The cost and shadow cost of credit

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Abstract

Using a novel micro-level dataset on firms' production and financing decisions, we estimate the distribution of firm—specific financial wedges in capital accumulation due to binding borrowing constraints—the shadow cost of credit—and compare these to observed market price of credit—the borrowing rate. We find that shadow costs are significantly higher, more dispersed, and more sensitive to variations in credit risk factors than borrowing rates. Our analysis also reveals a high sensitivity of firms' investment to shadow costs, indicating that credit rationing, rather than elevated borrowing costs, is the primary channel through which credit market frictions distort investment policies and capital allocation, particularly for small and medium enterprises.

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"Here we shall be concerned primarily with one argument which seems to have the greatest validity and general applicability: the proposition that interest rates charged to borrowers by financial intermediaries are largely controlled by institutional forces and slow to adjust at best; and that the demand for funds is accordingly limited not by the borrowers' willingness to borrow at the given rate but by lenders' willingness to lend -or, more precisely, by the funds available to them to be rationed out among the would-be borrowers."

— Franco Modigliani, 1963

1 Introduction

Credit market frictions are widely recognized as a key determinant of firms' investment decisions, affecting investments either by increasing borrowing costs per dollar borrowed or through shadow costs generated by credit quantity rationing. The idea that business capital spending decreases as interest rates rise is a central tenet of the monetary transmission mechanism. Previous contributions provided empirical evidence linking variations in borrowing rates—and, more broadly, the cost of capital to the user—to investment focusing on large firms with access to bond markets (Gilchrist and Zakrajšek 2007; Philippon 2009). However, evidence of this mechanism within private credit markets remains limited (Abel and Blanchard 1986). In contrast, credit constraints play a prominent role in theories connecting financial and business cycle fluctuations (Bernanke and Gertler 1989; Kiyotaki and Moore 1997) and long-term growth (Galor and Zeira 1993).

However, with some notable exceptions, evidence on the existence and magnitude of credit constraints remains scarce.[1] This scarcity is largely due to the difficulty of observing credit limits, and even when observed, it is uncertain whether these constraints meaningfully affect firms' investment decisions.

In this paper, we propose an empirical approach to measure the shadow price of credit in microdata and examine the sensitivity of firm investment decisions to variation in such costs. Toward this purpose, we assemble a longitudinal data set that provides us with a detailed account of financial and production choices of Italian corporations, including the very small ones. Importantly, the data allows us to observe firm-level information on borrowing rates—a fundamental component of the user cost of capital—and firm's credit applications—a measurable proxy of (unsatisfied) credit demand.

We find that, when borrowing constraints bind, shadow costs are substantially higher, more heterogeneous, and more sensitive to credit supply conditions and variation in credit risk factors than user costs. We then show that variation in shadow prices can explain the firm's investments (or their lack thereof). Taken together, our evidence suggests that credit-quantity constraints, rather than distorted borrowing costs, are the primary channel through which credit market frictions distort investment policies and generate capital misallocation. This is particularly true for small and medium enterprises. Consistent with the presence of size-dependent borrowing limits, the spread in shadow costs between small and large firms is much larger than the interest rate spread, and shadow costs are significantly more heterogeneous among small and medium-size firms.

To build intuition for our approach, consider investment decisions under a neoclassical benchmark. Standard optimality arguments suggest that firms should invest until the marginal revenue product of capital (MRPK) equals the user cost of capital:

$$\mathrm{IG} := MRPK - R. \tag{1}$$

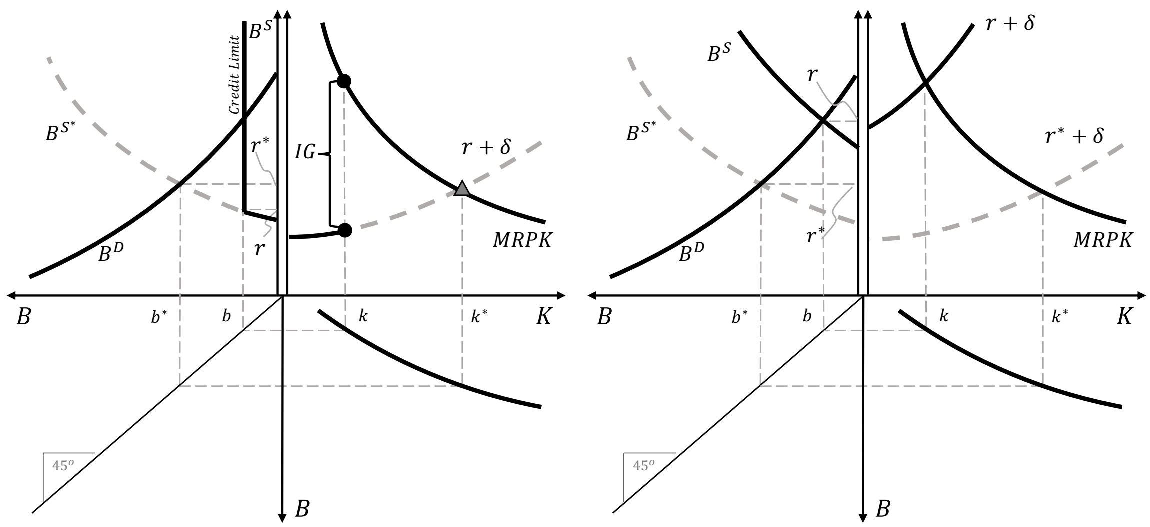
MRPK represents the incremental revenues generated by employing one additional unit of capital, holding other inputs constant. The user cost reflects the per-period cost of employing a unit of capital. When bank debt is the marginal source of finance, the user cost is the sum of the borrowing rate (the marginal cost of investment) and the capital

[1] See, for example, Banerjee and Duflo (2014) and Agarwal et al. (2018)

depreciation rate, $R := r+\delta$. Under this benchmark, a positive *investment gap* (IG) indicates an inefficiently low capital stock. When credit supply is inelastic, the size of the gap is proportional to the shadow cost of credit caused by binding borrowing constraints.

Figure 1: Credit rationing versus distorted borrowing rates

Panel a: Credit rationing | Panel b: Distorted borrowing rates



Notes. This figure illustrates the relationship investment and credit market frictions under two scenarios. Panel a depicts financial frictions as borrowing constraints (credit rationing), while panel b shows how frictions steepen the credit supply curve, distorting interest rates.

Figure 1, panel a, illustrates this concept graphically. B denotes the credit provided by the lender (based on its supply curve B^s) and r the borrowing cost paid by the firm. The credit market does not clear because the demand for loans (based on the demand curve B^D) at the borrowing rate r exceeds the supply. Although the firm may be willing to pay a higher borrowing rate (the Walrasian market-clearing level r^*) to secure more credit (B^*), restricting credit is the profit-maximizing choice for the lender.[2] Thus, credit rationing

[2] In a series of seminal contributions, Stiglitz and Weiss (1981, 1983, 1992) demonstrate how information frictions can lead lenders to impose quantity constraints—rather than adjusting interest rates—to adjust the credit supply. Additional factors preventing market clearing through price adjustments include imperfect competition (Petersen and Rajan 1995) and government interventions that restrict price discrimination, enforcing uniform credit costs across transactions of varying types (Benmelech and Moskowitz 2010; Banerjee and Duflo 2014).

emerges as an equilibrium outcome, causing the firm to operate at a scale below its efficient size ($K < K^*$). The investment gap IG, the vertical difference between the user cost of capital and the realized MRPK, reflects the economic costs of the forgone investment. The higher the firm's marginal value of investment, the greater the shadow cost of credit and the wider the investment gap. Note also that, when quantity constraints are binding, the *less* the loan rate increases, the *more* the shadow price rises, causing the investment gap to widen.

Panel b illustrates a different scenario, where financial frictions still lead to underinvestment but through distorted borrowing costs rather than restricted quantities. In this graph, the gray dotted line represents the credit supply schedule in the absence of frictions (B^{s^*}). The solid black line represents the supply schedule subject to credit market frictions (B^s), which implies that, for any level of credit supply, the bank requires a higher interest rate.[3] As in panel a, credit market distortions result in an equilibrium with lower credit and, a fortiori, a suboptimally low capital endowment. However, in this case, the borrowing rate is a Walrasian market clearing level (although higher than the efficient one). There is no excess credit demand and the investment gap is closed. Underinvestment arises because firms internalize the higher borrowing costs in their decisions. As interest rates rise, the firm's investment activity falls. Note that this might not happen in a setting with quantity rationing. There, as lenders tighten borrowing limits, rationed firms would face a raising shadow price for credit, causing investment to fall (and investment gaps to widen), even though market interest rates were stable or even fall.

We measure firm-specific, time-varying investment gaps using information on production decisions and user costs. Leveraging both cross-sectional and time-series variation in these gaps, we then estimate the shadow costs of credit arising from binding borrowing constraints. Our approach builds on the traditional Euler equation estimation framework (Whited 1992; Bond and Meghir 1994; Whited and Wu 2006), which we extend to account for demand-side heterogeneity—such as variations in productivity and markups—heterogeneous user costs of capital, borrowing constraints, and selection effects.

In Section 2, we introduce our data set and describe the institutional context. The data

[3] Of course, even in a frictionless market, it is sensible that the credit supply curve is upward sloping since the probability of default tends to be higher on a large loan. Here we envision a situation in which frictions, such as limited liability or information frictions, induce the lender to adjust the cost of credit more than it would be required by efficient risk pricing.

set covers nearly the entire Italian corporate sector for two decades and provides detailed information on both production and financing. The broad coverage and longitudinal nature make our data particularly well suited for studying credit market frictions, which are expected to have a greater impact on the real activity of small and young enterprises and gradually attenuate at later stages of their life cycle. Two key strengths of our microdata are detailed firm-level information on borrowing rates and credit application decisions. The borrowing rates enable us to produce a firm-level measure of the user cost of capital and construct investment gaps. Credit application data allows us to identify firms with unmet credit demand—those most likely facing binding borrowing constraints.

In Section 3 we outline the estimation and measurement strategy used to recover the two components of the firm-level investment gaps. We show that cross-sectional and time-series variation in investment gaps is closely related to credit market participation, common proxies of financial constraints, and financial variables. Investment gaps are, on average, twice as high for non-borrowers than for borrowers. Among borrowers, the investment gap is 1.5 times greater for firms that rely solely on working capital financing (revolving credit lines) compared to those that can also tap into long-term financing (term loans). Investment gaps decrease monotonically with the age and size of the firm—two commonly used proxies of financial constraints (Hadlock and Pierce, 2010)—and with the firm's leverage. The availability of individual firms' credit histories provides additional insight into the evolution of gaps over firms' life cycles and connect it to credit availability. Studying firms' credit histories, we document a sharp reduction of investment gaps upon access to credit and a steady convergence towards the frictionless benchmark as firms strengthen their lending relationships.

Although revealing, this descriptive evidence does not help quantify the magnitude of the implicit cost credit due by credit rationing. In fact, besides financial frictions, real adjustment costs and risks in capital accumulation can also explain why some firms with positive gaps underinvest even if they face an unconstrained supply of credit. Additionally, measurement and estimation errors might incorrectly suggest that some firms' investment policies are suboptimally low (or high) when they are not. To account for these forces, we estimate shadow prices associated with binding borrowing constraints using an intertemporal investment model.

In Section 4 we describe our theoretical framework. Building on Hennessy and Whited (2007), firms make production and financial decisions subject to financial frictions, including credit constraints, real frictions, and time-varying risks (both aggregate and idiosyncratic). To capture the strong link between credit access and firm size, the model incorporates size-dependent borrowing limits (Gopinath et al. 2017). Because of this dependency, when borrowing constraints bind, the shadow cost of credit directly enters the Euler equation for intertemporal investment, driving a wedge between the MRPK and the user cost of capital (i.e., the investment gap), formalizing the intuition developed above.

In Section 5, we outline the strategy for estimating firm-level shadow costs and present the results. We recover the shadow costs of credit for different sub-populations of firms via a Euler equation estimation. As in Whited and Wu (2006), we parameterize the shadow prices as functions of observables and use credit demand shifters to estimate the sensitivity of borrowing limits to firm size. Identification leverages the covariation between investment gaps, firm characteristics, and credit limits in the microdata, addressing issues related to selection, simultaneity, and measurement error. Importantly, we utilize data on firms' credit applications—a proxy for unmet credit demand—to separately estimate the Euler equation parameters (and the implied shadow costs of credit) for firms more or less likely to face constrained access to bank financing.

In Section 6, we use the fitted values of the Euler equation parameters to recover the distribution of shadow costs and compare their size and variation with the size and variation of borrowing costs observed in the data. Our estimates indicate that the average shadow price of credit is approximately 3 percent, with a 10–90 percentile range of approximately 8 percent. The distinction between firms with and without credit demand explains much of this heterogeneity. Among firms that submit credit applications, the average shadow cost is nearly 5 percent, while it is close to zero for those without credit applications. Equally important is the substantial heterogeneity in shadow prices between firms of different sizes. We estimate a shadow cost of credit of nearly 15 percent on average when we focus on the smaller firms in the economy.

