An American Call IS Worth More than a European Call:

The Value of American Exercise

When the Market is Not Perfectly Liquid

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A Familiar Classroom Demonstration

An American Call on a Non-Dividend Paying Stock

- Should never be exercised early
- Is therefore worth the same as a European call

To liquidate an American call position early, don't exercise

- sell it in the market for its fair value (the European call price)
- if there is no options market, sell short the stock and delta-hedge until expiration

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But in the Real World...

...the market's bid price for an in-the-money option is very rarely at fair value, and is usually lower than the option's intrinsic value.

Example: Exxon-Mobil, Monday, April 24, 2017, 1:57 P.M.

	BID	ASK	S-X	EBV
Exxon stock	81.16	81.17		
XOM MAY 75-strike Call (25 days)	5.85	6.40	6.16	0.31
XOM JUN 75-strike Call (53 days)	5.90	6.45	6.16	0.26

These bid prices are exercise boundary violations (EBVs).

Exercise is the only way to liquidate an American call position and recover even intrinsic value. (You still lose the option's time value.)

Unfortunately, EBVs in equity options are the norm, not the exception.

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What Can a Trader Do When the Best Bid is Too Low?

1. Sell in the market at C_{Bid}, below the intrinsic value (S – X)

2. Exercise (receive S – X)

3. Hold the option but exit the position synthetically by delta hedging to expiration.

In theory, the third strategy returns the full value C_{EUR}.

But...

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What Can a Trader Do When the Best Bid is Too Low?

...in practice, delta hedging to expiration requires

- borrowing and selling short the stock,
- paying transactions costs for hedge rebalancing
- earning interest on the short sale proceeds
- extra costs and risk if the stock is "on special"

Jensen and Pedersen (JFE 2016) show that this alternative is expensive and is easily dominated by early exercise. Using closing prices, they show that replication costs are frequently high enough that early exercise is rational, but they can't look at intraday bid quotes directly.

Selected Literature

Option Pricing Evidence (boundary violations, American option pricing): Bhattacharya (1983); Diz and Finucane (1993); Finucane (1997); Valkanov, Yadav, and Zhang (2011); and many others

Early Exercise (is irrational): Poteshman and Serbin (2003); Pool, Stoll, and Whaley (2008); Barraclough and Whaley (2012)

Early Exercise (might be rational): Jensen and Pedersen (2016)

Transactions Costs

Transactions costs differ widely between market makers and retail investors, and across retail brokerage firms.

But trading costs for "discount" retail brokers are typically well below levels that would make exercise unprofitable, or subject to significant timing risk.

Example: Interactive Brokers (a large, low-cost online brokerage)

- Option trades: $0.70 per contract
- Option exercise: No charge; stock is available in account immediately
- Stock trades: $0.005 per share plus $2.18 per $100,000 of trade value

IB's commission to exercise and liquidate an option contract on a $100 stock is

= 100 (0.005) + 10,000 (2.18/100,000)

= $0.72, less than 1 cent per share

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Results from Battalio, Figlewski and Neal (2017)

Battalio, Figlewski and Neal (2017). "Option Investor Rationality Revisited: The Role of Exercise Boundary Violations."

Examined intraday data for all single stock call and put option bid and ask quotes during March 2010.

- all in the money contracts
- ex-dividend days excluded – no "rational" call exercises in the sample
- intraday at one-minute intervals
- ~125,000 option contracts; ~670 million observations

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Results from Battalio, Figlewski and Neal (2017)

Battalio et al. find that

Early exercise boundary violations (EBVs) are

- common
- large
- persistent
- some investors trade options at EBV prices (~$39 million left on the table in a single month)
- some rationally exercise American options early, both calls and puts

Exercise Boundary Violations are Common and Large

Note: Deep ITM := X/S < 0.7; Mid ITM := 0.7 < X/S < 0.9; Near ITM := 0.9 < X/S < 1.0

Example: Consider a call with X = 25.

Deep ITM: S > 35.71
Mid ITM: 27.77 < S ≤ 35.71
Near ITM: 25.00 < S ≤ 27.77

Excerpt from Table 1 (Calls)

Maturity:	Nearby	Nearby	Nearby	2 - 4 Months	2 - 4 Months	2 - 4 Months
Moneyness:	Deep ITM	Mid ITM	Near ITM	Deep ITM	Mid ITM	Near ITM
Total Quotes	24,233,775	39,210,622	20,603,553	41,276,612	62,774,801	34,708,238
Total EBV Quotes	23,925,783	36,374,930	7,538,395	38,676,453	34,011,298	1,804,803
% Quotes with EBV	98.73%	92.77%	36.59%	93.70%	54.18%	5.20%
% Options with EBV	99.00%	98.21%	57.40%	98.13%	73.15%	12.67%

The XOM calls (~7.5% ITM) above would fall into these two categories:

