

Current Issues: Options

What Does an Option Pricing Model Tell Us About Option Prices?

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The story is told of a Seeker of Knowledge who sets off in search of the answer to a question that has troubled him for a long time. In his travels he hears of two wise men who are said by many to be very knowledgeable and experienced in such matters.

The first, a famous guru, lives at the top of a mountain, high above the hustle and bustle of everyday life. After a strenuous climb, the Seeker is able to pose his question: "What is a call option worth?"

The guru answers immediately, "It is not hard to prove that

$$C = S N[d] - X e^{-rT} N[d - \sigma \sqrt{T}],$$

where S is the stock price, X the strike price, T time to expiration, r the risk-free interest rate, σ volatility, $N[\cdot]$ denotes the cumulative normal distribution and

$$d = (\log(S/X) + (r + \sigma^2/2) T) / \sigma \sqrt{T}.$$

Of course, this has to be modified somewhat in practice to take into account dividends, the value of early exercise and a few other technical details (see the appendix)."

This answer seems pretty exact, if a bit complicated. The Seeker thanks the guru warmly and goes on his way.

The second wise man lives in the middle of a city, surrounded by a continuous swirl of noise and activity. Once the Seeker is able to get his attention, he poses the question again: "What is a call option worth?"

Again the answer is immediate:

"That depends. Are you buying them or selling them?"

Not knowing quite what to say, the Seeker starts to respond by repeating the words, and equations, of the first guru, but he is quickly interrupted with: "I don't care about all of that stuff. Tell him to make me a bid. Then we can talk about what a call option is really worth."

Somewhat confused and not at all sure the wise men's answers have brought him any closer to enlightenment, the Seeker goes away to meditate further on his question.

Valuation Models and Pricing Models

This parable offers two very different answers to the same basic question. The distinction between them reflects the not often recognized difference between theories of option *valuation* and option *pricing*.

The Black-Scholes model and others like it are theories that try to derive the *value* of an option so that it is consistent with the price of the underlying stock. They assume a market environment in which a dynamic riskless arbitrage strategy with the stock and the option is possible, and find the value of the option as a component of the arbitrage portfolio.

In this ideal market, if the option's price should differ from the model value, an arbitrageur can trade it against the correct number of shares of stock to produce a position that is riskless over the next instant of time. Continuous rebalancing keeps the hedged position riskless until the option expiration date. But its return will be higher than the risk-free interest rate by the exact amount by which the option was mispriced at the outset. As there are no constraints on the size of their positions, arbitrageurs will offer an unlimited number of options at any

price above the theoretical value and will have infinite demand at any price below it, so option prices in the market are driven to their model values. This is the reasoning behind the first guru's answer.

But in trying to apply a theoretical valuation model to the real world, it is immediately clear that none of the model assumptions actually holds. The arbitrage strategy, which is riskless and costless in theory, is neither in practice. There is risk because the position can't be rebalanced continuously when markets are closed, and there are costs because even less-than-continuous rebalancing can lead to large transaction costs. Even the theoretical option value itself is uncertain, because it depends on the volatility of the stock, which cannot be known exactly. Unlimited arbitrage does not dominate the market.

In actual markets, option *prices*, like prices for everything else, are determined by supply and demand. This includes supply and demand from non-arbitrageurs. Investors demand call options because the options offer participation in stock price movements on the upside, limit risk on the downside, and allow investors to control a large amount of stock with a small investment. Call writers supply call options to the market because it is a way to generate income when they expect stock price will not rise sharply in the near future.

Option demand and supply are also influenced by the market environment, which encompasses taxes, transaction costs, margin treatment of different securities, delivery features of option contracts, constraints on margin purchases and short sales of the stock, interaction between options and related futures contracts, and many other things. Anything that affects investors' trading decisions but is not in

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the model tends to push the market price away from the model value. As the second wise man indicated, a call option is worth exactly the price at which it can be traded in the market, and that does not depend on just the model.

A skeptic might ask: Without unlimited arbitrage between the option and its underlying stock as a foundation, how can a theoretical valuation model tell us *anything* about actual market prices?