Three key results emerge from studying the joint distribution of the market cost (borrowing rates) and the shadow costs of credit. First, in the subsample of firms for which the estimated shadow costs are positive, the cost of credit is comparable in magnitude to its shadow cost, but the latter is significantly more dispersed. This result speaks to the literature studying the welfare costs of resource misallocation to credit market frictions (e.g., Buera et al. 2011; Midrigan and Xu 2014; Gopinath et al. 2017; Bau and Matray 2023). Because the shadow cost of credit acts as an implicit tax on producers, these results indicate that some firms are too small and others too small relative to their "socially efficient" size (Restuccia and Rogerson 2008; Restuccia and Rogerson 2013). This issue is particularly acute for small and medium enterprises. In this subpopulation, shadow costs are not only significantly higher than interest rates (nearly 15 percent, on average) but also far more dispersed. These findings suggest that financial frictions that cause credit rationing, such as asymmetric information frictions, lead to more underinvestment and more capital misallocation in the left tail of the firm-size distribution.

Second, shadow costs and borrowing rates are positively correlated because both are sensitive to variation in credit risk factors. However, we document a much higher sensitivity of shadow costs to such a variation. The shadow cost gradient with respect to firm age, size, length of lending relationships, and firm credit rating is twice as steep (or more) than the interest rate gradient.

Relatedly, our third empirical result highlights that, while costs and shadow costs co-move, their relationship is not monotonic. Borrowing rates and shadow costs increase almost one-for-one in the left tail of the shadow cost distribution. However, further along the distribution, borrowing rates flatten and eventually decline, while shadow costs continue to rise. This non-monotonic relationship aligns with the theoretical predictions of credit rationing models (Stiglitz and Weiss 1981, Stiglitz and Weiss 1992). In these models, lenders may refrain from raising interest rates in response to excess credit demand because higher rates can reduce expected returns by increasing the likelihood of default. Two mechanisms drive this phenomenon: first, higher interest rates deter low-risk borrowers, leaving a riskier pool of applicants (the sorting effect); and second, higher rates incentivize borrowers to pursue riskier projects (the incentive effect). Together, these forces can explain the observed flattening—and eventual decline—of borrowing rates as shadow costs rise.

In Section 7, we examine the sensitivity of investment to the shadow price of credit. As predicted by theory, shadow costs are strongly positively correlated with investment gaps and even more so with investment rates. Evaluated at the mean of the distribution, a 1 percent decrease in shadow costs increases firms' investment rates by 50 basis points for firms that file credit applications. Once again, the average elasticity masks substantial heterogeneity across the firm size distribution. As discussed above, shadow costs are higher for smaller firms, reflecting size-dependent credit constraints that restrict access to profitable investment opportunities. Because the shadow value of a dollar of credit is high, these firms respond to reductions in shadow costs with investment increases ten times greater than those of large firms (elasticity of 0.12 vs. 0.01). These findings provide strong support to the hypothesis that quantity limits—rather than interest rates—are the primary margin of adjustment in bank credit markets, particularly for small and medium-sized firms. Section 8 concludes with final remarks.

2 Data

Data sources. We assemble a comprehensive firm-bank matched database that combines micro-level information on firm production, assets and liabilities, and credit market activity for the census of non-financial incorporated firms active in Italy between 1997 and 2013.

We gather detailed yearly information on balance sheets, income statements, and registry variables from Cerved Group S.p.A. (Cerved database). This data enables us to analyze firms' production decisions, including investments, employment, purchases of intermediate inputs, and sales, and to measure firms' fixed tangible assets. We complement these data with information on industry-specific price deflators, industry-specific depreciation rates of fixed assets, and socioeconomic indicators measured at the province level, all of which are collected from the publicly available archives of the Italian National Statistical Institute.[4]

Using unique firm identifiers, we merge the firm-level dataset with administrative data on firm credit balances, borrowing costs, and credit applications. Credit balance information is sourced from the Italian Credit Registry (CR) maintained by the Bank of Italy. This data set provides detailed, confidential information on the credit relationships of firms with all banks operating under Bank of Italy's supervision, disaggregated by credit

[4]Data available at https://www.istat.it/en/.

type: term loans (secured by assets or backed by account receivables) and revolving credit lines.[5]

Information on firms' borrowing rates comes from the TAXIA dataset, also maintained by the Bank of Italy. TAXIA covers the majority of firm-bank lending relationships and provides us with granular data on firms' borrowing costs (annual percentage rates, APRs) for different types of lending facilities (term loans or credit lines) used by the firm.[6] This information is essential for constructing our measure of firm-specific user costs of capital, as discussed below.

Our data also enable us to observe loan applications. When a firm applies for a new loan, Italian banks can freely access the Bank of Italy's credit registry to review the firm's credit history. These credit history checks are recorded in the Initial Information Service (IIS) dataset, which we use to measure loan demand at the firm level. By combining the IIS dataset with the CR one, we determine whether loan demand is met at the extensive margin. Loan applications to new lenders are classified as either successful—when a new loan is granted within three months—or rejected (Jimenez et al., 2012). For existing lenders, we infer new credit applications by analyzing outstanding credit balances. Specifically, we classify a credit application as accepted by a legacy lender if the firm-bank pair's credit balance either remains constant (indicating a rollover) or increases (indicating new credit) from one year to the next.

Sample selection. Appendix A.1 outlines the data cleaning filters applied to construct our final dataset, which we briefly summarize here. Observations missing information on age, number of employees, revenues, total assets, geographical location, or industry of operation are excluded. Additionally, we remove firms in the finance, insurance, and real estate industries, as well as industries with significant government ownership or subsidies (e.g., public administration, education, and health), those dominated by

[5]Intermediaries report to the Credit Registry any relationship with a client whose total amount of credit granted plus guarantees provided by the borrower exceeds 30,000 euro (75,000 euros before 2008).

[6]Until 2003, TAXIA included approximately 90 banks, representing over 80% of total bank lending. From 2004, coverage expanded to include 103 national banks and 10 foreign branches and subsidiaries, accounting for more than 90% of lending relationships. Data is reported for all firm-bank lending relationships where the total outstanding credit granted, plus guarantees provided by the borrower, exceeds 75,000 euros.

multinationals operating through local subsidiaries (e.g., pharmaceuticals and tobacco), and industries where measuring sales is particularly challenging (e.g., household activities and organizations).

Table 1: Summary statistics

	Mean	SD	10 pctile	Median	90 pctile
Assets (million Euros)	2.93	8.82	0.13	0.71	5.61
Age	13.00	12.42	2.00	9.00	28.00
ROA	0.05	0.13	-0.03	0.05	0.16
Bank Leverage	0.45	0.44	0.00	0.37	0.98
Number of lending relationships	3.70	3.40	1.00	3.00	8.00
Length of lending relationships	4.05	3.64	0.75	3.00	9.50
Borrower	0.79	0.40			
Ever borrower	0.91	0.29			
Borrower loans	0.55	0.50			
Credit applications	0.58	0.49			
Accepted credit applications	0.62	0.48			
Exit	0.07				
Firms	640128				
Observations	4174365				

Notes. This table presents summary statistics for key firm-level variables. Assets refers to total assets, and Age is measured in years. ROA is the ratio of earnings to assets, while Bank leverage represents bank credit over total assets. Number of lending relationships is the count of financial institutions from which the firm borrows, and Length of lending relationships measures the consecutive years of credit interactions with the firm's main lender. Borrower is a dummy variable equal to one if the firm has any bank credit, while Borrower loans is one if a bank term loan is observed. Credit applications is a dummy equal to one if the firm submits any application for credit, and Accepted credit applications is one if at least one application is approved. Exit equals one if the firm is no longer observed in the Cerved dataset in the following year.

Sample properties. Table 1 reports the summary statistics of the main variables used in our analysis. Our final data set includes over 4 million firm-year observations, 640 thousand firms, and over 13 million credit relationships. It amounts to 80–90 percent of the value added produced by the non-financial corporate sector in the selected industries.

Our sample is primarily represented by privately held small and medium enterprises

(SMEs). On average, total firm assets amount to approximately €3 million. Almost 90 percent of the firms in our sample qualify as SMEs (defined as firms with total assets below €50 million), while fewer than 0.1 percent (294 firms) are publicly listed.

Given the broad coverage of our data, attrition emerges as a notable feature, with an unconditional one-year probability of exit at approximately 6 percent. This is neither new nor surprising (Haltiwanger, 2012): some firms enter, thrive, and grow, while others decline, and exit. These patterns are particularly pronounced among privately owned SME—the vast majority in our dataset—that are more exposed and less resilient to local and aggregate shocks.

On the financing side, bank debt constitutes a significant portion of firms' assets—45 percent on average and more than 90 percent for firms in the top decile of the leverage distribution. More broadly, approximately 80 percent of observations engage in some form of credit market interaction in a given year (Borrower=1), and 90 percent have done so at least once during the sample period (Ever Borrower=1). About 55 percent of firm-bank observations finance operations through term loans (Borrower Loans=1). On the extensive margin, in any given year, there is a 58 percent probability of at least one credit application by a firm, with approximately a 60 percent likelihood of observing at least one application being accepted. These statistics underscore the critical role of credit markets as a source of external finance for SMEs.

3 Investment gaps and access to credit

In this section, we use our microdata to recover measurable counterparts of firm-level investment gaps (IG_{it}) introduced in Equation (1). We provide descriptive evidence linking variation in investment gaps to proxies for financial frictions and firms' credit market participation, which is consistent with the intuition developed in the context of a simple neoclassical framework discussed in Section 1.

3.1 Estimation strategy

Estimating MRPK. Without loss of generality, we can express a firm's marginal revenue product of capital as the product of the value of the marginal product (VMP_{it}^K) and the inverse markup (μ_{it}^{-1}):

$$MRP_{it}^K \equiv \frac{\partial pq_{it}}{\partial k_{it}} = \underbrace{p_{it}\frac{\partial q_{it}}{\partial k_{it}}}_{VMP_{it}^K}\underbrace{\left(1 + \frac{q_{it}}{p_{it}}\frac{\partial p_{it}}{\partial q_{it}}\right)}_{\mu_{it}^{-1}} = \theta_{it}^K \frac{pq_{it}}{k_{it}}\frac{1}{\mu_{it}}, \tag{2}$$

where the last equation decomposes the value of the marginal product into the output elasticity (θ_{it}^K) and the average product ($\frac{pq_{it}}{k_{it}}$) using the definition of the output elasticity.

We estimate MRPK by taking Equation (2) to the data. We use the perpetual inventory method (Hall and Jorgenson 1967) to construct a measure of firms' stock of fixed assets (tangible and intangible), k_{it}. We use this measure and firm-level sales to calculate firms' average product of capital, pq_{it}/k_{it}. We recover firm-time varying output elasticities through a production function estimation. Specifically, we consider a general gross output (log-)production function of the form:

$$\log(q_{it}) = \omega_{it} + f(\log(k_{it}), \log(l_{it}), \log(m_{it}), \gamma) + \epsilon_{it},$$

where l_{it} and m_{it} represent labor and intermediate inputs, respectively. ω_{it} denotes firm-level (log) productivity, observed by the firm at the time of its production decisions, while ϵ_{it} captures a production shock that occurs after input decisions are made. $\boldsymbol{\gamma}$ is a vector of structural parameters to be estimated.

We estimate the production function parameters $\boldsymbol{\gamma}$ using the structural approach proposed by Ackerberg et al. (2015), extended to account for selection bias arising from firm exit decisions (Olley and Pakes, 1996). To capture firms' production technologies $f(\cdot)$, we adopt a flexible Translog functional form, which allows us to recover firm-time-specific output elasticities: $\theta_{it}^K = \theta^K(\log(k_{it}), \log(l_{it}), \log(m_{it}); \boldsymbol{\gamma})$. To account for variation in production technologies across industries, we perform the production function estimation separately for each 3-digit NACE Rev. 2 code.

Finally, we recover firm-year markups using the production-side approach proposed by De Loecker and Warzynski (2012). In the spirit of Hall et al. (1986), this identification approach relies on the theoretical intuition that, conditional on the state variables of the problem, the first-order conditions of the cost-minimization problem for intermediate inputs provide an expression linking revenue to intermediate cost shares, output elasticities, and markups:

$$\hat{\mu}_{it} = \hat{\theta}^M_{it} \left(pq_{it}/p^M m_{it} \right).$$

Here $pq_{it}/p^M m_{it}$ represents the inverse of the expenditure share on intermediate inputs in revenues (directly observed in the data), and $\hat{\theta}^M_{it}$ denotes the output elasticity with respect to intermediate inputs, obtained from the estimation of production functions as described above.[7] Consistent with previous studies, our production function estimation yields average output elasticity estimates of $(\theta^L, \theta^K, \theta^M)$ = (0.21, 0.05, 0.75) and an average markup μ of 17 percent.[8]

Measuring user costs. We construct firm-time-varying user costs of capital as:

$$R_{it} = r_{it}(1 - z) + \delta.$$

We obtain industry-specific and time-varying depreciation rates of fixed assets, δ, from the Italian Statistical Agency (National Accounting Tables). Our baseline measure of borrowing costs is the average APR on term loans, calculated as the average across all outstanding term loans from the TAXIA dataset, adjusted to account for the interest tax shield granted to Italian corporations (stable at approximately $z = 0.24$ during our sample period).

The use of term loans as a measure of the marginal (borrowing) cost of fixed assets is justified by their prevalence in firm financing and by the sensitivity of firm investment to their changes. In our sample, term loans account for approximately three-quarters of total bank debt in the credit registry and are the primary credit product used for financing fixed asset expenditures.[9] Furthermore, in unreported regressions, we find that changes

[7]We follow De Loecker and Warzynski (2012) and adjust expenditure shares using the residuals from a regression of a polynomial function of deflated inputs on deflated revenues. This adjustment accounts for variation in firms' output unrelated to changes in their input utilization, such as those driven by demand, input prices, or productivity.

[8]See, e.g., De Loecker and Warzynski (2012), Ackerberg et al. (2015), and Lenzu et al. (2024).