Exercise Boundary Violations are Common and Large

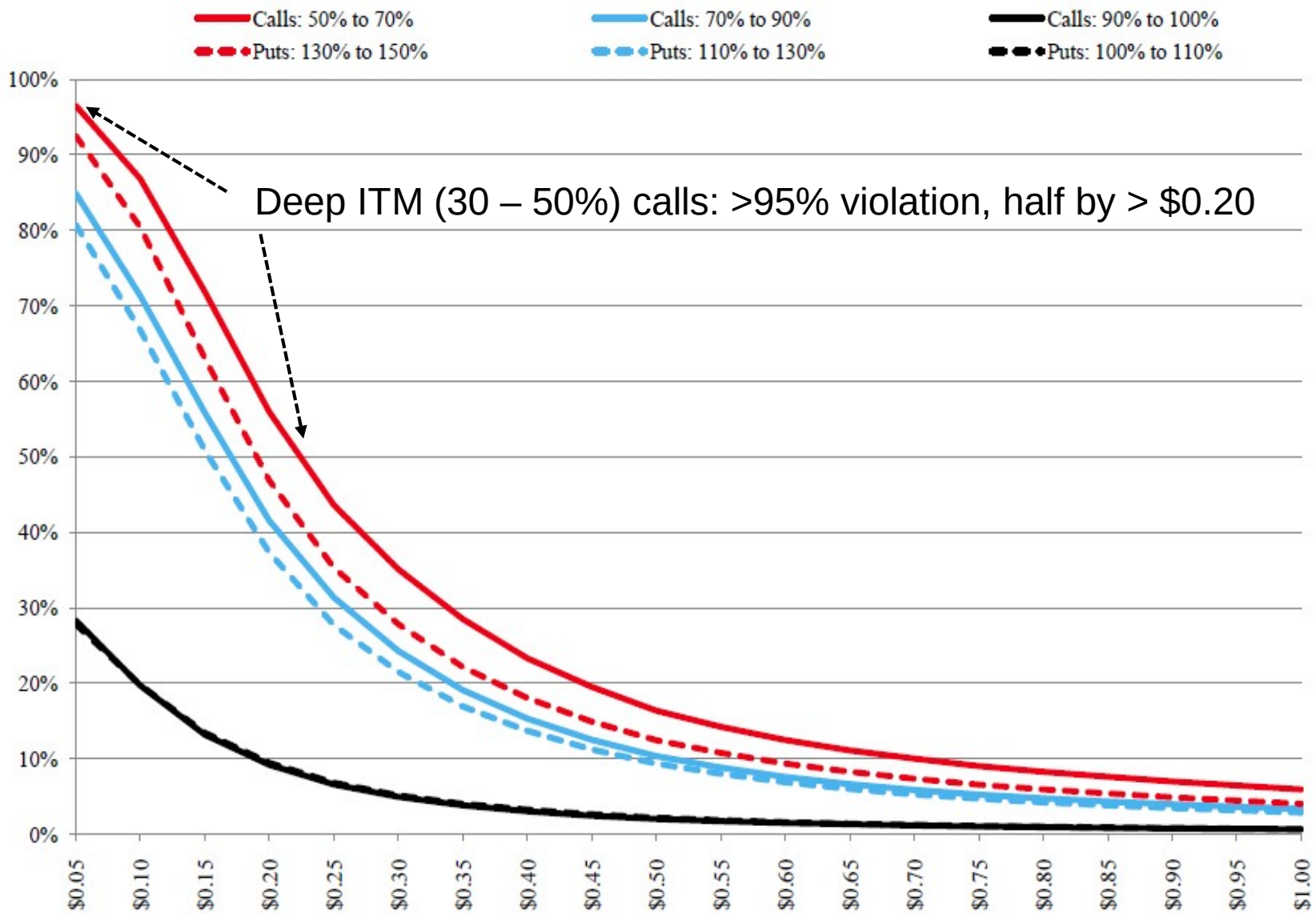
Note: Deep ITM := X/S < 0.7; Mid ITM := 0.7 < X/S < 0.9; Near ITM := 0.9 < X/S < 1.0

Excerpt from Table 1 (Puts)

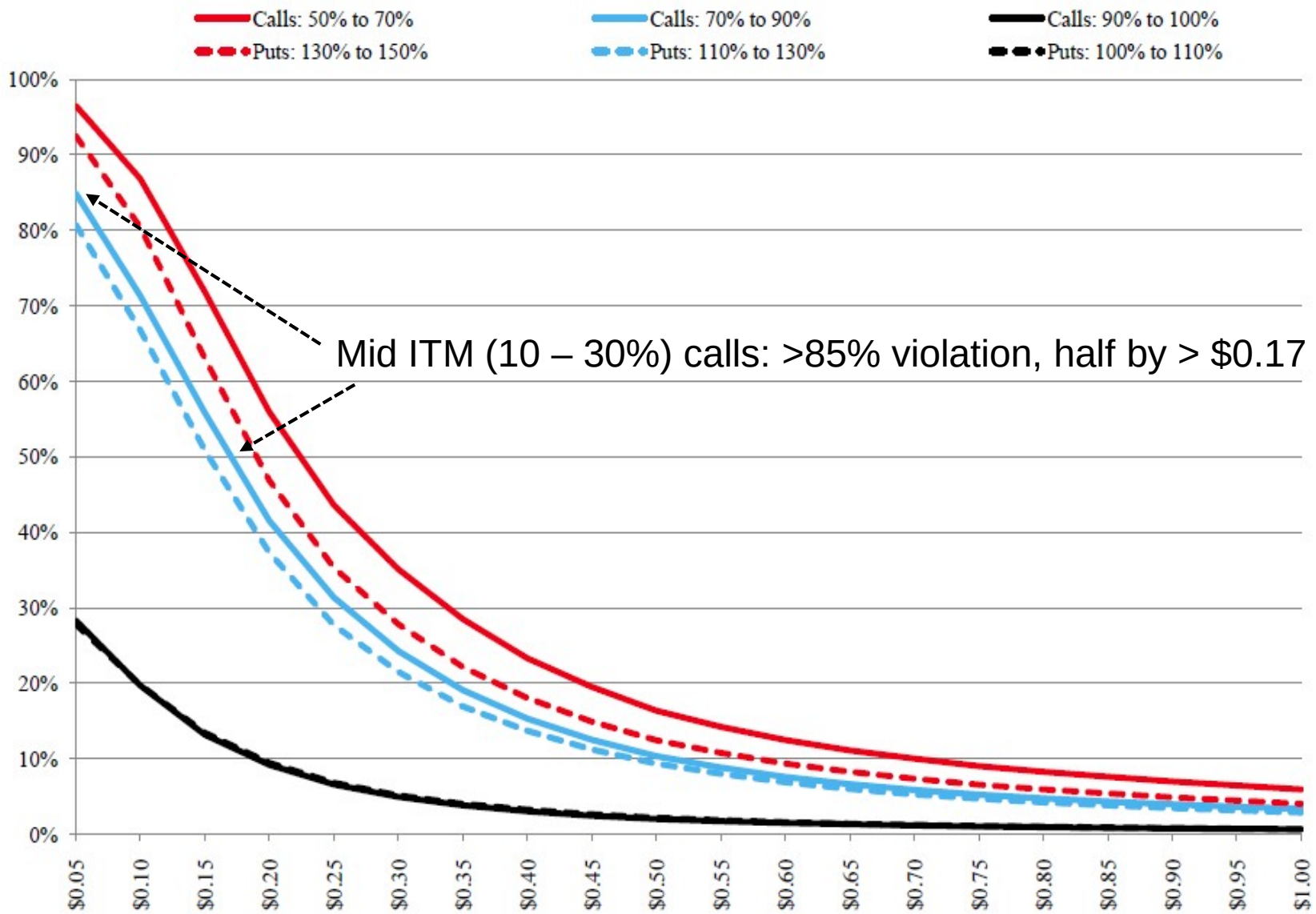
Maturity:	Nearby	Nearby	Nearby	2 - 4 Months	2 - 4 Months	2 - 4 Months
Moneyness:	Deep ITM	Mid ITM	Near ITM	Deep ITM	Mid ITM	Near ITM
Total Quotes	12,146,014	27,155,723	19,520,667	22,905,184	46,400,832	33,022,460
Total EBV Quotes	11,788,040	24,348,361	7,062,711	17,565,888	19,510,454	1,386,092
% Quotes with EBV	97.05%	89.66%	36.18%	76.69%	42.05%	4.20%
% Options with EBV	99.27%	97.14%	55.96%	90.09%	62.73%	10.35%

