Homework #6: Using the Option Pricing Model

Undergrad: DUE TUESDAY, APRIL 23
Langone: DUE MONDAY, APRIL 22

This homework uses the Option Calculator routine you have seen in class to solve a variety of option problems. A "demo" version of this assignment **HW6-Demo.xlsx** containing the "OptCalc" option calculation spreadsheet and templates for the answers should be downloaded from the website. Your answers don't have to follow the template, but you might find it convenient to enter the proper formulas in the right places and then solve the problems with the correct numbers. Be sure you update ALL of the relevant input cells.

These instructions are based on Excel 2013. Other Excel versions allow you to do the same things, but the menu structure may be different from what is described below. If your Excel is different, use the Help function to find the commands you need.

Question 1. Implied volatility Today is Nov. 15, 2016. The DEC S&P 500 index options expire on Dec. 16, 2016. The S&P 500 index is at 2050.00. The riskless interest rate is 2.00% and (assume) the dividend yield on the S&P is 2.00%. The DEC 2050-strike S&P 500 call option is trading at 83.00 in the market. What is the implied volatility?

To do this, you first put all the parameter values (S, X, Today's date, Option maturity date, riskless interest rate, and dividend into the Option Calculator. I normally just put the input parameters into ordinary cells on the spreadsheet and then modify the input cells in the Option Calculator to point to those cells. For example, I might put the stock price into cell D2, and then I would enter the formula { =D2 } in the "Asset Price" cell of the Calculator. If just one cell has the stock price, and more than one Option Calculator references that cell, then you can simultaneously change the stock price in all of the Calculators by just changing the single price cell.

Next you try a value for the volatility input. If the model value with that volatility is below the market option price, you have to raise the volatility; if the model value is above the market price, you have to lower the volatility input. Once you have one volatility that is too low and one that is too high, you can go back and forth and narrow the range until you have pinned down the implied volatility (where model and market prices are equal) to as close an approximation as you want.

Excel offers more efficient search techniques, including ones that search for constrained solutions with multiple variables. For this problem, "Goal Seek" is useful. It is available on the Data menu under What-If Analysis. If your Excel window is narrow, perhaps only the icon shows in the toolbar. It looks like

Question 2. Simulating Delta Hedges Today is Nov. 14, 2016. The DEC S&P 500 index options expire on Dec. 16, 2016. The S&P 500 index is at 2050.00. The riskless interest rate is 2.00% and (assume) the dividend yield on the S&P is 2.00%. The volatility is 16.0%

a. Compute the theoretical values and the deltas for a DEC 2050-strike call and a DEC 2075-strike call.

b. Suppose you buy one 2050-strike call and you want to delta hedge the position using the stock. Calculate the number of shares to trade and the total cost of the delta neutral hedged position.
c. Suppose instead, that you buy one 2050-strike call and want to delta hedge the position using the 2075-strike calls rather than the stock. How many 2075-strike calls do you trade and what is the total cost of that delta-hedged position?
d. You are now going to create a one-way Data Table, that will display the profit or loss on the 2050-strike call and the two delta-hedged positions for a range of stock prices. Here's how to do it.

0. First, read about how Data Tables work in Excel Help. Press the {F1} function key to bring up Excel Help, then enter "What-if" in the search box and follow the links to "data table".

1. Using the option theoretical values when the stock price is 2050, enter the initial prices for the 2050-strike call, the cost of the delta hedged position using the stock, and the cost of the call versus call delta hedge into cells in the spreadsheet. Also enter into cells the 2075-strike call delta and the hedge ratio you use in the call vs. call hedge that you calculated in parts b. and c. Be sure all of these are the actual dollar values, not formulas that reference cells in the Option Calculator (because those cells will change during the table calculation).
2. Construct a column of stock prices, from 1900 to 2200 in 25 point increments. See the table diagram below.
3. In the next column to the right of the Stock Prices column and one row above the first stock price, you enter a formula. It is going to calculate the (mark to market) profit or loss on the 1050-strike call at each of the stock prices in the column. The formula should be *{current call value (from the 1050-strike Option Calculator) - initial call price (from the cell where you saved it in step 1)}*.
4. In the next cell to the right of the one in step 3, enter the formula for the mark to market profit or loss on the delta hedge using the stock. The formula should be *{ (current call value - original delta x current stock price) -(initial hedge cost, as saved in step 1) }*.
5. In the next cell to the right of the one in step 4, enter the formula for the mark to market P/L on the call vs. call hedge. Again, use the current values of the two options and the original hedge ratio in calculating the mark to market value of the hedged position.
The skeleton of the table should now look basically like this:

Stock Prices	P/L 2050-strike call	P/L Delta Hedge w/ Stock	P/L Delta Hedge w/ 2075-strike call
	formula for {current C_{2050} call value - initial cost}	*formula for {current C_{2050} call - current S * orig. delta - initial hedge cost}*	*formula for {current call vs. call hedge value - initial cost}*
1900			
1925			
1950			
...			

6. Now tell Excel to make this a One-Way Data Table. First Select the table using the mouse. The four columns and all rows including the formulas (but not the column titles) should be selected. Then, on the *Data* menu, select *What-If Analysis* and then *Data Table...* . In the box that will appear, you tell it the cell into which you want the stock prices in the table to be entered one at a time. Since the table is set up with the stock prices in a column, this cell should be entered into the box labeled "Column input cell:" You must enter the cell that the Option Calculators for <u>both</u> options use for their ASSET PRICE input. Click OK and Excel will create the table of P/L values.

e. Now you are going to create a plot of the three P/L series as a function of the stock price. This will give a picture of the market risk exposures of the three positions. Here's how to do it.

1. On the *Insert* menu, in the Charts section, select *Scatter* then sub-type: *Scatter with Straight Lines* . This places a blank chart into the spreadsheet and opens the Design toolbar.
2. Click Select Data. Click Add to enter the first series. Click in the *Series name:* box and type in a name to identify the series. Click in the Series X values box and highlight the stock prices column. Then click in the *Series Y values* box, **delete the ={1} entry that is already there**, then highlight the first P&L column in the table. Click OK, which takes you back to the Select Data Source dialog. Repeat the process of clicking Add and entering the Name, X and Y values for the other two series.
3. Click on Layout on the Menu bar, which will allow you to enter titles for the chart and for the axes. Put in something appropriate. Click on Legend to put the legend at the bottom; click on Gridlines to show vertical gridlines. To adjust the scale on the X-axis (if necessary), click on Axes, then *Primary Horizontal Axis* then More options at the bottom of the sub-menu. This brings up a menu where you can set Fixed values for the Minimum and Maximum on the X-axis.
4. The completed chart should appear in the worksheet. Drag it into a convenient spot and resize it if you want. The plot is now perfect (in theory).

Question 3: Early Exercise of an American Call At the close of trading on Nov. 14, 2016 XYZ stock is going to go ex-dividend for a dividend payout of $2.00. The DEC XYZ call options will expire on Dec. 16, 2016. The riskless interest rate is 2.00% and the volatility is 25.0%. XYZ options are American, so there is a chance that the calls will be exercised early (i.e., today), just before the stock price drops as it goes ex-dividend. For a given American call, there is a critical level for the stock price S* such that if the market price (before it goes ex-dividend) is above S*, the call should be exercised, and if the stock price is below S*, the call should not be exercised early. The objective in this problem is to find the values of S* for the 100-strike and the 95-strike DEC XYZ calls.

a. What is the critical value S* for the DEC 100-strike XYZ calls?

Here's how to find it.

a. Enter Nov. 14 as Today and Dec. 16 as Maturity into an Option Calculator. Enter the Strike, Interest rate and Volatility in the Calculator. (Leave the Payout Rate set to 0, because the dividend is a discrete payout, not a continuous rate.)
b. Set up a cell to be used for trial values of the stock price before going ex-dividend. In one cell, enter the formula for the call's intrinsic value at this stock price. (This is the payoff if the American call is exercised.)
c. In another cell enter the ex-dividend stock price (i.e., the trial stock price minus the dividend.) Use this ex-dividend price as the Asset Price input to the Option Calculator, in order to compute the ex-dividend value of the option if you don't exercise it.
d. Now enter different values for the stock price (before going ex-dividend) and compare the Intrinsic value cell from step b. with the Call value from the Option Calculator using the ex-dividend stock price.
- If Intrinsic value > Call value (ex-div), the call should be exercised; the critical S* is lower than your trial value.
- If Intrinsic value < Call value (ex-div) the call shouldn't be exercised; the critical S* is higher than your trial value.
Keeping searching by narrowing the range of trial values until you pin down S* to within one penny.