[9]For firms borrowing from multiple banks, we calculate a value-weighted average APR by averaging the rates across lenders, weighted by the proportion of total loans granted by each institution, $r_{it} = \sum_b w_{ibt} r_{ibt}$, where $w_{ibt} = \frac{Loans_{ibt}}{\sum_b Loans_{ibt}}$.

in bank loans explain a larger share of variation in investment rates, with the elasticity of investment to changes in loans being three times greater than that for credit line draws.

Approximately 20 percent of the observations in our sample pertain to firms that do not actively engage in credit market transactions, leaving us without observable information on their borrowing costs. These firms, primarily SMEs, are of particular interest to our study, as credit market frictions may disproportionately distort their investment policies. As explained in Appendix A.2, we use firm-specific characteristics and geographical location to infer the interest rates that non-borrowers might have been charged had they engaged in credit market transactions. This approach is motivated by evidence showing that banks typically set borrowing rates using a limited set of observable characteristics (Crawford et al. 2018). Furthermore, previous studies emphasize that, particularly for SMEs, bank financing is strongly tied to local credit markets, where proximity between borrowers and lenders facilitates information acquisition (Petersen and Rajan 2002; Degryse and Ongena 2005). For each year and local credit market—defined by the boundaries of Italian provinces—we use the firm-bank matched dataset to estimate loan-pricing predictive regressions.[10] The set of predictors includes industry, age, assets, credit score, asset turnover, ROA, and whether the firm had any credit defaults during or prior to the year. These variables are chosen for two reasons. First, they provide a parsimonious set of firm characteristics that ensures a common support between borrowers and non-borrowers within each year-market combination. Second, they are observable indicators commonly used by banks to evaluate firms' riskiness and creditworthiness (Albareto et al. 2011). We estimate the pricing regressions focusing on the subsample of newly established relations. This is the most relevant comparison because non-borrowers would be new customers for the bank in case they approach them. Moreover, for new lending relationships, we do not have to account for the dynamics of firm-bank relationships and the acquisition of soft information and lower monitoring costs that repeated interactions bring about.

A second group of observations consists of firms that engage in credit market

[10]Italian provinces—108 in our dataset, spanning 22 different regions—are natural candidates for defining local credit markets for small-business lending (Guiso et al., 2013). They are administrative units comparable to US counties and are used by the Bank of Italy as proxies for local credit markets in regulatory and supervisory contexts.

transactions but for which we lack data on the interest rate for term loans. This occurs when firms rely solely on revolving credit lines, borrow from banks excluded from TAXIA, or have outstanding loan balances below the Credit Registry reporting threshold (see footnotes 5 and 6). For these cases, the missing price problem is less severe. In addition to firm-specific characteristics and geographical location, we can augment the loan-pricing regressions with variables such as bank leverage, the duration of each credit relationship, and the total number of lending relationships.

Table 2: Estimates or MPRK, user costs, and investment gaps

	Mean	10 pctile	Median	90 pctile	Mean	10 pctile	Median	90 pctile
	Panel a: Full sample							
MRPK	0.27	0.08	0.19	0.51				
r	0.06	0.03	0.05	0.08				
δ	0.10	0.06	0.11	0.12				
R	0.16	0.12	0.16	0.20				
IG	0.12	-0.07	0.05	0.36				
	Panel b: Borrowers				*Panel c: Non-borrowers*			
MRPK	0.25	0.08	0.18	0.46	0.33	0.10	0.23	0.68
r	0.06	0.03	0.05	0.08	0.06	0.04	0.06	0.09
δ	0.11	0.06	0.11	0.12	0.10	0.06	0.10	0.12
R	0.16	0.12	0.16	0.20	0.16	0.11	0.16	0.20
IG	0.10	-0.07	0.04	0.32	0.19	-0.05	0.10	0.53
	Panel d: Borrowers w/ Loans				*Panel e: Borrowers w/out Loans*			
MRPK	0.23	0.08	0.17	0.42	0.27	0.08	0.20	0.52
r	0.06	0.03	0.05	0.09	0.06	0.03	0.05	0.08
δ	0.11	0.10	0.11	0.12	0.10	0.06	0.10	0.12
R	0.17	0.13	0.16	0.20	0.16	0.11	0.16	0.19
IG	0.08	-0.08	0.02	0.27	0.13	-0.07	0.06	0.38

Notes. This table presents summary statistics for the distribution of investment gaps (IG) and their components. Panel a reports statistics for the full sample. Panel b and Panel c focus on firm-year observations for borrowers (firms with a positive credit balance) and non-borrowers (firms without credit market participation), respectively. Panel d and Panel e further divide the borrower sample into firms with outstanding bank loans and those relying exclusively on revolving credit lines.

Investment gaps. Table 2, panel a, reports descriptive statistics of the distribution of MRPKs, user costs, and investment gaps for the full sample. Based on our estimates, the median firm in our sample exhibits an MRPK of approximately 27 percent. However, the distribution is highly dispersed and right-skewed, with a 90th−10th percentile range of 0.08−0.51 percent. In contrast, the distributions of borrowing rates, depreciation rates, and user costs of capital are symmetric and exhibit modest variation around their central tendencies. The (before tax) cost of credit ranges between 3−9 percent while the depreciation rates range between 11−20 percent, depending on the industry.

Consistent with these figures, the distribution of firm-level investment gaps, IG, is centered around zero. Within the neoclassical framework, this suggests that investment policies are relatively undistorted. However, we observe substantial dispersion and right-skewness in the distribution of investment gaps, driven entirely by the distribution of MRPKs. Across the entire sample, the average investment gap is approximately 12 percent. Firms in the upper deciles of the gap distribution exhibit MRPK that exceeds the user cost of capital by 36 percentage points or more.

3.2 Investment gaps and credit market participation

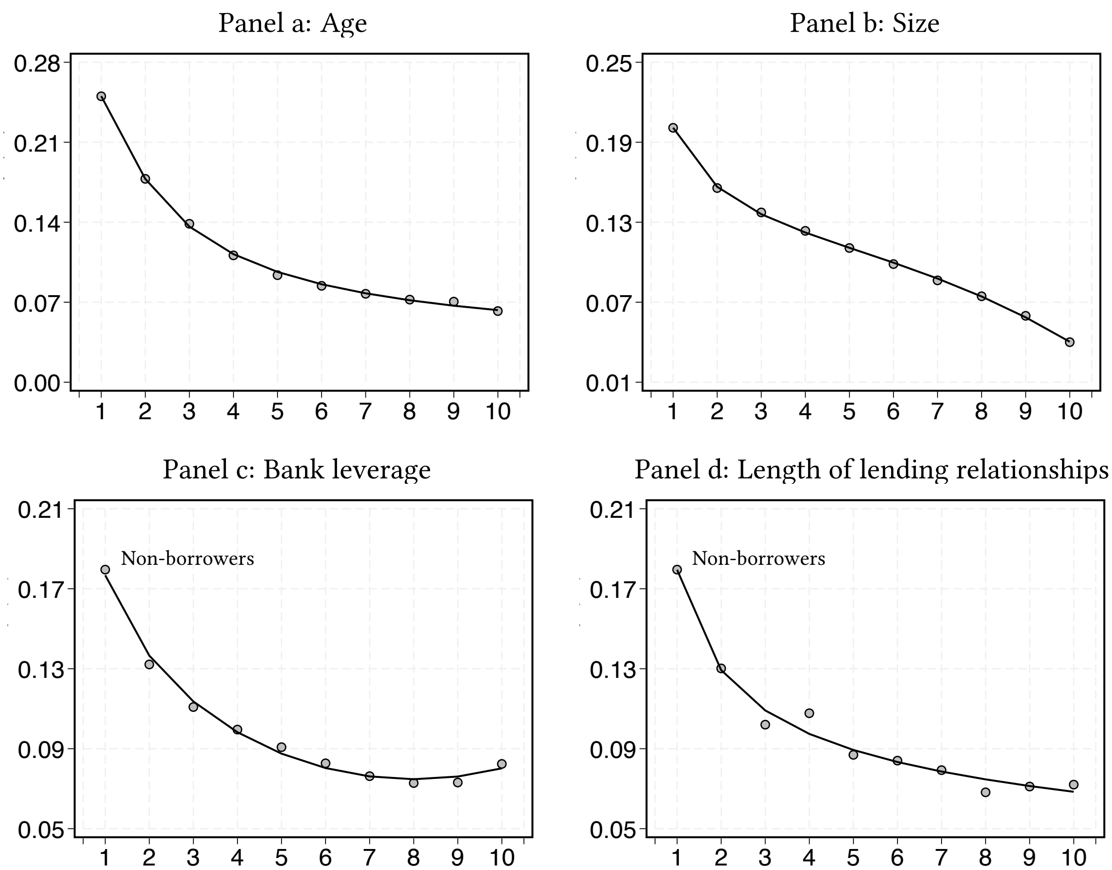
We now present descriptive evidence linking both cross-sectional and time-series variations in the estimated investment gaps to common proxies of financial frictions and to firms' credit market access and participation.

Access to credit. To begin, we analyze the distribution of investment gaps and its components for subsamples of our data. Specifically, we partition observations into groups based on whether we observe any credit market participation (borrowers vs. non-borrowers) and whether we observe long-term debt obligations (term loans versus credit lines).[11] As shown in panels b through e in Table 2, the estimated MRPK is, on average, twice as high for non-borrowers than for borrowers. Table 2 also highlights that investment gaps are over 1.5 times larger for those borrowers who have access only to

[11]Non-borrowers are observations with no bank credit reported, either in the Credit Registry (CR) or in liability statements.

credit lines compared to those with outstanding long-term financing. Because credit lines are a more expensive type of credit and can be revoked at a lender's discretion, firms rarely turn to credit lines to finance capital expenditures in fixed assets, unless the supply of bank loans is constrained or denied by credit institutions.[12]

Figure 2: Investment gaps and firm characteristics



Notes. These figures report the average investment gap (y-axis) sorting observations into ten groups based on the quantiles of the distribution of firm age, size (total assets), bank leverage (bank debt over total assets), and the length of lending relationship with their main lender. In panels c and d, the first quantile group collect observations referring to firms that do not borrow

[12]Consistent with this hypothesis, borrowers with only credit lines are younger and smaller, overrepresented in Southern regions of Italy, and in industries with lower tangible-to-intangible asset ratios (e.g., services). Not coincidentally, all these firm-specific variables are commonly regarded as proxies for credit constraints.

Firm characteristics. Evidence on the relationship between investment gaps and credit market frictions comes from their correlation with firm characteristics. Panels a and b of Figure 2 show that investment gaps decrease monotonically with firm age and size, two common proxies for financial constraints (Hadlock and Pierce 2010). Moreover, consistent with the descriptive statistics above, we find a strong inverse relationship between investment gaps and bank leverage (bank debt over total assets). Panel c displays the average investment gap across the quantiles of the distribution sorting forms based on their bank leverage ratios. The first quantile includes observations with no bank credit reported (non-borrowers). In line with the statistics reported in Table 2, firms in this group exhibit noticeably larger investment gaps. Importantly, we also find a relationship along the intensive margin, as investment gaps decrease monotonically with leverage also among borrowers. Consistent with the presence of quantity constraints, these findings suggest that access to external financing enables firms to undertake profitable investment opportunities, thus closing the gap between the marginal profitability of capital and its user cost.

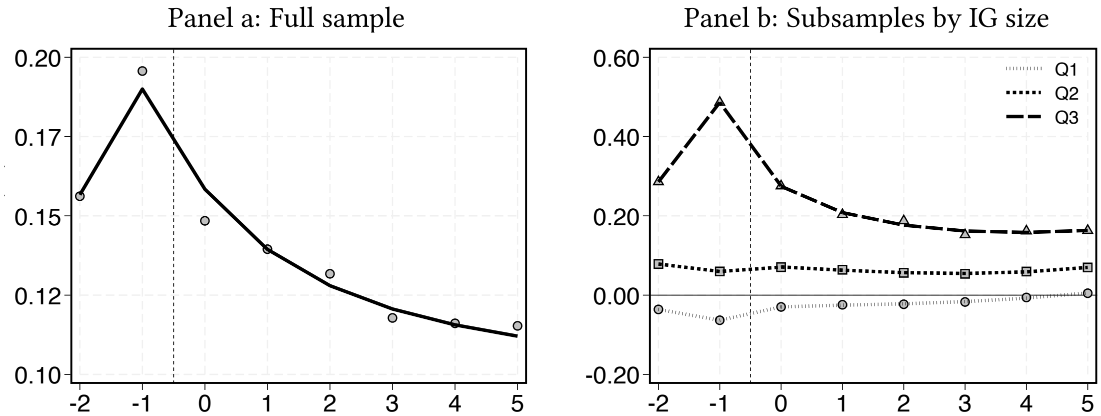
IG dynamics as lending relationships unfold. Panel d of Figure 2 uses data from firms' credit histories in the CR to plot average investment gaps against the length of a firm's lending relationship with its main lender. Enduring relationships reduce lenders' expected costs of credit provision, as prior interactions lower perceived loan risk (Diamond 1991). In addition, monitoring and screening costs are lower for existing customers as the information acquired at one point can be reused to assess future credit risk. In response to these reduced costs, lenders can adjust loan terms or relax credit limits over time.[13] Panel d shows that, consistent with a relaxation of borrowing limits as relationships unfold, firm-level investment gaps decline monotonically with the length of lending relationships, indicating that the marginal return on capital falls more than its user cost, aligning firms' capital endowment closer to the profit-maximizing level.