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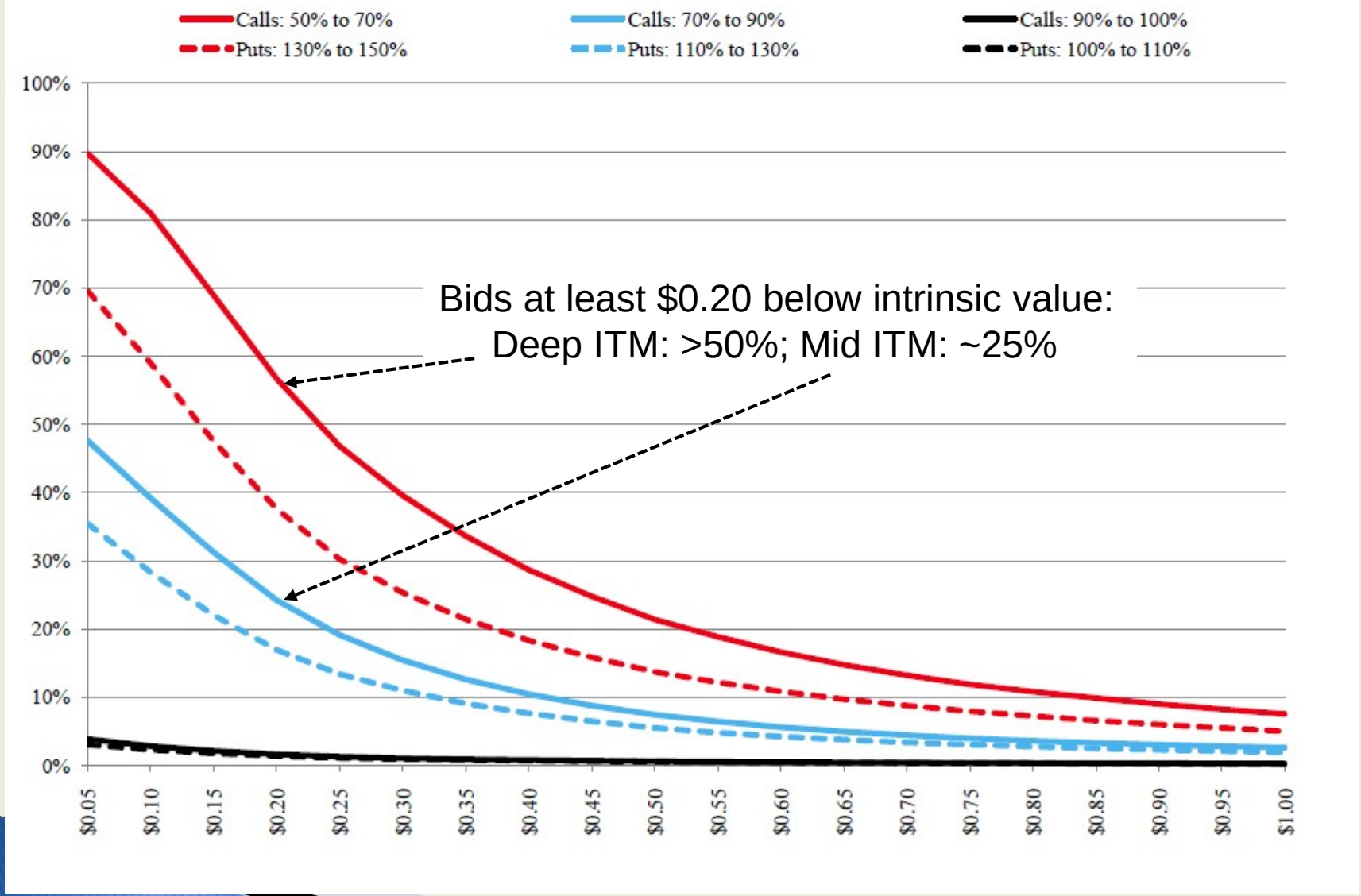
Percentage of Quotes below Intrinsic Value Nearby Maturity



Percentage of Quotes below Intrinsic Value Nearby Maturity



Percentage of Quotes below Intrinsic Value
2-4 Months to Maturity



Exercise Boundary Violations are Persistent

Can one avoid an EBV by patiently waiting for an opportunity intraday?

(excerpt from Table 2: Duration of Boundary Violations)

Maturity:	Nearby			2 - 4 Months		
Moneyness:	Deep ITM	Mid ITM	Near ITM	Deep ITM	Mid ITM	Near ITM
Panel (a): Calls						
Option Days	64,497	104,737	55,087	110,416	167,730	92,759
10:00am EBV Frequency	99.56%	93.43%	35.77%	93.96%	53.51%	4.92%
Avg Size of 10:00am EBV	$0.38	$0.29	$0.19	$0.40	$0.30	$0.28
Avg Duration of EBV	348	297	189	313	235	157
Panel (b): Puts						
Option Days	32,804	72,812	52,309	61,878	124,258	88,316
10:00am EBV Frequency	97.31%	90.46%	36.46%	77.41%	42.65%	4.35%
Avg Size of 10:00am EBV	$0.33	$0.29	$0.20	$0.34	$0.29	$0.30
Avg Duration of EBV	335	296	196	274	216	154

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Liquidity-Based American Call Early Exercise Premium

Consider an American call that must be liquidated before expiration, on a specific future date $t < T$.

