavior that the theorem asserts for the Country 1 boundaries.

IV. Economic Consequences of the Regional Shape

Inherent Conflict: Clearly, the best outcomes for Country 1 are always to the right of Z_C, those at or near the peak of the Country 1 boundary in Figure 3.5. Figure 3.5 also shows the corresponding region for Country 2, with the best equilibria for Country 2 near its peak to the left of the classical level. Because of the position of the peaks, the outcomes best for Country 1 are always poor for Country 2 and vice versa. A country that is successful in maximizing its utility does so at the expense of its trading partner. Thus the shape of the region, and the location of the peaks of the two countries shows that there is inherent conflict in the interests of the trading partners.

Trading with a Fully Developed Partner: Next we assert that if Country 1 trades with a fully developed Country 2, (i.e., $e_{i,2}=e^{max}_{i,2}$), it is confined to the outcomes to the left of Z_C.

Theorem 4.1- Fully Developed Partner Theorem: *If Country 1 (2) trades with a fully developed Country 2, the resulting equilibrium will always have $Z_1 \leq Z_C$. ($Z_1 \geq Z_C$).* Proof: Suppose there is an equilibrium with $Z_1 > Z_C$. If County 1 were fully developed (i.e., $e_{i,1}=e^{max}_{i,1}$), the classical assignment would already give Country 1 less revenue than its share Z_1 (Classical Assignment Theorem). If Country 1 is not fully developed, it will have fewer industries and less revenue than in the classical assignment. In either case its revenue does not match its share, equilibrium condition (1.1) does not hold, and there is no equilibrium.

As the figures clearly show all Country 1's best outcomes are to the right of Z_C, so trading with a developed partner confines it to relatively poor outcomes. So it is *not* good to be a partly developed country trading with a fully developed partner.

Trading with a Partly Developed Partner: Next we will show that it *is* good to be fully developed, especially if the trading partner is only partly developed.

Theorem 4.2 - Partly Developed Partner Theorem: *If Country 1 is fully developed, then the possible equilibria all lie to the right of Z_C and above the Country 1 autarky level.* The area of possible equilibria is area A in Figure 4.1. Proof : If Country 1 is fully developed, then from Theorem 4.1 the possible equilibria all have $Z_1 \geq Z_C$. Since there is trade, and hence gains from trade, the outcomes will be better than autarky.

Even within the region A there is a wide range of outcomes depending on the state of development of Country 2, i.e. its productivity parameters. In Figure 4.1, which is a typical example, the gains from trade to Country 1 at its peak P_1 exceed the gains from trade at the classical equilibrium C by more than the amount that the gains from trade at the classical equilibrium exceed autarky.

We know that a fully developed County 2 trading with Country 1 gives us the classical equilibrium. But what kind of a Country 2 will give Country 1 the equilibrium at P_1? We will call this Country 2 "The Ideal Trading Partner" and we will investigate its characteristics.

Characteristics of the Ideal Trading Partner: With what we already know we can find the ideal trading partner for any given Country 1. As an example we choose a 22 industry BP model with non-symmetric Cobb-Douglas utility, and with countries of almost equal size We assume that Country 1 is fully developed and look for its ideal trading partner. Using (3.2) we first calculate the regional frontier $B_1(Z_1)$and locate the Z_1 value of its peak. As Figure 4.2 shows, the peak is at Z_1=.74, so at the peak Country 1's wage is roughly 3 times that of Country 2. Because of its high wage, Country 1 would be the cheaper producer in only 4 of the 22 industries if Country 2 were fully developed. Next we follow our usual procedure, described in Section 3, and convert the assignment x of (3.3) into an equilibrium, reducing some of Country 2's productivities from their maximum values in the process. At the resulting equilibrium, the fully developed Country 1 is the sole producer in its 4 industries but, in addition, has almost the entire market in 13 other industries in which Country 2 is *not* fully developed. Country 2 is the sole producer in 5 industries and has a very small sliver of the market represented by the 13 other industries in which it would be the cheaper producer if fully developed. The incomplete development of Country 2 permits Country 1 to hold the market in those industries and to have its large relative national income of .74.

This example is typical, the partly developed productivities of the ideal trading partner usually allow Country 1 to make most of the world's goods while Country 2 produces at its maximum productivity only in the smaller share of industries that it has to itself. A high-technology country making most things for itself but trading for a few goods with an agricultural country can be an example. At the peak outcome *any* change in Country 2's productivities has a detrimental effect on Country 1. If Country 2's production parameters *increase*, this hurts Country 1, if Country 2's parameters *decrease*, that too hurts Country 1.

Peak Gains and History: Economic historians have long debated such questions as whether the U.K. lost out or benefitted from the relative rise in productivity since the 19th century in countries like Germany. Our analysis suggests that the effect cannot be determined simply from the change in German productivity, but requires knowing whether Germany, for example, moved closer to or further from being an ideal partner for the U.K.

Decreasing Productivities: In discussing departure from peak gains we mentioned the possibility of production parameters *decreasing* as well as *increasing*. It is natural to imagine a country's productivities *increasing* as it learns the latest methods of production or distribution, however *decreases* in productivity also have a realistic economic interpretation. We need only reinterpret the productivities of the BP family to allow for industry wide learning. We do this by allowing the *unit* of output in each industry to increases over time, while the $e^{max}_{i,j}$ remain constant. A country with fixed real output per labor hour in industry i, i.e. a country that does not keep up with the new methods, would move to new equilibria with smaller $e_{i,j}$.[6]

Quantifying Peak Gains: Our ability to compute using (3.2) and our ability to work out a rather complete theory of the symmetric demand case (see Gomory and Baumol [1998]) enable us to make quantitative statements about the importance of peak gains from trade. On the basis of both experiment and theory we assert that *over a wide range of country sizes and maximal productivity parameters the peak gains are a very considerable addition to the classical gains from trade, often exceeding the classical gains by more than the amount that the classical gains exceed autarky.*

The location of the peak also matters. As we will see in the next section, the location of any equilibrium relative to the peak indicates whether the further development of its trading partner is likely to be helpful or harmful to Country 1.

V. Maximal Productivity Equilibria and the Subregion of Maximal Productivity

The effect of limits on productivity is felt well into the region, not only near the boundary. These limits have some effect on all equilibria where there are producers who are producing at maximal productivity. They have the most effect at the equilibria at which the *producing* industries are all at their maximal productivities, i.e., $x_{i,j}>0$ implies $e_{i,j}=e^{max}_{i,j}$. We call these the *maximal productivity equilibria.*

If a country is an actual producer in an industry, it will tend to learn how to produce

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[6] More formally: Let the new unit in the ith industry be the old unit multiplied by $p_i(t) \geq 1$, $p_i(0)=1$. At any equilibrium (x,Z_1,ϵ), at time t, the productivities in new units are $e_{i,j}$, and in t=0 units, $p_i(t)e_{i,j}$. Utility has been increased by $\Pi_i p_i(t)$. Since we plot vertical position in autarky units, which are also multiplied by $\Pi_i p_i(t)$, the (Z_1,U_1) diagram does not change at all. But a country with fixed productivity in old units has productivity in new units of $e_{i,j}/p_i\{t\}$, a decrease.

better. Over time its productivity will tend to its maximal productivity. Therefore countries will tend to be near maximal productivity equilibria. For that reason we will focus our attention on the part of the region of equilibria which contains maximal productivity equilibria.

We might well expect the maximal productivity equilibria to lie in some sort of a band directly under the upper boundary, and indeed they do. We show these bands for each country as the subregions M_1 and M_2 in Figure 5.1. It is not hard to show that each band is the subregion that lies between the upper boundary and a lower boundary computed by minimizing instead of maximizing in (3.2) (Gomory and Baumol [1998]).

Conflict and Cooperation: In equilibria below the region of maximal productivity there are always some producers who can improve their productivity. In most cases they can do this without changing either x, the pattern of production, or relative national income Z_1. Such changes are quite benign, providing more output with fixed x and Z_1, benefitting both countries.

Within the region of maximal productivity, where we generally expect to find the trading countries, such benign changes are scarcer. At maximal productivity equilibria they are unavailable. There, increases in productivity are only possible for non-producers. These increases generally have no effect at first, since they are increasing the productivities of those who do not produce, but if the increase is sufficiently large a new equilibrium can emerge with the former non-producer becoming a producer. Then x and Z_1 must change, entailing migration of industries and change in relative national income.

The subregion of maximal productivity, then, separates the region of equilibria into two parts: (1) the part below the subregion, where increases in productivity usually benefit both countries, and where an analysis that does not consider technological limits can often be used, and (2) the subregion of maximal productivity itself where increases in productivity are usually constrained in one country or the other by maximal productivity limits and where industry productivity increases in one country will tend to cause shifts in industries, utility and relative national income.

The effect of these shifts depends strongly on relative income. Figure 5.1 shows that if Country 1 can *increase* its share anywhere within the region of maximal productivity to the left of its peak, it will generally increase its utility and *decrease* that of its trading partner. However if its relative national income is large enough that Country 1 is in the region to the right of its peak, *decreases* in its share generally *increase utility for both countries* since the regions of both countries slope down to the right.

So within the subregion of maximal productivity itself we have a further division (Figure 5.1) into the *region of conflict* and the *regions of cooperation*. The region of conflict lies between the two peaks, where gain in one country usually comes at the expense of the other. The regions of cooperation lie outside the two peaks, there shifts in relative national income tend to either benefit both countries or harm them both.

The effect of loss or gain of industries does not usually depend on the distinction between import and export oriented industries. In the region to the right of Country 1's peak, improvements in Country 2's productivities, that enable it to take over industries in which it formerly imported from Country 1, benefit both countries. The improvement in the cost of those goods, because they are now made by the low wage Country 2, overwhelms the effect on Country

1 of reduced share or, equivalently, shifting exchange rates. In contrast, between the two peaks the relationship is reversed. The increases in Country 2's productivity that enable it to export what it formerly imported still benefit it and still make those newly exported goods cheaper, but the effects on exchange rate, by making Country 2's other exports more expensive, are such as to produce a net loss in Country 1's welfare.

In the regions of cooperation, where one country tends to be highly developed while the other is relatively undeveloped, the development of the relatively undeveloped country helps both. Between the peaks however, where countries are more similar to each other in their stage of development, we see the inherent conflict in international trade.

VI. Some Underlying Economic Factors and a Connection With Economies Models

Most of the work we have done so far is quite quantitative. We calculate actual regional boundaries, we prove rigorously that the rise to the peak is monotone, we can calculate the size of peak gains from trade. In this section we will take a different tack. We will attempt some intuitive economic reasoning that brings our results in line with ordinary economic intuition.

The simplest intuitive explanation of the characteristic regional shape involves the generally dome-like shape of the world utility frontier. If we have a roughly dome-like world utility we can see why, as Country 1's share grows it should at first gain utility, it is getting a larger share of an increasing world output. We can see why it would later lose utility as it gets an ever larger share of a world output that is now decreasing toward the autarky level. But why, in economic terms, would we expect the world region to be domes shaped? One reason comes immediately to mind. The allocation of fair shares of industries to each country allows the exploitation of comparative advantage far better than when one country does most things for itself. We would expect this for any utility, not only Cobb-Douglas.

Certainly comparative advantage plays a role, but there is also something else at work. Both our formulas and our figures show that even when the two countries in the BP model are identical, i.e. $e^{max}{}_1(Z_1)=e^{max}{}_2(Z_1)$ we still have the same characteristic regional shape.

The second economic influence that contributes to the dome shape of the world utility frontier is the diminishing marginal rate of substitution usually assumed for consumer utilities. Because of this the world's consumers will have a relatively low valuation of a boundary equilibrium near the extreme right or extreme left of the graph. In such an equilibrium the country with the small share specializes in a very few goods, producing large quantities of this small number of items because it uses its entire labor force on them. The other country divides up its labor force giving a small amount of its labor to each of its many industries, so only small quantities of these goods will be produced. Diminishing marginal rate of substitution implies that consumers will prefer a more balanced bundle of outputs.

Neither of these arguments depends on special properties of Cobb-Douglas demand. So that it seems likely that these two economic factors, comparative advantage and diminishing marginal utility, will produce the same effect in models with other utilities.

However, there are limits to how far these general arguments can go. The general arguments make the dome shape, and, therefore, the general regional shapes of the two countries, seem plausible. But these arguments apply to small models as well as to large, and while it is true that the world frontier of Figure 3.1 first ascends and then descends, the economic consequence of that is not conflict but rather a central peak that is good for both countries. There is room for much more exploration of the demand structures and productivities that produce a dome shaped world utility with separated peaks for the two countries.

A third general economic influence that can push the world frontier toward a dome shape is economies of scale. Since we employ linear production functions, economies are absent from our models. But economies would only add further to the dome shape, since economies tend to be reduced when one country makes almost all the goods. There is in fact a very close tie between the region of equilibria that we have discussed and the multiple equilibria and the regional shape that emerges in the presence of scale economies Gomory [1994]. We now summarize that connection which is given in full in Gomory and Baumol [1998]

Economies of Scale and Linear BP Families: The connection, which we call the *correspondence principle,* is not between economies models and linear models, but between economies models and *linear BP families* The connection is along the following lines: A single economies model can yield many equilibria, we can always construct a BP family containing in its equilibrium region *all* those equilibria. Figure 6.1 shows the boundary for Country 1's region of equilibria together with the equilibrium points corresponding to a single scale economies model. The set of scale economies equilibria display the characteristic shape described in Gomory [1994], which mirrors the shape of the region itself, and tends to fill up a subregion of the linear BP family. For *sets* of equilibria in the region of a BP family we can often construct a *single* economies model with that set of equilibria. It is always true, for example, that all the specialized equilibria in the region of maximal productivity of a linear BP family correspond to the multiple equilibria of a *single* scale economies model (Gomory-Baumol [1997],[1998]).

VII. Summary and Conclusions

By taking explicitly into account the limits of productivity we have shown that the equilibria of a BP family of linear models form a well-defined region of equilibria with a robust shape. The shape of the region is such that the best outcomes for one country are always poor ones for its trading partner, so that policy or circumstances that yield the best outcomes for one country inherently involve conflict with the interests of the other. In fact all the best outcomes for a developed country require its trading partner to be in only a partly developed state.

We have shown how the productivity parameter values of a country's ideal trading partner can be determined. Any departure from these values, whether through increases or decreases in productivity of Country 2, the partner, will harm Country 1. Thus the welfare of a country is sometimes enhanced and sometimes reduced by a rise in productivity of its trading partner, but these outcomes follow a systematic pattern that is easily understood.

Within the region of equilibria there lies a maximal-productivity subregion. Within the region the interests of the two countries are in conflict over a wide range, but there is also a range where improvements in productivity in either country tend to benefit both. The beneficial range occurs when one trading partner is in the early stages of development, conflict occurs in the later stages of development.

Many extensions of this analysis based on regions of equilibria are possible, for example analyzing the effects of varying demand structures on the regional shape, or investigating models with more than two countries. We think that the results obtained so far indicate that such studies will be valuable.

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Figure 1.1 - Fourteen Point Pairs Showing Country 1 and Country 2 Utility

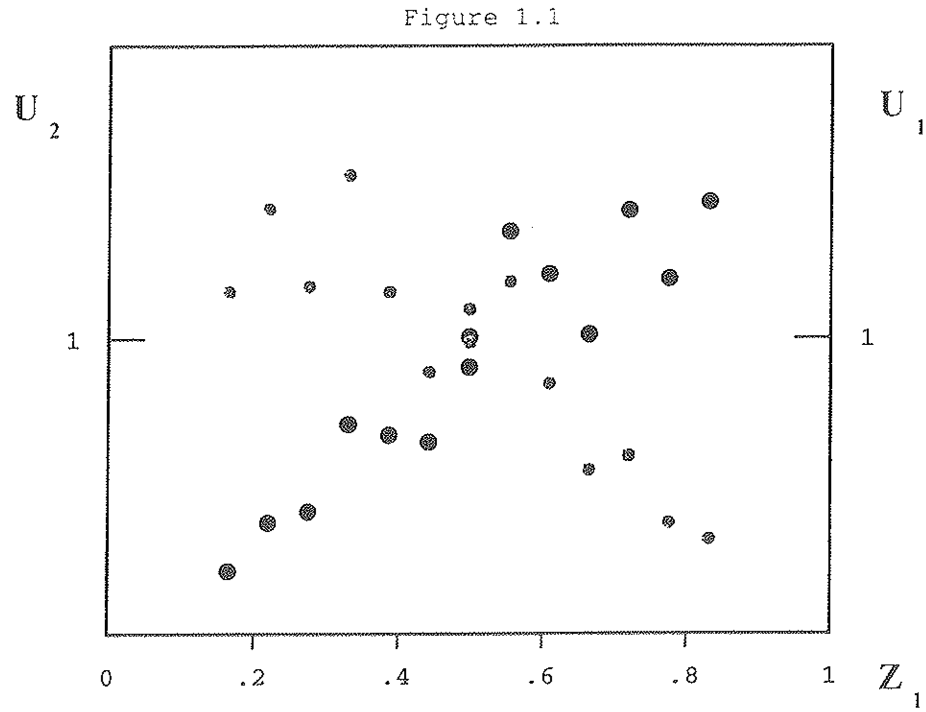


Figure 3.1 - Country 1, Country 2, and World Regional Boundaries

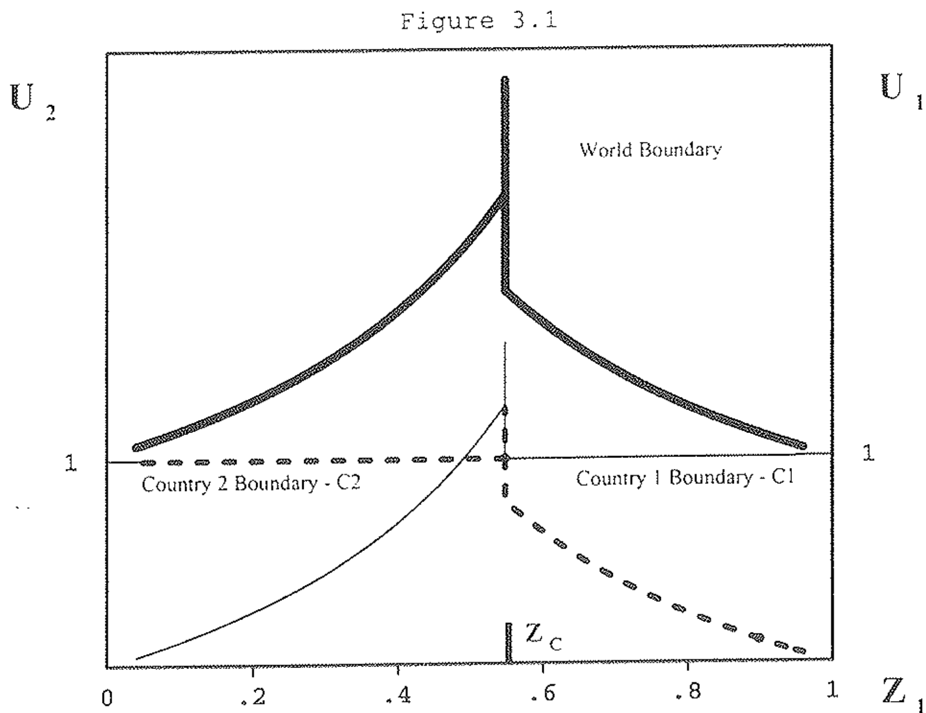


Figure 3.2 - Regional Boundaries for a Four Industry Model

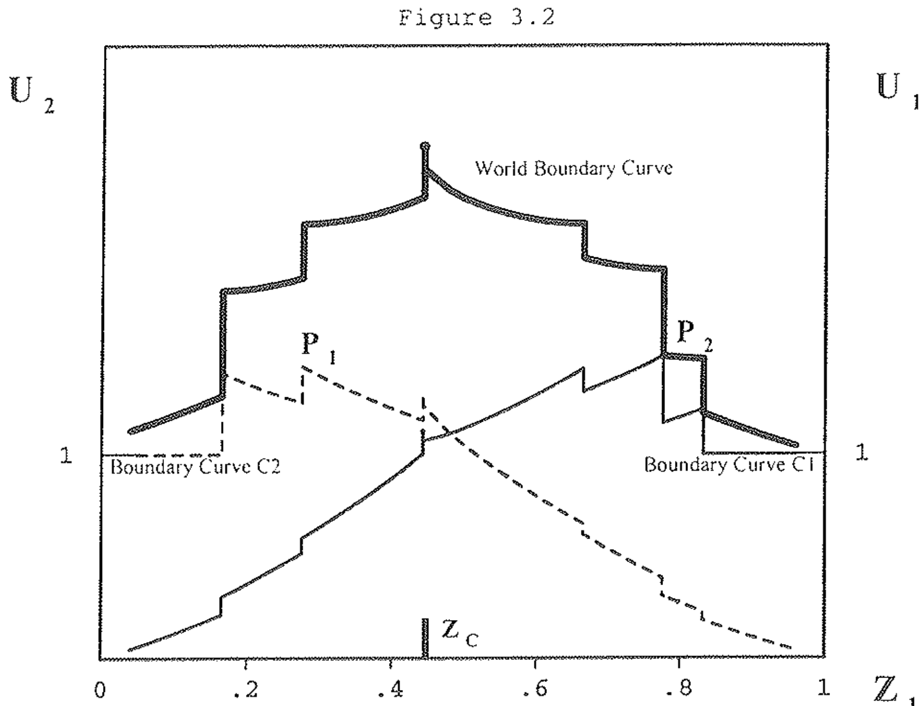


Figure 3.3 - A Six Industry Model with Boundaries and Approximating Curves

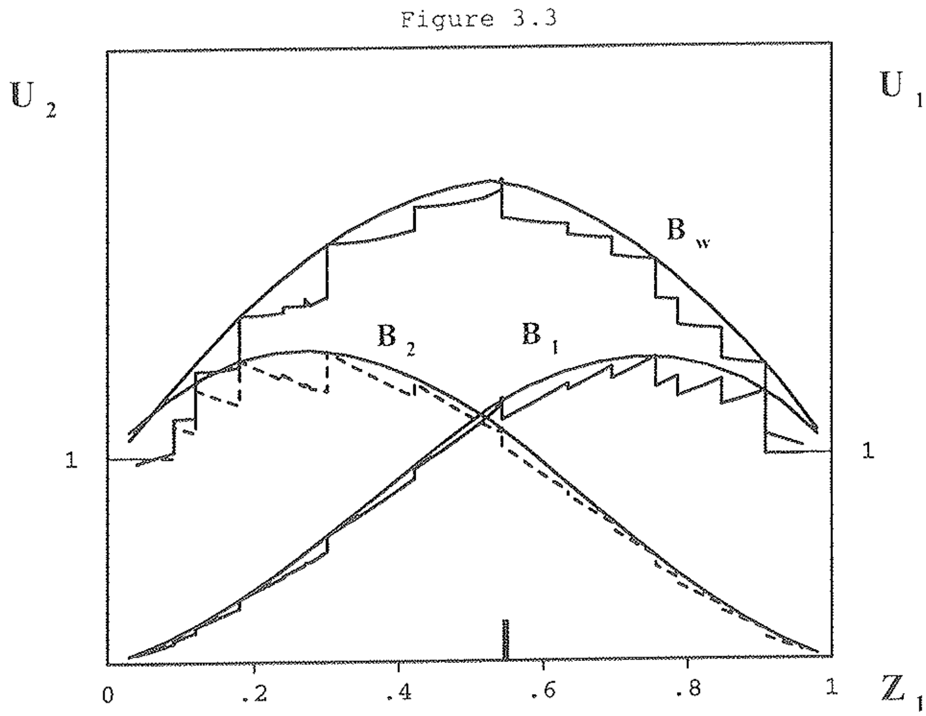


Figure 3.5 - A 22 Industry Model

Figure 3.5

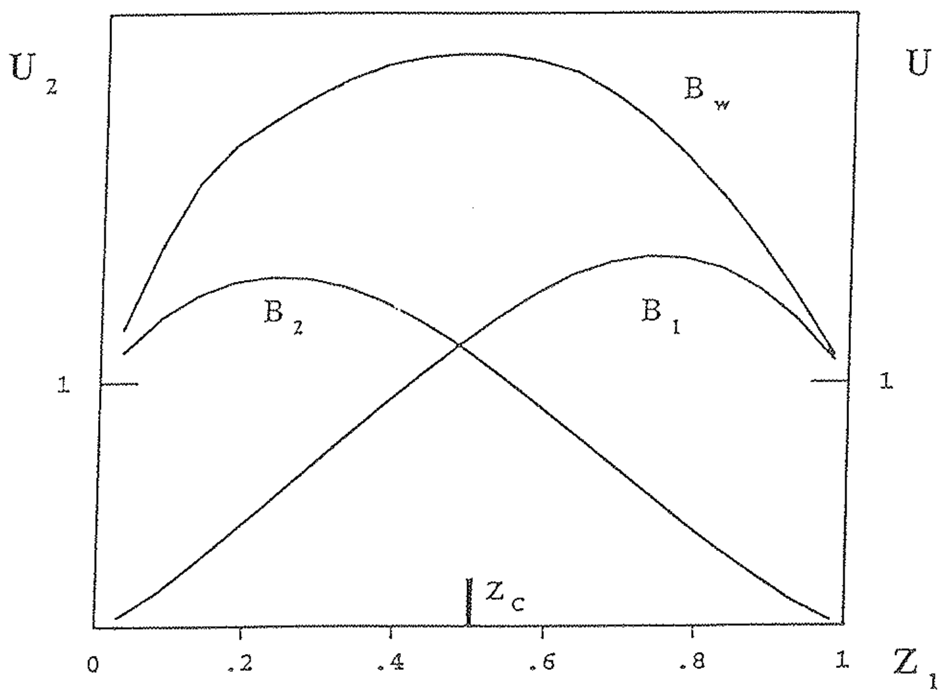


Figure 4.1 - Outcomes for a Fully Developed Country

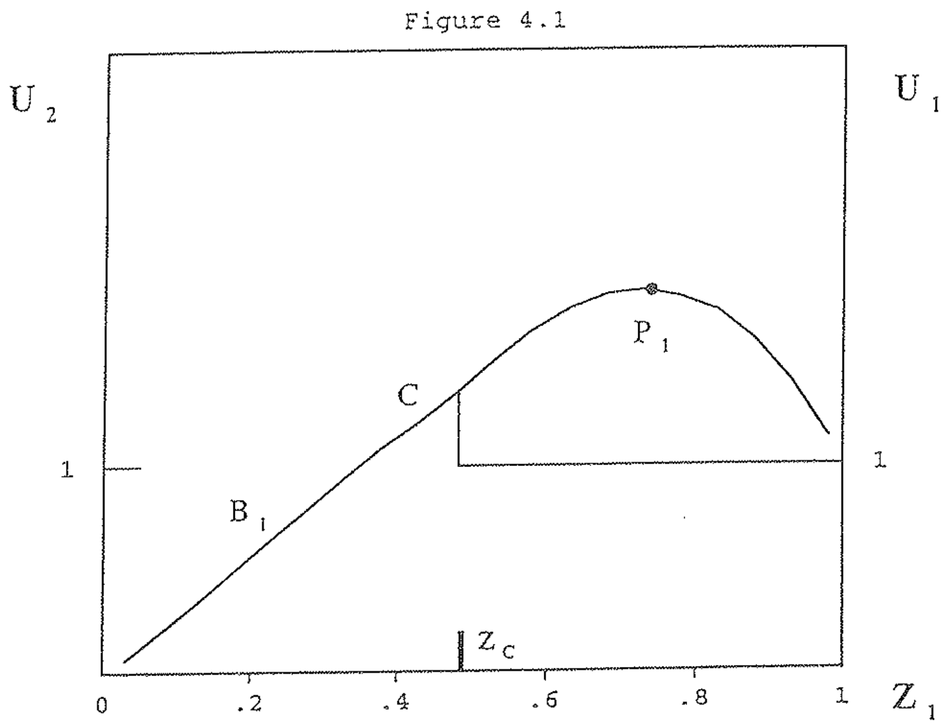


Figure 4.2 - The Ideal Trading Partner

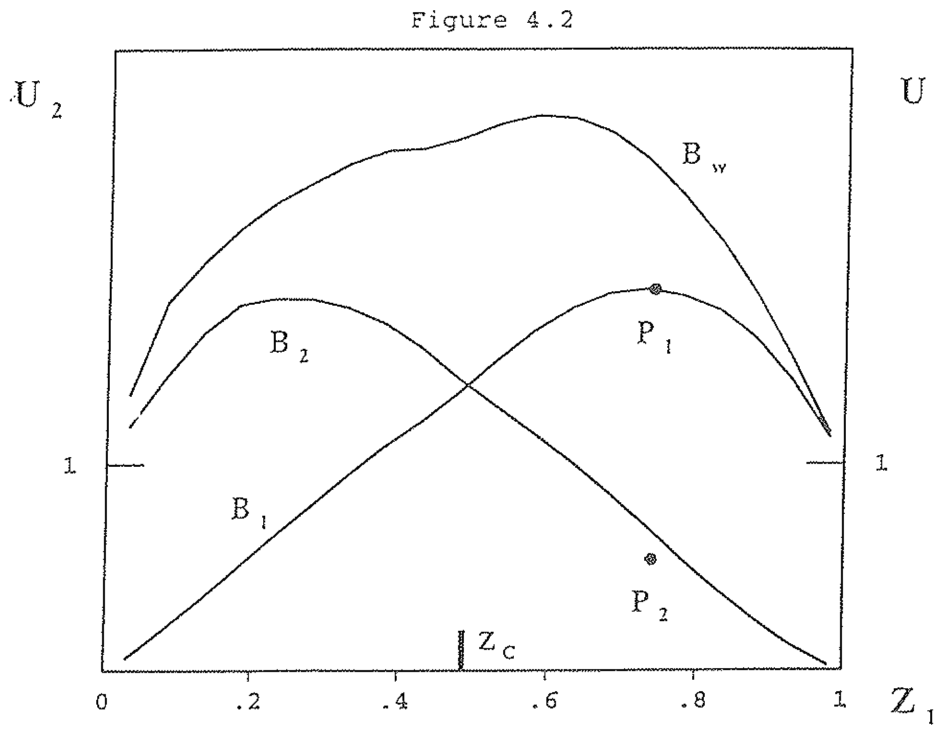


Figure 5.1 - Regions of Maximal Productivity and Ranges of Conflict and Cooperation

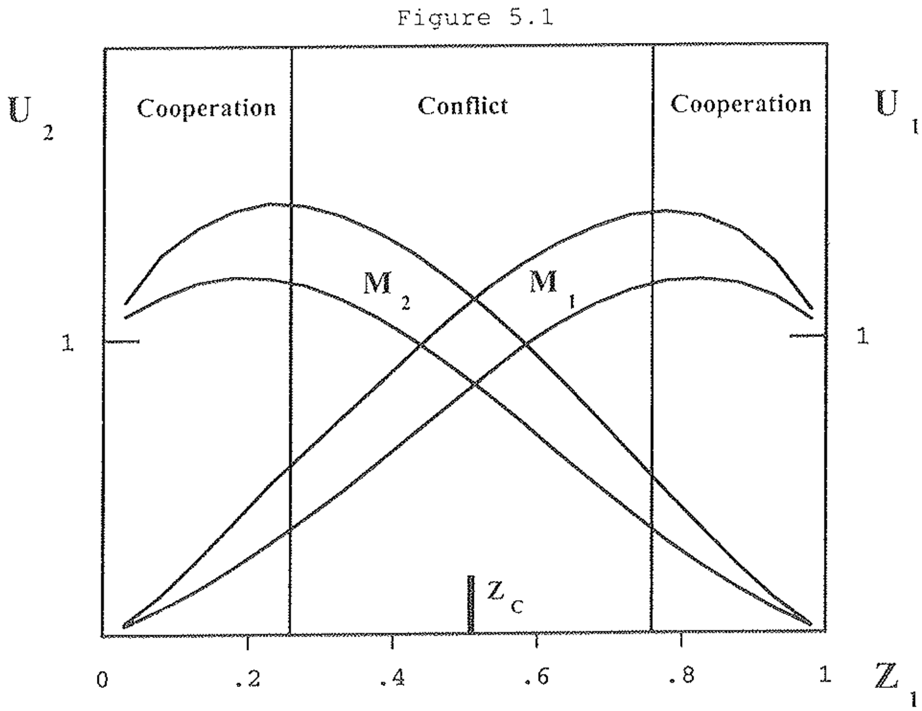
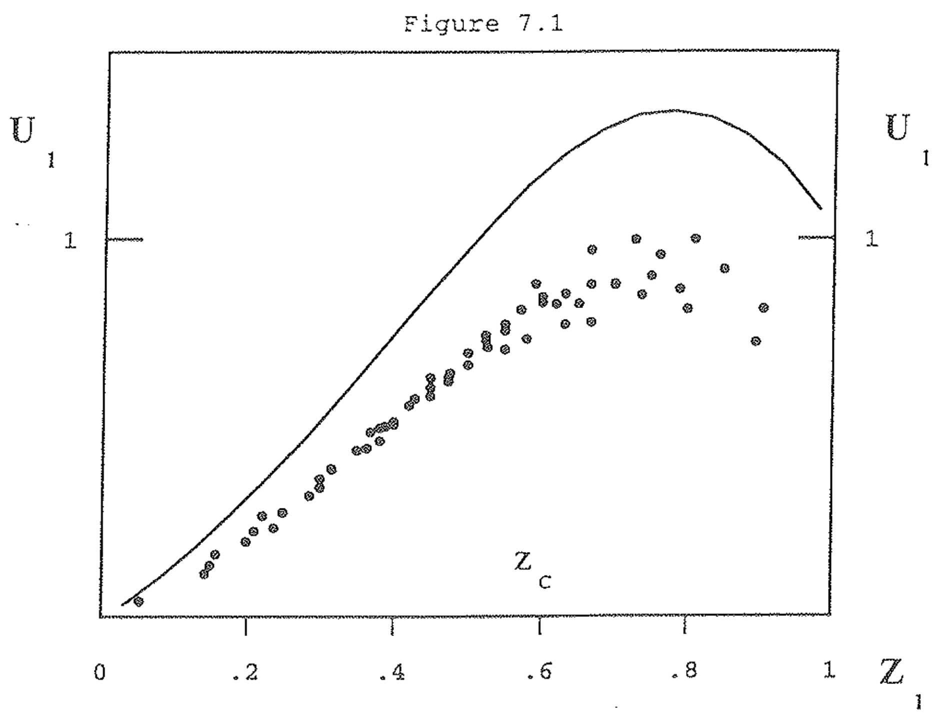


Figure 7.1 - Equilibia from an Economies Model Corresponding to Equilibria of a Linear Family



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