(This search can also be done using Goal Seek by finding the cum-dividend stock price that sets the difference between the intrinsic value at that stock price and the European call value at the ex-dividend stock price equal.)

b. What is the critical value S* for the DEC 95-strike XYZ calls?

Repeat the process for the 95-strike call. The easiest way to do this is just to copy all of the cells involved, change the strike price, and search for the new S*.

Question 4. Designing a Range Forward Contract A range forward is a combined position in which you buy a call and write a put, normally with strike prices set so that the out of pocket cost is zero (i.e., the call and put are priced the same). The objective is to set a maximum price you have to pay at maturity (the exercise price of the call) and pay for this protection by accepting to pay at least some minimum amount (the exercise price of the put). In between the two strike prices, neither option pays off and you will get whatever the price is in the market at option expiration. *[In many applications, this position is traded backward (write the call and buy the put) and combined with a long position in the stock. That position is called a "zero cost collar." You buy a put on the stock to protect against a big price drop, and pay for the put protection by writing the call. This gives up the chance to earn really high returns if the stock price jumps way up, but the downside is protected.*

In this problem you construct a range forward on the Euro exchange rate with a specified range, then figure out how to adjust the range to make the initial cost of the contract zero. Then you will simulate the value of the contract over a range of exchange rates at four different dates using a Two-Way Data Table, and plot the results.

Today's date is Nov. 14, 2016 and you want to price a range forward to Buy Euros for Dollars, which matures on May 15, 2017. The spot exchange rate is 1.0100 dollars per Euro, the interest rates are 2.00% in the U.S. and 4.00% in Europe. Exchange rate volatility is 12.00%

a. The upper bound on the exchange rate in the range forward should be set at 1.0300 and the lower bound is set at 0.9600. Compute the values for the call and the put that this range forward would involve. What would the range forward contract cost per Euro?

b. To make this zero cost, the upper bound should stay at 1.0300 but the lower bound must be adjusted. Find the strike for the put that makes the initial cost of the range forward equal to zero.

c. Assume the range forward is set up with the upper bound at 1.0300 and the lower bound equal to the zero-cost value you calculated in part b. Now you are going to construct a Two-Way Data Table to simulate the value of the range forward at four different dates over a range of exchange rates. A two-way data table lets you compute the value of a single formula (the value of the range forward, in this case) while the

values of two of the input parameters are varied (the spot exchange rate and the starting date).

Here's how to do it:

1. Construct a column of spot exchange rates, from 0.9500 to 1.0700 in intervals of 0.0100. In the four columns to the right of the spot rates column, enter these four dates in the row above the row with the first exchange rate: 11/14/2016, 2/14/2017, 4/14/2017, 5/14/2017. These are going to be the valuation dates for the contract. See the table below.

{formula for value of the range forward}	11/14/2016	2/14/2017	4/14/2017	5/14/2017
0.9500				
0.9600				
0.9700				
...				

2. Into the upper left corner of this table, enter the formula for the value of the range forward contract.
3. Select the entire table with the mouse. Then, on the *Data* menu, proceed as you did with the one-way data table in Question 2. You must tell it both the "Row input cell:" into which you want the four dates entered one at a time and the "Column input cell:" into which the spot exchange rates should go. There are two Option Calculators and you must tell them both to use the same cells for their TODAY'S DATE and ASSET PRICE inputs. Click OK and Excel will create the table of simulated values.

d. Now plot the simulated range forward values from the table, going through the same steps as in Question 2, with appropriate series names, etc.

Question 5. Relax. In this part of the exercise, you pat yourself on the back for the effort you put into Questions 1-4.

a. Get a cold drink.

b. Put your feet up.

c. Relax for a while.