Figure 3 further sheds light on these dynamics. Panel a shows the evolution of the average of the absolute investment gap around the year when it first established a lending relationship with a bank ($t = 0$), restricting our attention to firms that have been established during our sample period. We observe a sharp reduction in investment gaps

[13]See, e.g., Petersen and Rajan (1995), Berger and Udell (1995), and Bharath et al. (2011).

with access to credit and steady convergence toward the frictionless benchmark ($|\text{IG}_{it}| = 0$) as firms strengthen their lending relationships. On average, compared to the year before establishing a lending relationship, investment gaps are 30 percent lower one year after gaining access to credit and nearly 60 percent lower after 8 years.

Figure 3: Investment gaps and credit market participation



Notes. This figure shows the dynamics of investment gaps (y-axis) as firms gain access to credit. An event ($t = 0$) is defined as the year a firm first establishes a lending relationship. Panel a displays average absolute gaps across years relative to the event. Panel b shows average gaps over the same timeline, with observations grouped into terciles based on their investment gap distribution in $t - 1$.

As an additional empirical test, panel b divides the sample into terciles based on the size and sign of the investment gap observed in the period prior to establishing a credit relationship. Observations in the first (third) tercile correspond to firms with the largest negative (positive) gaps in $t = -1$. This sorting allows us to investigate whether firms with positive investment gaps exhibit sharper reductions upon access to credit, as predicted by theory. Consistent with this hypothesis, the average effect is entirely driven by firms in Q3 who show a significant initial reduction in IG immediately after gaining credit access, as well as the steepest gradient in subsequent periods. Notably, we also find a significant increase in investment gaps in the year prior to credit access, suggesting the presence of profitable investment opportunities that await financing.

3.3 Taking stock

The descriptive evidence presented so far shows that heterogeneity in investment gaps reflects, at least in part, access to credit and the intensity of credit market participation. We also documented a strong correlation of IG with common proxies for credit market frictions. However, financial frictions are not the only factors that can explain the sign and magnitude of investment gaps. First, investment gaps are derived from observed production and financing choices, and measurement or estimation errors may erroneously suggest that some firms' investment policies (e.g., small and young firms) are suboptimally low (or high) when they are not. Second, real capital adjustment costs, risks associated with capital accumulation, and aggregate uncertainty can explain why firms with positive investment gaps may not invest, even in the absence of credit constraints.

To address these considerations, the remainder of the paper adopts a structural approach, using variation in investment gaps to recover firm-specific financial wedges in optimality conditions. In the spirit of Hennessy and Whited (2007), we introduce a dynamic partial equilibrium investment model that incorporates credit constraints, real frictions, and risk. The model formalizes the relationship between investment gaps and the shadow costs generated by binding credit constraints in the Euler equation that governs intertemporal capital accumulation decisions. We parameterize shadow costs as a function of firms' observable credit demand and characteristics and estimate the model using the Generalized Method of Moments (GMM). Finally, we analyze the distribution of shadow costs across firms, compare these shadow costs to the observed borrowing interest rates, and examine their relationship to firms' investment behavior.

4 Economic model

The entrepreneur maximizes the expected present value of the discounted utility of net cash flows from the business, $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(d_{it})$, where the utility function is such that $U_d > 0$, $U_{dd} < 0$, and satisfies the Inada conditions. β is a risk-neutral discount factor.

We denote by $\mathcal{J}_{it}$ the entrepreneur's information set at the beginning of year t, which includes a set of firm-level state variables (detailed below), as well as information on product and factor markets and credit supply conditions. Given $\mathcal{J}_{it}$, the entrepreneur chooses between two options: (i) repaying any outstanding debt, refinancing, and continuing production, or (ii) exiting the market, potentially through default.

Product and factor markets. Conditional on production, the entrepreneur optimally chooses inputs and financial structure, subject to technological and financial constraints. Output is produced using a production function $q_{it} = e^{\omega_{it}} f(k_{it}, l_{it}, m_{it})$, which exhibits decreasing returns to scale for each input (capital, labor, intermediates). The term ω_{it} represents the entrepreneur's idiosyncratic log productivity, evolving as a first-order Markov process.

The entrepreneur faces a downward-sloping demand curve for their product, $q_{it} = q(p_{it}, \eta_i, \Omega_t)$, where p_{it} is the output price, η_i represents the residual demand elasticity and Ω_t is an aggregate demand shifter. It acts as a price taker in the input markets, hiring labor l_{it} at a wage rate w_t and purchasing intermediates m_{it} at a price p_t^M. The entrepreneur owns the capital stock, which is augmented through investments (i_{it}) at a normalized price of $p_t^K = 1$. Capital evolves dynamically according to the standard law of motion: $k_{it+1} = k_{it}(1 - \delta) + i_{it}$, where δ is the depreciation rate. Since current investments become productive only after a lag, the stochastic evolution of productivity and demand introduces risk into capital accumulation, resulting in uncertain realizations of the MRPK. The realizations of idiosyncratic productivity, residual demand elasticity, aggregate demand conditions, and input prices are known to the entrepreneur at the beginning of each period, $(\omega_{it}, \eta_i, \Omega_t, p_t^M, w_t, p_t^K) \in \mathcal{J}_{it}$.

Financing. Firms can finance capital purchases through internally generated revenues and bank debt, b_{it+1}. The credit contract offered by lenders to each firm i consists of a one-period debt contract specifying an interest rate and a borrowing limit $\{r_{it+1}, \theta_{it}\}$:

$$b_{it+1} \leq \theta_{it} k_{it+1}. \tag{3}$$

The interest rate, taken as given by firms, may vary across firms and over time based on market conditions (e.g., lenders' cost of funds) and a limited set of observable firm characteristics that broadly capture credit risk factors (e.g., the firm's industry, size, or credit score). The borrowing limit depends on two factors: local credit supply conditions s_{mt}, which affect all firms operating in a given credit market m, and the size of the firm. Specifically, we adopt the following functional form: $\theta_{it} := \left[\theta_{0,mt} + \theta_{1,mt}\frac{\Psi(k_{it+1})}{k_{it+1}}\right]$, where $\Psi(k_{it+1})$ is an increasing and convex function of firm assets, capturing the presence of size-dependent borrowing constraints (Arellano et al. 2012, Gopinath et al. 2017), consistent with the evidence presented below.[14]

This type of contract arises as a profit-maximizing choice for the lender when adverse selection and incentive problems are both present (Stiglitz and Weiss 1992).[15] In this context, credit rationing emerges as an equilibrium outcome for some firms whose credit demand at r_{it+1} exceeds the maximum credit offered. Note that when $\theta_{1,mt} = 0$, our formulation nests models with exogenous borrowing limits as a special case (e.g., Whited 1992, Whited and Wu 2006). Unlike these models, the dependency of borrowing constraints on firm size implies that debt and capital are not separable within the profit function, and, as we illustrate below, the shadow cost of credit directly enters the Euler equation governing firms' intertemporal decisions.

Firm problem. After observing the realization of the productivity shock and considering its capital, legacy debt, and aggregate state variables, the entrepreneur decides whether to continue operating or to default and exit.

We denote by $n_{it} := k_{it} - b_{it} > 0$ the firm's net worth. Using this, we reformulate the borrowing constraint (3) as:

$$k_{it+1} \leq \lambda_{it} n_{it+1}. \tag{4}$$

[14]Arellano et al. (2012) provide evidence of dependent size-dependent borrowing constraints and rs conclude that modeling financial frictions in this way is essential to understanding how borrowing constraints affect firm behavior across economies. To model frictions, they introduce scale-invariant component in the borrowing constraint that implies, which forces smaller firms to shrink their scale or avoid borrowing entirely. Gopinath et al. (2017) adopt a similar parameterization to model size-dependent borrowing constraints, assuming $\theta_{it} = \theta_0 + \theta_1 \frac{e^{k_{it+1}}-1}{k_{it+1}}$.

[15]Stiglitz and Weiss (1992) show that, despite a rich contract space might be available, a combination of a (partially) risk-adjusted borrowing rate and borrowing limit balances the impact of contract terms on the mix of applicants (adverse selection) and borrowers' risk-taking incentives (moral hazard) on lenders expected profits. This would be the case if the expected (certain equivalent) return received by the lender does not increase monotonically with the rate of interest charged and borrowers differ in multiple dimensions that affect the risk of credit so that a separation equilibrium à la Rothschild and Stiglitz (1971) cannot be achieved with {interest rate, collateral} contracts.

where $\lambda_{it} := \left[\lambda_{0,mt} + \lambda_{1,mt}\frac{\Psi(k_{it+1})}{n_{it+1}}\right] \geq 0$, with $\lambda_0 = \frac{1}{1-\theta_{0,mt}}$ and $\lambda_1 = \frac{\theta_{1,mt}}{1-\theta_{0,mt}}$ and the Bellman equation characterizing the entrepreneur's decision problem is:

$$V_t\left(n_{it}, k_{it}, \omega_{it}\right) = \max\left\{\Phi_t\ , \sup_{\{n_{t+1}, k_{it+1}, l_{it}, m_{it}, p_{it}\}} U(d_{it}) + \mathbb{E}\Big[M_{t,t+1}V_{t+1}(n_{it+1}, k_{it+1}, \omega_{it+1})|\mathcal{J}_{it}\Big]\right\} \tag{5}$$

$$\begin{aligned} s.t \quad & d_{it} = p_{it}q_{it} - w_t l_{it} - m_{it}p_t^M - (r_{it} + \delta)k_{it} + (1 + r_{it+1})n_{it} - n_{it+1} - c_{it} \\ & q_{it} = e^{\omega_{it}} f(k_{it}, l_{it}, m_{it}) \\ & q_{it} = q(p_{it}, \eta_{it}, \Omega_t) \\ & k_{it+1} \leq \lambda_{it} n_{it+1}, \end{aligned}$$

where the function $c_{it} := c(k_{it}, k_{it+1})$ captures real adjustment costs of capital. $M_{t,t+1} = \beta\frac{U_d(d_{it+1})}{U_d(d_{it})}$ denotes the stochastic discount factor (SDF) between t and $t+1$. Note that the value function is indexed by time because it depends on the market structure, factor prices, and financial conditions, which are assumed to be constant across agents in a given time period and omitted in Equation (5) to economize on notation. We relax this assumption in our empirical estimation by allowing production function parameters to vary by industry, markups to vary by firm and time, and financial conditions to vary by time and regions.

Equation (5) indicates that, if the entrepreneur chooses to continue, it determines its optimal input demand $(k_{it+1}, l_{it}, m_{it})$, production price, p_{it}, and net worth, n_{it+1}. If it opts to exit, it receives residual value of the firm's assets after debt repayments $\Phi_t \geq 0$, measured in utils. To formalize the firm's exit problem, we define the indicator function E_{it}, which takes the value of one if incumbent i exits at time t. As in Ericson and Pakes (1995) Olley and Pakes (1996), and Hennessy and Whited (2007), the solution to the discrete choice control problem in Equation (5) follows a threshold rule:

$$E_{it} = \begin{cases} 0 & \text{if } \omega_{it} \geq \underline{\omega}_t(n_{it}, k_{it}) \\ 1 & \text{otherwise.} \end{cases} \tag{6}$$

The threshold level, $\underline{\omega}_t(\cdot)$, is a time-varying function of the firm-specific state variables, n_{it} and k_{it}, as well as aggregate states.[16] Section 5 provides evidence that strongly supports

[16]The threshold $\underline{\omega}_t(\cdot)$ is decreasing in k_{it} and n_{it} because the dividend function (and thus the value function) is increasing in both capital and net worth. Intuitively, firms with a larger capital stock expect higher future returns for any given level of current productivity, making it optimal to continue operating even at lower realizations of ω_{it}. Similarly, holding capital constant, firms with greater net worth are less leveraged and

these theoretical predictions.

Capital Euler equation. Denote by χ_{it} the multiplier attached to the borrowing constraint (4), scaled by the shareholders' marginal utility from dividends in period t, and define $c_k := \frac{\partial c_{it}}{\partial k_{it+1}}$, $c'_k := \frac{\partial c_{it+1}}{\partial k_{t+1}}$, and $\Psi_k := \frac{\partial \Psi(k_{it+1})}{k_{it+1}}$. The firm's optimal investment policies are characterized by the following Euler equation governing capital accumulation decisions:

$$\mathbb{E}_{\omega_{it} > \underline{\omega}_{it}} \left[M_{t,t+1} \left(\mathrm{IG}_{it+1} - c'_k \right) \right] = \chi_{it} \left(1 - \lambda_{1,mt} \Psi_k \right) + c_k. \tag{7}$$

The left-hand side of Equation (7) represents the risk-adjusted present value of the expected investment gap $\mathrm{IG}_{it} := MRP^k_{it+1} - (r_{it+1} + \delta)$ introduced in the previous sections. The expectation operator reflects that firms maximize over the subset of future realizations of firm-specific and aggregate states where production is optimal.

The right-hand side of the equation formalizes the intuition discussed in Section 1 about the relationship between investment gaps and credit market frictions. A binding borrowing constraint distorts firms' intertemporal investment decisions, driving a wedge between the marginal product of capital and its observed user cost. The magnitude of such distortions is captured by the financial wedge:

$$\tau_{it} := \chi_{it} \left(1 - \lambda_{1,mt} \Psi_k \right). \tag{8}$$

We will refer to this financial wedge as the *shadow cost of credit*.

