The Cost of Shadow Cost of Debt

Simone Lenzu[†] Francesco Manaresi[‡]

August 2024

Abstract

Leveraging a novel micro-level database on firms' production and financing decisions, we recover the distribution of firm-specific *shadow costs* of capital due to binding borrowing constraints and compare them to the firm-specific *user costs* of capital observed in the data. Analyzing the distributions of costs and shadow costs, we show that the latter are substantially higher, more dispersed, and more sensitive to variation in credit supply conditions and financial frictions than the former. Our analysis suggests that quantity constraints, rather than distorted borrowing costs, are the most significant channel through which credit market frictions distort firms' investment policies and capital allocation.

Keywords: Bank Credit, Financial Frictions, Credit Rationing, Investment Policies.

† New York University. Email: slenzu@stern.nyu.edu; ‡ Bank of Italy. Email: Francesco.Manaresi@bancaditalia.it. We thank Holger Mueller and Quinn Maingi for helpful comments and suggestions. We are grateful to Cangyuan Lee for excellent research assistance. Any views expressed are those of the authors and not those of the Bank of Italy or the Eurosystem.

1 Introduction

Credit market frictions are widely acknowledged as a significant factor in explaining the variation in investment decisions of firms, both across different firms and over time. However, It remains an open question whether credit market frictions affect investments because they distort and heighten borrowing costs or because they generate heterogeneous shadow costs of finance by capping the amount of credit lenders are willing to provide at any given rate.

On the one hand, the idea that business fixed capital spending decreases when interest rates rise is a theoretically unambiguous relationship that is at the core of the monetary transmission mechanism. However, with the exception of studies focusing on large firms that access the bond market (Gilchrist and Zakrajsek, 2007), it has been challenging to document a robust relationship between variations in borrowing rates—and the user cost of capital more broadly—and investment expenditures in data (Abel and Blanchard, 1986). On the other hand, credit constraints feature prominently in theories of fluctuations in business cycles (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997) and long-term growth (Galor and Zeira, 1993). However, there is scant direct microeconomic evidence of the existence and magnitude of credit constraints (Banerjee and Duflo, 2014). Understandably, this is a daunting task because credit limits are rarely observable (Agarwal et al., 2018) and, even when they are, it is unclear whether they bind to the point of meaningfully affecting firms' investments.

In this paper, we utilize comprehensive administrative data on firms' financial and production choices to create firm-specific metrics for the user cost, which depend on borrowing rates, and estimate the firm-specific shadow costs of capital that result from borrowing constraints. We investigate how shadow costs due to credit rationing compare to the user cost of capital observed in the data and study the sensitivity of the two forms of costs to time-series variation in credit supply conditions and cross-sectional variation in the degree of financial frictions. Our analysis demonstrates that shadow costs may be significantly higher, more varied, and more responsive to credit supply conditions than user costs. We conclude that credit quantity constraints, rather than distorted borrowing costs, represent the primary channel through which financial frictions affect firms' investment policies.

Our approach builds on the traditional Euler equation estimation, which we extend to incorporate demand-side heterogeneity (heterogeneous productivity and markups), heterogeneous user costs of capital, borrowing constraints, and selection. Two notable features of our data are the availability of information on firm-specific borrowing rates and on firms' decisions to apply for new credit. The former allows us to directly account for the effect of heterogenous user costs on investment decisions; the latter puts us in the privileged position to be able to identify firms with unmet credit demand and incorporate this crucial piece of information in the estimation procedure that allows us to recover firm-specific shadow costs of capital.

We begin our empirical analysis by contrasting investment policies observed in the data against the predictions of a neoclassical benchmark. Standard optimality arguments suggest, as a first approximation, the efficiency of a firm's investment policies can be evaluated by looking at the gap between the realized marginal revenue product of capital (MRP^K) and its user cost, defined as the sum of the borrowing (or rental rate) and the depreciation rate of assets in place. Conceptually, a positive wedge between MRP^K and user cost reflects distortions in firms' capital accumulation decisions due to the pass-through of credit market frictions (e.g., asymmetric information frictions) in the form of quantity constraints (credit rationing).

Although firm-level borrowing constraints are generally unmeasurable, investment gaps are measurable quantities, and can be estimated using information on firms' production choices and user costs. We measure investment gaps at the firm-year level, characterize their distribution, and study their evolution before and after gaining access to credit markets and as they strengthen their relationships with lenders. This exercise provides prima facie evidence that financial constraints matter. We find that the average investment gap is 20 percent, but the the distribution is highly skewed. The median firm has investment gaps close to zero, indicating investment policies that broadly align with an undistorted benchmark. However, firms in the top deciles of the gap's distribution display a marginal of capital that exceeds the user cost of capital by 70 percentage points or more.

Observed deviations from the undistorted benchmark appear to be related to firms' credit market participation. The estimated marginal revenue product of capital is, on average, twice as high for non-borrowers than for borrowers. Among borrowers, the investment gap is 1.5 times higher for those firms that rely solely on working capital financing (revolving credit lines) compared to those that can also tap into long-term financing (term loans). Further descriptive evidence on the relationship between investment gaps and credit market frictions comes from their correlation with firm characteristics. Investment gaps monotonically decline with firm age and firm size—two commonly used proxies of financial constraints (Hadlock and Pierce, 2010)—and with firm leverage. Importantly, these patterns are driven by the variation of marginal revenue product of capital, whose gradient outweighs the variation in borrowing rates and in user costs of capital.

The availability of individual firms' credit histories provides further insights into the evolution of gaps over firms' life cycles and connect it to credit availability. We conduct an event study that illustrates how investment gaps change upon access to credit and how gaps evolve as firms develop tighter relationships with their lenders. We document a sharp reduction of investment gaps upon access to credit and a steady convergence towards the frictionless benchmark as firms strengthen their lending relationships. On average, investment gaps are 2.5 times lower one year after gaining access to credit and almost 4 times lower after 10 years. Events that mark the termination of ongoing lending relationships shed further light on the link between investment policy distortions and credit access. Investment gaps of firms with larger, positive gaps display a steady decline prior to the interruption of the lending relationship. In the five years before the termination of the lending relationship, we observe, on average, that the investment gaps of firms in the top quartile were reduced by about half. This trend halts once the lending relationship is severed, and gaps remain largely unchanged in the five years that follow. By contrast, we don't observe any significant change in investment gaps for firms with small or negative investment gaps. In other words, on average, the termination of lending relationships does not appear to affect the investment policies of firms classified as over-capitalized according to our metric.

While revealing, this reduced-form evidence does not allow us to quantify the implicit cost of capital driven by credit rationing. The reason is that financial frictions are not the only phenomena that can rationalize the sign and magnitude of investment gaps. For one, real adjustment costs of capital, risk in capital accumulation, and aggregate risk can explain why we observe that some firms with positive investment gaps do not invest sufficiently even if they face an unconstrained supply of credit. For another, measurement or estimation errors in the firm-level investment gaps might erroneously lead us to conclude that investment policies of some firms are sub-optimally low (or sub-optimally high), when in fact they are not.

To tackle these concerns, we implement a structural approach that allows us to leverage the covariation of investment gaps and credit constraints in the micro-data to estimate firm-time-specific shadow costs. We first introduce a partial-equilibrium inter-temporal investment model augmented to account for credit constraints, uncertainty, real frictions, and aggregate risk. The model formalizes the relationship between investment gaps and shadow costs generated by binding credit constraints. We then resort to structural methods to estimate the model, parametrizing shadow prices as a function of firms' observable demands for credit and firms' characteristics. We follow the approach adopted in the Euler equation literature (Whited, 1992; Bond and Meghir, 1994; Whited and Wu 2006) and estimate the parameters of interest via non-linear GMM. This approach allows us to recover firm-time specific shadow prices generated by binding borrowing constraints, while also dealing with sample selection, simultaneity, and measurement error. Crucially, in the estimation, we exploit information on firms' credit applications to separately estimate the Euler equation parameters (and thus the implied shadow costs due to binding credit constraints) for the subsample of firms with and without credit demand.

Our estimates suggest that, on average, the shadow cost of capital is about 13.3 percent (median 6.1 percent). We find substantial heterogeneity in both shadow values, as indicated by the 90/10 range. Distinguishing between firms with and without credit demand explains a substantial portion of this heterogeneity. The average shadow price among firms that put forward a credit application is 28.1 percent (median 17 percent), which is about eight times larger than the estimated shadow price of firms that do apply for credit. In line with the predictions of theories of credit rationing, we observe a monotone relationship between investment gaps and shadow prices and shadow costs. The elasticity of investment gaps with respect to the shadow cost of capital is 0.03 percent in the full sample, and 0.15 when restricted to the subsample of firms with positive credit demand.

We then formally study the relationship between user costs and shadow costs. We highlight four key facts regarding the relative magnitude and variation in the observable and implicit component of firms' costs of capital. First, for financially constrained firms, shadow prices and shadow costs are substantially larger than market prices and user costs. The shadow cost of capital is, on average, 35 percent higher and 6 times more dispersed than its user cost. Since higher shadow prices imply a lower capital accumulation and therefore forgone investment opportunities, these results suggest that the real costs due to credit rationing are substantial.

Second, among credit constrained firms, the dispersion in shadow prices swamps the dispersion in user costs. This results directly speaks to a large literature studying welfare losses due to resource misallocation (Midrigan and Xu, 2014). Binding borrowing constraints act as an implicit, heterogeneous tax on producers. The implication of these taxes is that some producers are too large whereas others are too small relative to their "socially efficient" size, thereby squandering resources and reducing aggregate productivity and economic growth (Restuccia and Rogerson, 2008; 2013).

Third, shadow costs and user costs are positively correlated because both move in tandem with credit risk factors. However, the shadow prices are far more sensitive to variation in risk factors than user costs are. Specifically, we compute the sensitivity of shadow costs and user costs to commonly used empirical proxies of credit market frictions: firm age, firm size, and the Kaplan-Zingales (1997) index of financial constraints. We find that both the observed user costs and the implicit cost of capital driven by binding credit constraints covary in the expected direction with all of the proxies. However, the sensitivity of shadow costs is orders of magnitude larger than the sensitivity of user costs. For example, a one standard deviation increase in firm size results in a reduction of 0.2 percent in the user cost of capital and a 2.8 percent reduction in the shadow cost of capital.

Finally, using variation in local credit supply shifters, we study how changes in local credit supply conditions affect the equilibrium user costs and shadow costs. This exercise provides a direct test of the channel of transmission of credit market frictions—heightened prices or constrained quantity—to firm's policies. We take advantage of the fact that Italian banks operate across multiple geographical locations and that firms simultaneously borrow from multiple banks located in their proximity to decouple the effect of credit demand and credit supply movements on equilibrium credit market outcomes. Specifically, in the spirit of Amity and Weinstein (2018), we run cross-sectional regressions of firm-year and bank-year fixed effects on the yearly growth rate credit growth of each firm–bank pair. We then construct local credit supply shifters for each municipality-year by averaging the estimated bank fixed effects, using local lagged market shares as weights. We show that both components of firms cost of capital—user costs and shadow costs—lessen following a local credit supply expansion. However, consistent with the predictions of the theories of credit rationing, we find that reduction in the shadow cost of capital is 40 times larger than the reduction in borrowing costs.

The remainder of the paper is organized as follows. In Section 2, we describe our data and outline a procedure to estimate micro-level investment gaps under a neoclassical investment benchmark. Section 4 provides reduced form evidence relating investment gaps to credit market access. Section 5 presents an inter-temporal investment model that formalizes the relationship between investment gaps and shadow costs generated by binding credit constraints. In Section 5 we describe the approach of the model parameters and present our estimates. In Section 6 we use the model estimates to recover firm-specific shadow costs of capital, study how they compare to the observed user cost of capital, and how the two measures of costs vary with measures of financial frictions. Section 7 concludes.

2 Micro-level investment gaps under a neoclassical benchmark

To begin our analysis, we use information on firms' production choices and user costs to estimate firm-year level investment gaps under a neoclassical theoretical benchmark. We outline a stylized framework illustrating how investment gaps relate to the presence of borrowing constraints. We then describe the estimation procedure to recover these investment gaps in the micro-data.

2.1 A neoclassical benchmark

Standard optimality arguments suggest that firms should invest up to the point where their marginal revenue product of capital equals the user cost of capital, defined as the sum of the borrowing (or rental rate) and the depreciation rate of assets in place. Thus, in this neoclassical investment framework, one can characterize the efficiency of firms' investment policies by examining the wedge between the realized marginal revenue product of capital and its user cost, which we refer to as *investment gap*:

$$\tau_{it}^{K} \equiv MRP_{it}^{K} - (r_{it} + \delta). \tag{1}$$

Given the user cost of capital, a positive gap ($\tau_{it}^{K} > 0$), signals inefficiently low investments. Figure 1, panel a, illustrates this intuition graphically.

In the top-right quadrant, the gray circle denotes the actual marginal revenues product of capital of firm i, associated to the observed capital endowment i; the gray triangle denotes the marginal revenue product of capital that firm i would have chosen had it be able to invest up to K_i^*. The investment gap τ^K—measured by the vertical difference between the observed user cost of capital and the MRP^K—is a manifestation of the forgone investment $K_i^* - K_i$.