Assume the price in the options market is the Black-Scholes value minus half of the bid-ask spread of 2B.

- Selling at date t yields: $C_{BS}(S_t,X,T-t) – B$.
- An American call can be exercised for: $S_t - X$

It is possible to replicate the payoff of the optimal sell or exercise decision with a portfolio of optional contracts that can all be priced at date 0.

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American Call Early Exercise Premium

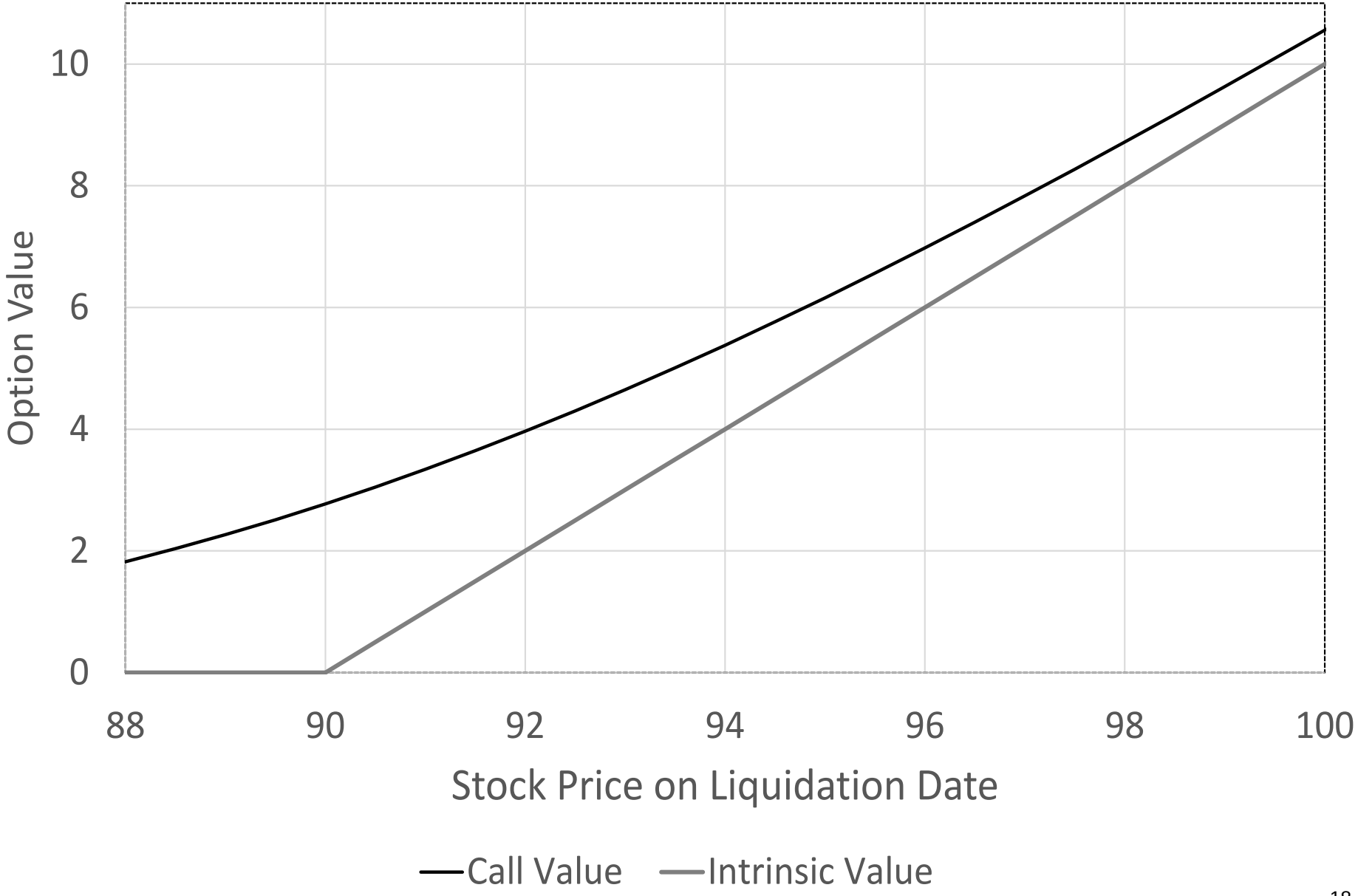
$C_{BS}(S,X,T)$ is the date 0 Black-Scholes price for a European call maturing at date T.

There is a level of the date t stock price S_t^* such that the proceeds from selling at the market's bid price or exercising are the same.