One possible answer is that, under certain conditions, the equilibrium option price is the Black-Scholes value even when there is no arbitrage, because when investors evaluate the option as if it were any other asset, that is what its payoff pattern is worth.¹ Unfortunately, one of the necessary conditions for this result is that all investors be identical, which is no more true of real markets than the continuous arbitrage assumptions it replaces.

An argument that is on stronger ground, but has weaker implications, is that as long as the arbitrage is possible, it will be done in spite of transaction costs and risk whenever the expected profit is large enough to compensate for them. This leads to arbitrage bounds around the model value. Within these bounds, there is no arbitrage and market price can move freely, but if the price strays too far from the model value, arbitrage becomes profitable and will tend to push price back into the bounded range.

How much information the model actually gives us about what the market price will be depends on how wide the arbitrage bounds are. This is not easy to determine, because the transaction costs and risk for the arbitrage are a function of which random path the stock price follows. In a recent paper, I simulated a large number of price paths and discovered that the arbitrage bounds are disturbingly wide, even for routine cases.² For example, the price of a one-month, at-the-money call with a Black-Scholes value of \$2.05 could be anywhere from \$1.74 to \$2.35 without giving an arbitrageur even a 50/50 chance of covering costs, or any compensation for

risk. Moreover, it is clear that, given the cost and uncertainty of the trade, arbitrageurs will not take unlimited positions, even at prices outside these wide bounds. This makes for something less than impenetrable barriers.

We are left with distressingly little in the way of a response to the skeptic's question.

Trading Real Options Using Model Prices

The problems associated with applying a theoretical option valuation model to the real world have obviously not prevented virtually every serious option trader from doing just that. But they use the model in ways that are more consistent with the second wise man's approach than with the model's theoretical underpinnings.

Few investors use the model mainly for finding mispriced options that can be arbitrated against the underlying stock. A major reason for this is that no one feels confident they know the true volatility. Estimating volatility from a sample of past prices is a routine exercise, but you can't be sure the future will be exactly like the past. Based on historical volatility, for example, the price change that occurred on October 19, 1987 was essentially impossible.³

Option traders normally pay more attention to the *implied* volatility that sets the model option value equal to the market price. This is easy to compute and, in principle, gives a direct reading of the market's volatility estimate. But it has the disadvantage that you have to assume the market is pricing the option according to the model, which rules out using implied volatility to detect mispricing.

Another problem is that, because volatility is the one input to the model that can't be directly observed, implied volatility actually serves as a free parameter. It impounds expected volatility *and everything else* that affects option supply and demand but is not in the model. If a change in the tax law makes writing options less attractive, for instance, the price effect will show up in implied volatility. Any time the market prices options differently from a given model, for any reason, the

implied volatility derived from that model is not going to be the market's true volatility estimate.

Generally, implied volatilities differ across strike prices in a regular way, even though it is logically inconsistent for a stock to have more than one volatility. Out-of-the-money options typically have higher implied volatilities than at and in-the-money options, puts are often priced on different volatilities than calls, and the patterns vary from time to time. This is evidence that the market does not price options strictly according to the model, or at least not according to the particular model that produces the differing implied volatilities.

Traders don't care much about these problems. In fact, they seldom think about the technical details of the computerized models they use, or even about whether they are pricing European or American options. Instead, they tend to take whatever theoretical model happens to be available on the computer and treat the implied volatility it produces for a particular option as a kind of index of how the option is currently being priced in the market relative to other options and relative to how it was priced at other times.

It is perfectly normal, for example, for an option trader to reason, "Yesterday the computer said these XYZ options were trading on a volatility of 20 per cent, but this morning the options market is a little soft, so I'll use 19.5 today and 21.5 for the out-of-the-money puts that always are a little richer."

These volatility "estimates" are then plugged into the computer's model, and bids and offers are based on the theoretical values that it produces. The idea behind this procedure is not that the trader thinks 19.5 is the best available estimate of XYZ's volatility until option expiration, but that it is a reasonable way to summarize the current state of supply and demand for XYZ options and that conditions will probably remain fairly stable for a while—until they change.

