

Exercise Set 5
October 4, 2006 (due October 11)

Two firms each produce pollution, from which both firms suffer damage costs. If, in a given period, firm i produces an amount a_i of pollution ($i = 1, 2$), then each firm incurs a damage cost equal to $b(a_1 + a_2)^2$. The "normal" amount of pollution produced by each firm is p , but a firm can reduce its pollution output by an amount $(p - a)$ at a cost $c(p - a)^2$. Thus if firm i produces an amount of pollution equal to a_i , its total pollution-related cost is

$$C^i(a_1, a_2) = b(a_1 + a_2)^2 + c(p - a_i)^2.$$

[For the purpose of this exercise, take all other costs and revenues of the firms as fixed.] Assume that the parameters b, c, p are > 0 , and that the amount of pollution produced by each firm must be nonnegative.

1. Formulate this situation as a game, and calculate a Nash equilibrium (NE) of the game. Is it unique?
2. Is the NE that you found in Part 1 Pareto-efficient, from the point of view of the 2 firms? Give a proof of your answer.
3. Suppose that this game is repeated. Find an equilibrium of the repeated game that is Pareto-superior (from the point of view of the two firms) to your equilibrium of Part 1. Make whatever further assumptions you need for your analysis.