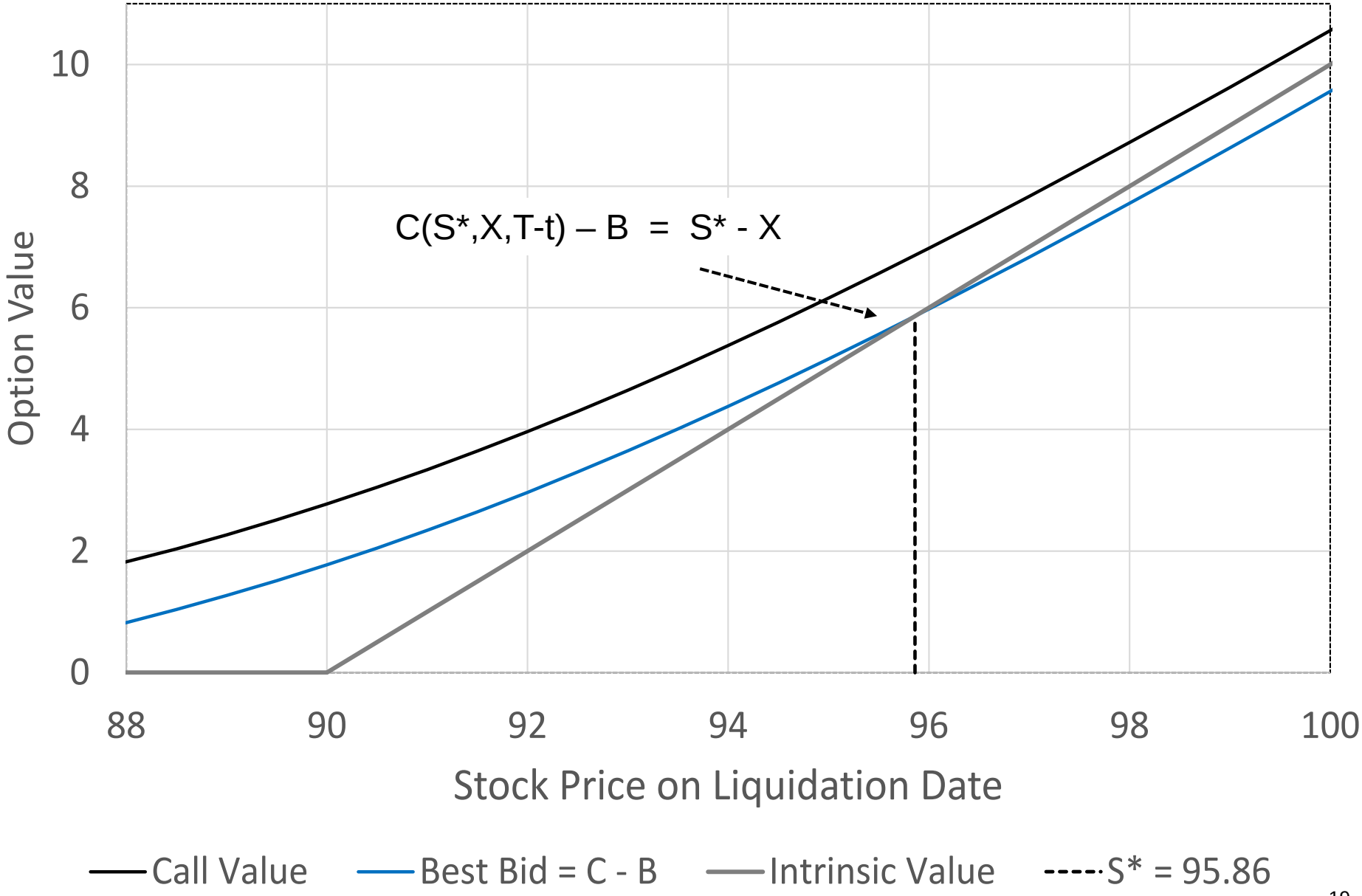
$$C_{BS}(S_t^*,X,T-t) - B = S_t^* - X$$

- Above S_t^* it is better to exercise (the option's remaining time value is less than B)
- Below S_t^* it is better to sell in the market

Finding the Early Exercise Boundary S* with X = 90



Finding the Early Exercise Boundary S* with X = 90



American Call Early Exercise Premium

Replicating Portfolio for "American" Call with Single Early Exercise Date t

1. Buy a European call with strike X and maturity T: $C_{BS}(S,X,T)$

2. Buy a European call with strike S_t^* and maturity t: $C_{BS}(S,S_t^*,t)$

3. Write a compound call on the call in step 1,
   with strike $S_t^* - X + B$ $- C_{call}(C_{BS}(S,X,T),S_t^* - X + B, t)$

4. Borrow present value of B with repayment
   at date t $- B e^{-rt}$

American Call Early Exercise Premium

These are the payoffs of this portfolio on date t for the two cases.

	Position	Liquidation value if $S_t \leq S_t^*$ Sell the American call	Liquidation value if $S_t > S_t^*$ Exercise the American call
1	$C_{BS}(S, X, T)$	$C_{BS}(S_t, X, T-t)$	*called by Option 3 exercise*
2	$C_{BS}(S, S_t^*, t)$	0	$S_t - S_t^*$
3	$-C_{Call}(C_{BS}(X,T), S_t^* - X + B, t)$	0	$S_t^* - X + B$
4	*Borrow* $B\, e^{-rt}$	-B	-B
Total		$\boldsymbol{C_{BS}(S_t, X, T-t)}$ - B	$\boldsymbol{S_t - X}$

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Closed form valuation

A similar replicating portfolio can be constructed for every possible early exercise date t.

Given a schedule of risk neutral probabilities $\{ p_1, p_2, \ldots , p_t, \ldots, p_{T-1} \}$ for liquidation on each future date, the American option can be valued in closed form.

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Closed form valuation

Example: Assume the probability of liquidating an existing option position early is given by a Poisson process with intensity λ. The probability that there is no exercise prior to date t is given by a survival function G(t):

$$G(t) = e^{-\lambda(t-1)}$$

The unconditional probability of a liquidation on date t is therefore

$$L(t) = G(t) - G(t+1) = e^{-\lambda(t-1)} - e^{-\lambda t}$$

To allow for options that are held to maturity, I assume λ is such that $G(T) = 0.25$, i.e., for a call purchased at time 0, there is 25% probability it will not be liquidated early.

$$L(T) = G(T) = e^{-\lambda T} = 0.25$$

American Call Early Exercise Premium

The European option value at time 0 for an investor subject to early liquidation at date t, where D(0,τ) is the price of a date t zero coupon bond:

$$C_0^E = E_0\left[\begin{array}{c}\sum_{\tau=1}^{T-1} D(0,\tau)\, L(\tau)(C_{Eur}(S_\tau, X, T-\tau) - B) \\ \\ +\ D(0,T)L(T)Max(S_T - X, 0)\end{array}\right]$$

The American option value at time 0 for an investor subject to early liquidation:

$$C_0^A = E_0\left[\begin{array}{c}\sum_{\tau=1}^{T-1} D(0,\tau)L(\tau)Max(C_{Eur}(S_\tau, X, T-\tau) - B, S_\tau - X) \\ \\ +\ D(0,T)L(T)Max(S_T - X, 0)\end{array}\right]$$

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American Call Early Exercise Premium

Additional Assumptions

The continuously compounded riskless rate is r: $D(s,t) = e^{-r(t-s)}$

The stock price dynamics under risk neutrality are given by:

$$\frac{dS}{S} = \mu dt + \sigma dz$$

with volatility σ a known constant.

The American call valuation equation becomes

$$C_0^A = \sum_{\tau=1}^{T-1} e^{-r\tau}\left(e^{-\lambda(t-1)} - e^{-\lambda t}\right)E_0[Max(C_{BS}(S_\tau, X, T-\tau) - B, S_\tau - X)]$$
$$+ e^{-\lambda(T-1)}C_{BS}(S_0, X, T)$$

Early Exercise Premium with a Realistic Bid-Ask Spread

In the real world, the bid-ask spread on an option is not a fixed constant. It varies with liquidity and moneyness.