Equation (7) illustrates how firms' production decisions and user cost of capital embedded in the investment gaps can help recover the distribution of firm's shadow prices and shadow costs for the firms via the Euler equation. It also stresses how the identification of shadow prices requires one to account for other economic forces that drive variation in investment gaps. First, firms' MRPKs may co-move with the stochastic discount factor (SDF), $M_{t,t+1}$ (David et al. 2020). For example, due to differences in size or financial structure, some firms might be more exposed to fluctuations in aggregate risk.[17]

therefore face lower debt repayment burdens. As a result, for any given k_{it}, firms with a higher net worth are more likely to continue operating at lower productivity realizations.

[17]David et al. (2020) emphasize that cross-sectional variation in the SDF can explain part of the dispersion in MRPK between firms. Heterogeneous exposures to aggregate risk imply that the SDF becomes firm-specific. Although we do not explicitly model this heterogeneity, we account for time series variation in aggregate risk in our empirical model.

Furthermore, accounting for SDF variation is also critical because periods of increased risk aversion often coincide with exacerbated credit market frictions, such as during financial crises. Second, even in the absence of aggregate and idiosyncratic risk, real adjustment costs and capital accumulation risk due to time to build introduce dispersion in realized MRPK between firms (Asker et al. 2014).[18] Consequently, part of the dispersion in MRPK between firms would persist even in an undistorted economy, where capital is chosen under uncertainty, it is costly to adjust, and becomes productive in the subsequent period.

Finally, Equation (7) highlights that part of the cross-sectional dispersion in MRPK arises from differences in firm-specific user costs. Since borrowing costs reflect (at least in part) variation in idiosyncratic credit risk, some dispersion in MRPK is actually efficient, even within narrowly defined industries. This observation is important because within-industry MRPK dispersion is often interpreted as evidence of inefficient capital allocation, potentially driven by financial frictions. Investment gaps address this by "risk-adjusting" MRPK, netting out firm-specific user costs.

5 Structural estimation of shadow prices

In this section, we describe how we map the theoretical components of the Euler equation to their empirical counterparts and present the identification strategy to estimate the pin down the shadow shadow cost of credit.

5.1 Measurement

Size-dependent borrowing constraint. We require a proxy of $\lambda_{1,mt}\Psi_k$, the size-dependent component of the financial wedge. To obtain a measurable counterpart of this object, we make a functional form assumption for $\Psi(k_{it+1}) = k_{it+1} \ln k_{it+1}$. Using this and Equation (3), we obtain the following increasing relationship between leverage and firm size:

$$\frac{b_{it+1}}{k_{it+1}} \leq \theta_{0,mt} + \theta_{1,mt} \ln k_{it+1}$$

[18] Time to build introduces capital accumulation risk because a capital stock determined in a previous period may become suboptimal ex-post, after productivity shocks are realized.

We estimate the parameters $\theta_{1,mt}$ and $\theta_{0,mt}$ using our micro data, allowing the coefficients across Italian regions (North, Center, and South) and years. This choice allows us to capture cross-sectional differences in local credit supply conditions as well as its cyclical movements.[19] To do so, we need to address two identification issues. First, the above equation holds with equality only for firms whose constraint is binding. To address this, we use information on credit applications to restrict the estimation sample to firms that (i) submitted at least one credit application in year t and (ii) had at least one credit application rejected. Second, we are interested in estimating the sensitivity of supply-driven borrowing limits to firm size. However, the firm leverage observed in the data is determined by a combination of both supply- and demand-side factors. To achieve identification, we adopt an instrumental variable strategy, relying on variation in firm's assets that is driven by changes in firm-level productivity, $\Delta\omega_{it}$, which we view as a shifter of credit demand. Consistent with this hypothesis, we find that changes in firm-level productivity are a strong predictor of firm's investment rates, even after controlling for firm's observable characteristics, firm-level unobservables, and shifts in local credit supply. Given the estimates $\widehat{\theta}_{1,mt}$ and $\widehat{\theta}_{0,mt}$, we recover the slopes $\widehat{\lambda_{1,mt}\Psi_k} = \frac{\widehat{\theta}_{1,mt}(1+\ln k_{it+1})}{1-\widehat{\theta}_{0,mt}}$.

Table 3 presents summary statistics for the distribution of $\widehat{\lambda_{1,mt}\Psi_k}$ for the entire sample and the splitting of the firm sample into sub-samples based on the size of the firm.[20] The estimated slope coefficients are all positive and generally bounded within the unit interval, suggesting that, on average, firms can borrow up to 60 cents against an additional euro of fixed assets. Importantly, consistent with the presence of size-dependent borrowing constraints, our estimates indicate that the ability to collateralize assets in order to lever up is increasing monotonically in firm size. To account for variation in borrowing limits due to firm and changing local credit market conditions, we assign each firm in the estimation

[19] Guiso et al. (2004) identify significant regional disparities in credit supply throughout Italy, particularly between the north and the south. These stem from two key interrelated factors: stronger balance sheets between financial intermediaries in the North and Central regions, and regional differences in socioeconomic and legal institutions. As a result, leverage rates are higher and the cost of capital is lower in the North compared to the Center, and especially in the South.

[20] The size groups correspond approximately to quintiles of the asset distribution. Group 1: total assets $\leq 250,000$ euros; Group 2: $250,000 <$ total assets $\leq 500,000$ euros; Group 3: $500,000 <$ total assets ≤ 1 million euros; Group 4: 1 million $<$ total assets ≤ 10 million euros; Group 5: total assets > 10 million euros. A coarser (terciles) finer (deciles) grouping yields similar results.

Table 3: Slopes of size-dependent borrowing constraints

	Full	Group 1 [Small]	Group 2	Group 3	Group 4	Group 5 [Large]
mean	0.59	0.50	0.55	0.58	0.65	0.72
p10	0.39	0.36	0.40	0.43	0.49	0.56
p50	0.58	0.51	0.55	0.59	0.67	0.70
p90	0.80	0.65	0.67	0.72	0.83	0.90

Notes. This table reports the estimates of the sensitivity of borrowing constraints to firm size for the full sample and for different size groups.

sample a value of $\lambda_1 \Psi_k$ equal to the average $\widehat{\lambda_{1,mt} \Psi_k}$ of firms that belong to the same (region $\times$ year) $\times$ size group.

Shadow prices. Next, we construct a measurable counterpart for firm-time-specific shadow prices, χ_{it}. We extend our model by parameterizing the shadow price as an exponential function of a parsimonious set of observable characteristics:[21]

$$\chi_{it} = \exp\Big\{\chi_0 + \chi_1 \cdot \text{ROA}_{it} + \chi_2 \cdot \text{g(sales)}_{it} + \chi_3 \cdot \text{g(sales)}_{ind,t} + \chi_4 \text{ age}_{it} + \chi_5 \text{ TFP}_{it}\Big\}. \tag{9}$$

We expect shadow prices associated with binding constraints to be higher for more profitable firms. Accordingly, our model includes sales growth and industry sales growth to reflect that only firms with good investment opportunities are likely to invest enough to be constrained. These firms are expected to belong to high-growth industries but exhibit low individual sales growth (Whited and Wu 2006). Age is included as a determinant of shadow prices because younger firms typically have more abundant growth opportunities than older firms, consistent with the observed dynamics of investment gaps and age, where younger firms exhibit larger gaps. As a result, we expect the marginal valuation of a dollar of investment to be higher for younger, borrowing-constrained firms. Note that firm size is excluded from the variables affecting shadow prices, as size effects are captured by heterogeneity in the slope $\lambda_1 \Psi_k$, as explained above.

[21] Whited (1992); Hubbard et al. (1995); Love (2003); Whited and Wu (2006) adopt similar parametric approaches to estimate shadow costs arising from equity constraints.

5.2 Estimating equation

To arrive at the estimating equation, we write the Euler equation (7) at realized values and rearrange:

$$M_{t,t+1}\left(\mathrm{IG}_{it+1} - c'_k\right) - \chi_{it}\left(1 - s_t\Psi_k\right) - c'_k + h(\omega_{it}, P_{it}) + \epsilon_{it} = 0. \tag{10}$$

We assume a standard quadratic functional form for the real adjustment costs function, $c_{it} = \frac{\alpha}{2}\left(\frac{(k_{it+1}-k_{it})}{k_{it}}\right)^2$, and substitute for its derivatives in the Euler equation (c'_k and c_k). Following Whited and Wu (2006), we adopt a reduced form specification for the SDF using the three-factor model of Fama and French (1993):

$$M^{-1}_{t,t+1} = \Big(m_0 + m_1 \cdot MKT_{t+1} + m_2 \cdot SMB_{t+1} + m_3 \cdot HML_{t+1}\Big),$$

where MKT is the market excess return; SMB is the return on a size-based arbitrage portfolio (long on small firms, short on large firms); and HML is the return on a value-based arbitrage portfolio (long on high book-to-market firms, short on low book-to-market firms). Finally, we use the parametric form in Equation (9) to replace for χ_{it}, and insert the estimated slopes $\widehat{s_t\Psi^g_k}$ and investment gaps IG_{it+1} discussed above. We estimate the 12 parameters which govern our Euler equation, $\{m_0 \ldots m_3, \alpha, \chi_0 \ldots \chi_6\}$, via Generalized Methods of Moments (GMM).

Selection. As discussed in Section 2, our data exhibit considerable churning, with an unconditional one-year exit probability of approximately 8 percent. Crucially, as our model suggests, exit events are not random, but systematically linked to firm characteristics such as current and expected productivity, capital stock, debt levels, and external factors, including current and anticipated credit supply conditions. This implies that the expectation errors ϵ_{it} appearing in Equation (7) are drawn from a truncated distribution, potentially biasing the estimation of the coefficients.

To address selection effects, we augment our estimation equation with a control function, captured by the term $h(\omega_{it}, P_{it})$ in Equation (7). The control function is a second-order polynomial in firm-level productivity, ω_{it}, and $P_{it} \equiv Pr\{E_{it+1} = 0\}$, the probability that the firm remains operational in $t + 1$, given the firm's choices and information set at time t. Intuitively, the control function serves the same purpose as the inverse Mills ratio in a standard two-step sample selection model (e.g., Heckman 1974). By controlling for selection probabilities, the expectational error ϵ_{it} satisfies $\mathbb{E}\left[\epsilon_{it}|h(\omega_{it}, P_{it})\right] = 0$ and $\mathbb{E}\left[(\epsilon_{it})^2|h(\omega_{it}, P_{it})\right] = \sigma_\epsilon^2$. However, the econometric problem is somehow more complex in our context than standard selection models because selection depends on two unobservable firm-specific variables, ω_{it} and $\underline{\omega}_{it}$, rather than a single variable.[22] Appendix A.3 provides additional details on the control function approach and the estimation of P_{it}.

Identification. We estimate model (7) in first differences to address potential sources of bias due to firm-specific unobservable characteristics. Consequently, identification requires orthogonality between the first differenced error term and a set of instruments ($\boldsymbol{z}_{it-2}$):

$$\mathbb{E}\left[\boldsymbol{z}_{it-2} \otimes (\epsilon_{it} - \epsilon_{it-1})\right] = 0.$$

Under our timing assumptions and rational expectations, double-lagged values of the regressors in the Euler equation serve as valid instruments.

To enhance inference, we require two additional moment restrictions to be satisfied. First, as in Whited and Wu (2006), we discipline the estimation of the SDF parameters by imposing the unconditional moment restriction that the expected value of the SDF equals $E\left[M_{t,t+1}\right] = (1 + r_f)^{-1}$, where r_f denotes the risk-free rate.Second, the shadow price parameters are identified by the (co-)variation in firm characteristics and the slopes of borrowing constraints, $\lambda_1\Psi_k$. To improve identification, we augment the set of instruments with a set of local credit supply shifters. Following an approach similar to Amiti and Weinstein (2018), we use firm-bank matched microdata from the CR to estimate a series of cross-sectional regressions (one for each year t):

$$\text{g(Credit)}_{ibt} = \boldsymbol{\iota}_{it} + \boldsymbol{\iota}_{bt} + e_{ibt}, \tag{11}$$

projecting the annual growth rate of credit granted by bank b to firm i, g(Credit)_{ibt}, onto bank-time and firm-time fixed effects, $\boldsymbol{\iota}_{bt}$ and $\boldsymbol{\iota}_{it}$, respectively. We construct the local credit

[22]Note that the control function approach is valid as long as the difference between the value of continuing operations and the sell-off value of the firm is increasing in ω, k, and n. When this condition holds—which is generally the case in our setting—it does not depend on whether the sell-off value is independent of firm-level states (as assumed for simplicity in our behavioral model) or not.

supply shifters as the weighted average of the bank-time fixed effects:

$$S_{mt} = \sum_{b \in \mathcal{B}^m_{t-1}} w_{mbt-1} \widehat{\iota}_{bt}, \tag{12}$$

where $\mathcal{B}^m t-1$ is the set of banks with outstanding corporate lending positions in municipality m in year t, and w_{mbt-1} denotes bank b's lending share in municipality m in year t.[23]

Credit demand and size-dependent coefficients. Finally, the complementary slackness condition, $\chi_{it}(k_{it} - \lambda_{it})$, implies that the shadow price distribution has a mass point at zero when the constraint is not binding. However, our parameterization of shadow prices implicitly assumes that the data-generating process follows a continuously differentiable distribution. To address this, we use information on firms' credit applications, interpreting applications as evidence of unmet credit demand. Accordingly, we estimate the Euler equation separately for firms that submit credit applications and those that do not. If our conjecture is correct, we should observe a compact distribution of $\widehat{\chi}_{it}$ clustered near zero for non-applicant firms and a smooth distribution with mostly positive values for applicants.

