Microeconomics
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Exercise Set 4. "Voluntary Contributions"

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(due October 4)

1. A certain firm has two divisions, which are relatively autonomous profit centers. The two divisions are invited by the HQ to make voluntary contributions of personnel to a company project that will produce additional revenue to the company as a whole. Let A and B be the respective contributions of the two divisions. (These can be any nonnegative numbers.) The gross revenue from the project will be

$$Y = 4(A + B).$$

This revenue will be shared equally by the 2 divisions. The cost to Division 1 (i.e., loss in profit) of contributing A days of employee time will be A^2, and the corresponding cost to Division 2 will be B^2. Therefore, the respective net benefits to the 2 divisions will be

$$M(A,B) = 2(A + B) - A^2,$$

$$N(A,B) = 2(A + B) - B^2.$$

All the data are known to both divisions. Formulate the situation as a game between the divisions, and characterize the Nash equilibrium contributions of the two divisions.

2. Is the equilibrium (or equilibria) you described in your answer to Exercise 1 (Pareto) efficient? Explain your answer

3. Extend the analysis of Exercises 1 and 2 to cover the case in which Division 1 gets a share sY of the gross output and Division 2 gets (1-s)Y, where s is some given fraction between 0 and 1.