For many option traders, having exactly the right model and volatility may not be of such great importance, because what they really care about is the *delta*, which tells how to hedge an

1. Footnotes appear at end of article.

option, and delta is much less sensitive to these things than theoretical value. Also, market-makers and active traders frequently hedge options against each other, rather than against the stock, so the effects of changing volatility and other model inaccuracies on the different options partly offset each other. In other words, it may not matter so much if your model misprices an option as long as the option will continue to be mispriced in the same way when the stock price changes, or as long as it is hedged by other options that are similarly mispriced.

Still, when a stock has a large price move, implied volatilities also typically change, meaning that the actual change in the option's price is not what was predicted by the original delta. The skeptic might wonder if we can even be sure that the apparent stability of implied volatility under normal circumstances isn't partly due to a kind of self-fulfilling prophecy, what with so many option traders using the model to change their bids and offers as the stock price moves.

While it was conceived as an arbitrage-based valuation theory, the Black-Scholes model is actually used by option traders as a pricing equation, to predict how the option price will change when the underlying stock moves. But traders treat the model almost as if it were a rule of thumb, rather than a formula that gives the true option value with confidence.

Testing Models with Real Option Prices

In a sense, the mirror image of an option trader evaluating market prices using the model is the academic theorist "testing" a model on market data. The standard procedure is to compute theoretical values for a set of actual options and compare them with market prices. Small random differences can be explained as "noise," but systematic deviations are viewed as evidence of problems with the model. The common finding that deep-out-of-the-money options seem to be priced higher in the market than Black-Scholes would suggest has given rise to any number of increas-

ingly complex alternative models to explain the phenomenon.

Tinkering with a model to try to make it fit market prices better confuses valuation and pricing. What an arbitrage-based option model says is that buying the option at its fair value and following the arbitrage strategy until expiration will return the risk-free rate of interest. If empirical tests show that this would happen in practice, *assuming* you could buy the option for its model value and had no transaction costs, then the valuation formula is correct. How those options might be priced in the market has nothing to do with whether the model *values* them correctly!

What model-testers have in mind, of course, is that the theoretical valuation model should also be the market's pricing equation. As we have seen, in the real world an arbitrage-based valuation model produces a band around the theoretical option value, within which the excess profit that could be made is smaller than the cost to set up the arbitrage trade. In this context, the model only says that the price should be *within* the bounds. It is perfectly consistent with the model for the market price to be always at the lower bound or the upper bound, or following any kind of pattern in between. The finding of a consistent "bias" in market prices does not indicate a rejection of the model, unless the bias is large enough that prices lie well outside the arbitrage bounds. And those bounds may be very wide.

Is the Skeptic Right?

A true skeptic might argue that this discussion has shown two things. First, option valuation models don't give correct fair values because the continuous arbitrage can't be done in practice. Second, even if models did give correct values, option prices in the market would not equal those values because of all of the other factors affecting supply and demand.

I would not go so far, although I think some skepticism is a healthy, and risk-averse, attitude in this case. A model does not have to be exactly right for it to be of use. The general acceptance of option models in the real world, and the fact that extensive em-

pirical examination of those models has not led to widespread rejection of them, suggests that, at least for short-maturity, exchange-traded option contracts, the models work acceptably well.

But some situations warrant a greater degree of skepticism than others. There are problems of two kinds: The model can be wrong, so that the theoretical value is not the true option value, or the model may give the correct value, but the market price the option differently.

In the first category we include situations in which the model assumptions are violated (to a larger extent than normal). This is true for long-maturity options, where parameters such as volatility and interest rates (which one can treat as being fairly constant over a month) can vary widely and unpredictably over longer periods. We can have confidence in a valuation model only to the extent that we are confident of our forecasts of its parameters out to option expiration, which may be many years in the future.

We can also expect inaccuracies arising from errors in modeling security prices as geometric brownian motion. There is considerable evidence that actual price changes have "fat tails"—that is, there is a greater probability of a large change in a short interval than the model assumes, leading to hedging problems and undervaluation of short-maturity options. There is also growing evidence that over both long and short horizons, there is some non-randomness in stock price movements.⁴

There is certainly reason to doubt an arbitrage-based model's valuation of an option on an underlying asset that is not traded, even though theorists routinely apply the Black-Scholes formula to all manner of cases in which the arbitrage is impossible, or essentially so. For example, it is an important theoretical insight that limited liability in bankruptcy makes the equity of a firm with outstanding debt similar to a call option to buy the firm's assets from the bondholders by paying the debt. But the firm as a whole is not an asset that can be traded independently from its securities, so there is no way investors could arbitrage the stock

against the underlying firm if they thought the market was mispricing its option value.