To get a more accurate view of option market liquidity, I looked at 1-month call options on 10 stocks and fitted the following model to the quoted bid-ask spreads (adjusted to $S_\tau = 100$):

$$B \equiv (B_0 + B_1 Max(S_\tau - X, 0))/2$$

B_0 is the minimum spread and the slope B_1 determines how fast the spread widens as the option goes deeper in the money. B is the half-spread, as before.

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Early Exercise Premium with a Realistic Bid-Ask Spread

Replicating Portfolio for "American" Call with Single Early Exercise Date t with Bid-Ask Spreads that Depend on Moneyness

Let $B \equiv (B_0 + B_1 Max(S_\tau - X, 0)/2$

1. Buy a European call with strike X and maturity T $C_{BS}(S,X,T)$
2. Buy (1+ $B_1/2$)European calls with strike S_t^* and maturity t $C_{BS}(S,S_t^*,t)$
3. Write a compound call on the call in step 1, with strike $S_t^* - X + B$ $- C_{call}(C_{BS}(S,X,T),S_t^* - X + B, t)$
4. Borrow present value of $B_0/2$ with repayment at date t $- B_0 e^{-rt}/2$
5. Write $B_1/2$ European calls with maturity t and strike X. $-B_1 C_{BS}(S, X, t)/2$

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American Call Early Exercise Premium

These are the payoffs of this portfolio on date t for the two cases.

	Position	Liquidation value if $S_t \leq S_t^*$ Sell the American call	Liquidation value if $S_t > S_t^*$ Exercise the American call
1	$C_{BS}(S, X, T)$	$C_{BS}(S_t, X, T-t)$	*called by Option 3 exercise*
2	$(1+\frac{B_1}{2})C_{BS}(S, S_t^*, t)$	0	$(1+\frac{B_1}{2})(S_t - S_t^*)$
3	$-C_{Call}(C_{BS}(S, X, T-t), S_t^* - X + B(S_t^*), t)$	0	$S_t^* - X + (B_0 + B_1(S_t^* - X))/2$
4	*Borrow* $B_0\, e^{-rt}/2$	*-B_0/2*	*-B_0/2*
5	$-B_1 C_{BS}(S, X, t)/2$	$-B_1 Max(S_t - X, 0)/2$	$-B_1(S_t - X)/2$
Total		$\boldsymbol{C_{BS}(S_t, X, T-t) - B(S_t)}$	$\boldsymbol{S_t - X}$

American Call Early Exercise Premium

I used a ranking of about 3000 option stocks by trading volume from the OCC and selected:

- Four with high option trading volume (in top 10):
  AAPL, FB, BAC, XOM
- three with medium option trading volume (ranked 200-300):
  AIG, COST, GT
- three with low option trading volume (ranked 500-600):
  TRIP, TMUS, CAKE

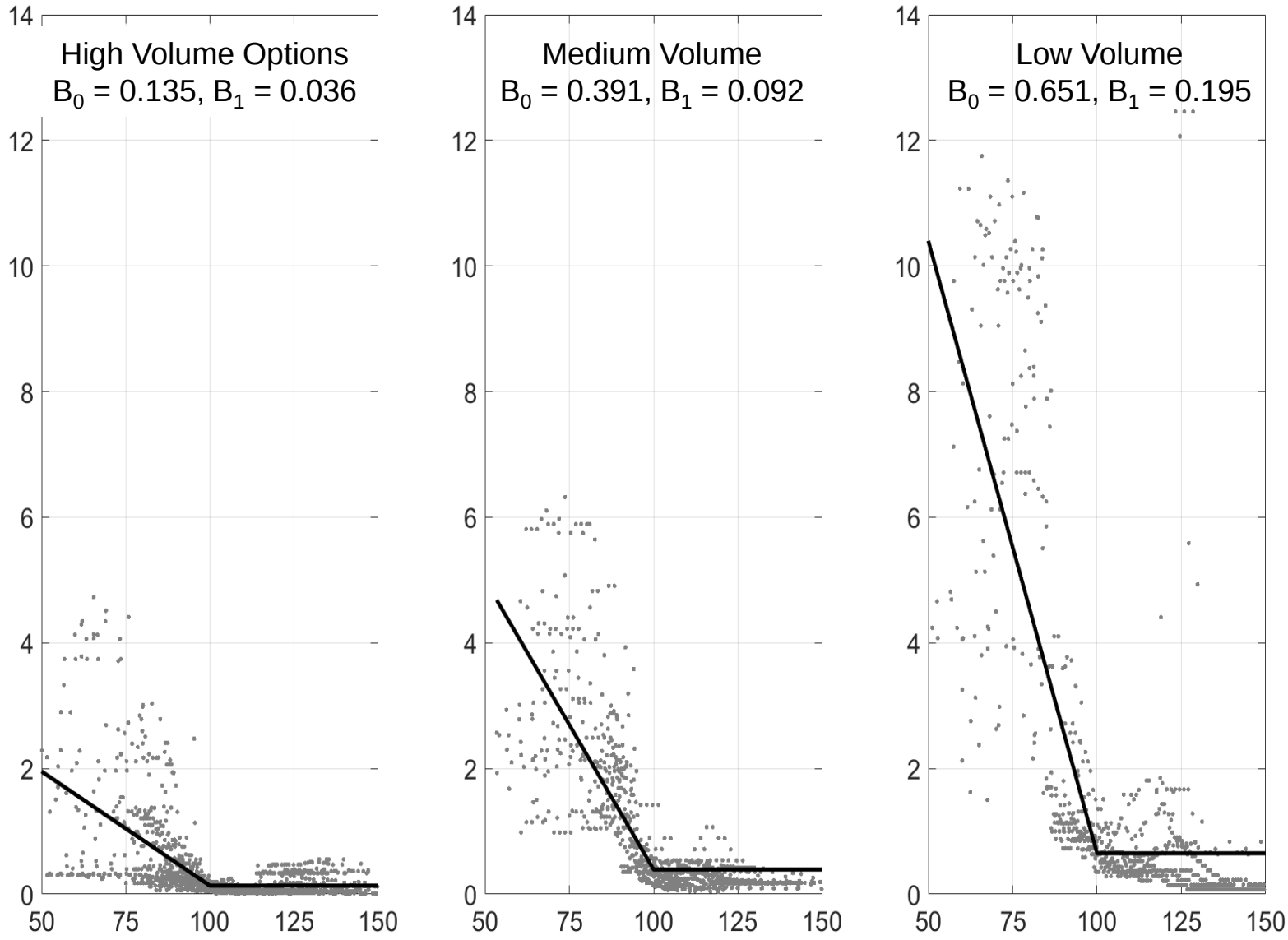
I picked two weeks with low and high market volatility:

