Microeconomics R. Radner 11/01/2006

Exercise Set 8 (due 11/08/06) Learning from a Prototype

General Setting. A firm is about to try to develop a new software package. The development project is either feasible (G) or is is not (B). If the project is undertaken and is feasible, then the firm will make a profit of \$90 million; if it is undertaken and turns out not to be feasible, then it will lose \$10 million. A priori, the probability of feasibility is 1/2.

One option in the development process is to try first to build a prototype. For the protopye, the possible outcomes are success (G) and failure (B). If the project is feasible, then prototype is sure to be successful, i.e., the conditional probability that the the prototype is successful is 1. On the other hand, if the project is infeasible, then the the conditional probability that the prototype is a failure is c, where 0 < c < 1. In other words, the prototype might "succeed" even though the project is infeasible. The cost of trying to build the prototype equals \$3 million. After building the prototype, the firm decides, depending on the prototype outcome, whether to try to complete the development project. Assume that if the firm decides to try to complete the project, then its subsequent *additional* profit or loss will be the same as in the first paragraph. If the firm decides not to try to complete the project, it loses the \$3 million that it spent on the prototype.

[Assume that the firm is risk-neutral.]

1. Derive a formula for the conditional probability that the project is feasible given that the prototype is successful, as a function of the parameter c. Give an intuitive interpretation of your formula.

2. Given that the prototype is built, derive the optimal decision rule for deciding whether to try to complete the project, and describe how it depends on the parameter c.

3. Derive a formula for the value of building the prototype, as a function of the parameter c. For what values of c should the prototype be built?