The same problem applies to options on any nontraded asset, or on anything that is not an asset at all, such as the "option" to abandon an investment project. Even valuing an option on a marketable but indivisible asset, such as a unique piece of property, causes difficulty when there is no way to form a hedge portfolio that can be rebalanced. There are clearly many cases in which one must be skeptical about deriving an option's value from an arbitrage strategy that cannot be done.

The second category of problems pertains when the model gives the true value of an option, but the market prices it differently. The more difficult and costly the arbitrage trade is to do, the greater the scope is for factors not covered in the model to move the market price away from its theoretical value. Several situations have proved particularly difficult for arbitrage-based models.

Out-of-the-money options: People particularly like the combination of a large potential payoff and limited risk and are willing to pay a premium for it. That is why they buy lottery tickets at prices that embody an expected loss. Out-of-the-money options offer a similar payoff pattern. At the same time, the writers of those options are exposed to substantial risk because it is hard to hedge against large price changes. Why should we *not* expect out-of-the-money options to sell for a premium over fair value?

American options: The possibility of early exercise makes American options hard to value theoretically, especially because the early-exercise provision is seldom exercised optimally according to the theory. This is an enormous problem with mortgage-backed securities because of the homeowner's op-

tion to prepay the mortgage loan, but all American options share it to some extent. We should not be surprised if the market prices American options differently from their model values because of the uncertainty.

Embedded options: Valuation models treat a security with embedded option features, such as a callable bond or a security with default risk, as if it were simply the sum of a straight security and the option. But the market does not generally price things this way. For example, when coupon strippers unbundle government bonds, or when mortgage pass-throughs are repackaged into CMOs, the sum of the parts sells for more than the original whole. Why should we expect the market to price embedded options as if they could be traded separately when this is not true of other securities?

Times of crisis: The period around the crash of October 1987 showed that in times of financial crisis, arbitrage becomes even harder to do and option prices can be subject to tremendous pressures. At such times, we should not expect to be able to explain market prices well with an arbitrage-based valuation model.

Where Do We Go From Here?

If what is really wanted is a model to explain how the market prices options, it doesn't make sense for academics and builders of option models to restrict their attention entirely to elaborating arbitrage-based valuation models in an ideal market. They should at least examine broader classes of theories that include factors such as expectations, risk aversion and market "imperfections" that do not enter arbitrage-based valuation models but do affect option demand and supply in the real world.

For those who would use theoretical models to trade actual options, it is safer to use models for hedging than

for computing option values; furthermore, the harder the arbitrage is to do, the less confidence these investors can have that the model is going to give either the true option value or the market price. Hedging options with options, rather than with the underlying stock, can provide some defense against inaccurate volatility estimates and model misspecification.

In general, investors who are not trading as market-making arbitrageurs should be less concerned with valuation models than with using options to produce overall payoff patterns that suit their market expectations and risk preferences. When they think the market might drop sharply, it makes sense for them to buy put options, even if they have to pay more than "fair" value.

Footnotes

1. See M. Rubinstein, "The Valuation of Uncertain Income Streams and the Pricing of Options," *Bell Journal of Economics and Management Science*, Autumn 1976, or M. J. Brennan, "The Pricing of Contingent Claims in Discrete Time Models," *The Journal of Finance*, March 1979.
2. S. Figlewski, "Options Arbitrage in Imperfect Markets," *The Journal of Finance*, forthcoming 1989.
3. And including that day's price change in volatility estimates after the event meant that it dominated the calculation. There was then a spurious sharp fall in estimated volatility months later, on the day October 19 dropped out of the data sample.
4. See, for example, E. Fama and K. French, "Permanent and Temporary Components of Stock Prices," *Journal of Political Economy*, April 1988, or J. Poterba and L. Summers, "Mean Reversion in Stock Prices," *Journal of Financial Economics*, October 1988.