June 22-26 (VIX = 13.23 on average)

Aug. 24-28, 2015 (VIX = 31.85 on average)

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American Call Bid-Ask Spreads in High Volatility Regime (σ=0.75)



American Call Early Exercise Premium
Low Volatility ($\sigma = 0.25$) Regime

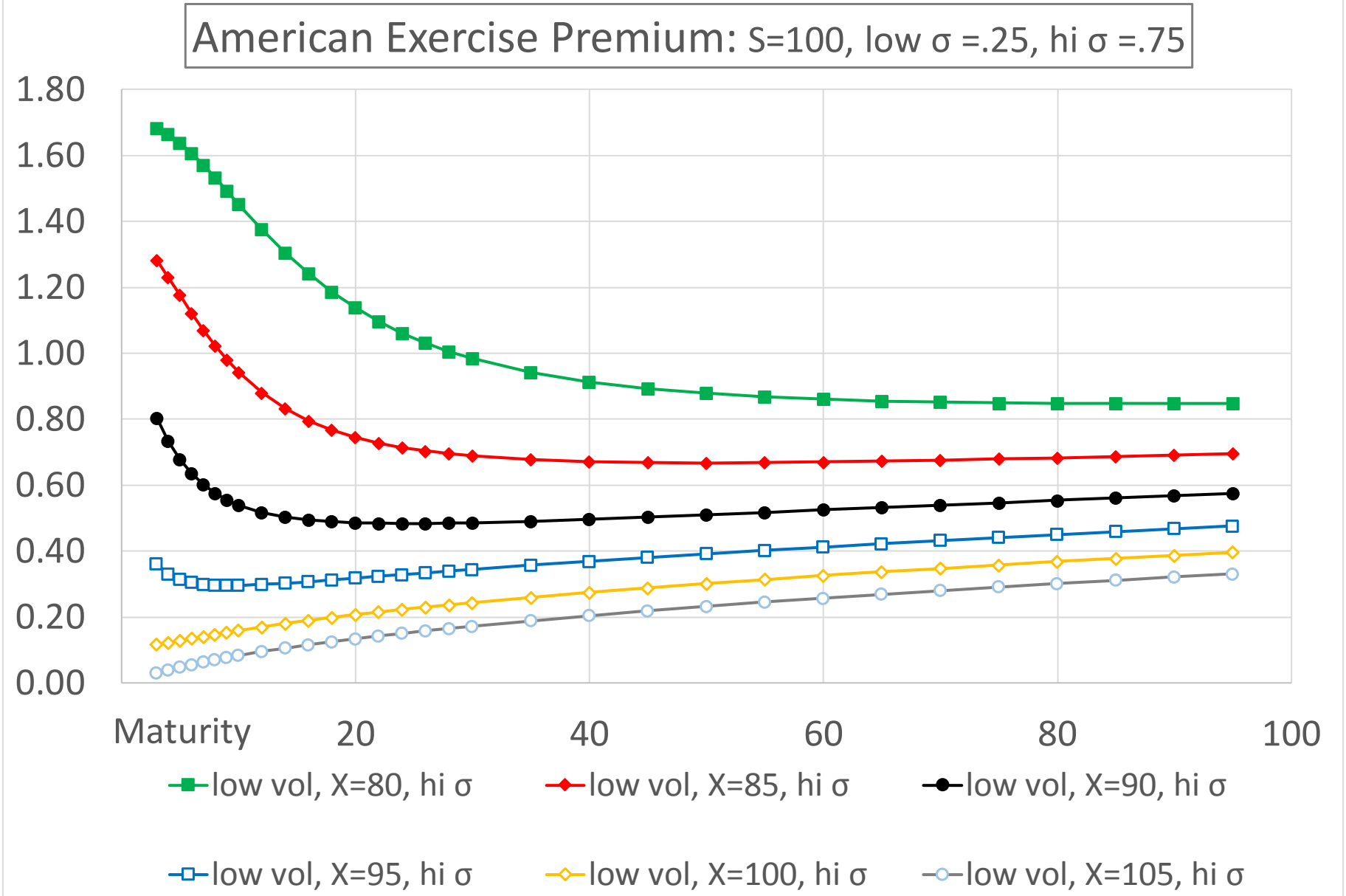
Strike Price	1 week	2 week	1 month	2 months
80	0.183	0.133	0.051	0.024
	0.633	0.583	0.465	0.265
	1.166	1.117	1.000	0.750
90	0.100	0.044	0.020	0.011
	0.302	0.222	0.128	0.090
	0.640	0.550	0.357	0.243
100	0.037	0.020	0.011	0.007
	0.103	0.072	0.057	0.051
	0.297	0.203	0.151	0.132
105	0.001	0.002	0.002	0.002
	0.001	0.004	0.008	0.013
	0.004	0.010	0.021	0.034

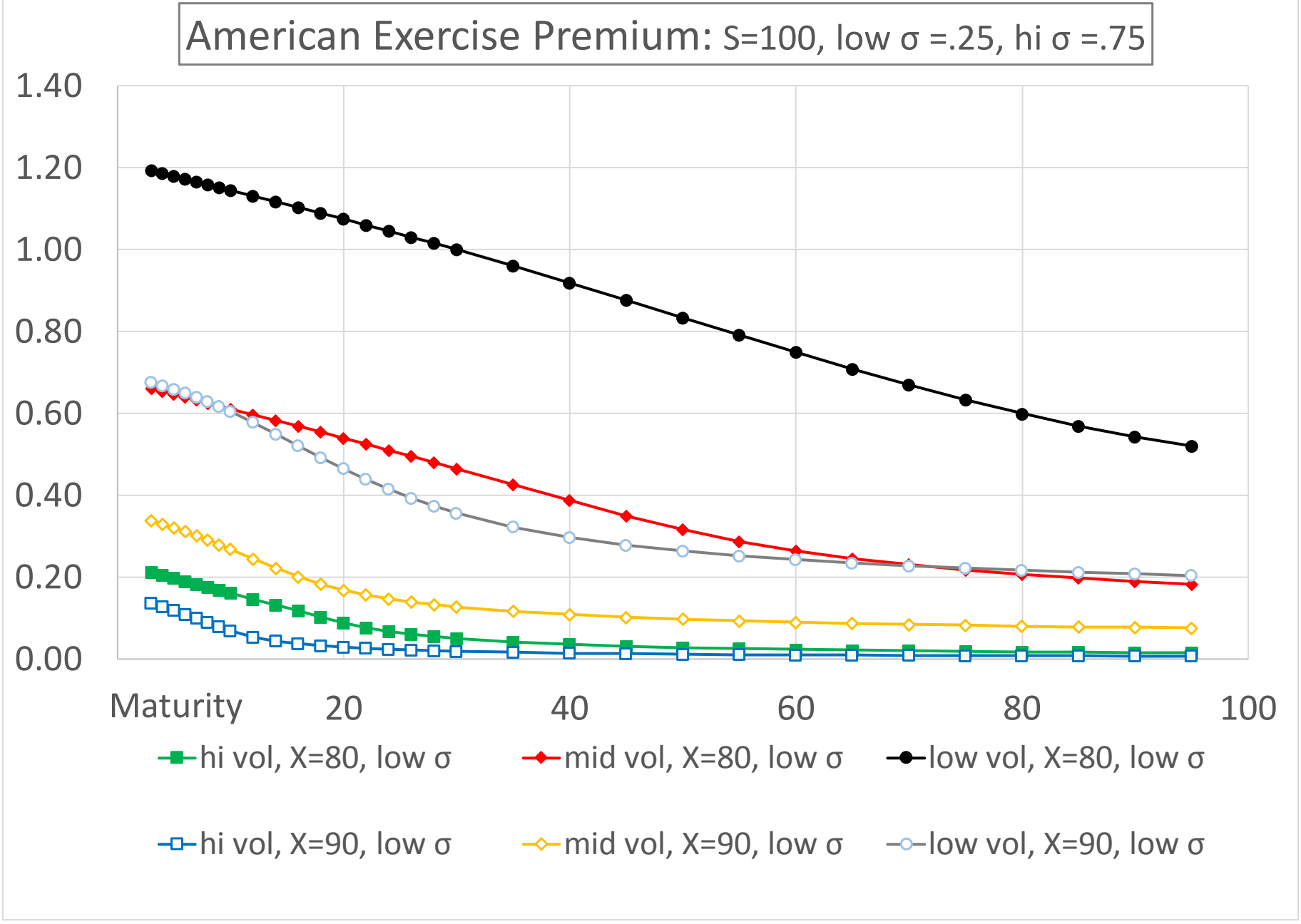
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American Call Early Exercise Premium
High Volatility ($\sigma = 0.75$) Regime

Strike Price	1 week	2 week	1 month	2 months
80	0.220	0.144	0.110	0.098
	0.711	0.518	0.377	0.335
	1.570	1.305	0.984	0.861
90	0.073	0.063	0.061	0.064
	0.242	0.203	0.197	0.211
	0.602	0.504	0.487	0.525
100	0.039	0.040	0.045	0.051
	0.124	0.126	0.142	0.168
	0.300	0.303	0.345	0.413
105	0.009	0.015	0.024	0.033
	0.028	0.046	0.073	0.107
	0.064	0.106	0.172	0.258

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American Exercise Premium: S=100, low σ =.25, hi σ =.75
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
Maturity
20
40
60
80
100
low vol, X=80, hi σ
low vol, X=85, hi σ
low vol, X=90, hi σ
low vol, X=95, hi σ
low vol, X=100, hi σ
low vol, X=105, hi σ



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Comparison with Standard Early Exercise Values

The American Put:

In theory, an American put should be optimally exercised when the extra interest gained by investing the strike price offsets the optionality component of time-value that is lost.

The maximum value of early put exercise is the interest that could be earned on the strike over the remaining life of the option: $X(e^{r(T-t)} - 1)$, treating the lost optionality as having zero value.

At the 5% interest rate we have assumed, these amounts for 1 week or one month are, respectively, \$0.116 and \$0.495 for a 120-strike put.

The liquidity-based early exercise premia are the same order of magnitude and in many cases larger than these maximum exercise premia for American puts in the standard framework.

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Comparison with Standard Early Exercise Values

An American Call on a Dividend-Paying Stock:

In theory, an American call should be exercised early, just before the stock goes ex-dividend.

The dollar amount of the dividend is the maximum value of early exercise. This can only occur if the call that would be left if the option were not exercised would have zero remaining time value.

Example: A 30-day call on a $100 stock that pays a quarterly dividend of $0.50. Ex-dividend is in 15 days. 40% volatility

Strike Price	American exercise for dividend	American exercise for liquidity		
		Hi volume	Mid volume	Low volume
80	0.228	0.053	0.394	0.893
90	0.192	0.024	0.133	0.301
95	0.044	0.015	0.075	0.167

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Conclusion

Real world option markets are not perfectly liquid, unlike the "perfect" markets of option theory.

The best bid in the market for an in the money option with less than a few months to expiration is below intrinsic value much of the time.

In that case, the holder of an American call who liquidates prior to expiration will typically do better to exercise than to sell in the market.

It is possible to value the right to exercise the American call and recover intrinsic value under standard Black-Scholes assumptions.

An American call IS worth more than a European call, by an amount that can easily exceed the value of theoretically correct early exercise of an American put or an American call on a dividend-paying stock.

Clearly, More Research Is Needed

Several major questions remain:

1. Why are the bids so low in the options market?

2. Why do option holders sell American options at prices below intrinsic value instead of exercising?

3. Why do we finance professors persist in telling our students that an American call on a non-dividend paying stock should never be exercised early, and is worth the same as a European call?

THANKS!