Additionally, motivated by the evidence of size-dependent borrowing constraints presented in Sections 3.2 and 5.1, and incorporated into our theoretical framework, we divide the sample into five size-based groups (see footnote 20) and perform the GMM estimation separately for each group, both for firms with and without credit applications. This approach mirrors Hennessy and Whited (2007), who analyze size-based subsamples to quantify differences in financing costs and constraints between small and large firms.

5.3 Estimation results

Table 4 presents the estimates of the parameters governing the Euler equation for different sub-populations of firms. We report in parentheses cluster bootstrapped standard errors, treating each firm as a cluster, and resampling the entire history of individual firms to

[23]The credit supply shifters in Amiti and Weinstein (2018) vary at the firm-year level but are computable only for firms with positive leverage in $t-1$. In contrast, our shifters vary at the municipality-year level and can be computed for all firms, including those without credit market transactions.

account for estimation error in the construction of investment gaps and to address serial correlation of these errors within firms due to firm-level unobserved heterogeneity.

First, consider the estimates for the sub-sample of firms that submit credit applications (panel a). Across all sub-samples, the J-test of overidentifying restrictions indicates that we cannot reject the joint validity of the exclusion restrictions of our model. The estimates of the shadow price parameters are in line with our priors. They indicate higher shadow prices for more profitable firms and those in industries with greater investment opportunities, as reflected by the positive coefficients on ROA and firm-level sales, and the negative coefficient on industry sales. These findings are consistent with Whited and Wu (2006), whose financial constraint index loads positively on firm-level and negatively on industry-level sales growth. Furthermore, shadow prices increase with firm-level productivity but decrease with firm age. Intuitively, more productive firms incur larger (forgone) revenue losses from suboptimal investments due to their higher capital productivity, while older firms are less likely to face highly profitable growth opportunities compared to younger ones. Importantly, consistent with theoretical predictions, the estimated parameters differ significantly between size groups. Evaluating Equation (9) at the estimated parameter values, the last row of Table 4 reports the average and 10−90 range (in squared brackets) shadow prices for firms in each size group. In the subsample of firms that do not report any credit application, we estimate an average shadow price of 0.29 percent for small firms but close to zero for firms with assets close to zero, on average, providing strong evidence of size-dependent borrowing constraints.

Next, we turn to the estimates for the subsample of firms without credit applications, reported in panel b. The signs and magnitudes of the Euler equation coefficients are generally comparable to those for firms with credit applications, with two exceptions. First, for firms without credit applications, the estimates of the constant terms of the parametrized shadow costs ($\hat{\chi}_0$) are negative and large, effectively shifting the entire distribution of shadow prices towards zero. Second, we observe a significantly steeper (negative) relationship between shadow prices and both firm revenues and age. This likely reflects lower growth opportunities in this subpopulation, particularly among older firms, where internal financing may suffice to fund profitable investments.

Table 4: Euler equation parameter estimates

	Panel a: Firms w/ credit applications					Panel b: Firms w/out credit applications				
	Group 1 [Small]	Group 2	Group 3	Group 4	Group 5 [Large]	Group 1 [Small]	Group 2	Group 3	Group 4	Group 5 [Large]
m_0	0.969	0.942	0.953	0.970	0.900	0.973	1.018	1.004	0.981	0.854
	(0.023)	(0.046)	(0.035)	(0.007)	(0.037)	(0.030)	(0.037)	(0.047)	(0.036)	(0.083)
m_1	-0.278	-0.125	-0.548	-0.468	-0.004	-0.578	-0.550	-0.603	-0.643	-0.305
	(0.075)	(0.108)	(0.158)	(0.050)	(0.107)	(0.102)	(0.139)	(0.158)	(0.126)	(0.404)
m_2	0.795	0.642	1.000	0.897	0.991	0.426	0.338	0.286	0.974	1.000
	(0.425)	(0.458)	(0.595)	(0.118)	(0.440)	(0.468)	(0.526)	(0.629)	(0.625)	(0.794)
m_3	1.149	1.905	1.828	0.999	1.049	1.971	0.816	1.462	1.584	1.990
	(0.279)	(0.561)	(0.581)	(0.105)	(0.182)	(0.320)	(0.427)	(0.438)	(0.286)	(0.545)
α	0.014	0.124	0.176	0.087	0.026	0.030	0.083	0.114	0.119	0.197
	(0.016)	(0.024)	(0.036)	(0.002)	(0.009)	(0.012)	(0.014)	(0.018)	(0.014)	(0.075)
χ_0	-1.282	-1.299	-1.331	-0.470	0.960	-3.836	-4.453	-4.452	-4.522	-4.563
	(0.233)	(0.171)	(0.281)	(0.140)	(0.509)	(0.205)	(0.446)	(0.552)	(0.943)	(2.341)
ROA	0.260	0.001	0.153	0.200	0.016	0.137	0.041	0.195	0.141	0.156
	(0.072)	(0.073)	(0.042)	(0.016)	(0.055)	(0.030)	(0.043)	(0.047)	(0.030)	(0.090)
$g(\text{sales})$	-2.128	-2.282	-3.350	-1.786	-0.589	-2.465	-4.601	-4.684	-4.100	-4.073
	(0.422)	(0.394)	(0.783)	(0.131)	(0.350)	(0.452)	(0.669)	(0.612)	(0.391)	(1.865)
$g(\text{sales}_{ind})$	0.250	0.026	0.226	0.823	0.805	0.126	0.628	0.301	0.127	0.157
	(0.194)	(0.225)	(0.181)	(0.087)	(0.138)	(0.145)	(0.156)	(0.154)	(0.162)	(0.393)
Age	-0.400	-0.433	-0.889	-1.285	-3.000	-0.899	-1.572	-1.622	-1.354	-1.649
	(0.280)	(0.273)	(0.157)	(0.148)	(0.382)	(0.226)	(0.321)	(0.350)	(0.136)	(0.554)
TFP	0.440	0.514	0.426	0.489	0.373	0.922	0.950	0.726	0.355	0.328
	(0.135)	(0.134)	(0.237)	(0.090)	(0.235)	(0.165)	(0.258)	(0.162)	(0.158)	(0.583)
$\bar{\chi}$	0.292	0.240	0.091	0.054	0.004	0.023	0.005	0.002	0.001	0.000
	[0.09–0.51]	[0.07–0.43]	[0.01–0.17]	[0.01–0.12]	[0.00–0.01]	[0.00–0.05]	[0.00–0.01]	[0.00–0.00]	[0.00–0.00]	[0.00–0.00]
J-test	0.114	0.483	0.156	0.245	0.832	0.122	0.482	0.608	0.484	0.533

Notes. This table reports the estimates of the Euler equation parameters. Shadow prices are parametrized according to Equation (9). $m_0 - m_3$ parametrize the SDF according to Equation (5.2). The parameter α captures the importance of real adjustment costs. Panel a shows the parameter estimates obtained from the GMM model applied to the sub-sample of firms filing credit applications (panel a) or not (panel b). Within each panel, we report the estimates for different sub-samples defined by firm size. Clustered bootstrap standard errors are reported in parentheses. The p-values of the J-test of overidentifying restrictions are reported in the last row.

These estimates suggest that, on average, shadow prices are near zero for firms that do not file credit applications, indicating that credit rationing is unlikely to significantly distort their investment policies. These findings support our conjecture and highlight the importance of credit application data to account for credit demand and quantify the extent of credit rationing in microdata. A notable exception is small firms (Group 1 in panel b). For these firms, we estimate positive shadow prices on average, though these are ten times smaller than those for similarly sized firms that apply for credit. Why do these firms exhibit any positive shadow prices at all? Two plausible explanations emerge. The first is related to estimation and measurement error. The second is a "discouragement effect": applying for credit entails effort and cost, so firms that have repeatedly but unsuccessfully applied in the past may stop applying despite remaining credit-constrained. In support of this interpretation, we find that shadow prices are about 20 percent higher for firms that do not file credit applications but have previously had credit applications rejected.

Finally, the estimates for the other Euler equation parameters are reasonable and consistent with prior research. The adjustment cost parameter, α, lies within the range reported in previous studies (e.g., Cooper and Haltiwanger 2006; Whited and Wu 2006). Likewise, the SDF parameters, m_0–m_3, yield an average discount rate of approximately 2 percent annually, consistent with the risk-free rate observed during our sample period. Across population subgroups, we robustly reject the hypothesis that our parametrized SDF is zero, with p-values below 1 percent.

6 The distribution of the cost and shadow cost of credit

Combining the estimates of the Euler parameters with the slopes of the size-dependent borrowing constraint according to Equation (8), we recover the distribution of financial wedges (τ_{it}), our measure of the shadow cost of credit. Table 5 reports summary statistics (mean and 10–90 percentile range) for the distributions of the shadow cost of credit by size groups in the full sample (panel a) and separately for firms with and without credit applications (panels b and c). The table also presents summary statistics for the distributions of the cost of credit (the borrowing rate r_{it}).

Our estimates indicate that the average shadow price of credit is approximately 3 percent, with a 10–90 percentile range of about 8 percent. The distinction between firms with and without credit demand explains much of this heterogeneity. Among firms that submit credit applications, the average shadow cost is nearly 5 percent, while it is close to zero for those without credit applications. As discussed earlier, shadow costs also exhibit substantial cross-sectional heterogeneity, which is strongly linked to firm size. The smallest firms in the sample not only face the highest average shadow cost of credit (nearly 15 percent) but also display the greatest dispersion. These findings suggest that financial frictions that cause credit rationing, such as asymmetric information frictions, lead to more underinvestment and more capital misallocation within this group.

How do shadow costs relate to borrowing costs? Three key findings emerge from the comparison of the magnitude and variation in the costs and shadow costs of credit in different sub-populations of firms.

1. *Among firms with positive credit demand, the cost of credit is comparable in magnitude to its shadow cost, but the latter is significantly more dispersed, especially among SME.*

The 10–90 interest rate spread is approximately 5 percentage points, compared to a 12 percentage point spread for shadow costs. The shadow costs generated by binding borrowing constraints act as implicit, heterogeneous taxes on producers, distorting capital accumulation. These distortions result in some firms being too large and others too small relative to their "socially efficient" size, which imply a suboptimal resource allocation and lower aggregate productivity (Restuccia and Rogerson 2008, 2013).

The limited variation in borrowing rates alongside the substantial variation in shadow costs aligns with a credit assessment process where lenders classify borrowing firms into broad categories based on observable factors, such as size or industry affiliation. Banks set uniform interest rates and lending limits for each group, optimizing expected profit based on available information, even though firms within a group may differ significantly in loan demand or risk. As emphasized by Jaffee and Modigliani (1969), this approach inevitably leads to credit rationing within groups: Some firms receive loan offers that fall short of their demand and would willingly pay higher interest rates for larger loans. However, accommodating these differences would undermine the bank's classification system and its objectives. Table Table 5 shows that the distinction between the cost and shadow cost of credit is particularly pronounced among small firms. In this subpopulation, shadow costs are not only significantly higher than interest rates (15 percent vs. 6 percent) but also far more dispersed (a 10–90 range of 21 percentage points vs. 4 percentage points), reflecting more depressed investment and greater capital misallocation among SMEs.

Table 5: The distribution of the cost and shadow shadow costs of debt

	All size groups	Group 1 [Small]	Group 2	Group 3	Group 4	Group 5 [Large]
			Panel a: all firms			
Shadow cost of debt	0.031 [0.00–0.08]	0.060 [0.00–0.16]	0.056 [0.00–0.15]	0.023 [0.00–0.05]	0.014 [0.00–0.04]	0.001 [0.00–0.00]
Cost of debt	0.057 [0.03–0.08]	0.060 [0.04–0.08]	0.060 [0.04–0.09]	0.059 [0.03–0.09]	0.054 [0.03–0.08]	0.047 [0.02–0.07]
Spearman corr.	0.088					
Spearman p-val	0.000					
			Panel b: Firms w/ credit applications			
Shadow cost of debt	0.049 [0.00–0.12]	0.150 [0.03–0.28]	0.114 [0.03–0.22]	0.041 [0.00–0.08]	0.020 [0.00–0.05]	0.002 [0.00–0.00]
Cost of debt	0.056 [0.03–0.08]	0.058 [0.04–0.08]	0.061 [0.04–0.09]	0.060 [0.03–0.09]	0.055 [0.03–0.08]	0.047 [0.02–0.07]
Spearman corr.	0.144					
Spearman p-val	0.000					
			Panel c: Firms w/out credit applications			
Shadow cost of debt	0.004 [0.00–0.01]	0.012 [0.00–0.03]	0.003 [0.00–0.00]	0.001 [0.00–0.00]	0.000 [0.00–0.00]	0.000 [0.00–0.00]
Cost of debt	0.058 [0.03–0.09]	0.061 [0.04–0.09]	0.060 [0.04–0.09]	0.057 [0.03–0.09]	0.053 [0.03–0.08]	0.046 [0.02–0.07]
Spearman corr.	0.097					
Spearman p-val	0.000					

Notes. This table presents summary statistics for the distributions of shadow costs (τ_{it}) and borrowing costs (r_{it}). Statistics are reported for the full sample and subsamples defined by firm size.

2. *Shadow costs and borrowing rates co-move as both reflect variation in credit risk factors, but shadow costs are significantly more sensitive to these variations.*

Table 5 reports the Spearman rank correlation coefficients between the two variables. The coefficient is 0.09 in the full sample and 0.14 among firms that submit credit applications, with all p-values below 1 percent. This positive relationship arises because, consistent with theoretical predictions, both cost measures capture variation in credit risk factors. However, shadow costs are far more sensitive to such variation.