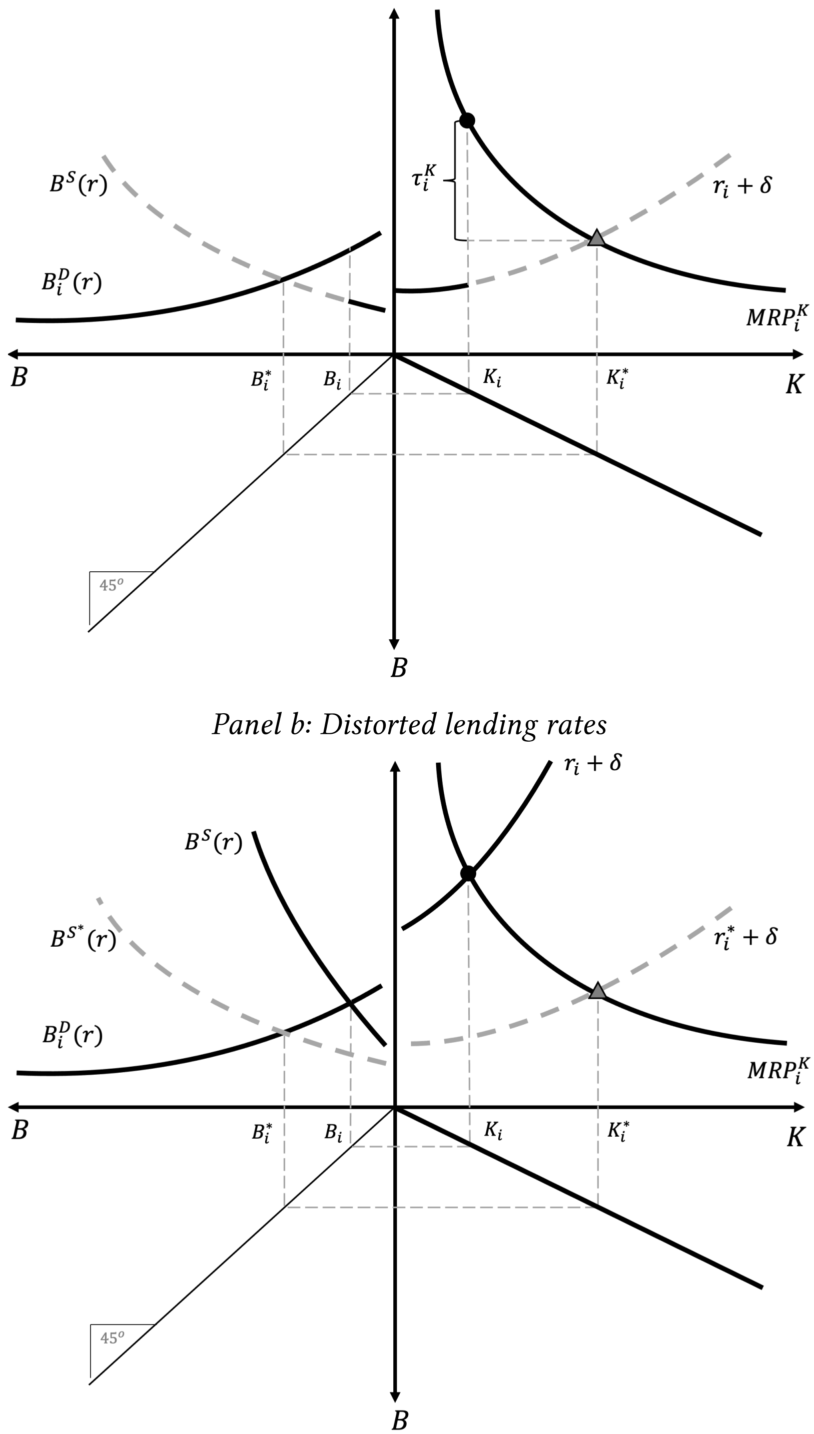
The top-left quadrant links the investment gap to the presence of borrowing constraints. B_i denotes the actual amount borrowed by the firm at rate r_i. As we will illustrate formally in Section 4, binding borrowing constraints imply that firms face a shadow cost of capital, which contributes to explaining the gap between the realized marginal product of capital and its user cost. It has been argued that interest rates need not adjust to clear the market. A prominent explanation for the lack of price adjustments is asymmetric information frictions (Stiglitz and Weiss, 1981, 1992).[1] Firm i would be willing to pay higher borrowing costs in order to receive more credit ($B_i^* > B_i$), but the credit supply schedule is capped. However, a higher interest rate might draw riskier applicants (adverse selection) or induce borrowers to choose riskier investments (moral hazard). As a result, lenders may choose to ration the quantity of credit offered rather than raise the rate to clear the market. Thus, a wedge between MRP^K and user cost reflects the distortions in firms' capital accumulation decisions caused by the pass-through of credit market frictions in the form of quantity constraints.

To illustrate different implications of transmission of financial frictions to market prices (distorted borrowing rates) versus quantity constraints, Figure 1, panel b depicts a situation where financial frictions shift and steepen the credit supply schedule, but *do not*

[1]Other contributions emphasized the role of imperfect competition (Petersen and Rajan 1995) and government interventions that prevent or limit price discrimination and force lenders to charge the same cost of credit in types of transactions that are intrinsically different (Benmelech and Moskowitz 2010; Banerjee and Duflo 2014).

Figure 1: Credit rationing versus distorted borrowing rates

Panel a: Credit rationing



Notes: This figures illustrates the relationship between investment gaps and borrowing constraints. Panel a depicts a situation where financial frictions manifest themselves as borrowing constraints and credit rationing. Panel b depicts a situation when financial frictions manifest themselves as distorted interest rates.

lead credit rationing. Specifically, the gray dotted line depicts the credit supply schedule absent credit frictions (e.g., without information frictions). The black solid line illustrates the supply schedule with credit frictions. While the observed amount of credit used by the firm and its capital endowment is the same as that in panel a ($B_i < B_i^*$ and $K_i < K_i^*$), in panel b underinvestment occurs because firms internalize higher borrowing costs in their decision-making. That is, given r_i, when financial frictions manifest as distortions in the price of credit, there is no excess credit demand at B_i. In this scenario, we would not observe a gap between the marginal revenue product of capital and its observed user cost.

2.2 Data and institutional context

We assemble a comprehensive firm-bank matched database that combines micro-level information on firm-specific borrowing costs, firms production and balance-sheet data, and bank credit for and bank credit for the majority of non-financial incorporated firms active in Italy between 1997 and 2013. We collect detailed information on yearly balance sheets, income statements, and registry variables from Cerved Group S.p.A. (Cerved database), restricting our attention to the non-financial and non-public industries.[2] Thus, unlike data sets compiled from census sources, our data contains information on both production and balance sheet variables. Moreover, compared to other publicly available datasets providing information on firm balance sheets (e.g., Orbis and Amadeus by Bureau van Dijk Electronic Publishing), our database has the advantage of having no selection bias, no issues with merging different data vintages, and a substantially richer set of balance sheet, income statement, and registry variables.

We merge the firm-level dataset with administrative data from the Bank of Italy, which gives access to firm-bank matched information on credit balances (Credit Registry) and borrowing rates (Taxia) for different bank-firm credit relationships, which we use for

[2]Our database includes only incorporated businesses (limited liability companies), and excludes sole proprietorships and other unincorporated firms. The unit of observation is a firm-year. No plant-level information is available. We drop the following industries: Agriculture, Mining and quarrying, Utilities, Public administration and National defense, Education, Health services, Activities of membership organizations, Activities of households as employers, and Activities of extraterritorial organizations and bodies to avoid dealing with firms with complete or partial government ownership, or firms that are heavily subsidized by the government. We drop Financial and insurance activities and Real estate activities because firms operating in these industries are themselves credit providers.

the construction of firm-specific user costs of capital.

Our data also enables us to observe loan applications. When a firm applies for a new loan, Italian banks can freely access the Bank of Italy's credit registry in order to learn about the firm's credit history. When the credit history is accessed, the request is recorded in the initial information service (IIS) dataset and helps us measure loan demand at the firm level. We combine the IIS dataset with the CR to determine whether loan demand is met at the extensive margin. All loan applications directed to *new* lenders are classified as either successful—a new loan was granted over the next three months in response to the application—or the application was rejected (Jiménez et al 2012). We infer credit applications directed to *existing* lenders by looking at outstanding credit balances. Because loans are amortized over time, we infer that a new credit application has been put forward by the firm and accepted by its legacy lender if the credit balance of a firm-bank pair either stays constant (roll-over) or increases (new credit) from one year to the next.

We complement these data with information on industry-specific price deflators, industry-specific depreciation rates of fixed assets, and socioeconomic indicators measured at the province level, all of which are collected from the publicly available archives of the Italian National Statistical Institute (ISTAT).[3]

Our final dataset includes over 5.9 million firm-year observations, 871 thousand firms, and 13.3 million credit relationships. It amounts to 80−90 percent of the value added produced by the non-financial corporate sector in the selected industries, and over 70 percent of the total value added produced by the whole Italian corporate sector. To the best of our knowledge, ours is the first longitudinal dataset that provides information on both production and financing, as well as firm-specific borrowing costs for the corporate sector of a country. Thus, it is particularly suited for the purpose of studying credit market frictions, which are expected to have a greater impact on the real activity of small and young enterprises.

Table 1 reports the summary statistics of the main variables used in our analysis. On average, firm total assets amount to 2.71 million euros, indicating that our sample is predominantly composed of privately held small and medium enterprises, precisely matching the size and industry distribution of Italian firms. 28 percent of the observations refer to firms operating in manufacturing; 25 percent of firms operating in the service

[3]Data available at https://www.istat.it/en/.

Table 1: Summary statistics

	Mean	SD	10 pctile	Median	90 pctile
Assets (Million Euros)	2.71	6.89	0.11	0.67	5.68
Age	13.10	11.00	3.00	10.00	28.00
Assets Turnover	1.48	1.12	0.41	1.24	2.69
ROA	0.02	0.18	-0.10	0.04	0.16
Cash Flows / Assets	0.04	0.16	-0.05	0.05	0.17
Exit	0.08	0.27	0.00	0.00	0.00
Bank Leverage	0.38	0.40	0.00	0.29	0.89
Number of Relations	2.40	3.03	0.00	1.00	6.00
Length Lending Relations	4.11	3.58	0.75	3.00	9.50
Borrower	0.78	0.42			
Ever Borrower	0.79	0.40			
Borrower Loans	0.50	0.50			
Credit Applications	0.54	0.49			
Accepted Credit Applications	0.59	0.49			
Manufacturing	0.28				
Services	0.25				
Construction	0.149				
Firms	871,307				
Observations	5,974,036				

Notes: This table reports the summary statistics of the main variables used in the paper. Assets is total assets. Age is firm age, measured in years. Assets turnover is revenues over assets. ROA is earnings over assets. Cash flows over assets it cash flows over total assets. Exit is a dummy variable taking value one if the following year we don't observe the firm in the Cerved dataset. Bank leverage is bank credit over total assets. Length of lending relationships are measured as the number of years of consecutive credit market interactions with the main lender of the firm. Borrower is a dummy variable taking value one if any bank credit is observed. Borrower loans is a dummy variable taking value one if any bank (term) loan is observed. Credit applications is a dummy taking value one if any credit application to new or existing lenders is observed. Accepted credit applications is a dummy equal one if, conditional on submitting a credit application, at least one is accepted.

sector; 14.9 percent of firms in construction industry; the remaining observations refer to firms that operate in the transportation and trade sector.

Attrition is an important feature of the data. The unconditional one-year probability of exit is about 8 percent. This is, of course, neither a new nor surprising fact. It is inherent in any dynamic capitalist economy that some firms enter, thrive, and grow, while others decline and sometimes exit. These patterns are particularly marked among small, privately owned enterprises who tend to be more exposed and less resilient to local and aggregate shocks (Haltiwanger 2012).

On the financing side, bank debt represents a significant portion of firms' assets—38 percent on average (almost 90 percent conditional on having any debt), and over 90 if we consider firms in the top decile of the leverage distribution. More broadly, 77.6 percent of our observations engage in some form of credit market interactions in a given year (BORROWER=1) and almost 80 percent engaged at least once during our sample period (EVER BORROWER=1). Almost 50 percent of the firm-bank observations finance their operations through term loans (BORROWER LOANS=1). On the extensive margin, in any given year, there is a 54 percent probability of observing one or more credit applications by a firm, with a 59 percent likelihood that at least one application is accepted. These facts stress the importance of credit markets as a source of external finance for SMEs.

2.3 Estimation of investment gaps

We compute micro-level investment gaps under the neoclassical benchmark (equation (1)) as the difference between an estimate of firms' marginal revenue product of capital and a measure of their user cost.

Estimating marginal revenue products. Without loss of generality, we can express a firm's marginal revenue product of capital as the product of the value of the marginal product (VMP_i^K) and the inverse-markup (μ_i^{-1}):

$$MRP_i^K \equiv \frac{\partial\left(p_i(q_i)q_i\right)}{\partial k_i} = \underbrace{p_i\frac{\partial q_i}{\partial q_i}}_{VMP_i^K}\underbrace{\left(1+\frac{q_i}{p_i}\frac{\partial p_i}{\partial q_i}\right)}_{\mu_i^{-1}} = \theta_i^k\frac{pq_i}{k_i}\frac{1}{\mu_i}\,. \tag{2}$$

where the last equation decomposes the physical value of the marginal product into the output elasticity (θ_i^K) and average product ($\frac{pq_i}{k_i}$) using the definition of output elasticity.

We estimate marginal revenue products taking equation (2) to the data. We measure average products of capital (pq_i/k_i) directly in the data as the ratio of total sales to fixed assets (tangible and intangible).[4] We estimate firm-time varying output elasticities via production function estimation. Specifically, we consider a general gross output (log-)production function of this form:

$$\log(q_{it}) = \omega_{it} + f(\log(k_{it}), \log(l_{it}), \log(m_{it}), \gamma) + \epsilon_{it},$$

[4]We construct the firm-level stock of fixed assets adopting the perpetual inventory method (PIM).

where k_{it}, l_{it}, and m_{it} denote the logarithm of capital, labor, and intermediate inputs. ω_{it} is firm-level (log) productivity, observed by the firm at the moment of its production decisions. ϵ_{it} is a production shock taking place after input decisions have been made. $\boldsymbol{\gamma}$ is a vector of structural parameters to be estimated. We employ a translog functional form for production technologies $f(\cdot)$, which allows us to recover firm-time specific estimated of the output elasticity of interest: $\theta^K_{it} = \theta^K(\log(k_i), \log(l_i), \log(m_i); \gamma)$. We estimate production function parameters $\boldsymbol{\gamma}$ following the structural approach proposed in Ackerberg, Caves, and Frazer (2015), extended to account for selection bias due to firm-exit decisions (Olley and Pakes 1992). We implement the production function estimation separately for every three-digit industry (NACE, rev.2 industry classification system), thereby allowing the structural technology parameters γs to vary across narrowly defined industries that encompass both the manufacturing and non-manufacturing sectors of the economy.

We estimate firm-year markups following the production-side approach in De Loecker and Warzynski (2012). The identification rests on the theoretical intuition that, conditional on the state variables of the problem, the first-order conditions of the cost-minimization problem for intermediate inputs provides an expression relating revenue cost shares and output elasticities to markups:

$$\hat{\mu}_i = \hat{\theta}^M_i \left(pq_i / p^M m_i \right) , \tag{3}$$

where $pq_i/p^M m_i$ is the inverse of the expenditure share on intermediate inputs in revenues (directly observed in the data) and $\hat{\theta}^M_i$ is the output elasticity with respect to intermediate inputs (obtained via production function estimation as described above).[5]

Measuring user costs. We construct a measure of firm-time-varying user costs of capital as the sum of borrowing costs, r_{it+1}, and depreciation rates of fixed assets, δ. Industry-specific depreciation rates are collected from the Italian Statistical Agency (National Accounting Tables). We use the annual percentage rate (APR) on firm-bank

[5]Taking equation (3) we follow De Loecker and Warzynski (2012) and adjust expenditure shares using the residuals of a regression of a polynomial function of deflated inputs on deflated revenues. This adjustment helps to net out variation in output not correlated with changes in input utilization (such as the one due to demand, inputs prices, or productivity).

matched loans from the credit registry as a benchmark borrowing rate.[6,7]. This information allows us to overcome an important limitation of previous studies which relied on time- and firm-specific effects in the empirical specifications to control for the variation in unobserved borrowing rates.

About 20% of the observations in our sample are made up of firms that do not actively engage in credit market transactions. These observations are of interest because they allow us to investigate the relationship between credit market participation and firm investment policies. At the same time, they pose an empirical challenge because we need to infer the borrowing rate they would have paid had they borrowed. Ample empirical evidence suggests banks set their rates based on a limited number of observable characteristics (Crawford, Pavanini, and Schivardi 2018). Moreover, it is well established that financing of small and medium firms—the lion's share in our data—is tied to their local credit markets, as proximity between borrowers and lenders facilitates information acquisition (Petersen and Rajan 2002; Degryse and Ongena 2005). We thus use firm-specific characteristics and geographical location to estimate the interest rate that non-borrowers could have been plausibly charged had they engaged in credit market transactions. Specifically, within each year and local credit market—defined by the perimeter of an Italian province—we use the firm-bank matched dataset to estimate loan-pricing predictive regressions.[8] The set of predictors includes industry, age, assets, credit score, asset turnover, ROA, and whether the firm has any credit in default during or previous to that year. These variables are selected to meet two criteria. First, they represent a parsimonious choice that ensures the existence of a common support between the group of borrowers and non-borrowers for every year-market combination. Second,

[6]Although alternative credit products are available to firms, bank loans represent around three-quarter of total bank debt and they are the typical credit product used to finance expenditures in fixed assets. In unreported regressions we found that changes in bank loans can explain a larger share of the variation in investment rates and that the elasticity of investment with respect to changes in loans is three times as large as the elasticity with respect to changes in credit line draws.

[7]When multiple banks extend bank loans to a given firm, we compute the weighted average APR with weights equal to the fraction of total loans granted by each institution. That is, we calculate the value-weighted average APR for each firm-year as $r_{it+1} = \sum_b w_{ibt} r_{ibt+1}$, where $w_{ibt} = {}^{Loans_{ibt}}/_{\sum_b Loans_{ibt}}$. When we observe multiple APRs for the same firm-bank pair, we calculate the weighted average using as weights the share of interest expense imputable to each loan.

[8]Italian provinces are the natural candidates for the definition of local credit markets for small-business lending (see Guiso et al, 2012). They constitute administrative units comparable to US counties. The Bank of Italy uses the administrative boundaries of provinces as a proxy of local credit markets for regulatory and supervisory purposes.

they are observable indicators commonly used by banks to assess firms' riskiness and creditworthiness. We estimate the pricing regressions focusing on the subsample of newly established relations. This is the most relevant comparison because non-borrowers would be new customers for the bank in case they approach them. Moreover, for new lending relationships, we do not have to account for the dynamics of firm-bank relationships and the acquisition of soft information and lower monitoring costs that repeated interactions bring about.

A second group of observations is represented by firms that engage in credit market transactions, but for which we are unable to observe the interest rate on term loans because they only have access to revolving credit lines, because they borrow from banks that are not required to report information on lending rate to the Bank of Italy, or because their outstanding loan balance is below the CR reporting threshold. For these observations, the missing price problem is less severe because, in addition to firm-specific characteristics and geographical location, we can augment the loan-pricing regressions with information about bank leverage, the length of each individual credit relation, and the total number of lending relations.

3 Investment gaps and credit market access

We now present descriptive evidence linking both cross-sectional and time-series variations in the estimated neoclassical investment gaps to firms' credit market access.

3.1 Correlation between investment gaps and firm characteristics

Table 2 reports descriptive statistics of the distribution of marginal revenue products, user costs, and investment gaps for the full sample (panel a) and for different subsamples defined by firms' credit market participation (panels b and c) and type of credit products (panels d and e).[9] Over the 1997-2013 period, the median firm in our dataset has a marginal product of capital of 18 percent, which is approximately equal to the median value of the user cost (panel a). As a result, the distribution of investment gaps is centered around zero, which is consistent with the neoclassical benchmark and suggests that investment policies are relatively undistorted. However, we find substantial dispersion and right-skewness

[9] Estimates of the components of MRP^k and firm-level productivity are reported in Appendix ??.

Table 2: Estimates or marginal revenue products, user costs, and investment gaps

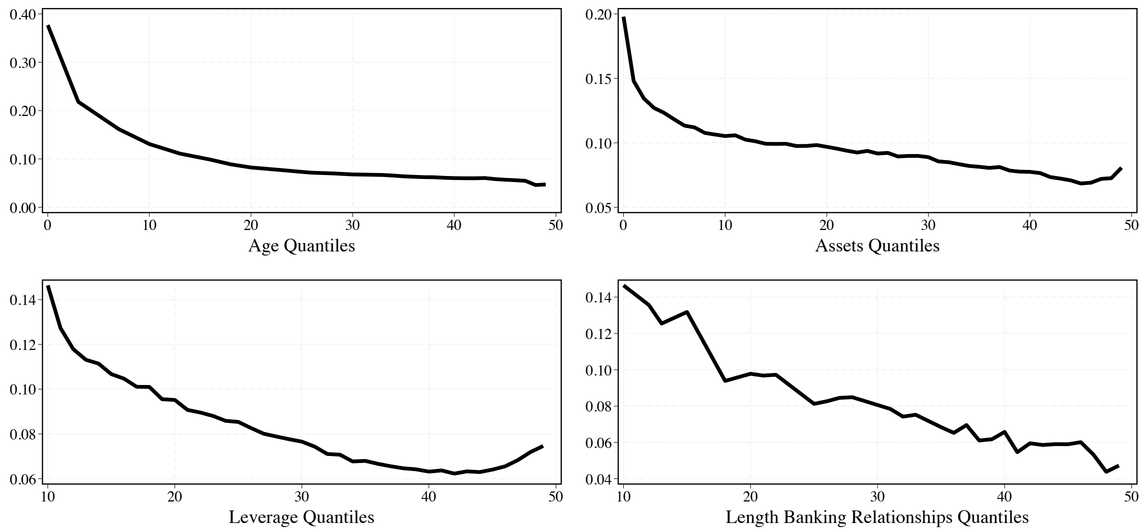
	Mean	10 pctile	Median	90 pctile	Mean	10 pctile	Median	90 pctile
	Panel a: Full sample							
MRP^k	0.373	0.023	0.182	1.050				
r	0.060	0.035	0.058	0.085				
δ	0.104	0.056	0.114	0.123				
R	0.164	0.119	0.166	0.200				
τ^k	0.201	-0.132	0.016	0.700				
	Panel b: Borrowers				*Panel c: Non-borrowers*			
MRP^k	0.344	0.022	0.170	0.930	0.488	0.029	0.252	1.600
r	0.058	0.033	0.056	0.083	0.067	0.046	0.065	0.089
δ	0.105	0.057	0.114	0.125	0.100	0.056	0.103	0.119
R	0.163	0.120	0.164	0.199	0.167	0.118	0.170	0.204
τ^k	0.162	-0.133	0.004	0.606	0.364	-0.125	0.083	1.110
	Panel d: Borrowers w/ Loans				*Panel e: Borrowers w/out Loans*			
MRP^k	0.323	0.021	0.164	0.844	0.383	0.023	0.185	1.100
r	0.056	0.030	0.053	0.084	0.061	0.042	0.060	0.082
δ	0.107	0.057	0.114	0.125	0.101	0.056	0.101	0.123
R	0.163	0.124	0.164	0.200	0.162	0.114	0.166	0.197
τ^k	0.134	-0.134	-0.003	0.540	0.215	-0.131	0.022	0.739

Notes: This table reports summary statistics of the distribution of MRP-cost gaps and their components. Panel a looks at the full sample. Panel b and panel c focus on firm-year observations for we observe a positive credit balance or no credit market participation, respectively. Panel d and panel e further split the sample of firms for which we observe a positive credit balance into observations with outstanding bank loans and those for which we only observe revolving credit lines.

in the distribution of investment gaps. This is entirely driven by the dispersion and right-skewness of the distribution of marginal revenue products, whereas the distribution of user costs is compact and symmetric. In the full sample, the average investment gap is 20 percent. Firms in the top deciles of the gaps distribution display a marginal product of capital that exceeds the user cost of capital by 70 percentage points or more.

The observed deviations from the neoclassical investment benchmark appear to be related to firms' credit market participation. In panels b through d, we partition observations into groups based on whether we observe any credit market participation (borrowers vs. non-borrowers) and whether we observe long-term debt obligations (term loans versus credit lines). Table 2 highlights that the estimated marginal revenue product of capital is, on average, twice as high for non-borrowers than for borrowers. That is,

Figure 2: Investment gaps and firm characteristics



Notes: This figures correlates investment gaps with firm's characteristics, reporting the average investment gap of firms of different age, size, leverage, and length of lending relationships.

---

as expected, access to bank credit is likely a crucial source of finance for private firms. Table 2 also highlights that investment gaps are 1.5 times larger for those borrowers that have access only to credit lines compared to those with outstanding long-term financing. Because credit lines are a more expensive type of credit and they can be revoked at a lender's discretion, firms rarely turn to credit lines to finance capital expenditures in fixed assets, unless the supply of bank loans is constrained or denied by credit institutions.[10]

Further descriptive evidence on the relationship between investment gaps and credit market frictions comes their correlation with firm characteristics. Figure 2 shows that investment gaps (dark blue bars) monotonically decline with firm age and firm size, two commonly used proxies of financial constraints (Hadlock and Pierce, 2010), and firm leverage. Once again, these patterns are driven by the variation in the marginal revenue product of capital, whose decline far exceeds that of user costs of capital.

[10]Consistent with this hypothesis, borrowers with only credit lines are younger and smaller, overrepresented in Southern regions of Italy, and in industries with lower tangible-to-intangible asset ratios (e.g., services). Not coincidentally, all these firm-specific variables are commonly regarded as proxies for credit constraints.

3.2 Dynamics of investment gaps and credit market access

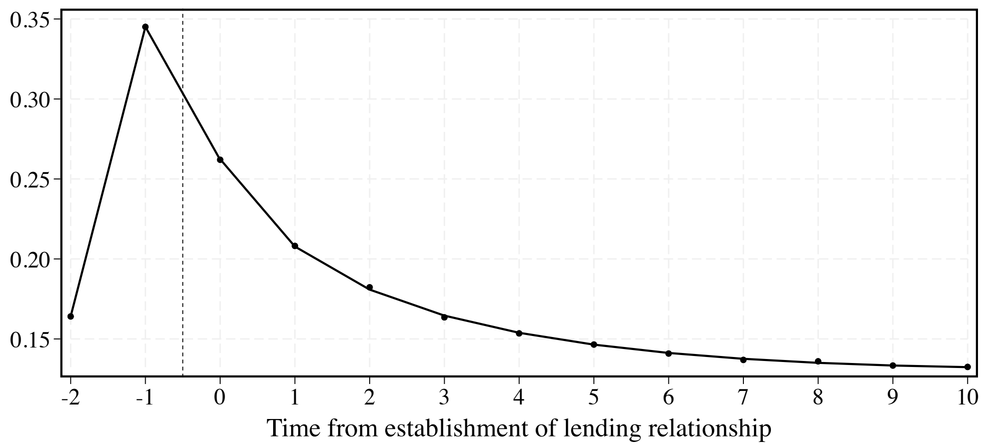
An extensive body of research in corporate finance has highlighted the importance of "relationship lending" for borrowers' access to credit, with banks gradually expanding their credit supply as they develop stronger relationships with their borrowers (Petersen and Rajan 1994; Berger and Udell 1995; Bharath et al., 2011). Repeated interactions with financial intermediaries allow firms to overcome possible asymmetric information frictions and gradually accumulate a capital endowment that is more aligned with profit maximization (Botsch and Vanasco, 2015). Enduring bank-firm relations typically lead to a reduction in the expected costs of credit provision for lenders, because, conditional on past experience with a borrower, the lender now expects loans to be less risky (Diamond 1991). Moreover, monitoring and screening costs related to information acquisition are generally lower for existing customers because information obtained at one date may also be used to assess credit risk in the future. In principle, lenders could respond to the decline in the expected cost of credit provision as relatinships unfold by adjusting the price term of the loan contract or by relaxing the credit limits that might be in place. The dynamics of investment gaps as lending relanships ufold suggests quantity adjustments are the most salient margin of adjustment.

The bottom right panel of Figure 2 plots the average investment gaps against the length of the firm's longest-lasting lending relationship. It shows how firm-level investment gaps monotonically decrease with the length of lending relationships, suggesting that firms implement more efficient investment policies as they develop tighter relationships with their lenders. To further shed light on the evolution of gaps over firms' life cycles and connect it to credit availability, we conduct two event studies that illustrates how investment gaps change upon access to credit and upon termination of existing lending relationships.

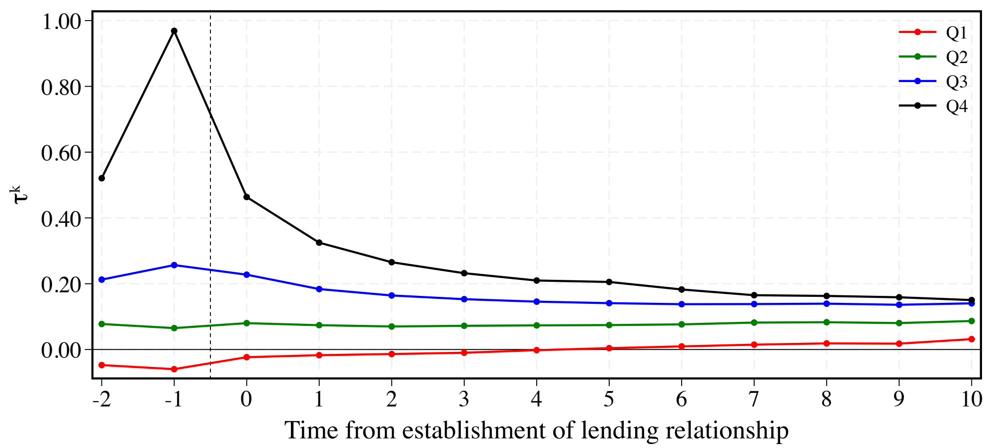
For each firm in our sample, we identify the year when they first establish a bank lending relationship ($t = 0$) and plot the average of the absolute investment gap ($|\tau_{it}^{k}|$) across firms around this event. Figure 3, panel a, shows a sharp reduction of investment gaps upon access to credit and a steady convergence towards the frictionless benchmark as firms strengthen their lending relationships. On average, compared to the year prior to establishing a lending relationships, investment gaps are 60 percent lower one year after gaining access to credit and almost 3 times lower after 10 years.

Figure 3: Investment gaps and credit market participation

Panel a: Full sample



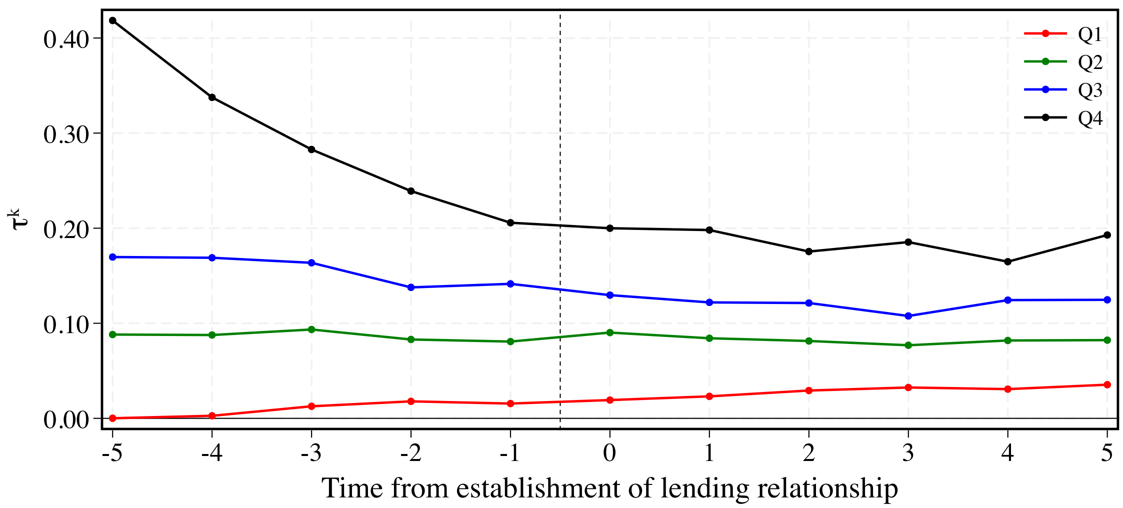
Panel b: Subsamples bases on pre-event gaps



Notes: This figure shows the dynamic of investment gaps as firms establish and tighten lending relationships. We define an event ($t = 0$) the year when a firm first establishes a lending relationship and gains access to credit markets. Panel a plots average absolute gaps in different years since the event. Panel b plots average gaps for different years since the event, sorting observations into sub-samples based on the quartile of the distribution of the investment gap to which the firm belonged the year prior to gaining access to credit.

To further appreciate the information content of investment gaps, we sort firms into five different groups based on the quartile of the investment gap distribution to which they belonged in $t = -1$, one year prior to establishing a lending relationship. The rationale behind this sorting is that, to the extent that positive gaps reflect variation in the shadow price of credit due to binding borrowing constraints, we expect to find firms with the largest, positive gaps to display a greater reduction of gaps upon access to credit as well as a steady convergence of their investment policies as tighter lending relationships relax credit constraints. Figure 4, panel b, depicts the average investment gaps of the different groups of firms over event time. As we can see, consistent with our hypothesis, the sharp reduction and subsequent gradual convergence observed on average is entirely driven by firms with positive investment gaps (black and blue lines) and, in particular, those with large investment gaps (black lines) are the ones that display a greater drop in gaps shortly after gaining access to credit.

Figure 4: Investment gaps and termination of lending relationships



Notes: This figure shows the dynamic of investment gaps before and after the termination of the lending relationship with their main lender. We define an event ($t = 0$) the year when a firm first interrupts a lending relationship with its main lender. The figure plots average gaps in different years since the event, sorting observations into sub-samples based on the quartile of the distribution of the investment gap to which the firm belonged the year prior to the event.

Next, we study how investment gaps evolve once the lending relationship with a firm's main lender ends. Ideally, one would like to focus on cases when the bank unilaterally terminates an ongoing relationship for reasons unrelated to changes in a firm's credit worthiness. Unfortunately, we are unable to isolate these events. However, the theory behind investment gaps suggests firms with investment positive gaps are more likely in need of external finance, and therefore are the firms whose capital accumulation should be the most affected by a termination of an ongoing lending relationship, especially the one with their main lender. With these predictions in mind, we define an "event" as the interruption of the lending relationship with the main lender of firm ($t = 0$). We then sort firms into four different groups based on the quartile of the investment gap distribution prior to the termination event ($t = -1$). Figure 4 plots the average investment gaps of the different groups of firms in event-time.

Consistent with the theoretical predictions, firms with larger, positive investment gaps are the most affected by the termination of a lending relationships. Their gaps display a steady decline prior to the interruption of the lending relationship. In the five years prior to the termination of the lending relationship, on average, we observe that the investment gaps of firms in the top quartile reduced by about half. This trend visibly halts once the lending relationship breaks, with gaps remaining estially unchanged in the five years that follow. By contrast, we don't observe any significant change in investment gaps for firms with small or negative gaps. That is, on average, a termination of lending relationships does not seem to affect investment policies of those firms that, according to our metric, would be classified as "over-capitalized".

3.3 Taking stock

The descriptive analysis presented so far provides several pieces of evidence connecting the firm-level investment gaps to credit market frictions. Specifically, we argued that the heterogeneity in investment gaps reflects the shadow price of capital generated by borrowing constraints. We also emphasized, however, that financial frictions are not the only phenomena that can rationalize the sign and magnitude of gaps. For one, investment gaps are estimates obtained from firms' observed production and financing choices. It is therefore possible that measurement or estimation errors might erroneously lead us to conclude that investment policies of some firms are sub-optimally low (or sub-optimally high) when in fact they are not. For another, real adjustment costs of capital, risk in capital accumulation, and aggregate risk can explain why we observe that some firms with positive investment gaps do not invest sufficiently even if they face an unconstrained supply of credit.

In the remainder of the paper, we adopt a structural approach to leverage variation in the micro-data in order to estimate firm-specific shadow costs from variation in firm-specific investment gaps. In the spirit of Hennessy and Whited (2007), we first introduce a partial-equilibrium inter-temporal investment model with credit constraints, real frictions, and aggregate risk. This model formalizes the relationship between investment gaps and shadow costs generated by binding credit constraints. We then resort to structural methods to estimate the model, parametrizing shadow prices as a function of firms' observable demands for credit and firms' characteristics. This approach allows us to recover firm-time-specific shadow prices generated by binding borrowing constraints, while also dealing with sample selection, simultaneity, and measurement-error. Finally, we shed light on the distribution of shadow costs of debt in the cross-section of firms, contrasting this distribution of shadow costs with the distribution of borrowing interest rates observed in the data.

4 Model

Firms maximize the expected discounted value of future net cash flows. Every period, the manager observes the realization of firm's productivity, and then decides whether (i) to repay their outstanding debt (if any) or (ii) to exit and possibly default. In the first scenario, firms optimally choose production factors and their financial structure.

Both the exit and production decisions depend on the firm's perception of the distribution of future market structures given current information. We denote by J_{it} the information set of firm i at the beginning of year t, before making exit and production decisions. The information set includes all firm-level state variables (discussed below), information on product and factor markets, as well as information on credit supply conditions.

Product and factor markets. Firms produce output with a production function $y_{it} = e^{\omega_{it}} f(k_{it}, l_{it}, m_{it})$, that displays decreasing returns to scale in each input (capital, labor, intermediates). ω_{it} denotes a firm's idiosyncratic productivity, which evolves as a first-order Markov process. Firms face a downward-sloping demand curve for their product, $q_{it} = q(p_{it}, \eta_i, \Omega_t)$, where p_{it} is the firms' output price, and η_i the residual demand elasticity faced by firm i, and Ω_t an aggregate demand shifter. We assume that the realization of the firm's idiosyncratic productivity, residual demand elasticity, and aggregate demand conditions are known to the firm (but not the econometrician) at the beginning of period t, before exit, production, and financing decisions have taken place. Formally, $\{\omega_{it}, \eta_i, \Omega_t\} \in \mathcal{J}_{it}$.

Firms are price takers in input markets. They hire labor l_{it} and buy intermediates m_{it}

paying a wage rate w_t and the intermediates' price p_t^M. They own their capital stock and augment it through investments (i_{it}) whose price, p_t^K, normalized to one. Capital evolves dynamically according to the standard law of motion: $k_{it+1} = k_{it}(1-\delta) + i_{it}$, where δ denotes the depreciation rate of current assets. Note that because today's investments become productive with a lag, the stochastic evolution of firms' productivity generates risk in capital accumulation that translates into uncertain realizations of the marginal return on capital.

Financing. Firms can finance capital purchases by internal finance or bank debt, b_{it+1}. The credit contract offered by lenders to firm i consists of a one-period debt contract that specifies a fixed interest rate (r_{it+1}) and a borrowing limit ($\bar{b}_{it+1}$):

$$b_{it+1} \leq \bar{b}_{it+1} := \theta(s_t, k_{it+1})k_{it+1}. \tag{4}$$

The credit limit $\theta(s_t, k_{it+1}) := s_t \frac{\Psi(k_{it+1})}{k_{it+1}}$ captures the severity of firm i's borrowing constraint $\theta(s_{it}, k_{it+1})$, which depends on two factors: Local credit supply conditions $s_t \in \mathcal{J}_{it}$—affecting all firms that operate in a given local credit market—and firm size. We assume that $\frac{\partial \Psi(k_{it+1})}{\partial k_{it+1}} > 0$, capturing the empirically established relation between firm size and bank leverage.[11] Contrary to the case with an exogenous borrowing limit (e.g., Whited 1992; Whited and Wu 2006), the dependency of borrowing constraints on firm size implies that debt and capital are not separable within the profit function. In turn, this implies not only that credit market frictions affect firms' inter-temporal transfers of resources, but also that the shadow cost of debt generated by binding borrowing constraints directly enters the Euler conditions that characterizes firms' investment policies.

Firm problem. After observing the realization of the productivity shock and given its capital, legacy debt, demand conditions, credit supply conditions, and factor prices, a firm chooses whether to operate or to default and exit ($E_{it} = 1$). If it continues, it chooses investment (i_{it}), labor (l_{it}), the output price (p_{it}), and debt (b_{it+1}) to maximize shareholders' expected present value of discounted utility of dividend flows:

[11]The parametrization of size-dependent borrowing constraints is similar in spirit to the one proposed by Gopinath et al (2017), who assume $\theta(k_{it+1}) = \theta_0 + \theta_1 \frac{\Psi(k_{it+1})}{k_{it+1}}$, with $\Psi(k_{it+1}) = e^{k_{it+1}} - 1$. See, e.g., Arellano, Bai, and Zhang (2012) and Gopinath et al (2017). We provide direct evidence consistent with size-dependent borrowing constraints in Section 3.

$$\max_{\{E_{it},b_{it+1},k_{it+1},l_{it},m_{it},p_{it}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(d_{it})$$

where the utility function is such that $U_d > 0$, $U_{dd} < 0$, and satisfies the Inada conditions. β is a risk-neutral discount factor. If the firm chooses to exit, it receives the sell-off value of $\Phi_t \geq 0$ (measured in utils) and never reappears again.

Define firm's net worth as $n_{it} := k_{it} - b_{it} > 0$ and reformulate the borrowing constraint (4) as:

$$k_{it+1} \leq \lambda_t(k_{it+1}, n_{it+1})n_{it+1}, \tag{5}$$

with $\lambda_t(k_{it+1}, n_{it+1}) = \left[1 + \frac{s_t \Psi(k_{it+1})}{n_{it+1}}\right] \geq 0$. The Bellman equation that characterizes the firm's problem is:

$$V_t\left(n_i, k_i, \omega_i\right) = \max\left\{\Phi_{it}\, , \sup_{\{n'_t, k'_i, l_i, m_i, p_i\}} U(d_i) + \mathbb{E}\left[M'V\left(n'_i, k'_i, \omega'_i\right)\right] |\mathcal{J}_t\right\} \tag{6}$$

$$\begin{aligned}
s.t \quad & d_i = p_i y(p_i, \epsilon_i) - w_i l_i - m_i p^M - (r_i + \delta)k_i + (1 + r_i)n_i - n'_i - c(k_i, k'_i) \\
& q_i = e^{\omega_i} f(k_i, l_i, m_i) \\
& q_i = q_t(p_i, \epsilon_i) \\
& k'_i \leq \lambda_t(k'_i, n'_i)n'_i
\end{aligned}$$

where d_i denotes cashflows to shareholders and the function $c(k_i, k'_i)$ captures real adjustment costs of capital, with $\frac{\partial c(\cdot)}{\partial k_{it+1}} > 0$. We denote by $M' = \beta \frac{U_d(d')}{U_d(d)}$ the stochastic discount factor (SDF) between t and $t+1$, such that $\mathbb{E}[M'] = \frac{1}{1+r^f}$. Two remarks are in order. First, note that the value and the profit functions are indexed by time because they depends on the market structure and on factor prices, which are are assumed to be constant across agents in a given time period and omitted in (6) to economize on notation. We relax this assumption in our empirical estimation by allowing production function parameters to vary by industry, markups to vary by firm and time, and financial conditions to vary by time, industry, and time.

Second, the max operator in (6) indicates that a firm compares the sell-off value of the firm to the expected discounted return of staying in business, and chooses to shut down if the current state variables indicate that continuing operation is not worthwhile. In this case, we assume the firm hands out ownership and control to creditors and shareholders collect the sell-off value $\Phi_t \geq 0$. For simplicity, we assume that the sell-off value is independent of firms' state variables but allow it to depend on market conditions. As we discuss below, this assumption is not necessary for our estimation strategy and we relax it when we take the model to the data.

To formalize the firm's exit problem, we define the indicator function E_i, which takes value one if incumbent i exits at time t. The solution to the discrete choice control problem in (6) takes the form of a threshold rule (Ericson and Pakes, 1995; Olley and Pakes, 1996; Hennessy and Whited 2007):

$$E_i = \begin{cases} 0 & \text{if } \omega_i \geq \underline{\omega}_t(n_i, k_i) \\ 1 & \text{otherwise.} \end{cases} \tag{7}$$

The notation emphasizes that the threshold level is a time-varying function $\underline{\omega}_t(\cdot)$ of the firm-specific states, n_i, k_i, and aggregate states. $\underline{\omega}_t(\cdot)$ is decreasing in k and n because the dividend function (and thus the value function) is increasing in both capital and net worth. Intuitively, firms with larger capital stock expect larger future returns for any given level of current productivity, and hence will continue operation at lower realizations of ω. Similarly, holding capital constant, firms with greater net worth are less leveraged and therefore have a lower amount of debt coming due. Therefore, for any given k, firms with larger net worth will continue operation at lower productivity realizations. Section 5 shows that these predictions find strong support in the data.

Euler equation. Denote by χ_i the multiplier attached to the borrowing constraint (5), scaled by the shareholders marginal utility from dividends in period t. Given the state vector $\{\omega_i, k_i, n_i\}$ and the borrowing constraint, firms' optimal investment policies are characterized by the following Euler equation:

$$\mathbb{E}_{\omega_i' > \underline{\omega}_t}\left[M' \left(\underbrace{MRP^{k'}{}_i - (r_i' + \delta)}_{\tau_i^{k'}} - c_k' \right) \right] = \chi_i \left(1 - s\Psi_k(k_i')\right) + c_k. \tag{8}$$

where $c_k = \frac{\partial c}{\partial k'}$ and $c_k' = \frac{\partial c'}{\partial k'}$ are the derivatives of the adjustment cost functions and $\Psi_k(k_i')$ the derivative of the function $\Psi(k_i')$ with respect to k'. The left-hand side of the equation contains the risk-adjusted, present-value of the expected investment gap $\tau_i^{k'}$—the difference between the Marginal Revenue Product of capital and its user cost—introduced in the previous section. The expectation operator makes explicit that firms take expectations over the subset of future productivity realizations where production is optimal, which depends on the information set $\mathcal{J}_{it}$ and, in particular, the current state variables (see equation (7)).

A number of observations are in order. Dispersion in observed Marginal Revenue Products is often viewed as manifestation of investment policy distortions and inefficient capital allocation, possibly driven by financial frictions (Hsieh and Klenow, 2009; Bau and Matray, 2022). Equation (8) highlights that part of this variation is driven by differential borrowing costs. Because, as the data shows, heterogenous interest rates capture (at least in part) heterogenous default risk some dispersion in *MRPK* is "efficient", even within narrowly defined industries.[12]

Second, even in a world without default risk, capital accumulation risk translates into dispersion in realized marginal revenue products across firms. Capital accumulation risk arises because a capital stock determined in some previous period may not be optimal ex-post, that is, after productivity shock is realized. As a result, part of the dispersion of the *MRPK* across firms would also arise in an undistorted economy in which the capital stock is chosen under uncertainty and becomes productive in the next period. Third, and related, real adjustment costs are captured by the derivatives of the adjustment cost function c_k and c_k' (Asker, Collard-Wexler and De Loecker, 2014).

Fourth, to the extent that firms' *MRPK*s co-move differently with the SDF M' (e.g., firms of different sizes), their expected MRPKs will differ. [13]Accounting for variation in the SDF is important because periods when investors discount future consumptions more heavily often coincide with episodes that exacerbate credit market frictions (e.g., during financial crises).

Finally, equation (8) elucidates how financial frictions, in the form of size-dependent borrowing constraints, can distort firms' inter-temporal decisions, thereby generating

[12]Heterogeneous interest rates can also capture variation in banks' market power as suggested, e.g., by Petersen and Rajan (1995).

[13]David, Smidth, and Zeke (2020) point out that there could be cross-sectional correlation between the stochastic discount factor and marginal products of individual firms. In other words, the discount factor becomes heterogeneous. If this is the case, part of the variation in MRPK could be explained by the heterogenous exposure of different "types" of firms to aggregate risk. We do not model this cross-sectional heterogeneity and restrict our attention to controlling for time-series variation in aggregate risk.

dispersion in marginal revenue products. When borrowing constraints are binding, the multiplier χ_i (the *shadow price of capital*) is positive. A positive shadow price, together with the sensitivity of the borrowing constraint to firms' collateral, drives a wedge in firms' Euler equation, $\chi_i\left(1 - s\Psi_k(k_i')\right)$, which captures the *shadow cost of capital.* We resort to structural methods to estimate the distribution of firm-time specific shadow costs. As we will show, for some firms, the shadow cost of capital might be substantially higher than the *user cost of capital* $(r_t' + \delta)$ observable in the data, suggesting that borrowing constraints create distortions in investment policies beyond what heterogeneity in market prices (interest rates) might suggest.

5 Structural Estimation

To estimate the model, we follow the approach adopted in the Euler equation literature (Whited, 1992; Bond and Meghir, 1994; Whited and Wu 2006). Under the assumption firms' rational expectations, we evaluate the expectations in (8) at their realized values. In doing so, however, we need to account for two important facts. First, we observe significant churning in our data. As discussed above, the unconditional one-year probability of exit is about 8 percent. Second, exit events are non-random, but rather related to firm characteristics, including current productivity, their capital, and debt, as well as environmental factors and their perceptions of their future productivity. These forces are captured by our firm-dynamic model, as equation (7) shows. Together, the two facts imply that our sample is selected and that the selection process, if not addressed, can introduce bias in the estimation of the Euler equation coefficients. To address this self-selection bias, we nest a control function approach within a more standard Euler equation estimation framework.

Selection equation. To illustrate this approach, first note that the marginal product of capital $MRPK_i' = MRPK(\omega_i', k_i', n_i')$ is increasing in productivity. Therefore, we have that:

$$\begin{aligned}\mathbb{E}_{\omega_i'>\bar{\omega}_i}\left[MRPK(\omega_i', k_i', n_i')\right] &= \mathbb{E}\left[MRPK(\omega_i', k_i', n_i')|\omega_i', k_i', n_i', E'=0\right] \\ &= \int_{\underline{\omega}_i}^{\infty} MRPK(\omega_i', k_i', n_i')\frac{F(d\omega_i'|\omega_i)}{\int_{\bar{\omega}_i}^{\infty} F(d\omega_i'|\omega_i)},\end{aligned}$$

which tells us that, given $k_i', n_i', \mathbb{E}_{\omega_i'>\bar{\omega}_i}\left[MRPK(\omega_i', k_i', n_i')\right]$ is a function of two firm-specific state variables: ω_i' and $\underline{\omega}_i$. Thus, to control for the impact of unobservables on selection we need a measure of the cut-off value $\underline{\omega}_i$, which makes the firm indifferent between continuing operation and exiting. Next, consider the probability of survival:

$$\begin{aligned} Pr\{E' = 0|\underline{\omega}_i(k_i', n_i'), \mathcal{I}_t\} =& Pr\{\omega_i' \geq \underline{\omega}_i(k_i', n_i')|\omega_i, \underline{\omega}_i(k_i', n_i'), \mathcal{I}_t\} \\ =& \varphi\{\omega_i, \underline{\omega}_i(k_i', n_i'), \mathcal{I}_t\} \\ =& \varphi\{\omega_i, i_i, k_i, b_i, \mathcal{I}_t\} \equiv P_{it} \end{aligned} \tag{9}$$

Equation (9) is the selection equation. The third line follows from: (i) the fact that k' and b' are chosen by the firm as a function of current states; (ii) the investment rule ($i = f(\omega, k, n)$), (iii) the capital accumulation rule ($k' = (1-\delta)k + i$), and (iv) the net worth rule ($n' = k' - b'$).

Finally, we note that, provided that the density of ω' conditional on ω is positive in the region about $\underline{\omega}'$, the selection equation in (9) can be inverted to express $\underline{\omega}'$ as a function of $h(\omega_i, P_i)$ (Olley and Pakes, 1996).[14] This allows us to condition on the the selection probability (or "propensity score"). We can then write the Euler equation (8) at realized values and rearrange:

$$M'\left(\tau_i' - c_k'\right) = \chi_i\left(1 - s\Psi_k(k_i')\right) + c_k' + h(\omega_i, P_i) + \epsilon_i'. \tag{10}$$

By controlling for the propensity score, the expectational error ϵ_i' has the standard properties $\mathbb{E}\left(\epsilon_i' \mid h(\omega_i, P_i)\right) = 0$ and $\mathbb{E}\left((\epsilon_i')^2 \mid h(\omega_i, P_i)\right) = \sigma_\epsilon^2$. We approximate the function $h()$ as a second-order polynomial in ω_i and P. We obtain estimates of firm-level productivity, $\hat{\omega}_{it}$, via the production function estimation, as described in Section 2.3. We estimate the probability of exit, $\hat{P}_{it}$, fitting a probit model of productivity, investment, capital, and debt on an indicator variable of exit. Mindful of the skewed distributions of debt, fixed assets, investments, and productivity, we apply the following transformations to our regressors. We scale bank debt by total assets (aka, leverage), apply a logarithmic transformation to the book value of assets and to firm-level TFP, and

[14] The function $h()$ serves the same purposes of the inverse of Mills' ratio that is included in two-step sample selection models (e.g. Heckman, 1974). However, the econometric problem but is somewhat more nuanced in our case. First, in our case the sample selection bias depends on two variables (ω_i and $\underline{\omega}_i$) rather than on just one. Second, both variables are unobservable; while a measurable proxy of former can be recovered via a production function estimation (ω_i), the latter is unknown ($\underline{\omega}_i$).

Table 3: Probability of Exit

	Exit	
	Coefficient	SE
Leverage	-0.084	0.002
Ln(Fixed Assets)	-0.097	0.000
Net Investments rate	-0.681	0.002
TFP	-0.081	0.001
Age	-0.009	0.000
Market Share	-0.119	0.006
g(sales industry-year-provice)	-0.178	0.002

Notes: This tables reports the estimates of a probit model of productivity, investment, capital, and debt on an indicator variable of exit.

scale net investments (investments minus divestments in fixed assets) by the book value of fixed assets. Moreover, we augment the regression with the firm's market share in the industry-province, firm's age, and the industry×year×province growth rates of sales. We include these variables because the probability of exit depends on time-varying factors related to product market structure and demand conditions, as highlighted by equation (9). Additionally, these variables also account for differences in exit rates due to firms' lifecycle stages. Table 3 reports the estimates. Consistent with the predictions of the model, larger and more productive firms are less likely to exit the dataset. Higher leverage increases the probability of exit. Intuitively, conditional on size and productivity, highly leveraged firms are less likely to be able to honor their obligations following adverse demand shocks. High investment rates also correlate with lower likelihoods of default, as firms' actions contain information regarding their rational perceptions of the future.

Finally, note that the assumption that the sell-off value is independent of the firm's state variables is not a necessary condition for our approach. In fact, the control function approach described above is valid as long as the difference between the value of continuing operation and the sell-off value of the firm is increasing in ω, k, and n. When this condition is met, it does not matter whether the sell-off value is independent from the firm-level states (as we assumed for simplicity in our behavioral model) or not.

Real frictions and SDF. We assume a standard quadratic functional form for the real adjustment cost, $c = \frac{\alpha}{2}\frac{(k_i'-k_i)}{k_i}$.[15] As in Whited and Wu (2006), we adopt a reduced form specification for the SDF using the three-factor model of Fama and French (1993):

$$M' = (l_0 + l_1 \cdot MKT' + l_2 \cdot SMB' + l_3 \cdot HML')^{-1} \tag{11}$$

where MKT' is the return on the market; SMB' is the return on an arbitrage portfolio that is long on small firms and short on large firms; and HML' is the return on an arbitrage portfolio that is long on firms with high book to market ratios and short on firms with low book to market ratios.

Credit constraints. We need a measurable proxy of $s_i\Psi_k(k_i')$, which is the key term in the Euler Equation of our model. The variable s_i is a firm-specific level of the credit supply that is independent of the firm size / collateral. For example, it reflects the degree of financial development or willingness to lend of the local banking system as well as the strength of lending relationships between a firm and its current lenders. The function $\Psi_k(k_i')$ measures the steepness of the borrowing constraint with respect to the "collateral" provided or size-dependent collateral constraints more generally.

We proceed as follows. First, we approximate the possibly non-linear function $\Psi(k'_i)$ using a piecewise linear approximation. Formally, we sort firms into g groups based on the distribution of assets. For every group g, we approximate the function $\Psi^g(k'_i)$ as an affine function: $\Psi^g(k'_i) = \bar{\Psi}^g + \Psi_k^g k_i'$. This implies that, for every group g, we have:

$$s_i\Psi^g(k_i') = s\bar{\Psi}^g + s\Psi_k^g k_i',$$

where the second term represents the slope coefficient of interest ($s\Psi_k^g$). Plugging this expression inside the collateral constraint (5) and re-arranging:

$$\frac{b_i'}{k_i'} \leq s\Psi_k^g + s\bar{\Psi}^g \cdot \frac{1}{k_i'}.$$

Assuming for a moment the borrowing constraint is binding for all firms, for each

[15] We also experimented with higher-order polynomials and obtained similar results.

Table 4: Slopes of size-dependent borrowing constraints

	$\widehat{\Psi_k^g s}$				
	Group 1 [Small]	Group 2	Group 3	Group 4	Group 5 [Large]
p10	0.083	0.090	0.145	0.301	0.372
mean	0.177	0.215	0.302	0.447	0.509
median	0.168	0.205	0.305	0.473	0.530
p90	0.287	0.346	0.450	0.564	0.635

Notes: This tables reports the estimates the sensitivity of borrowing constraints to firm size. Estimates vary by firm size (quintiles of firm asset distribution), year, and industry (1 digit NACE code).

group of firms g the equation above has the following regression counterpart:

$$Leverage_{it+1} = \beta_0^g + \beta_1^g \cdot \frac{1}{k_{it+1}} + u_{it+1}, \tag{12}$$

We then use $\hat{\beta}_0^g$ as our empirical approximation of $\widehat{s\Psi_k^g}$. We separately estimate (12) for different subsamples of firms formed based on size × year × region × industry combinations (a total of 14,405 subsamples).[16] This allows us to capture not only size-specific variation, but also incorporate cyclicality movements in credit supply as well as local credit market conditions and industry-specific factors which impact the level of credit supply, such as differences in assets tangibility.

In taking equation (12) to the data, we need to identify a subset of firms for which borrowing constraints are more likely binding. We take two steps in this direction. First, we leverage the information on credit applications available through the credit registry and estimate (12) focusing on the subsample of firms for which (i) we observe at least one credit application in year t and (ii) not all credit applications are accepted. Second, to further ensure that variation in $\widehat{s\Psi_k^g}$ picks up credit-supply factors and not heterogeneous credit demand, we instrument $\widehat{s\Psi_k^g}$ with local credit supply shifters in the GMM estimation.

[16]We define $g = 1, \ldots 5$ size groups. Group $g = 1$ includes firms with total assets<= 250 thousand euros; Group $g = 2$ firms with 250 thousand < total assets <= 500 thousand euros; Group $g = 3$ firms with 500 thousand < total asset <= 1 million euros; Group $g = 4$ firms with 1 million < total asset <= 10 million euros; Group $g = 5$ firms with total asset > 10 million euros; These groups approximately correspond to the quintile of the asset distribution of the firms in our sample. We experimented with coarser (terciles) and more granular sorting (deciles) and obtained similar results.

The following section provides a description of these shifters.

Table 4 reports the summary statistics on the distribution of $\widehat{s\Psi_k^g}$. The estimated slope coefficients are well-behaved and, consistent with the predictions of the theoretical model, bounded within the interval $\in [0, 1)$. Importantly, consistent with the presence of size-dependent borrowing constraints, our estimates indicate that $\widehat{s\Psi_k^g}$ increases with firm size, suggesting that larger firms can borrow more against each unit of assets.

Shadow prices. Finally, firm-time-specific shadow prices χ_i are unobservable. To address this issue, we extend our model and parameterize χ_i as an exponential function of observable firm characteristics, which can drive variation in shadow prices conditional on being constrained:[17]

$$\begin{aligned}\chi_{it} = exp\{&\chi_0 + \chi_1 \cdot ROA_{it} + \chi_2 \cdot g(sales_{it}) + \chi_3 \cdot g(sales_{ind,t}) \\ &+ \chi_4 \cdot g(assets\,turnover_{it-1}) + \chi_5 \cdot cash\,assets_{it-1} + \chi_6 \cdot age_{it}\} \end{aligned} \tag{13}$$

We expect the shadow price associated to a binding constraint to be higher for more profitable firms. We include sales growth and industry sales growth to capture the intuition that only firms with good investment opportunities are likely to want to invest enough to be constrained. We expect these firms to belong to high-growth industries but have low individual sales growth (Whited and Wu, 2006). The availability of internally generated resources can mitigate the real effects of financial constraints. Therefore,given the same borrowing constraint, we expect firms with higher asset turnover and those entering year t with larger cash holdings to display lower shadow prices. We do not include firm size in the set of variables affecting shadow prices because, as explained below, we non-parametrically account for size effects.

5.1 Estimation

To arrive at the estimation equation we compute the derivatives of the adjustment cost function ($c_{k'}$ and c_k) and substitute them into equation (10) together with the estimated investment gaps $\tau_i^{k'}$. We then use equation (11) to substitute for the SDF, replace firm-specific shadow prices with their parametric expressions (equation (13)), and insert

[17] Whited (1992), Hubbard, Kashyap, and Whited (1995), Love (2003), and Whited and Wu (2006) adopt a similar parametric approach to estimate shadow costs due to equity constraints.

the estimated slopes $\widehat{\Psi_k^g s}$. We estimate equation (10) using non-linear GMM, and including the propensity score ($g(\hat{P}, \omega)$) to control for selection due to endogenous exit decisions. We perform the estimation in first differences to eliminate possible firm-specific unobservable effects. This procedure requires us to use instruments lagged by two periods. Thus, the moment conditions have the following form:

$$\mathbb{E}\left[z_{it-2} \otimes (\epsilon_{it} - \epsilon_{it-2})\right] = 0.$$

A few remarks are in order. First, we separately estimate the parameters of interest for for different subsamples of firms defined by firm size, using the five groups g outlined above. By allowing parameters to vary by firm size, we can better capture the effects of size-dependent credit constraints on firms' investment policies.

Second, the parametrization of shadow prices in equation (13) implicitly assumes that the data generating process is drawing from a continuous distribution. However, the complementary slackness condition—$\chi_i(k_i' - \lambda_t(k_i', n_i')n_i')$—indicates that their distribution has a mass point at zero whenever the constraint is not binding. Our data allows us to directly tackle this concern thanks to the availability of information on firms' credit applications. We interpret credit application as a manifestation of unmet credit demand. Accordingly, we separately perform the estimation for firms that put forward credit applications and for those that do not. If this conjecture is right, the estimation should return a compact distribution of $\hat{\chi}_i$ clustered around zero values for firms without credit applications, and a smooth distribution with mostly positive values for firms with positive applications. As we discuss below, this is precisely what we find.

Finally, we need to comment on the set of instruments z_{it-2}. Leveraging the rational expectation assumption, all double-lagged values of regressors in the Euler equation are valid instruments. We include two additional moment conditions. First, it is important for our purpose that our specification explicitly controls for time-series variation in aggregate risk factors that affect the discount rate of cash flows across periods. We discipline the estimate of the SDF parameters by imposing in the estimation the additional unconditional moment restriction that the expected value of the SDF is equal to $(1+r_f)^{-1}$, where r_f is the risk-free rate. Secondly, we are particularly interested in the identification of the shadow prices, which are identified by the (co)variation of firm characteristics in equation (13) and the slopes of borrowing constraints $s\Psi_k^g$. As noted above, the latter is estimated via a regression which can potentially pick up both credit-supply and credit-demand factors. We therefore improve the identification by augmenting the set of instruments with a local credit supply shifter, which we construct as follows. In the spirit of the Amity and Weinstein (2018), we run the following cross-sectional regression using the firm-bank matched micro-data from the credit registry:[18]

$$\text{g}(\text{Credit}_{ibt}) = \boldsymbol{f}_{it} + \boldsymbol{b}_{bt} + \varepsilon_{ibt}\ . \tag{14}$$

We then construct an local credit-supply shifter as a weighted average of the bank fixed effects:

$$shifter_{imt} = \sum_{b \in \mathcal{B}^m_{t-1}} w_{mbt-1} \hat{b}_{bt}, \tag{15}$$

where $\mathcal{B}^m_{t-1}$ denotes the set of banks with outstanding corporate lending position in municipality m in year t, and the weight w_{mbt-1} denotes bank b's lending shares in municipality m in year t. The credit shifter in (15) is essentially an local aggregation of the firm-level shifters in Amity in Weinstein (2018). Such shifters have the advantage of varying at the firm-year level, however, by construction, they are computable only for the firms with positive leverage in $t-1$.[19] This subset of firms does not include start-ups firms and any other firms that would like to borrow but are unable to, which are obviously subpopulations of interest for our analysis. Appendix B shows that these shifters have strong predictive power on a firm's credit growth.

5.2 Estimation Results

Table 5 presents the estimates of the Euler equation parameters. Panels a and b report the estimates for the subsamples of observations for which we observe and do not observe credit applications, respectively. Within each panel, we report the estimated coefficients for different subsamples based on firm size. Because the estimation routine involves several steps, deriving the appropriate analytic standard errors is nontrivial. We therefore compute clustered bootstrap standard errors, treating all observations for a single firm as

[18]See Alfaro, Garcia-Santana, and Moral-Benito (2021) and Berg et al. (2021) for recent applications of the Amity and Weinstein (2018).

[19]The firm-level shifter in Amity in Weinstein (2018) which is constructed as a follows: $shifter_{it} = \sum_{b \in \mathcal{B}^i_{t-1}} w_{ibt-1} \hat{b}_{bt}$, where w_{ibt-1} is the share of firms' i credit granted by bank b in year $t-1$.

one cluster.

Overall, the more standard Euler Equations parameter estimates are sensible. For example, the adjustment cost parameter is largely consistent with the range of estimates found by previous papers (see, e.g., Cooper, Haltiwanger, and Willis (2006) and Whited and Wu (2006)). The estimates of the risk factor are also reasonable and in line with the level of the risk-free rate during our sample period (about 2 percent annually).

Moving to the parameter of interest (χ_0 to χ_6), we can see that the estimates are largely in line with the theoretical predictions. First, consider the estimates for the subsample of firms that put forward credit applications (Table 5, panel a). We find that shadow prices associated to a binding constrain are higher for more profitable firms and for constrained firms that operate in industries with high investment opportunities, as indicated by the positive sign of the coefficient associated with ROA and firm-level sales and the negative sign of coefficient of industry sales.[20] We also estimate lower shadow prices for firms that experience high turnover and collected cash flows in the previous fiscal year, suggesting that the availability of internally generated resources can mitigate the real effects of financial constraints. We also note variation in the magnitude of the estimated coefficients across size-groups. For example, the coefficient associated with firm's profitability is almost twice as large for small firms than it is for large firms, whereas the absolute value of the coefficient associated to past cash flows is six times smaller. While these differences partially reflect a differential support of the regressions' distributions across size-groups, we will see that the difference in the coefficient estimates translates into significant heterogeneity in the shadow prices and shadow costs, which is consistent with the presence of size-dependent borrowing constraints.

Before moving to shadow prices and costs, it is important to highlight similarities and differences in the coefficient estimates obtained for the subsample of firms for which we do not observe credit applications. The sign and magnitude of the estimates are similar, with one important exception. For this group of firms, the estimated value of the constant term of the parametrized shadow costs ($\hat{\chi}_0$) is negative and large. This practically shifts the entire distribution of shadow prices (and therefore shadow costs) towards zero, indicating that credit rationing is not a salient distortion for these firms. This result validates our conjecture and highlights the value of information on credit application for the purposes

[20] The Whited and Wu (2006) measure of financial constraints also features an opposite sign for the coefficient of firm-level and industry sales growth.

of quantifying the extent of credit rationing in the data.

6 The Shadow Costs of Capital

6.1 The Distribution of Shadow Prices and Shadow Costs

Using the estimated parameters, we recover the distribution of firm-time specific shadow prices, χ_{it} (equation (13)) . Combining shadow prices with the estimated slopes of borrowing constraints, we recover the distribution of firm-time specific shadow costs $(\chi_{it}\,(1 - s_t \Psi_k(k_{it+1})))$ due to the presence of binding borrowing constraints. Table 6 presents summary statistics for both distributions; Figure 5 plots the cumulative distribution functions.

Table 5: Parameter estimates

	Panel a: With Credit Applications				
	Group 1 [Small]	Group 2	Group 3	Group 4	Group 5 [Large]
l_0	1.03	1.04	1.03	1.03	1.05
	(0.053)	(0.052)	(0.055)	(0.052)	(0.051)
l_1 [MKT]	-0.011	-0.209	-0.162	-0.219	-0.302
	(0.009)	(0.098)	(0.081)	(0.108)	(0.143)
l_2 [SMB]	-1.130	-1.130	-1.150	-1.170	-1.200
	(0.117)	(0.105)	(0.111)	(0.118)	(0.124)
l_3 [HML]	1.670	1.570	1.590	1.600	1.590
	(0.150)	(0.142)	(0.145)	(0.147)	(0.148)
α	0.178	0.094	0.045	0.031	0.012
	(0.021)	(0.013)	(0.006)	(0.004)	(0.002)
χ_0	-0.429	-0.175	0.105	0.226	0.306
	(0.049)	(0.020)	(0.013)	(0.025)	(0.033)
χ_1 $[ROA]$	0.485	0.222	0.226	0.243	0.258
	(0.046)	(0.019)	(0.021)	(0.024)	(0.026)
χ_2 $[g(sales)]$	-0.833	-1.430	-1.250	-1.160	-1.090
	(0.120)	(0.180)	(0.160)	(0.150)	(0.140)
χ_3 $[g(sales_{ind})]$	0.443	0.274	0.337	0.376	0.400
	(0.055)	(0.035)	(0.042)	(0.047)	(0.050)
χ_4 $[g(assets\,turnover)]$	0.009	-0.104	-0.240	-0.096	-0.032
	(0.004)	(0.051)	(0.057)	(0.020)	(0.010)
χ_5 $[cash\,assets]$	-0.204	-1.200	-1.250	-1.240	-1.230
	(0.025)	(0.142)	(0.150)	(0.147)	(0.145)
χ_6 $[age_{it}]$	-0.040	-0.083	-0.088	-0.107	-0.084
	(0.008)	(0.017)	(0.020)	(0.024)	(0.020)
Observations	489,956	496,379	586,487	1,370,968	301,024

Notes: This tables reports the estimates of the Euler Equation parameters. The Euler Equation we take to the data is reported in equation (8). We plug in (8) the derivatives of the adjustment cost function ($c_{k'}$ and c_k) and the estimated investment gaps ($\tau_i^{k'}$); we substitute equation (11) to replace the SDF, replace firms-specific shadow prices with their parametric counterparts (equation (13)), and plug in the estimated slopes $\widehat{\Psi_k^g s}$. We estimate equation (10) via non-linear GMM, performed in first differences. Panel a reports the parameter estimates obtained estimating the GMM model on the subsamples of firms with credit applications. Panel b reports the estimated obtained estimating the GMM model on the subsamples of firms without credit applications. Within each panel estimates are reported for different subsamples defined by firm size. Cluster bootstrap standard errors are reported in parenthesis.

Table 5: Parameter estimates (Continued)

	Panel b: Without Credit Applications				
	Group 1 [Small]	Group 2	Group 3	Group 4	Group 5 [Large]
l_0	1.010	1.030	1.040	1.070	1.060
	(0.392)	(0.526)	(0.472)	(0.415)	(0.540)
l_1 [MKT]	-0.022	-0.107	-0.175	-0.369	-0.259
	(0.011)	(0.055)	(0.089)	(0.188)	(0.132)
l_2 [SMB]	-0.868	-0.918	-0.932	-0.964	-0.946
	(0.337)	(0.469)	(0.477)	(0.502)	(0.490)
l_3 [HML]	0.940	0.916	0.904	0.888	0.901
	(0.365)	(0.467)	(0.550)	(0.579)	(0.548)
α	0.082	0.044	0.038	0.024	0.007
	(0.032)	(0.022)	(0.020)	(0.014)	(0.004)
χ_0	-1.440	-1.390	-1.370	-1.240	-1.320
	(2.748)	(2.780)	(2.740)	(2.480)	(2.640)
χ_1 $[ROA]$	0.291	0.287	0.289	0.303	0.291
	(0.419)	(0.415)	(0.417)	(0.432)	(0.415)
χ_2 $[g(sales)]$	-1.160	-1.160	-1.150	-1.100	-1.140
	(1.854)	(1.859)	(1.841)	(1.760)	(1.820)
χ_3 $[g(sales_{ind})]$	0.389	0.392	0.395	0.411	0.400
	(0.841)	(0.849)	(0.856)	(0.889)	(0.865)
χ_4 $[g(assets\ turnover)]$	0.259	0.025	0.053	0.233	0.083
	(0.158)	(0.015)	(0.031)	(0.141)	(0.050)
χ_5 $[cash\ assets]$	-1.110	-1.100	-1.100	-1.090	-1.100
	(2.229)	(2.200)	(2.195)	(2.180)	(2.200)
χ_6 $[age_{it}]$	-0.533	-0.187	-0.159	-0.345	-0.147
	(1.196)	(0.419)	(0.359)	(0.780)	(0.335)
Observations	991,738	539,394	494,022	660,948	43,120

Notes: See notes of Table 5.

Table 6: Shadow prices and Shadow Costs

Panel a: With Credit Applications

	Mean	p10	p25	Median	p75	p90
Shadow Prices	0.198	0.001	0.016	0.096	0.305	0.529
Shadow Costs	0.133	0.001	0.011	0.061	0.195	0.365

Panel b: With Credit Applications

	All	Group 1 [Small]	Group 2	Group 3	Group 4	Group 5 [Large]
	Shadow Prices					
Mean	0.331	0.507	0.363	0.338	0.280	0.267
p10	0.050	0.288	0.105	0.081	0.033	0.021
Median	0.275	0.486	0.296	0.275	0.210	0.178
p90	0.652	0.741	0.657	0.638	0.610	0.640
	Shadow Costs					
Mean	0.218	0.411	0.283	0.235	0.155	0.132
p10	0.027	0.228	0.079	0.053	0.017	0.010
Median	0.170	0.390	0.228	0.186	0.113	0.084
p90	0.460	0.612	0.521	0.456	0.344	0.319

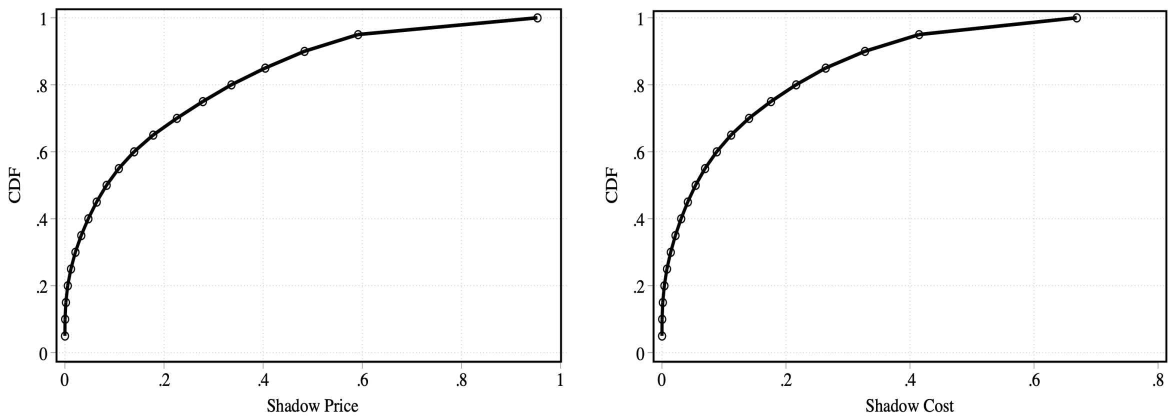
Panel c: Without Credit Applications

	All	Group 1 [Small]	Group 2	Group 3	Group 4	Group 5 [Large]
	Shadow Prices					
Mean	0.037	0.028	0.057	0.059	0.017	0.034
p10	0.000	0.000	0.003	0.003	0.000	0.000
median	0.014	0.009	0.041	0.040	0.002	0.012
p90	0.106	0.080	0.130	0.137	0.057	0.100
	Shadow Costs					
Mean	0.028	0.024	0.045	0.042	0.010	0.018
p10	0.000	0.000	0.002	0.002	0.000	0.000
median	0.010	0.007	0.032	0.028	0.001	0.006
p90	0.079	0.066	0.104	0.099	0.032	0.052

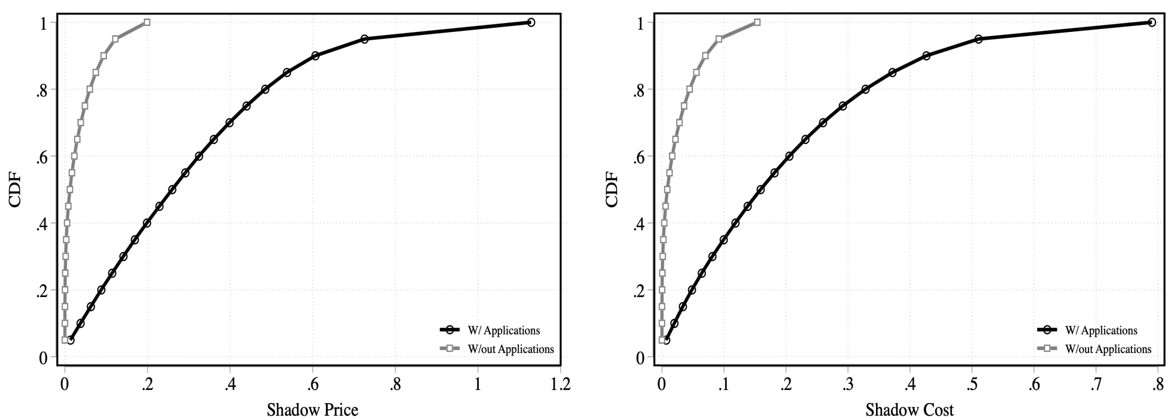
Notes: This tables presents summary statistics of the distribution of the recovered shadow prices (χ_{it}) and shadow costs ($\chi_{it}\,(1 - s_t \Psi_k(k_{it+1}))$). Firm-time specific shadow prices are recovered evaluating equation (13) at the estimated values of χ_0–χ_6 (reported in Table 5). Shadow costs are obtained combining shadow prices and the estimates of the size-specific slopes of borrowing constraints. Panel a reports statistics of shadow values and shadow costs for the full sample; Panels b and c for the subsample of firms with and without credit applications, respectively. Within Panels b and c, statistics are also reported for different subsamples defined by firm size.

Figure 5: Shadow Prices and Shadow Costs

Panel a: Full sample



Panel a: Firms with and without credit applications



Notes: This figure displays the cumulative distribution function of the recovered shadow prices (χ_{it}) and shadow costs ($\chi_{it}\,(1 - s_t \Psi_k(k_{it+1}))$). Firm-time specific shadow prices are recovered evaluating equation (13) at the estimated values of χ_0–χ_6 (reported in Table 5). Shadow costs are obtained combining shadow prices and the estimates of the size-specific slopes of borrowing constraints. Panel a includes all observations in our dataset; Panel b splits the sample into firms with and without credit applications.

Our estimates suggest that, on average, the shadow price of debt is 19.8 percent (median 9.6 percent) and the shadow cost is about 13.3 percent (median 6.1 percent). We find substantial heterogeneity in both shadow values, as indicated by the 90/10 range. Distinguishing between firms with and without credit demand explains a substantial portion of this heterogeneity (panel b and panel c). The average shadow price among firms that put forward a credit application is 33.1 percent (median 33.8 percent), which is about 10 times larger than the estimated shadow price of firms that do not apply for

Table 7: Elasticity of Investment Gaps to Shadow Costs of debt

	Full Sample	W/ Applications	W/out Applications
Shadow Costs	.029***	0.157***	0.126***
	(0.001)	(0.002)	(0.001)

Notes: This tables reports the elasticity of investment gaps to shadow prices and shadow costs of debt. Elasticities are computed for the full sample, and for the subsamples of firms with and without credit applications. Robust standard errors are reported in parenthesis. *** denotes that the mean difference is significance at the 1% level, ** denotes significance at the 5% level, * denotes significance at the 10% level.

credit. Figure 5 (panel b) makes this point clear.

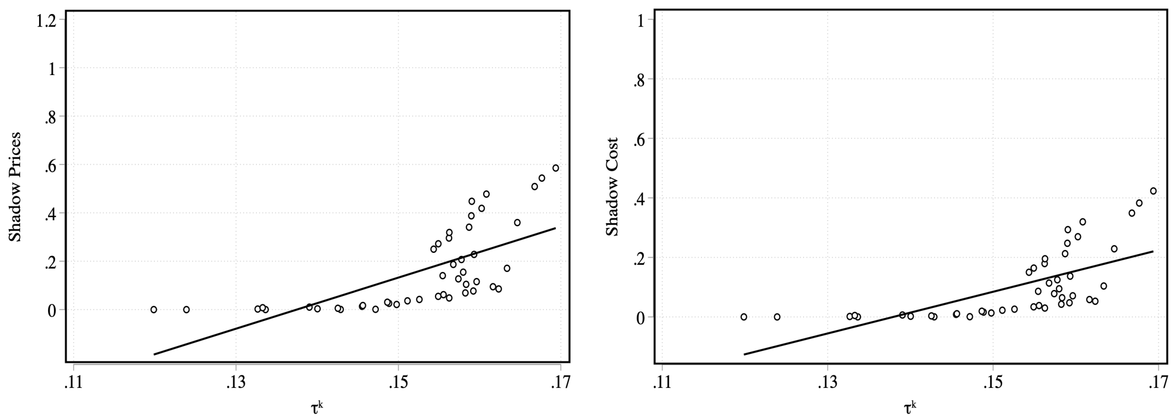
A natural question arises: Why do firms that do not apply for credit show any positive shadow price at all? The answer is twofold. The first is estimation and measurement error. The second explanation is a "discouragement effect". Searching and applying for credit is not effortless; thus, it is possible that some ffirms that repeatedly but unsuccessfully applied for credit in the past stop doing so despite remaining credit constrained. Consistent with this, shadow prices are about 20 percent lower for firms that had at least some applications rejected in the past.

Shadow costs and investment gaps. In Section 3, we introduced the neoclassical investment gaps (τ^k) as a metric to evaluate the efficiency of firms investment policies. We argued that credit rationing is one explanation for the deviations of firms' investment policies from the neoclassical benchmark and provided some descriptive evidence in this direction. We can now directly ask: Can the presence of binding credit constraints explain the observed variation in investment gaps? Figure 6 shows that the answer is yes. In line with the predictions of theories of credit rationing (Stiglitz and Weiss, 1981), we observe a monotone relationship between investment gaps and shadow prices and shadow costs. The elasticity of investment gaps to shadow costs of debt is 0.03 percent in the full sample and 0.15 if we restrict attention to the subsample of firms with positive credit demand (Table 7).

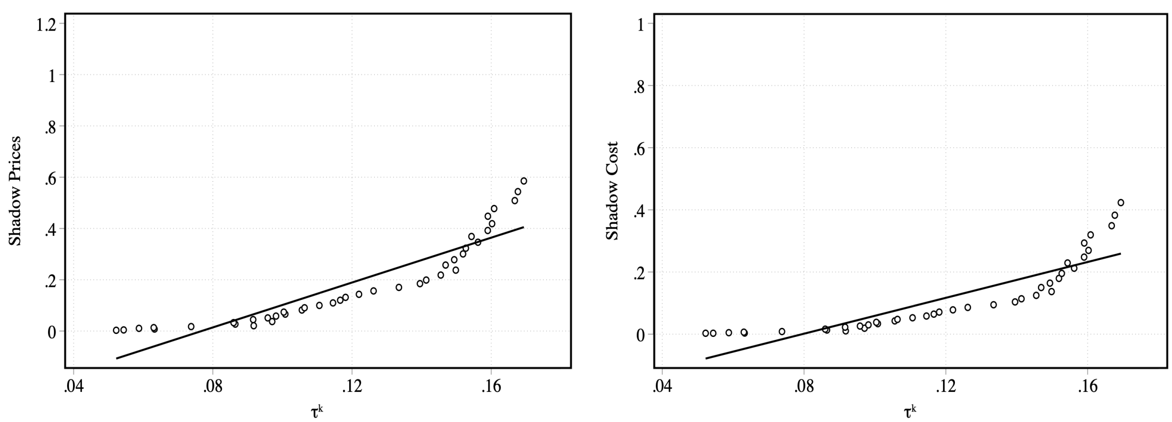
Correlation with firm characteristics. Next, we study how shadow prices and shadow costs vary across firms. Because shadow costs are estimated to be small (or zero) for firms that don't demand credit, we restrict our analysis on the sample of firms that

Figure 6: Shadow Prices, Shadow Costs, and Investment Gaps

Panel a: Full sample



Panel b: Firms with credit applications



Notes: This binned-scatter plot shows the correlation between investment gaps (τ^k), shadow prices (left panel) and shadow costs (right panel).

put forward a credit application.[21] We begin by studying heterogeneity along the age and size dimensions. These characteristics are perhaps the most common proxies for financial constraints (see, e.g., Hadlock and Pierce, 2010). Figure 7, panel a, shows that the entire distribution of shadow costs shifts to the left as firms grow in size and become older. This result is consistent with small and young firms facing more severe financial frictions due, to the lack of pledgeable assets and greater opacity, among other factors. To further explore the relationship between credit frictions due to asymmetric information, Figure 7. Panel b, shows the distribution of shadow costs, splitting firms into subsamples based on the length of their lending relationships with banks. Mirroring previous results for investment gaps, we find that tighter relationships allow firms to overcome asymmetric information frictions and gradually relax existing credit constraints. Finally, we show that shadow prices strongly correlate with local financial development. Comparing economic outcomes across Italian regions, Guiso, Sapienza and Zingales (2004) show that local financial development is an important determinant of the economic success of an area. The distribution of shadow prices across Northern regions (the most financially developed), Center regions, and Southern regions (the least financially developed), offers strong evidence in the same direction (Figure 7, panel b).

6.2 Shadow costs and user costs

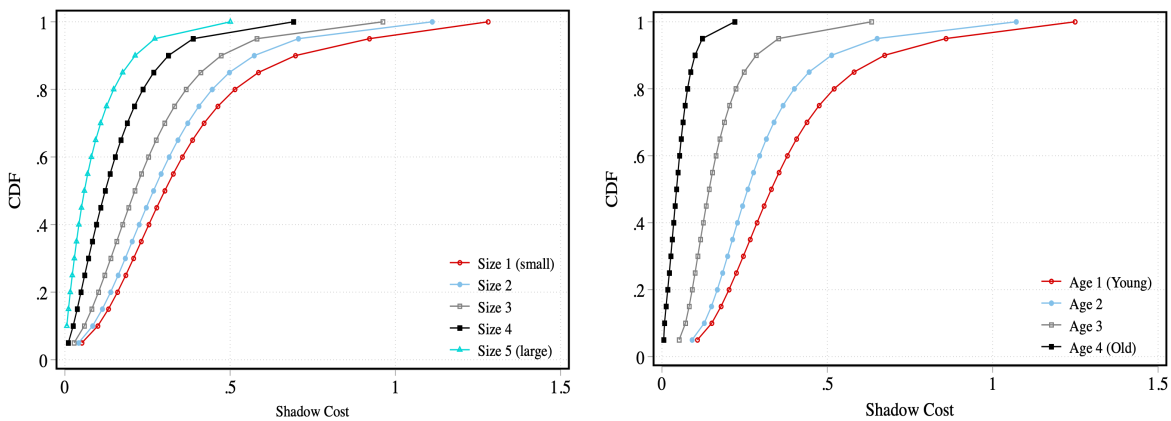
In Section 3, we highlighted the compactness of borrowing rates and user costs, as opposed to the large dispersion in shadow values. We now formally study the relationship between user costs and shadow costs. We highlight four key facts regarding the relative magnitude and variation in the observable and implicit component of firms' cost of capital.

First, for financially constrained firms, shadow prices and shadow costs are substantially larger than market prices and user costs. Figure 8 presents a binned scatter plot where we sorted observations into percentiles, based on the unconditional distribution of shadow costs. For each bin we compute the average shadow cost and average user cost. As we can see, in the subsample of firms that reveal a credit need (the subsample with credit applications), the shadow cost of capital is, on average, 35 percent higher and six times more dispersed than its user cost (left panel). Since higher shadow

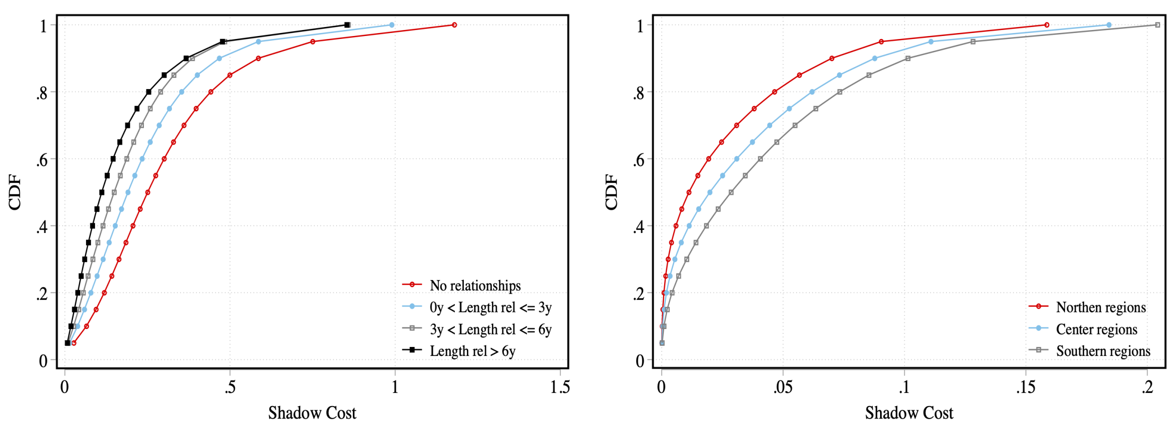
[21]Results are consistent if use the full sample of firms (include in the analysis firms with very small shadow prices). As we discussed above, including observations without credit demand adds a cluster of observations at the bottom of the distribution of shadow values.

Figure 7: Correlation between Shadow Prices, Shadow Costs, and Investment Gaps

Panel a: Size (left) and Age (right)



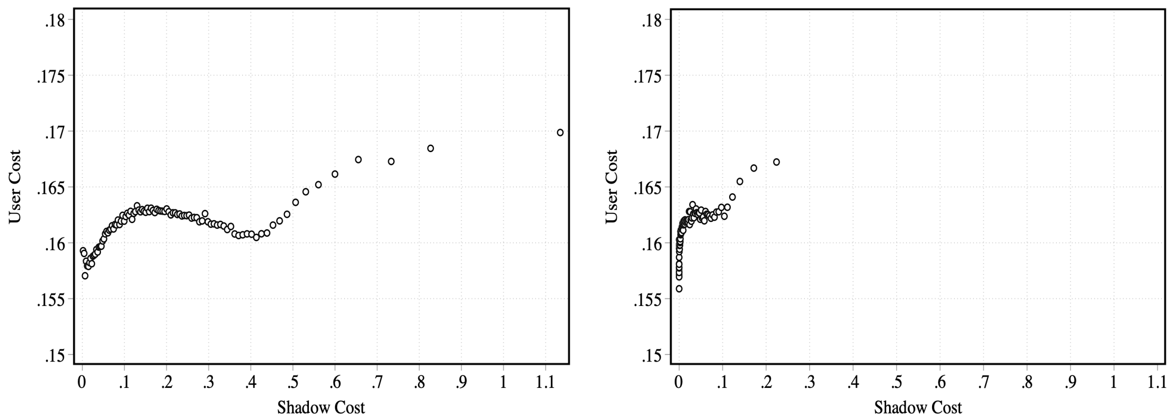
Panel a: Length of Lending Relationships (left) and Local Financial Development (right)



Notes: This figure displays the cumulative distribution function of the recovered shadow costs splitting the observations into subsamples defined according to firm size and age (panel a) and length of lending relationships and local financial market development (panel b).

Figure 8: Shadow Costs and User Costs

Firms with credit applications (left) and without credit applications (right)



Notes: This figure presents a binned scatter plot where we sorted observation into percentiles based on the unconditional distribution of shadow costs. For each bin we compute the average shadow costs and average user cost. The left panel focuses on the subsample of firms for which we observe a credit applications. The right panel on the subsample of firms for which we don't.

---

prices imply lower capital accumulation and therefore forgone investment opportunities, these results suggest that the real costs due to credit rationing are substantial.

Second, among credit constrained firms, the dispersion in shadow prices swamps the dispersion in user costs. This result directly speaks to a large literature studying welfare losses due to resource misallocation (Midrigan and Xu, 2014). Binding borrowing constraints act as an implicit, heterogeneous tax on producers. The implication of these taxes is that some producers are too large whereas others are too small relative to their "socially efficient" size, thereby squandering resources and reducing aggregate productivity and economic growth (Restuccia and Rogerson, 2008; 2013).

Third, shadow costs and user costs are positively correlated because both co-move with credit risk factors. However, the shadow prices are far more sensitive to variation in risk factors than user costs are. To make this point, we compute the sensitivity of shadow costs and user costs to commonly used empirical proxies of credit market frictions: firm age, firm size (the logarithm of firm's total assets), and the Kaplan-Zingales (1997) index of financial constraints. The latter is an industry-based measure of reliance on external financing. Companies with higher KZ-Index scores are more likely to experience difficulties when financial conditions tighten since they may have difficulty financing their ongoing operations or new investments. We control for firm profitability (ROA), bank

Table 8: Sensitivity of User Costs and Shadow Costs

	User Cost	Shadow Cost	User Cost	Shadow Cost
	(1)	(2)	(3)	(4)
Age_{t-1}	-2.593E-04***	-0.089***		
	(0.000)	(0.000)		
$Ln(Assets)_{t-1}$	-0.002***	-0.029***		
	(0.000)	(0.001)		
RZ Index	0.003***	0.034***		
	(0.000)	(0.000)		
Credit Supply Shifter			-3.4E-04***	-0.016***
			(0.000)	(0.002)
R^2	0.0143	0.8002	0.266	0.828
Observations	2,572,538	2,572,538	2,572,538	2,572,538

Notes: This table studies the relation between user and shadow costs of capital, proxies of credit constraints and changes in credit supply conditions. The dependent variables are the natural logarithm of the user cost and the natural logarithm of the shadow cost. All regressions control for lag profitability (ROA), bank leverage, and firm's credit score. Columns (3) and (4) also include year×industry×fixed effects. Standard errors are reported in parenthesis. *** denotes that the mean difference is significance at the 1% level, ** denotes significance at the 5% level, * denotes significance at the 10% level.

leverage, and the firm's credit score in order to restrict comparisons to observationally similar firms in terms of credit risk, and focus on the subsample of firms that reveal a credit demand through their credit applications. To avoid simultaneity issues, all firm-specific regressors enter the regression model lagged by one year. Finally, to facilitate the interpretation and comparison of the coefficients across columns, we apply a logarithmic transformation to the dependent variables and standardize all independent variables so that each coefficient measures the percentage change in the dependent variable associated to a one-standard deviation increase of the regressor. Standard errors are clustered at the firm-level.

Columns (1) and (2) in Table 8 report regression estimates.[22] As we can see, both the observed and the implicit cost of capital covary in the expected direction with all

[22]We note that all of the observations that follow hold true if we study each measure in isolation. They also hold true if we restrict the regression sample to firms with outstanding term loans, for which the borrowing rate is directly observed in the data.

of the proxies. However, the sensitivity of shadow costs is orders of magnitude larger than the sensitivity of user costs. For example, a one-standard deviation increase in firm size translates into a 0.2 percent reduction in the user cost of capital and a 2.8 percent reduction in the shadow cost of capital.

Finally, using variation in local credit supply shifters (introduced in Section 5), we examine how changes in credit supply conditions affect the equilibrium user costs and shadow costs. This exercise provides a direct test of the channel of transmission of credit market frictions—prices or quantity—to firm's policies. Our credit shifter varies at the year-by-municipality level. Thus, we can include in our specification year-by-industry-by-province fixed effects, which allow us to decouple changes in credit supply and changes in credit demand driven by aggregate and local demand conditions. Moreover, because our estimates of depreciation rates vary at the industry-level, the inclusion of this set of fixed effects implies that variation in user costs is only driven by variation in borrowing rates, which is the key variable of interest. Columns (3) and (4) in Table 8 report the estimated effects. As one would expect following a credit supply shift, we find a significant reduction in both components of firms' costs of capital. Once again, however, the reduction in the shadow cost of capital is fourty times larger than the reduction in borrowing costs: a one-standard deviation difference in the exposure to a local credit supply shock leads to a reduction of 1.6 percent of the shadow costs associated to binding borrowing constraints in borrowing costs.

7 Concluding remarks

We started by asking how shadow costs of debt generated by credit rationing compare to the borrowing costs observed in the data. Our analysis suggests that, in the subsample of firms that demand credit, the shadow costs generated by binding credit constraints can be substantially higher than the observable user cost of capital.

Leveraging detailed, administrative data on firms' production, financial decisions, and costs of debt, we estimate the distribution of shadow costs of capital for the Italian corporate sector over a 17-year time span. We documented substantial deviations in firms' investment policies from an unconstrained benchmark and shown that such deviations can be explained, at least in part, by heterogenous shadow costs of debt due to credit constraints. The size and dispersion of shadow costs is substantially larger than the size and dispersion of the user cost. Shadow costs also display a stronger correlation with measures of financial frictions and are more sensitive to changes in credit supply conditions.

These results suggest that credit rationing is the most salient feature of credit markets for SME firms. Financial frictions distort investment policies of firms that rely on bank credit as a primary source of finance (e.g., private corporations, especially those of small size) mostly through credit quantity rationing rather than heightened borrowing costs. This implies that infering the degree of financial frictions for SME from variation in borrowing costs would substantially underestimate the extent of financial frictions and their consequences on a firm's real activity.

Our findings speak to the discussion regarding the design of policy aimed at stimulating corporate lending. They suggest that policies providing direct or indirect interest rate subsidies might have muted effects on equilibrium credit outcomes relative to interventions that directly relax quantity constraints, such as credit guarantee programs.

References

Abel, A.B. and Blanchard, O.J., 1986. The Present Value of Profits and Cyclical Movements in Investment. Econometrica, 54(2), pp.249.

Ackerberg, D. A., Caves K., and Frazer G., 2015. Identification Properties of Recent Production Function Estimators. Econometrica, 83 (6), pp.2411–2451.

Agarwal, S., Chomsisengphet, S., Mahoney, N. and Stroebel, J., 2018. Do banks pass through credit expansions to consumers who want to borrow?. The Quarterly Journal of Economics, 133(1), pp.129–190.

Alfaro, L., Garcia-Santana, M., and Moral-Benito, E., 2021. On the direct and indirect real effects of credit supply shocks. Journal of Financial Economics, 139(3), pp.895–921.

Amiti, M. and Weinstein, D. E., 2018. How much do idiosyncratic bank shocks affect investment? Evidence from matched bank-firm loan data. Journal of Political Economy, 126(2), pp.525–585.

Asker, J., Collard-Wexler, A. and De Loecker, J., 2014. Dynamic inputs and resource (mis) allocation. Journal of Political Economy, 122(5), pp. 1013-1063.

Banerjee, A. V. and Duflo E., 2014. Do firms want to borrow more? testing credit constraints using a directed lending program. The Review of Economic Studies, 81 (2), 572–607.

Bau, N. and Matray, A., 2022. Misallocation and capital market integration: Evidence from India.

Berg, T., Saunders, A., Schafer, L., and Steffen, S., 2021. Brexit and the contraction of syndicated lending. Journal of Financial Economics, 141(1), pp.66–82.

Berger, A.N. and Udell, G.F., 1995. Relationship lending and lines of credit in small firm finance. