CLASS MATERIALS

INTERNATIONAL PARITY CONDITIONS

- -

1. Key Interest Rate-Exchange Rate Linkages: The Parity Framework

   - Parity conditions are useful when parity holds
   - Parity conditions are useful when parity *does not* hold

2. Purchasing Power Parity: Theory and Evidence

   - Theory of purchasing power parity
   - PPP and real exchange rates
   - Empirical methods effect our empirical findings about PPP
   - Evidence about PPP in the short-run
   - Evidence about PPP in the long-run (Is there *mean reversion*?)

3. Interest Rate Parity: Theory and Evidence

   - Theory of Interest Rate Parity
   - Interest Rate Parity and covered interest arbitrage flows
   - Empirical methods effect our empirical findings about IRP
   - The choice of securities effects our empirical findings about IRP
   - Opportunities for 'one-way' arbitrage even when IRP does not hold
   - Interest Rate Parity for long-dated maturities

4. Uncovered Interest Parity (Fisher International Effect): Theory and Evidence

   - Theory of interest rates and exchange rate changes in the short run
   - Empirical methods effect our empirical findings about UIP
   - Empirical evidence on UIP in the long-run and in emerging markets

5. Forward Rate Unbiased Property: Theory and Evidence

   - Theory of forward rates and expected future spot rates
   - Empirical evidence on unbiasedness

- -

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FIGURE 4.1

PARITY RELATIONSHIPS IN INTERNATIONAL FINANCE

KEY ASSUMPTIONS: 1. No Transaction Costs, 2. No Taxes, 3. Complete Certainty

THEORY	IN WORDS	IN SYMBOLS	DRIVING FORCES
Purchasing Power Parity			
Absolute Version	The price of a market basket of US goods equals the price of a market basket of foreign goods when multiplied by the exchange rate	$P_{\$} = P_{DM} \times Spot$	Arbitrage in goods
Relative Version	The percentage change in the exchange rate equals the percentage change in US goods prices less the percentage change in foreign goods prices	$?Spot = ?P_{\$} - ?P_{DM}$	Arbitrage in goods
Interest Rate Parity	The forward exchange rate premium equals (approximately) the US interest rate minus the foreign interest rate	$(F - S)/S = i_{\$} - i_{DM}$	Arbitrage between the spot and forward exchange rates
Fisher Parities			
Fisher Effect (Fisher Closed)	For a single economy, the nominal interest rate equals the real interest plus the expected rate of inflation	$i_{\$} = r_{\$} + E(?P_{\$})$	Desire to insulate the real interest against expected inflation. Arbitrage between real and nominal assets.
International Fisher Effect (Fisher Open)	For two economies, the US interest rate minus the foreign interest rate equals the expected percentage change in the exchange rate	$i_{\$} - i_{DM} = E(?Spot)$	Arbitrage in bonds denominated in two currencies
Forward Rate Unbiased	Today's forward premium (for delivery in *n* days) equals the expected percentage change in the spot rate (over the next *n* days)	$(F_t - S_t)/S_t = (E(S_{t+n}) - S_t)/S_t$	Market players monitor the difference between today's forward rate (for delivery in *n* days) and their expectation of the future spot rate (*n* days from today)

Explanation of symbols

Delta (?) in front of a variable means "percentage rate of change".

Tilde (~) indicates a random variable whose precise value is determined in the future.

All exchange rates S (for spot) and F (for forward) are in $/DM terms.

Why Parity Conditions Are Useful - Part I

When a Parity Condition *Holds*:

It shows the condition under which two strategies lead to similar results

For example:

When **Absolute PPP** holds:

Cost of a product in one country is identical to cost in another country

When **Relative PPP** holds:

Higher inflation in one country is fully offset by a depreciation of that country's currency.
$\Rightarrow$ relative prices between countries are not effected by currency depreciation

When **Interest Rate Parity** holds:

The cost of borrowing (or return on investment) is identical (after forward covering for exchange risk) no matter which currency is chosen.
$\Rightarrow$ No superior investments or low cost source of funds on a covered basis

When the **International Fisher Effect** (*Uncovered Interest Parity*) holds:

The *expected* cost of borrowing (or *expected* return on investment) is identical no matter which currency is chosen.
$\Rightarrow$ No superior investments or low cost source of funds (if investors and borrowers are willing to ignore currency risk)

When the **Forward Rate Unbiased** principle holds:

The *expected* return from speculating in a forward contract is zero
$\Rightarrow$ No expected return from speculation, no expected cost to hedging risk, and the forward rate is an unbiased forecaster of the future spot rate. (Even so, the forward rate may not be a very good predictor, in terms of accuracy.)

Why Parity Conditions Are Useful - Part II

When a Parity Condition *Fails*:

It shows the condition under which two strategies are *not* similar
⇒ the structure of international financial markets may tilt a firm's strategy for borrowing, investing, hedging, speculating, or selecting a country for manufacturing, sourcing or sales

For example:

When **Absolute PPP** fails:

The cost of identical products differs across countries.
⇒ Sellers may have the power to price discriminate across countries, buyers have incentives to overcome barriers to access lower cost goods

When **Relative PPP** fails:

Inflation in one country is not matched one-for-one with an offsetting depreciation of that country's currency.
⇒ relative prices between countries (i.e. real exchange rates) are subject to change

When **Interest Rate Parity** fails:

The cost of borrowing (or return on investment) differs (after forward covering for exchange risk) depending upon which currency is chosen.
⇒ It is possible to find superior investments or lower cost funds *without* taking on exposure to exchange risk

When the **International Fisher Effect** (*Uncovered Interest Parity*) fails:

The *expected* cost of borrowing (or *expected* return on investment) differs depending upon which currency is chosen.
⇒ Investment with higher *expected* return and funds with lower *expected* cost may be predicted. *However*, these strategies carry exposure to risk!

When the **Forward Rate Unbiased** principle fails:

The *expected* return from speculating in a forward contract is *non-zero*
⇒ Expected returns from speculation are *positive*, and the forward rate is an *biased* forecaster of the future spot rate, *however*, these strategies carry exposure to risk!

Purchasing Power Parity

1. Law of One Price

$$P[US,wheat] = Spot[\$/£] \times P[UK,wheat]$$

example: $\frac{\$4.50}{\text{bushel}} = \frac{\$1.50}{£} \times \frac{£3.00}{\text{bushel}}$

2. Absolute PPP

$$P[US] = Spot\ [\$/£] \times P[UK]$$

where $P[US] = \sum_i w[US,i]\ P[US,i]$

where $P[UK] = \sum_i w[UK,i]\ P[UK,i]$

and $i = 1, \ldots n$ goods

example: $\frac{\$150}{\text{US market basket}} = \frac{\$1.50}{£} \times \frac{£\ 100}{\text{UK market basket}}$

3. Relative PPP

Suppose that absolute PPP is violated -- the US market basket costs only $100, so absolute PPP holds only if a new term, K=0.67, is introduced into the equation.

Absolute PPP at time (t+1): $P[US,t+1] = K \; Spot[t+1] \; P[UK,t+1]$

Absolute PPP at time (t): $P[US,t] = K \; Spot[t] \; P[UK,t]$

Taking the ratio of P[US,t+1]/P[US,t] we have:

$$(1 + p[US]) = (1 + s)(1 + p[UK])$$

or $$p[US] = s + p[UK] + s\,p[UK]$$

where x is the percentage change in X

Example: P[US] increases from $100 to $132, or 32%

P[UK] increases from £100 to £120, or 20%

S increases from $1.50/£ to $1.65/£, or 10%

For small percentage changes, $s\,p[UK] \rightarrow 0$, so we can write

$$p[US] = s + p[UK], \quad \text{or}$$

$$\boxed{s = p(US) - p(UK)}$$

% exchange rate change = % change in US prices - % change in UK prices

Purchasing Power Parity - Continued

Things to be concerned about when thinking about PPP

How to test PPP:

Regression model: $s = a + b (p[US] - p[UK]) + e$

Average deviation: $d = s - (p[US] - p[UK])$

Reversion to PPP: $d = 0$ {after some period of time}

Selection of an appropriate base period

Selection of an appropriate price index

Is PPP a useful construct even if PPP does not always hold?

The 'Units' of the Real Exchange Rate

Real is a short-hand way of saying "real goods and services."

Real magnitudes are constructed from nominal magnitudes adjusting for prices levels or inflation.

Example 1-- Nominal and Real Income

Nominal income = \$55,000/year

Real income = $\frac{\$55,000/\text{year}}{\$250/\text{market basket}}$ = 220 market baskets / year

Usually expressed as an index relative to a base year.

For example, if real income in 1990 is 220 market baskets/year, define this as 100. In 1991, if nominal income rises by 10% to \$60,500, and the price of a market basket rises by 8% to \$270, then real income is now 60,500/270 = 224.07.

Expressed as an index,

Index (1991) = Real Income (1991) / Real Income (1990)

= 224.07 / 220

= 1.0185

$\Rightarrow$ Real income has risen by 1.85%

Example 2 -- Nominal and Real Exchange Rates

Nominal exchange rate = \$0.60/DM (rate of exchange between the currencies of two sovereigns)

When Purchasing Power Parity holds

\$0.60/DM = (\$600 Price/US good) / (DM 1,000 Price/German good)

Dividing the left hand side of above by the right hand side

$$\frac{\$0.60/DM}{\frac{\$\ 600\ /\ US\ good}{DM\ 1{,}000\ /\ German\ good}} = 1\ US\ good\ /\ German\ good$$

In other words, when PPP holds, identical US goods and German goods exchange for each other on a one-for-one basis.

The units of the real exchange rate are

$$\frac{US\ goods}{German\ goods} \quad \text{or} \quad \frac{German\ goods}{US\ goods}$$

The real exchange rate measures the change in competitiveness of domestic versus foreign products after adjusting for exchange rate changes and inflation.

The 'Units' of the Real Exchange Rate - continued

The real exchange rate is usually expressed as an index, relative to a base year.

$$\text{Spot (Real, t)} = \frac{\text{Spot (Nominal, t)}}{\text{Spot (PPP, t)}}$$

For example, if today's spot exchange rate is \$0.60/DM and the PPP spot rate is \$0.50, the real exchange rate is 1.20 or 120. At this rate, the DM is 'overvalued' on a PPP basis. A German could exchange 1 German good for 1.2 US goods. If today's spot exchange rate were \$0.45/DM, the real exchange rate is 0.90, or 90. At this rate, the DM is undervalued on a PPP basis.

Spot (PPP, t) is the spot exchange rate that would establish PPP; that is, the exchange rate that would offset the relative inflation between a pair of countries since the base period.

For example, assume that the nominal exchange rate in the base period was \$0.60 and that prices of US goods has risen by 8%, and that prices of German goods has risen by 4%. Then the Purchasing Power Parity exchange rate is:

$$\text{Spot (PPP,t)} = \text{Spot (Nominal, base period)} \times \frac{\text{Prices (\$,t) / Prices (\$,base period)}}{\text{Prices (DM,t) / Prices (DM,base period)}}$$

$$= \text{\$0.60/DM} \times \frac{1.08}{1.04}$$

$$= \text{\$0.6231/DM}$$

A nominal exchange rate of \$0.6231/DM would re-establish PPP in comparison to the base period. With this exchange rate, the real exchange rate remains at 1.0 or 100, and competitiveness is unchanged. Nominal exchange rates greater than \$0.6231/DM represent DM 'overvaluation' (\$ undervaluation), and nominal exchange rates less than \$0.6231/DM represent \$ 'overvaluation' (DM undervaluation).

Figure 4.3

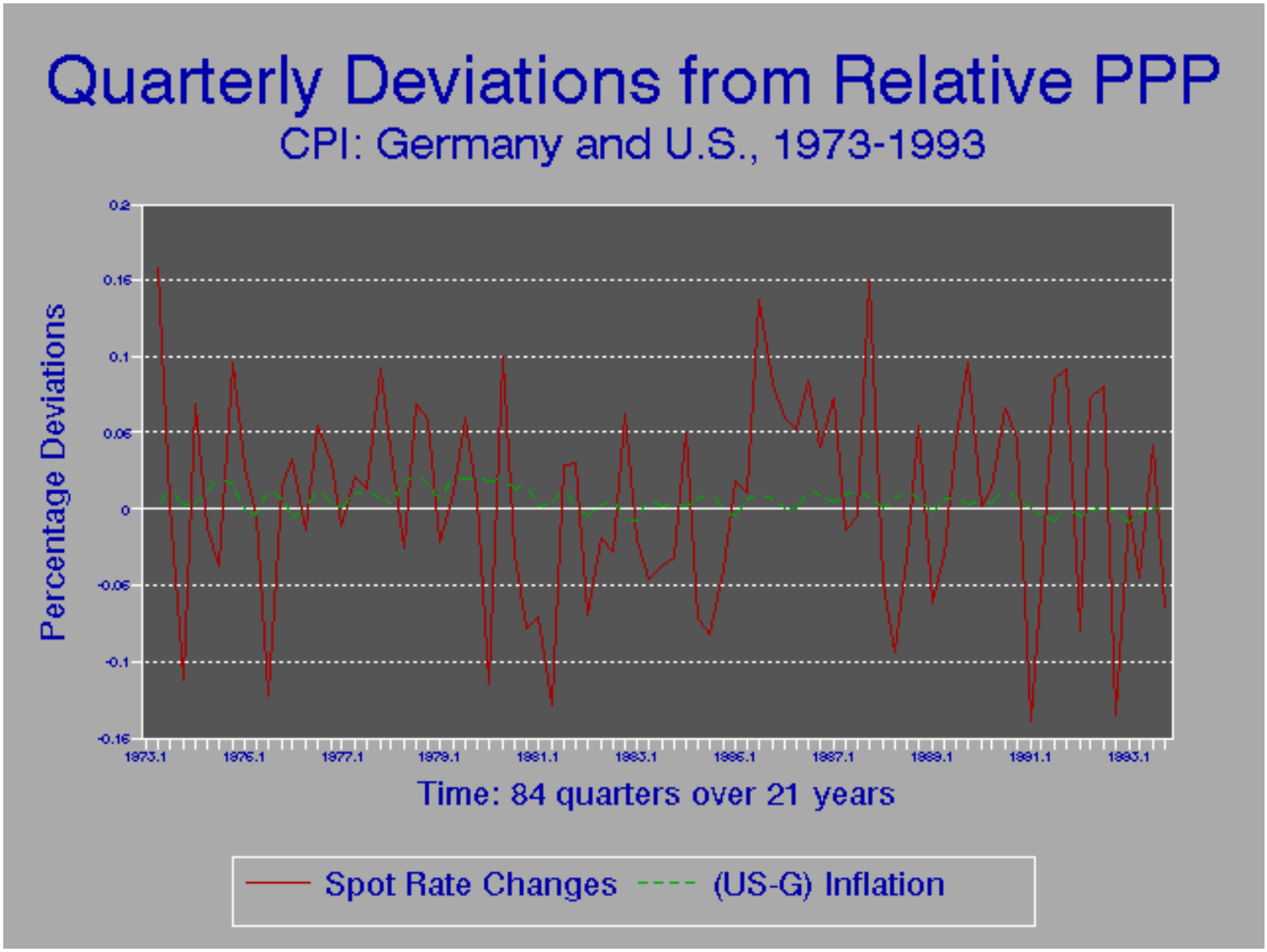

Quarterly Deviations from Relative PPP
CPI: Germany and U.S., 1973-1993
Percentage Deviations
0.2
0.16
0.1
0.06
0
-0.06
-0.1
-0.16
1973.1
1975.1
1977.1
1979.1
1981.1
1983.1
1985.1
1987.1
1989.1
1991.1
1993.1
Time: 84 quarters over 21 years
Spot Rate Changes
(US-G) Inflation

Figure 4.4

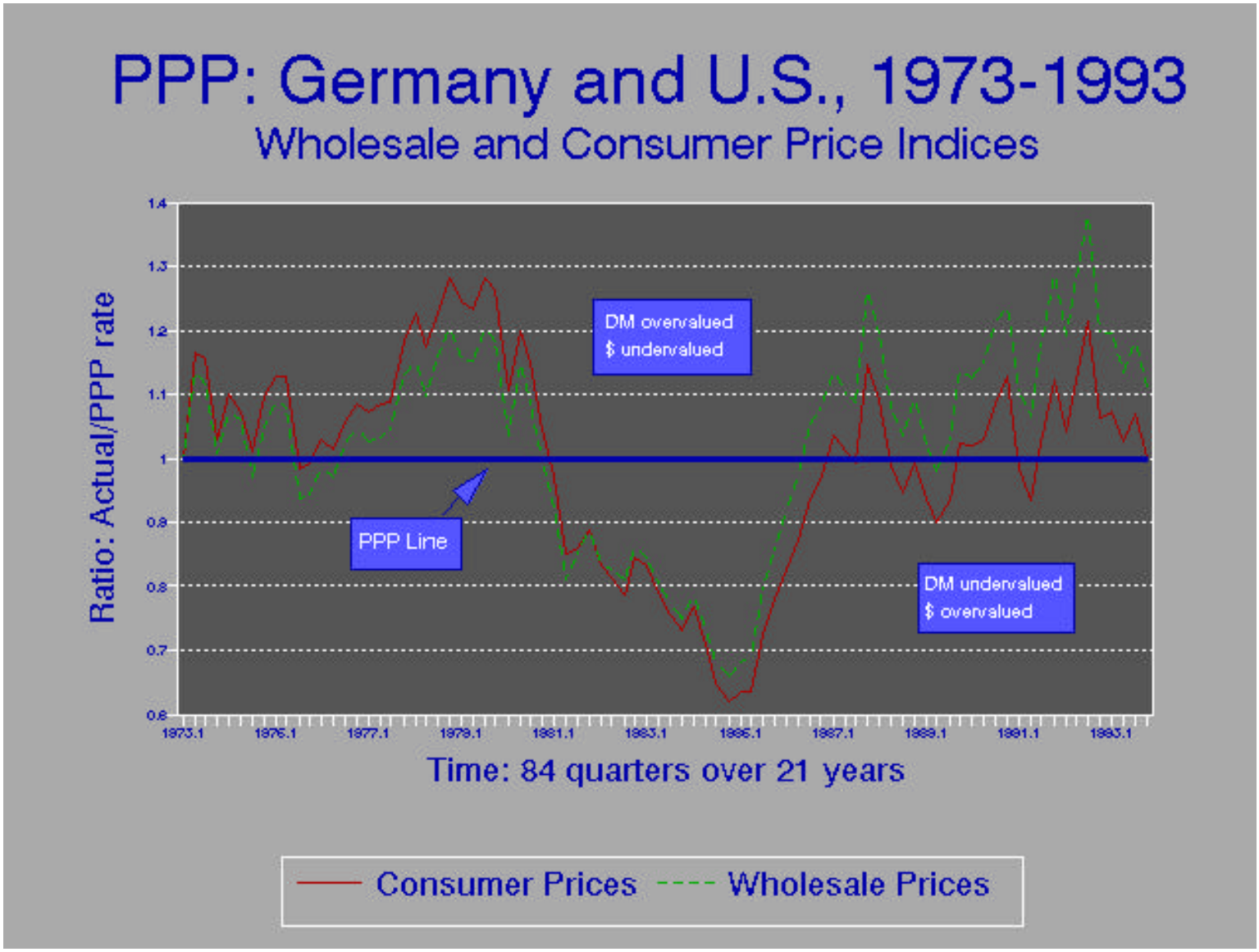

PPP: Germany and U.S., 1973-1993
Wholesale and Consumer Price Indices
Ratio: Actual/PPP rate
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
DM overvalued
$ undervalued
PPP Line
DM undervalued
$ overvalued
1973.1
1975.1
1977.1
1979.1
1981.1
1983.1
1985.1
1987.1
1989.1
1991.1
1993.1
Time: 84 quarters over 21 years
Consumer Prices
Wholesale Prices

Figure 4.3 -- Updated

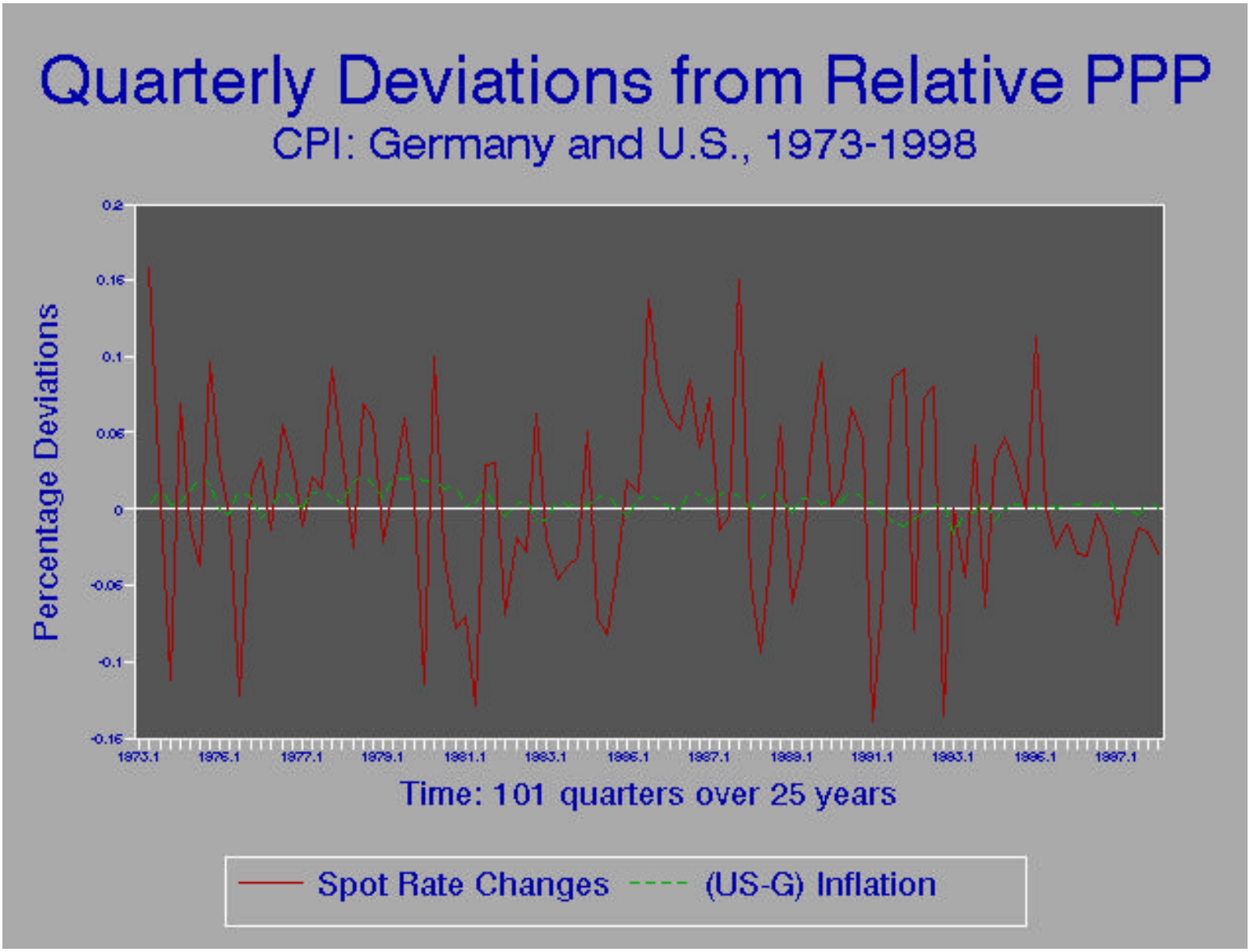
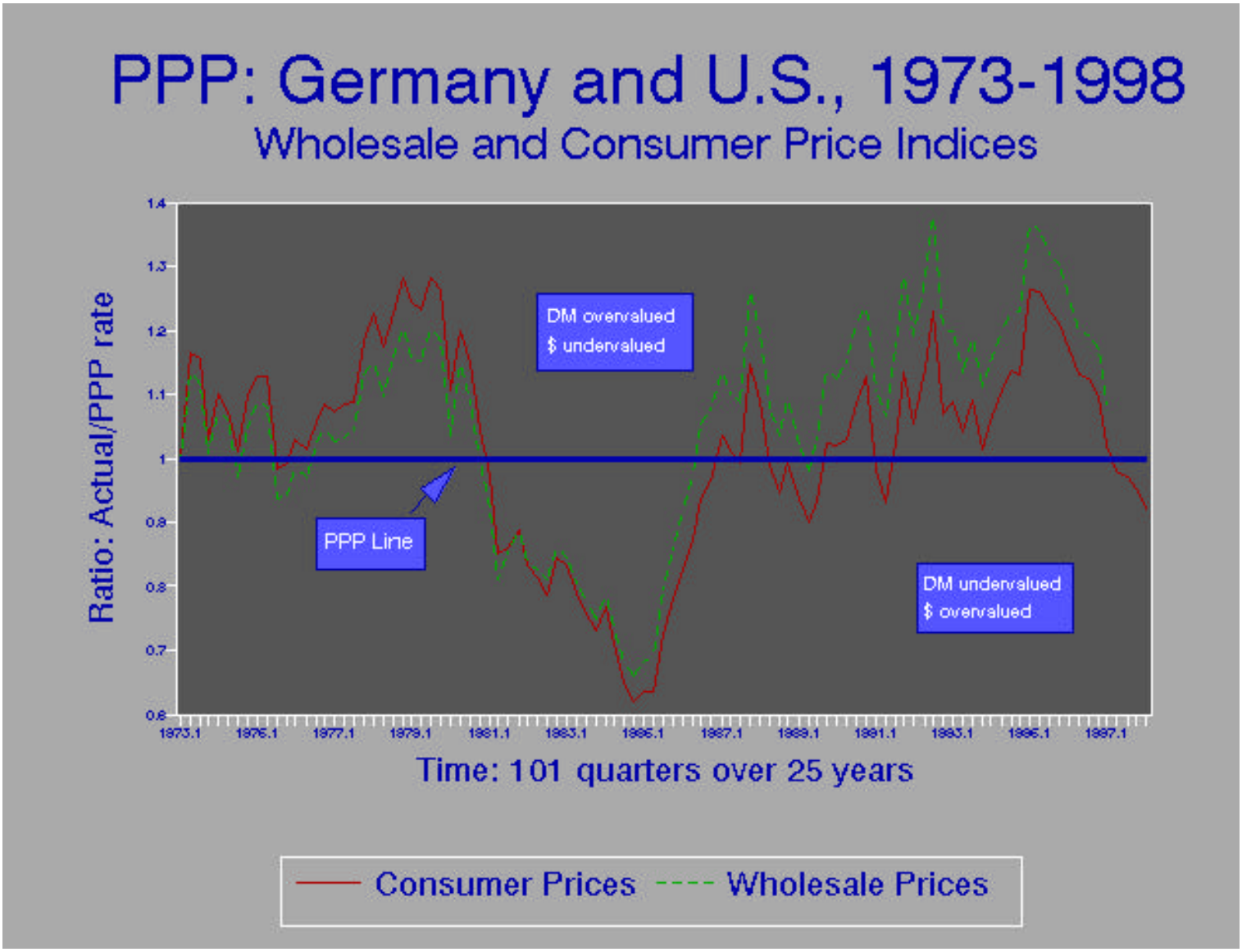
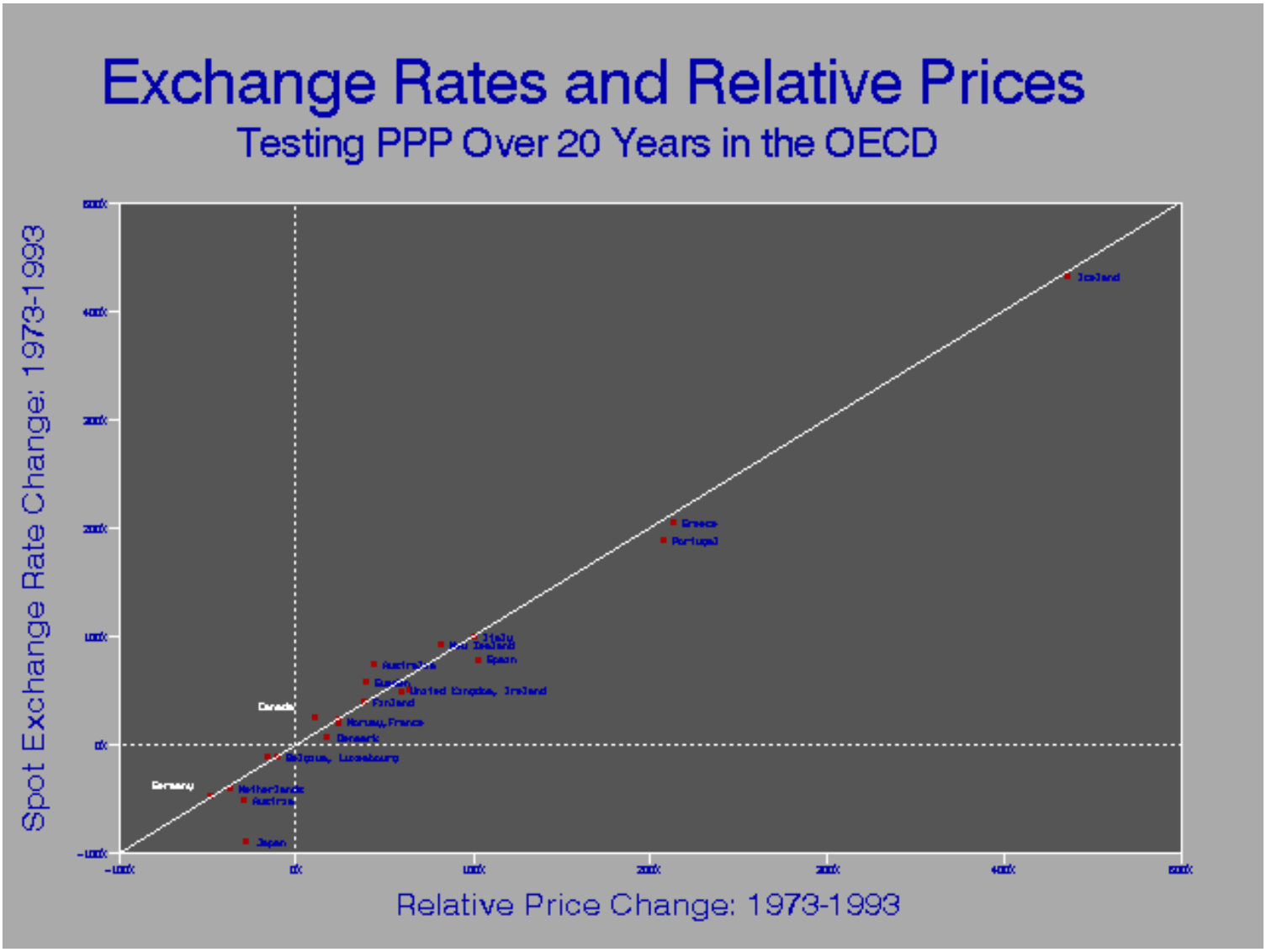


Figure 4.4 -- Updated



PPP Evidence from Long-Run Data



Looking at spot exchange rate changes and relative price changes over 20 years shows a close correspondence with relative PPP. In a cross-sectional regression with the above 22 countries, Obstfeld (1995) reports an intercept term α=-0.066 (0.193); a slope term β=1.011 (0.038); and an R^2=0.97. (Standard errors in parentheses)

Interest Rate Parity

The Basic Investment Arbitrage

Yield on a $ investment = Yield on a £ investment {*covered* against exchange risk}

$$\$1\,(1 + i[\$]) = \$1\,\frac{1}{S[\$/£]}\,(1 + i[£])\;F[\$/£]$$

$$\frac{(1 + i[\$])}{(1 + i[£])} = \frac{F[\$/£]}{S[\$/£]}$$

$$\boxed{\frac{(i[\$] - i[£])}{(1 + i[£])} = \frac{F - S}{S}}$$

% interest differential = % forward premium

example: i[$] = 12.2% for one-year
i[£] = 10.0% for one-year
S = $1.50/£, spot rate

then F = $1.53/£, one-year forward rate

This forward rate equalizes the return on a covered £ investment and a $ investment.

if F < $1.53 then $ investments have a higher yield than covered £ investments (and capital will flow from £ to $)

if F > $1.53 then covered £ investments have a higher yield than $ investments (and capital will flow from $ to £)

Interest Rate Parity - Continued

Continuous compounding of interest (removes the nuisance term in denominator)

$$\$1\, e^{(i[\$])} = \$1\, \frac{1}{S[\$/£]}\, e^{(i[£])}\, F[\$/£]$$

$$e^{(i[\$] - i[£])} = \frac{F[\$/£]}{S[\$/£]}$$

taking logarithms,

$$i[\$] - i[£] = \ln (F/S)$$

% interest differential = % forward premium

Things to be concerned about when thinking about Interest Rate Parity

How to test IRP:

Regression model: $i[\$] - i[£] = a + b \{\ln (F/S)\} + e$

Average deviation: $d = i[\$] - i[£] - \ln (F/S)$

Neutral band: $d <$ transaction cost measure

Selection of appropriate interest rates (comparable risk)

Impact of taxes on interest earnings and capital gains (losses)

How to modify IRP for periods longer (or shorter than) one year

How to modify IRP for non-zero coupon investments

Figure 5.1

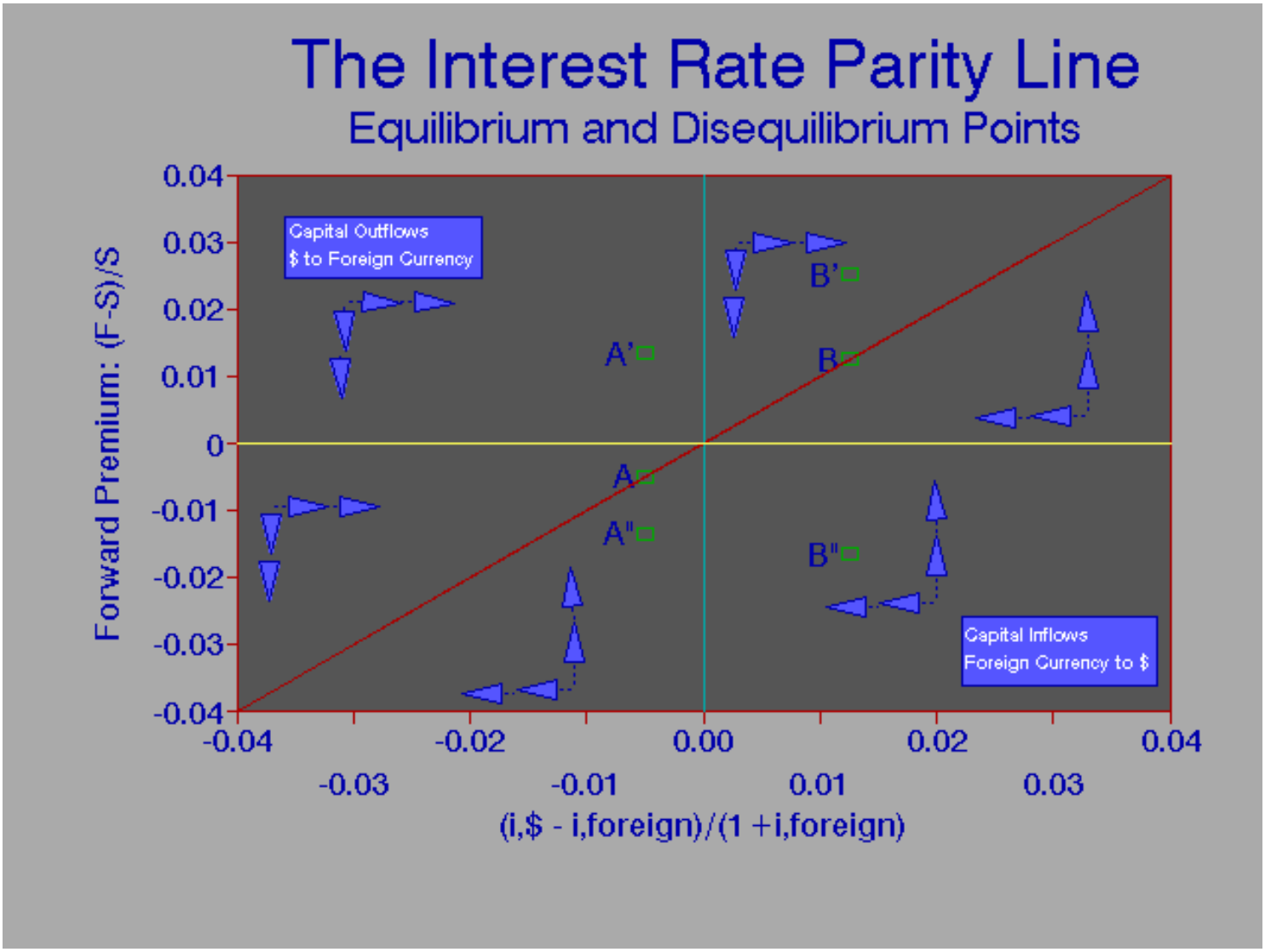

The Interest Rate Parity Line
Equilibrium and Disequilibrium Points
Forward Premium: (F-S)/S
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
Capital Outflows
$ to Foreign Currency
A'
B'
B
A
A"
B"
Capital Inflows
Foreign Currency to $
-0.04
-0.02
0.00
0.02
0.04
-0.03
-0.01
0.01
0.03
(i,$ - i,foreign)/(1 +i,foreign)

Figure 5.2

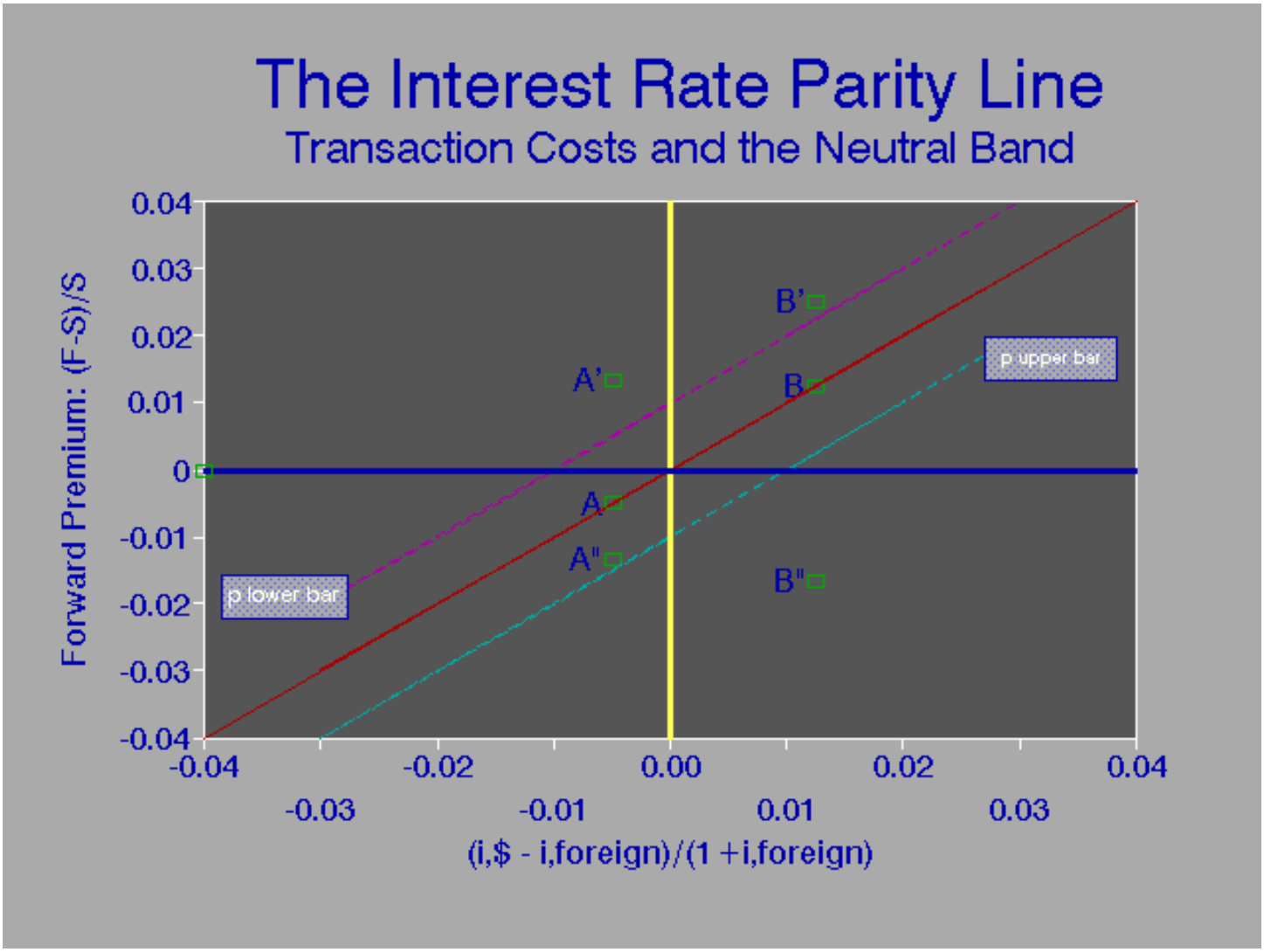
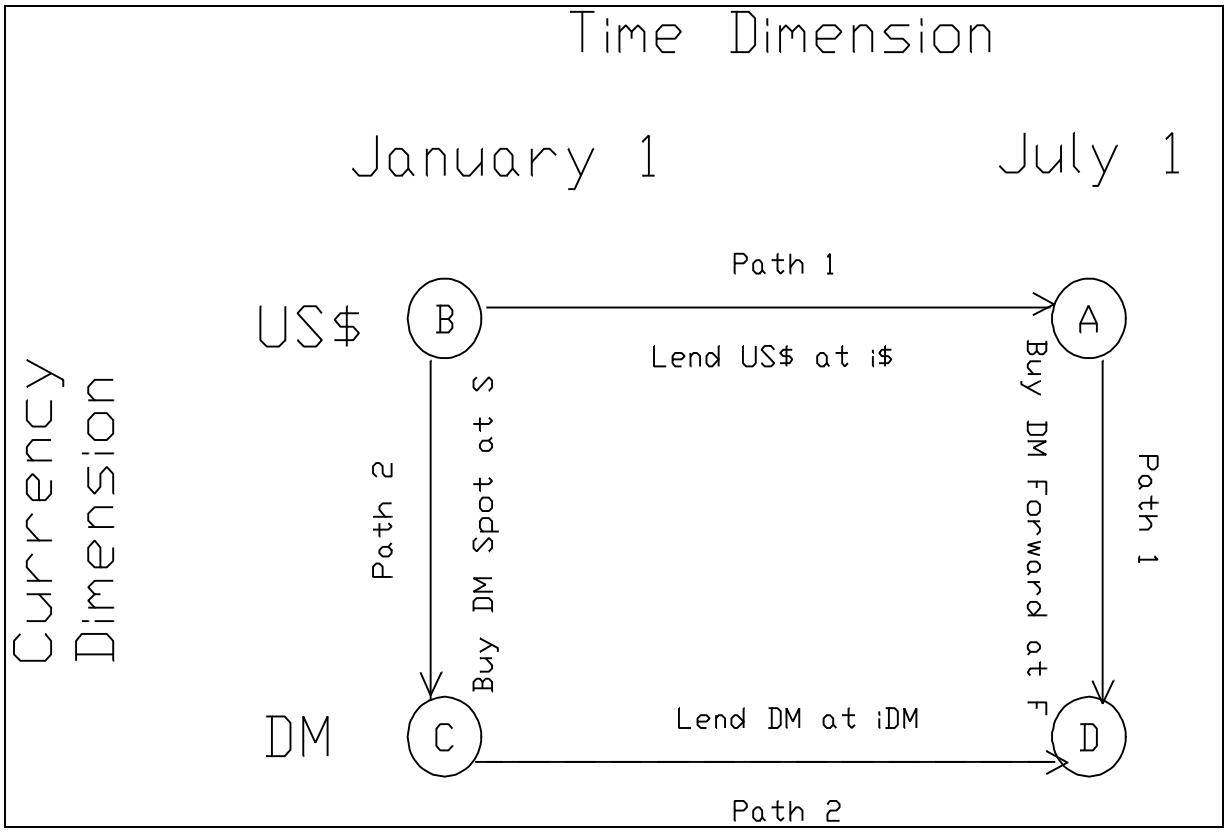

The Interest Rate Parity Line
Transaction Costs and the Neutral Band
Forward Premium: (F-S)/S
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
A'
B'
B
A
A"
B"
p upper bar
p lower bar
-0.04
-0.02
0.00
0.02
0.04
-0.03
-0.01
0.01
0.03
(i,$ - i,foreign)/(1 +i,foreign)

Figure 5.9

Example of One-Way Arbitrage in Foreign Exchange
and Security Markets



Calculate the cost of paying for future delivery of DM with present holding of US$ using two alternative paths. By taking the low cost path, the manager is engaging in "one-way arbitrage."

Path 1: Invest US$ for 6 months, buy DM forward on January 1 for July 1 delivery

Cost = $F_{\text{Jan 1, July 1}} / (1 + i_{\$, \text{ 6 months}})$

Path 2: Buy DM spot on January 1, invest DM for 6 months

Cost = $S_{\text{Jan 1}} / (1 + i_{\text{DM, 6 months}})$

The Fisher Effects

Fisher Closed ⇒ Closed economy

Nominal interest rates reflect *inflationary expectations*

i	=	r	+	E(p)
nominal interest rate	=	real interest rate	+	expected price inflation

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Fisher Open ⇒ Open economy [a.k.a. International Fisher Effect, or Uncovered Interest Parity]

Nominal interest differential reflects *exchange rate expectations*

Derivation 1: Using Fisher Closed and PPP

$$i[\$] = r[\$] + E(p[\$])$$
$$i[£] = r[£] + E(p[£])$$

subtract

$$i[\$] - i[£] = r[\$] - r[£] + E(p[\$]) - E(p[£])$$

Assume that real returns are equal: r[$] = r[£] , then

$$i[\$] - i[£] = E(p[\$]) - E(p[£])$$

Then assume that expected PPP holds. We get

$$\boxed{i[\$] - i[£] = E(s)}$$

% interest differential = % expected exchange rate change

The Fisher Effects - Continued

Derivation 2: Using the arbitrage principle, under certainty

What expected future spot rate, E(S[t+1]), will equalize the returns on a $ bond and a £ bond given their current interest rates?

$$\$1\, e^{(i[\$])} = \$1\, \frac{1}{S[t]}\, e^{(i[£])}\, E(S[t+1])$$

$$e^{(i[\$] - i[£])} = \frac{E(S[t+1])}{S[t]}$$

taking logarithms,

$$i[\$] - i[£] = \ln (E(S[t+1])/S[t]) = E(s)$$

% interest differential = % expected exchange rate change

Things to be concerned about when thinking about the International Fisher Effect

How to test Uncovered Interest Parity:

Regression model: $i[\$] - i[£] = a + b \{E(S)\} + e$

Average deviation: $d = i[\$] - i[£] - E(s)$

Serial correlation of deviation: $Corr (d[t] , d[t \pm k])$

How to modify for periods longer (or shorter than) one year

How to modify for coupon paying investments

The break-even exchange rate, E(S[t+1]), assumes that the exchange rate moves at a linear trend given by the interest differential.

How to modify if investors require a risk premium on investments

How to modify if investments in a particular country offer non-cash benefits, such as political safe-haven, investor anonymity, etc.

Deviations from Uncovered Interest Parity [Box 5.1]

How to Pay Very High Interest on $ Borrowing

Example: First Quarter of 1973 *[With Euro-$ interest rate at 5.875%]*

Borrow £ 425,188.15 on January 5, 1973

(at 2.3519 $/£ this is $1,000,000)

Pay interest of 2.25% per quarter (or 9.00% per annum)

Pay back £ 434,754.88 on March 30, 1973

(at 2.4755 $/£ this is $1,076,235.70)

Cost of $ funds

= ln (1,076,235.70 / 1,000,000) = 7.35% per quarter

= 29.39% per annum

How to Pay Very Low Interest on $ Borrowing

Example: Fourth Quarter of 1973 *[With Euro-$ interest rate at 10.375%]*

Borrow DM 2,414,292.61 on September 28, 1973

(at 0.4142 $/DM this is $1,000,000)

Pay interest of 1.53% per quarter (or 6.125% per annum)

Pay back DM 2,451,261.47 on December 28, 1973

(at 0.3702 $/DM this is $907,457.00)

Cost of $ funds

= ln (907,457.00 / 1,000,000) = -9.71% per quarter

= -38.84% per annum

Figure 5.5

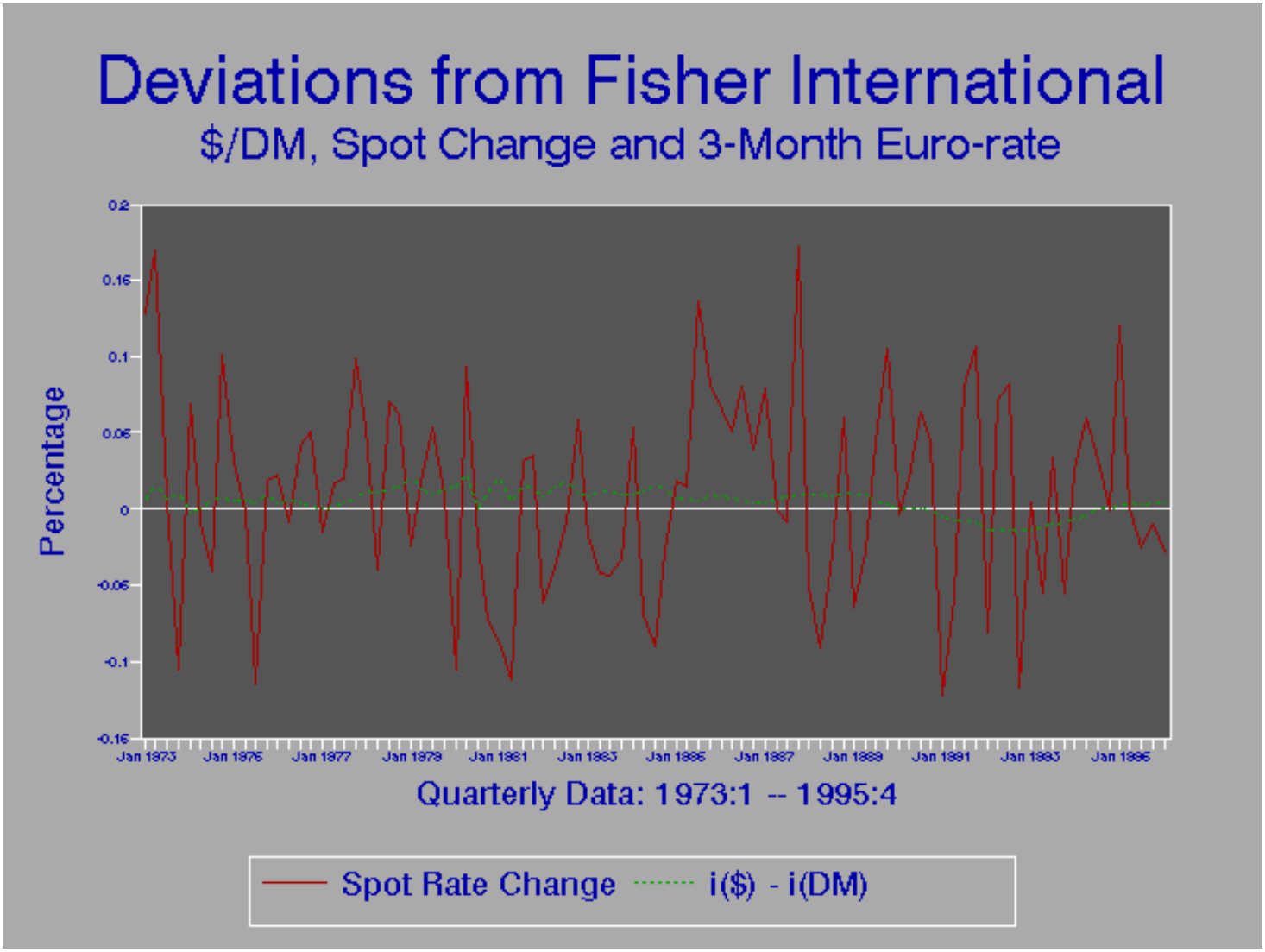
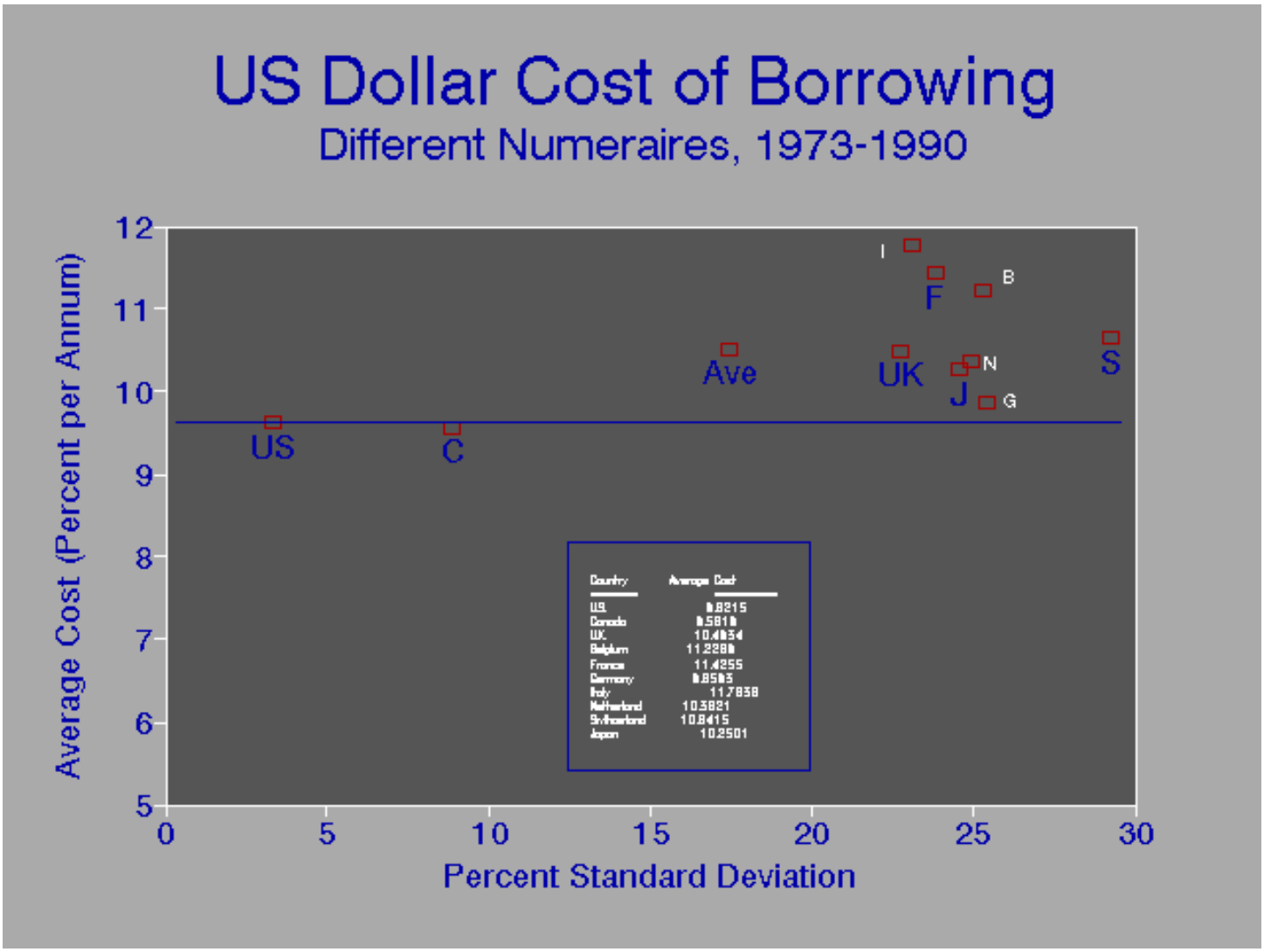

Deviations from Fisher International
$/DM, Spot Change and 3-Month Euro-rate
Percentage
0.2
0.16
0.1
0.05
0
-0.05
-0.1
-0.16
Jan 1973
Jan 1975
Jan 1977
Jan 1979
Jan 1981
Jan 1983
Jan 1985
Jan 1987
Jan 1989
Jan 1991
Jan 1993
Jan 1995
Quarterly Data: 1973:1 -- 1995:4
Spot Rate Change
i($) - i(DM)

Figure 5.10



Note:US$ cost of borrowing is computed as the cost of borrowing in foreign currencies on an uncovered basis using 3-month Eurocurrency interest rates. Average and standard deviation are computed over 72 quarterly observations. "Ave" represents the average result from an evenly weighted portfolio of all nine currencies.

Figure 5.6

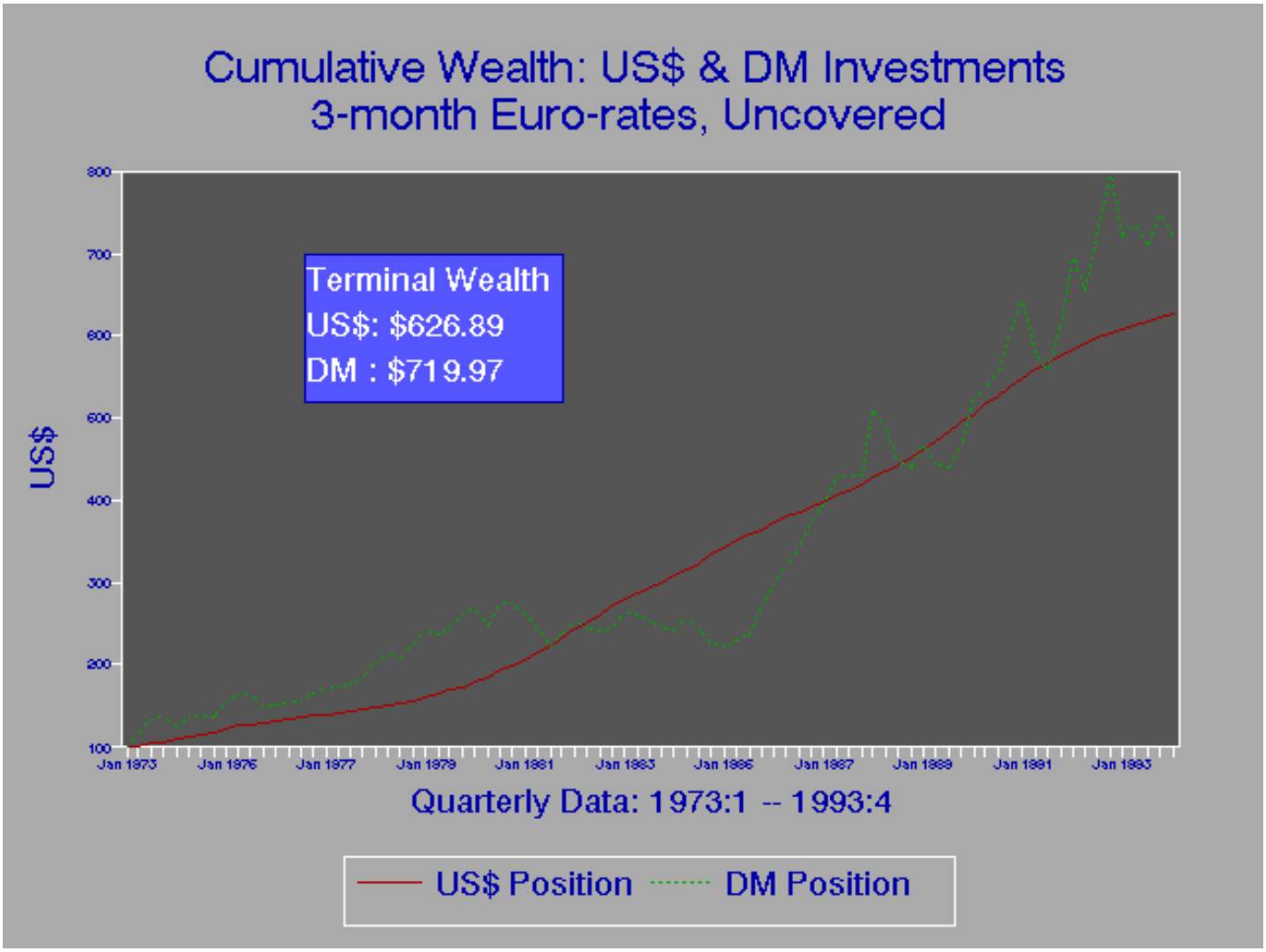

Cumulative Wealth: US$ & DM Investments
3-month Euro-rates, Uncovered
Terminal Wealth
US$: $626.89
DM : $719.97
US$
800
700
600
500
400
300
200
100
Jan 1973
Jan 1975
Jan 1977
Jan 1979
Jan 1981
Jan 1983
Jan 1985
Jan 1987
Jan 1989
Jan 1991
Jan 1993
Quarterly Data: 1973:1 -- 1993:4
US$ Position
DM Position

Figure 5.6 -- Updated

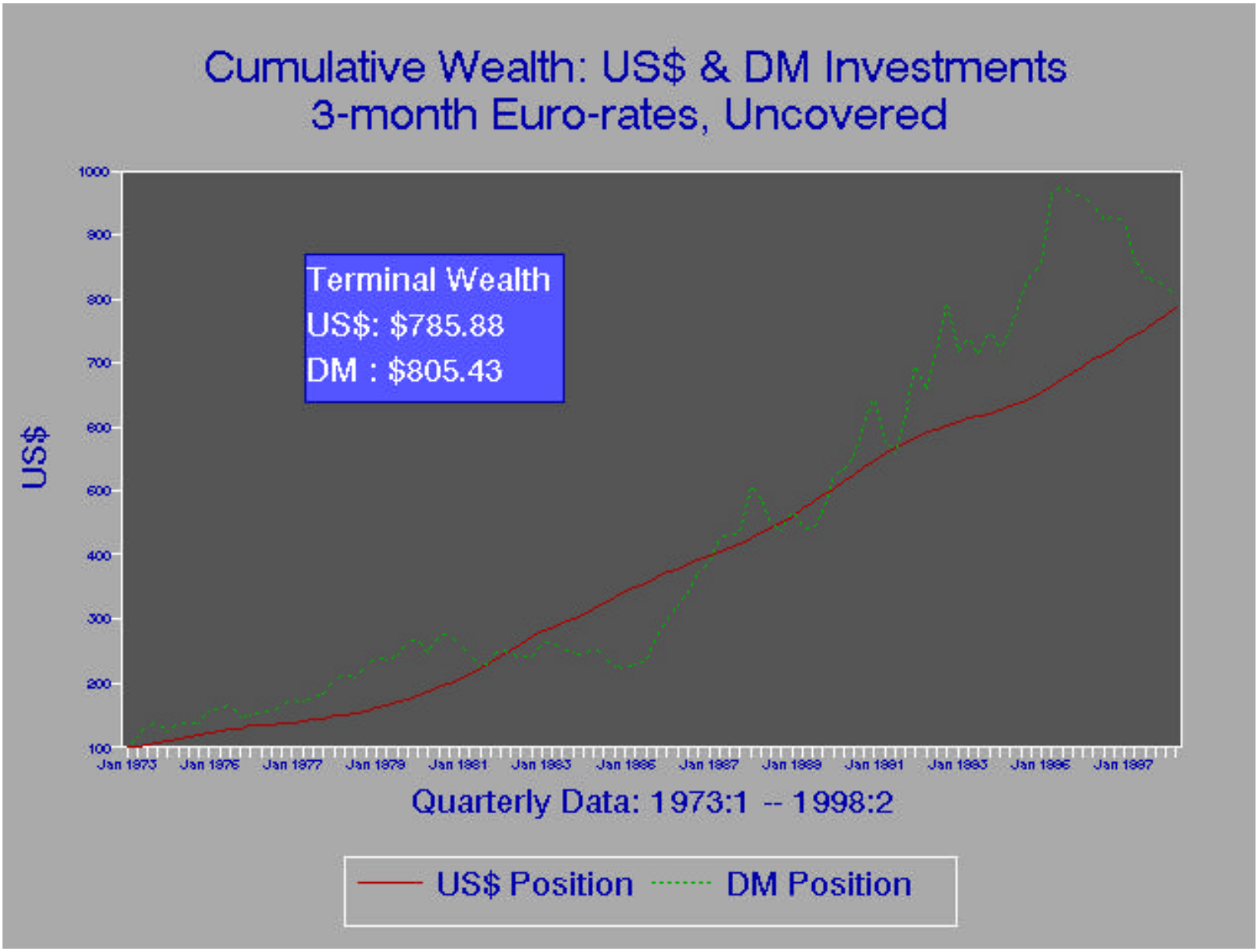
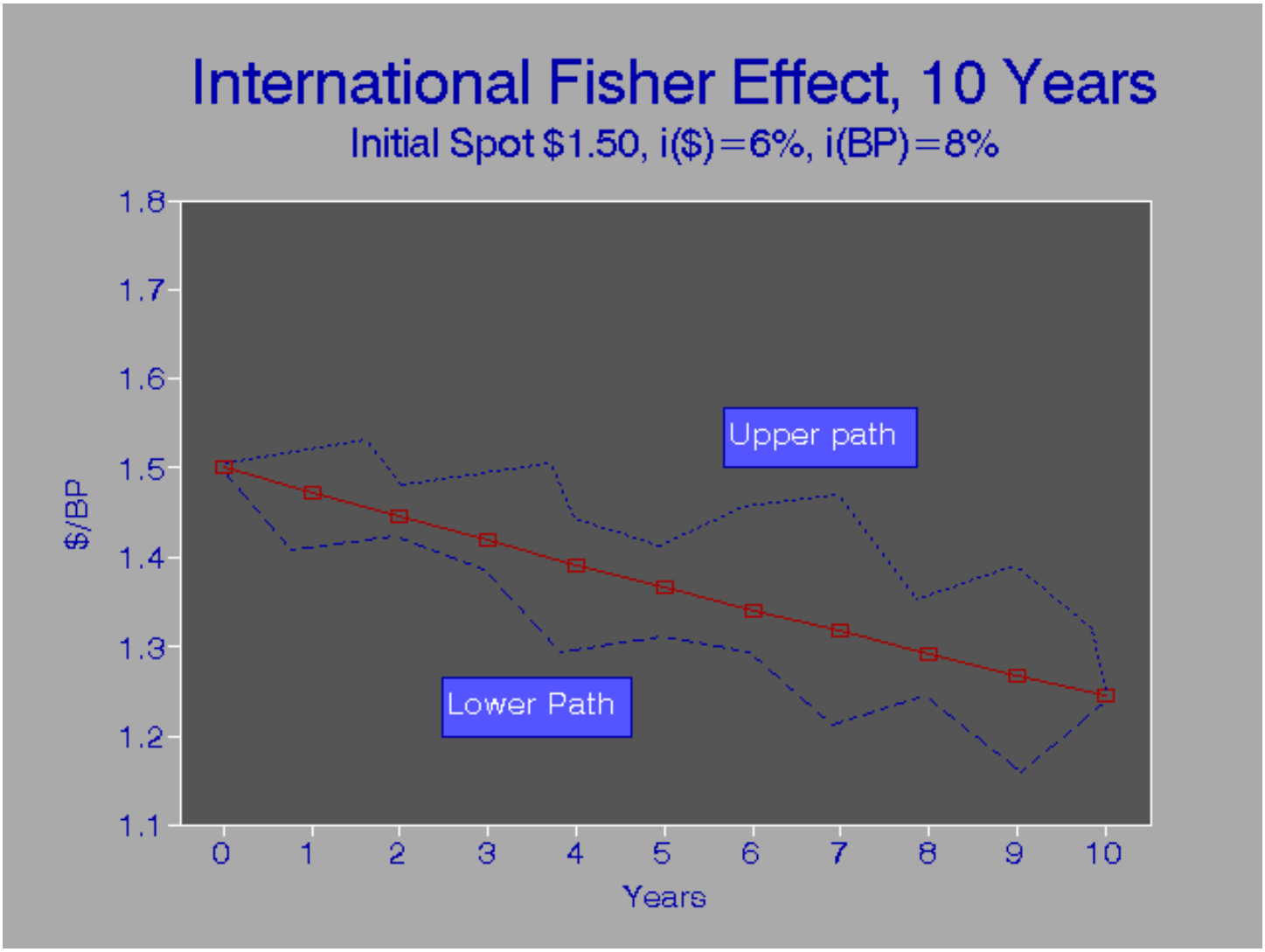


Figure 5.11



Note: The path of the exchange rate according to the International Fisher Effect is $E(S_{t+n}) = S_t [(1+i_{\$})/(1+i_{£})]^n$. The other paths are hypothetical, but constrained to have the same starting and ending values, and the same average rate of change over the 10 year period.

Forward Rate Unbiased

Given the initial assumptions of (1) No transaction costs, (2) No taxes, and (3) Complete certainty, the forward rate unbiased principle follows directly from Interest Rate Parity and the International Fisher Effect.

$$\frac{F[t] - S[t]}{S[t]} = \frac{E(S[t+1]) - S[t]}{S[t]}$$

% Forward Premium = % Expected Exchange Rate Change

If the average deviation between the today's forward rate and the future spot exchange rate is small and near zero, we say that the forward rate is an unbiased predictor of the future spot rate.

Unbiased prediction relies on two assumptions:

(1) Market Efficiency: $E(S[t+1]) = S[t+1]$

(2) Forward Pricing: $F[t] = E(S[t+1])$

(1) implies that the market can generate unbiased expectations

(2) implies that forward prices are set based on expectations

If the forward rate is a biased predictor of the future spot rate, it may be because

(1) The market is inefficient $\Rightarrow$ Pure profit opportunities

(2) Forward prices reflect a risk premium $\Rightarrow$ Profits from forward speculation may only reflect a fair return for risks incurred.

How to Test Forward Unbiased:

Regression model: $(S[t+1] - S)/S = a + b\{(F-S)/S\} + e$

Average deviation: $d = (S[t+1] - S)/S - (F-S)/S$

Serial correlation of deviation: $Corr(d[t], d[t \pm k])$