To illustrate, we compute the sensitivity of shadow costs and borrowing rates to common empirical proxies for credit market frictions. Figure 4 shows the average costs and shadow costs by firm age and size groups (panels a and b). As expected, borrowing rates decline with firm age and size. Shadow costs also decline, reflecting the parameterization of shadow costs (which depend on age) and their strong heterogeneity across size groups. These findings support the idea that smaller and younger firms face more severe financial frictions due, for example, to their limited availability of pledgeable assets and greater opacity. However, the size gradient is notably weaker for borrowing rates than for shadow costs. Panels c and d explore the relationship between two measures of credit risk that are not directly included in our measure of shadow costs. The first is the length of a firm's lending relationship with its main lender, a proxy for the strength of information frictions between borrowers and lenders. The second is credit risk, measured by the Altman Z-score.[24] Both costs and shadow costs show a strong monotonic relationship with these risk factors, but shadow costs display significantly higher sensitivity. For example, borrowing rates increase by about 25 percent from the safest to the riskiest firms, while average shadow costs nearly triple. Similarly, borrowing rates decline by about 40 percent for firms with longer lending relationships compared to newly established ones, while shadow costs drop by approximately 80 percent.

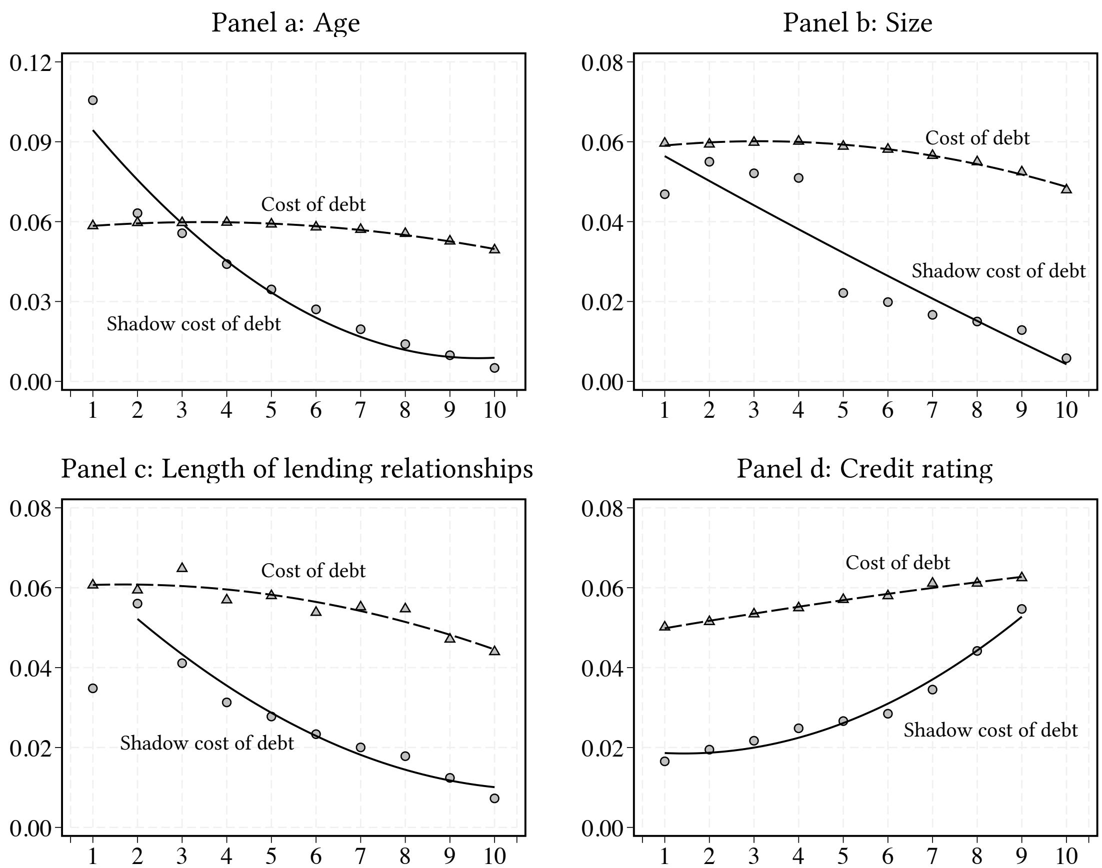
3. *While costs and shadow costs co-move, their relationship is not monotonic.*

The binned scatter plot in Figure 5 shows the average borrowing cost across quantiles

[24] The Altman Z-Score assesses a firm's likelihood of financial distress. It is readily available to lenders through the Cerved data set and is part of the criteria used by Italian banks for the risk assessments of borrowers.

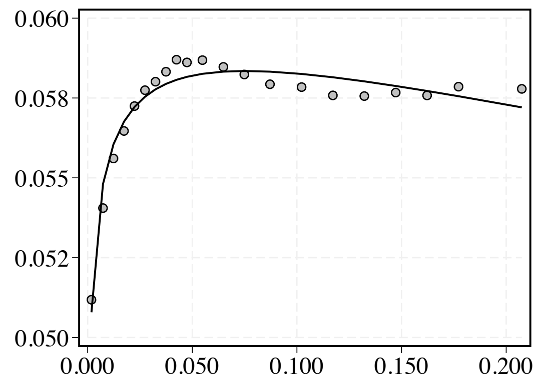
of the shadow cost distribution for the full sample of firms (panel a) and firms that file credit applications (panel b). Borrowing rates and shadow costs increase almost one for one in the left tail of the shadow cost distribution. However, further along the distribution, borrowing rates flatten and eventually decline, while shadow costs continue to rise. This non-monotonic relationship aligns with the theoretical predictions of credit rationing models (Stiglitz and Weiss 1981, Stiglitz and Weiss 1992).

Figure 4: Correlation between the cost and shadow cost of credit and proxies of credit risk



Notes. These binned scatter plots illustrate the correlation between borrowing costs (triangles and dashed lines) and shadow costs of credit (circles and solid lines) on the y-axis and proxies for credit risk on the x-axis: firm age, size (total assets), length of lending relationships with the main lender, and credit rating (Altman Z-score).

Figure 5: Costs and shadow costs of credit



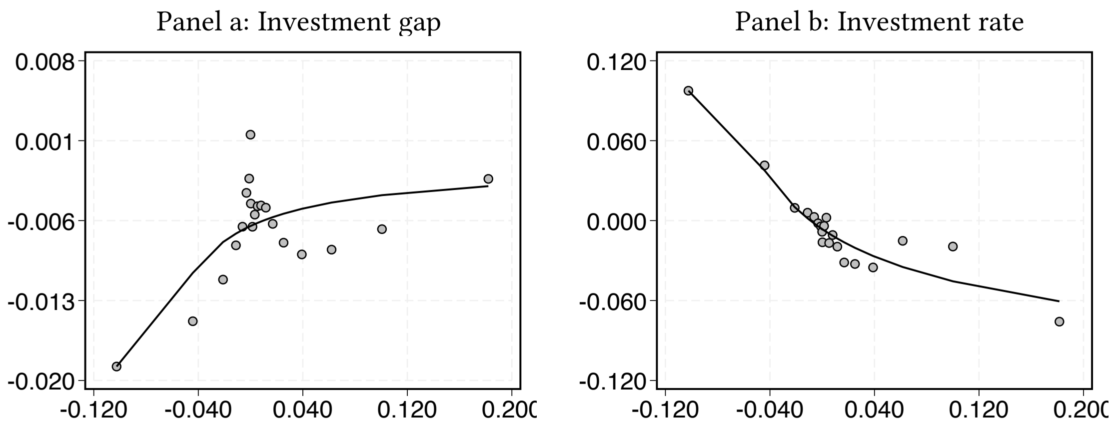
Notes. This binned scatter plot shows the correlation between borrowing costs (r, y-axis) and shadow costs of credit (τ, x-axis) focusing on the subsample of firms filing credit applications.

In these models, banks may refrain from raising interest rates in response to excess credit demand because higher rates can lower expected returns by increasing default risk. Two mechanisms underpin this phenomenon: first, higher interest rates deter low-risk borrowers, leaving a riskier applicant pool (the sorting effect); second, higher rates encourage borrowers to undertake riskier projects (the incentive effect). Together, these effects help explain the observed flattening—and eventual decline—of borrowing rates as shadow costs rise.

7 Investment sensitivity to the shadow cost of credit

In Section 1, we introduced the concept of investment gaps (IG) as a metric to evaluate the efficiency of firms' investment policies. We argued that credit rationing is a key driver of deviations from the neoclassical benchmark and provided descriptive evidence supporting this claim in Section 3.2. We now ask: Can binding credit constraints explain the observed variation in investment gaps and, *a fortiori*, the cross-section of investments? Our analysis suggests that the answer is yes.

Figure 6: Sensitivity of investment gaps and investment rates to changes in shadow costs



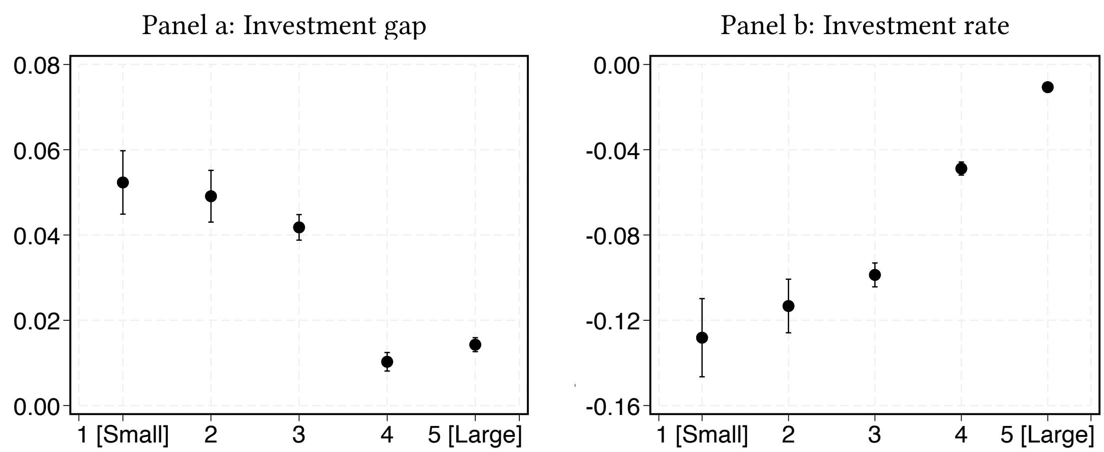
Notes. This binned-scatter plot shows the the relation between our estimated shadow cost of credit (x-axis), investment gaps (panel a, y-axis), and investment rates (panel b, y-axis).

We partition the data into quantiles of the shadow price distribution. For each quantile, the binned scatter plot in Figure A.4 shows the average change in shadow prices against the average change in investment gaps (panel a) and investment rates (panel b). Investment rates are defined as the ratio of current net investments in fixed assets to the previous period's capital stock. This analysis focuses on firms filing credit applications, as they are the most likely to face binding constraints and positive shadow costs. (Results for the full sample are provided in Appendix A.4.)

Consistent with the theoretical predictions, we find a strong positive relationship between shadow costs and investment gaps. On average, a 10 percent reduction in shadow costs is associated with a 25 basis point decline in investment gaps (elasticity of 0.025, SE 0.001). The relationship between shadow prices and investment rates is also strong and monotonic: a 10 percent reduction in shadow prices is associated with a 50 basis point increase in investment rates (elasticity of -0.053, SE 0.001).

Finally, Figure 7 examines the elasticity of investment gaps and investment rates to the shadow price of credit across firm size groups. As shown above, shadow costs are higher for smaller firms, reflecting size-dependent credit constraints that restrict access to profitable investment opportunities. The results in Figure 7 reinforce this point: *because* the shadow value of a dollar of credit is high, SMEs respond to reductions in shadow costs with investment increases ten times larger than those of large firms. This effect likely also reflects the greater financial flexibility of larger firms, which can access alternative funding sources, making their investment policies less sensitive to bank credit supply conditions.

Figure 7: Elasticity of investment gaps and investment rates to changes in shadow costs



Notes. This figure shows the elasticity of investment gaps (Panel A) and investment rates (Panel B) to the shadow cost of credit across firms of different sizes. Vertical bars represent 95 percent confidence intervals.

8 Concluding remarks

By studying firms' production decisions and credit market interactions, we examined how the shadow costs of credit generated by quantity rationing compare to borrowing costs and whether these costs explain the large dispersion in investment rates observed in the data. Our analysis reveals that, for firms demanding credit, shadow costs can be substantially higher and more heterogeneous than the market price of credit.

Our findings indicate that credit rationing—rather than elevated borrowing costs—is the most significant manifestation of financial frictions in the private corporate credit market, particularly for small and medium enterprises. Among them, we observe greater underinvestment and capital misallocation. This suggests that relying on borrowing cost variations to infer the extent of financial frictions for SMEs would severely underestimate their impact on firm activity. These results also suggest that policies aimed at reducing interest rates, such as direct or indirect subsidies, may have limited effects on equilibrium credit outcomes. Instead, interventions that aim at relax quantity constraints, such as credit guarantee programs, are likely to be more effective in addressing the needs of bank-dependent but underserved sectors of the economy.

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Appendix

A.1 Sample selection procedure

To construct our final dataset, we apply the following filters and data cleaning steps:

Continuity of Data: We exclude firms with two or more years of discontinuous (missing) balance sheet information. This ensures that our sample includes firms with uninterrupted time-series data and reliable accounting records.

Non-Positive or Missing Values: We drop observations with non-positive revenues or total assets and those missing information on firm age, industry, or location (province of headquarters). A review of these cases revealed that they pertain to firms already in default or liquidation, which, despite filing balance sheets, no longer engage in real economic activity.

Employment Data: We exclude firm-year observations with missing or zero employment, including firms whose only workers are owners or those exclusively using contracted labor.

Industry: We remove firms operating in the following NACE Rev. 2 2-digit industries:

- Agriculture, Mining and Quarrying, Utilities (1–3, 5–9, 35–39)
- Postal Services and Courier Activities (53)
- Scientific Activities and R&D (72)
- Public Administration and National Defense, Education, Health Services (84–88)
- Sport, Arts, Entertainment, and Membership Organizations (90–94)
- Activities of Households as Employers and Extraterritorial Organizations (97–99)
- Financial and Insurance Activities, Real Estate (64–66, 68)
- Tobacco and Pharmaceuticals (12, 21)

These exclusions are due to difficulties in measuring output, potential government ownership or subsidies, or because these industries primarily operate as credit providers. Tobacco and pharmaceutical firms are excluded as they are often subsidiaries of multinationals.

Marginal Revenue Product of Capital: We retain only firm-year observations for which we can compute the marginal revenue product of capital.

A.2 Inferring missing interest rates

We estimate predictive pricing regressions to infer borrowing costs for firms without observable data in the TAXIA dataset. Using information from firms with available borrowing cost information (APR on term loans), we project rates onto a set of firm characteristics and fixed effects that capture variation in local credit market conditions. As discussed in the paper, this approach is motivated by the well-documented reliance of small and medium firms on local credit markets, where proximity between borrowers and lenders facilitates information acquisition, and by evidence that banks base credit costs on a limited set of observable borrower characteristics.

Depending on the "type" of firm, we use our bank-firm matched data set (CR + TAXIA) to predict missing interest rates by estimating a variant of the following regression model:

$$r_{ibt} = \boldsymbol{\beta} \boldsymbol{X}_{it} + \Gamma \mathbf{Z}_{ibt} + \iota_{ist} + \iota_{ipt} + \iota_{bt} + \epsilon_{it}$$

Here, i denotes a firm, b a bank, s an industry, and t a year. The vector ι_{st} captures industry (2-digit NACE code) by year fixed effects. Following Guiso et al. 2013, we define the boundaries of the local credit market using Italian provinces and include fixed effects by province by year (ι_{ipt}). A vector of bank by year fixed effects is denoted by ι_{bt}, which captures variation in banks' cost of capital and other time-varying variables that influence loan pricing.

The vector $\boldsymbol{X}_{it}$ includes firm-specific observable characteristics. These include a quadratic polynomial in age, the natural logarithm of total assets, ROA, asset turnover (revenues/assets), cash over assets, a dummy variable for discrete credit score values (9 Altman Z-score categories), and a dummy indicating current or past debt obligations in default, restructured, or under restructuring (as recorded in the CR). These covariates are chosen to ensure two criteria. (i) common support between borrowers and non-borrowers within each year-market combination, and (ii) representation of the key indicators used by Italian banks to assess firms' riskiness and creditworthiness (Albareto et al., 2011).

The vector $\mathbf{Z}_{ibt}$ includes relationship-specific covariates derived from firm-bank information in the CR: the number of ongoing relationships, the duration of each loan relationship, and bank leverage. Note that banks regularly query this type of (anonymized) information in the CR to assess the overall risk profile of both existing customers and new firms seeking credit. The specific variables in $\mathbf{Z}_{ibt}$ depend on the subset of observations for which we aim to estimate the APR of term loans.

There are four "types" of observations for which APR information is missing:

- *Type 1 - Borrowers with only credit lines.* Firms that have outstanding credit lines with banks in the TAXIA database but no outstanding bank loans.

  For these observations, in addition to firm-specific characteristics and geographic location (ι_{is}), the predictive pricing regression includes bank-year fixed effects (ι_{bpt}) and a vector $\mathbf{Z}_{ibt}$ capturing firm leverage and the number and length of existing lending relationships.

- *Type 2 - Borrowers not in TAXIA.* Borrowers that appear in the Credit Registry but for whom no APR on loans is observed because their lender does not belong to the group of banks in the TAXIA database or because the credit has been granted but not yet utilized by the firm.

  The pricing equation for this group is similar to that for firms of Type 1. The difference is that, for firms borrowing from lenders outside the TAXIA database, we cannot estimate the bank-year fixed effects ι_{bt}. Instead, we construct a province-year fixed effect as a weighted average of ι_{bt}:

  $$\iota_{\bar{b}t} = \sum_{b \in \text{TAXIA}} w_{bpt}\ \iota_{bt},$$

  where $w_{bpt} = \frac{\text{Credit}_{ibt}}{\sum_{b \in \text{TAXIA}} \text{Credit}_{ibt}}$ is the market share of bank b in province p at time t.

- *Type 3 - Borrowers not in CR.* Borrowers with no outstanding debt obligations recorded in the CR but reporting positive bank debt on their balance sheet.

  Similar to Type 2, the pricing equation for this group includes bank-year fixed effects and computes the average vector ($\iota_{\bar{b}t}$) varying across province-years. However, we are unable to include any of the co-variates in $\mathbf{Z}_{ibt}$.

- *Type 4 - Non-borrowers.* Firms with no outstanding debt obligations (neither in the CR nor on their balance sheets). The pricing equation for this group is similar to that of Type 3. However, the estimation sample is restricted to firm-bank observations representing newly established credit relationships (Duration of lending relationship $\leq$ 1 year). Focusing on new relationships is crucial, as non-borrowers would be new customers for the bank if they sought credit.[25] This filter also reduces the information gap between what the econometrician knows about the firm and what the bank knows (Crawford et al., 2018).

We predict the APR on term loan using the coefficients and fixed effects estimated from the pricing regressions. For firms of Type 1 and Type 2, the prediction is performed at the relationship level, generating $\hat{r}_{ibt}$. We then calculate the firm-year weighted average APR on term loans, $\hat{r}_{it}$, as described in the paper. The distinction between Type 1 and Type 2 firms lies in the weighting: for Type 1 firms, we use the share of credit lines as the weight for each ongoing relationship. For firms of Type 3 and Type 4, where no information on lenders is available, we directly predict $\hat{r}_{it}$ at the firm-year level.

A.3 Control function estimates

We augment our estimating equation with a control function $h(\omega_{it}, P_{it})$ to address a possible bias due to selection. To illustrate the approach, note that MRPK as increasing in firm-level productivity, capital, and net-worth and that the expected MRPK (conditional on $E_{it+1} = 0$),

$$\mathbb{E}_{\omega_{it+1} > \underline{\omega}_{it+1}} \left[MRPK(\omega_{it+1}, k_{it+1}, n_{it+1}) \right] = \int_{\underline{\omega}_{it+1}}^{\infty} MRPK(\omega_{it+1}, k_{it+1}, n_{it+1}) \frac{F(d\omega_{it+1}|\omega_{it})}{\int_{\underline{\omega}_{it}}^{\infty} F(d\omega_{it+1}|\omega_{it})},$$

is a function of two firm-specific variables, ω_{it} and $\underline{\omega}_{it+1}$. These variables determine the probability that the entrepreneur operates the firm in $t + 1$, $P_{it} \equiv Pr\{E_{it+1=0}\}$, which is given by:

$$P_{it} = Pr\left\{\omega_{it+1} \geq \underline{\omega}_{it}(k_{it+1}, n_{it+1}) | \omega_{it}, k_{it+1}, n_{it+1}, \mathcal{J}_{it}\right\} = \varphi\{\omega_{it}, i_{it}, k_{it}, b_{it}, \mathcal{J}_{it}\} \tag{A.1}$$

The equality follows from the fact (i) that (k_{it+1}, n_{it+1}) are predetermined states, determined by the capital accumulation equation ($k_{it+1} = (1 - \delta)k_{it} + i_{it}$) and the net worth dynamic

[25] We exclude the initial year in our dataset (1997), as we cannot distinguish between new and ongoing relationships for these observations.

equation ($n_{it+1} = k_{it+1} - b_{it+1}$), (ii) that firm-level productivity follows a Markov process, and (ii) that b_{it+1} and i_{it} are chosen by the firm based on current states.

Thus, to control for selection, we need measures of ω_{it} and $\underline{\omega}_{it+1}$. Provided that the density of ω_{it+1} conditional on ω_{it} is positive in the region around $\underline{\omega}_{it+1}$, we can invert equation (A.1) to express $\underline{\omega}_{it+1}$ as a function of $\hat{\omega}_{it}$ and $\hat{P}_{it}$ (Olley and Pakes, 1996). We obtain a proxy firm-level productivity, $\hat{\omega}_{it}$, via the production function estimation, as described in Section 3.1.

We estimate the probability of exit, $\hat{P}_{it}$, fitting a probit model of firm-level variables and shifters on an indicator variable of exit. Consistent with the predictions of the model, the set of firm-level control variables includes measures of bank leverage, firm size (log-assets), net investment (investments minus divestments in fixed assets), and log-productivity. Stepping outside the model's boundaries, we the set of explanatory variables also include firm age and market share, and the growth rate of sales in the industry-province-year. We include these variables because the probability of exit depends on time-varying factors related to product market structure and demand conditions, as highlighted by equation (A.1). Additionally, these variables also account for differences in exit rates due to firms' life-cycle stages.

Table A.1: Probability of Exit

	Exit	
	Coefficient	SE
Leverage	-0.084	0.002
Ln(Fixed Assets)	-0.097	0.000
Net Investments rate	-0.681	0.002
TFP	-0.081	0.001
Age	-0.009	0.000
Market Share	-0.119	0.006
g(sales industry-year-provice)	-0.178	0.002

Notes. This tables reports the estimates of a probit model of productivity, investment, capital, and debt on an indicator variable

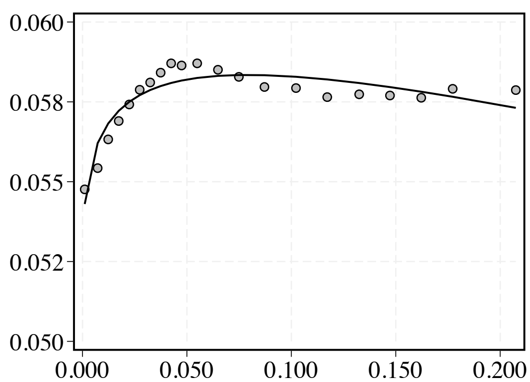
Table A.1 reports the estimates. Consistent with the predictions of the model, larger and more productive firms are less likely to exit the data set. Higher leverage increases the probability of exit. Intuitively, conditional on size and productivity, highly leveraged firms are less likely to be able to honor their obligations following adverse demand shocks. High investment rates also correlate with lower likelihoods of default, as firms' actions contain information regarding their rational perceptions of the future.

Given our estimates of firm-level productivity and probability of exit, we construct the control function as a second-order polynomial in $\hat{\omega}_{it}$ and $\hat{P}_{it}$: $h(\hat{\omega}_{it}, \hat{P}_{it}) = \hat{\omega}_{it} + \hat{P}_{it} + \hat{\omega}_{it} \cdot \hat{P}_{it}$.

A.4 Additional graphs

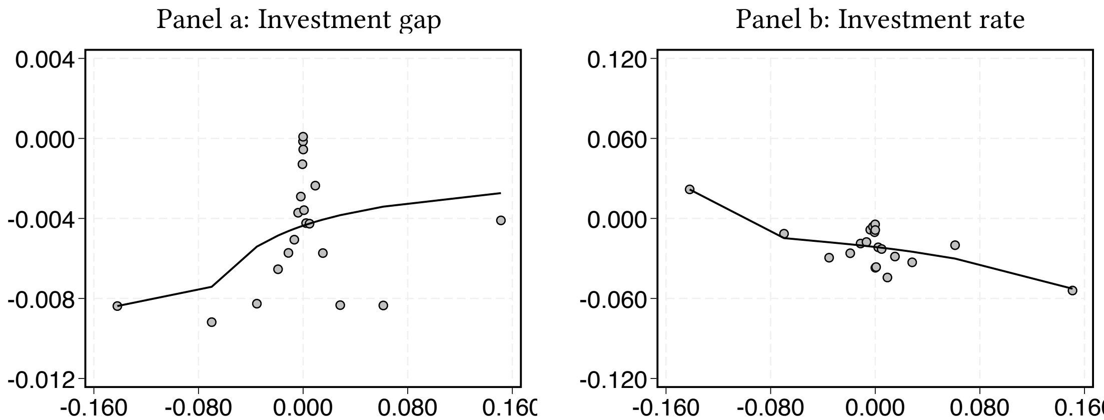
Mirroring Figure 5, Figure A.1 presents the relationship between shadow prices and borrowing rates for the full sample, pooling together firms that file a credit application and those who don't. Mirroring Figure , Figure presents the relationship between changes in shadow costs and changes in investment gaps and investment rates.

Figure A.1: Costs and shadow costs of credit—Full sample



Notes. This binned-scatter plot shows the the correlation between borrowing costs (r, y-axis) and our estimated shadow cost of credit (τ, x-axis) for the full sample (panel a) and for the subsample of firms that file a credit application (panel b). This figure includes all observations in our dataset.

Figure A.2: Sensitivity of investment gaps investment rates to changes in shadow costs—Full sample



Notes. This binned-scatter plot shows the the relation between our estimated shadow cost of credit (x-axis), investment gaps (panel a, y-axis), and investment rates (panel b, y-axis). This figure includes all observations in our dataset.

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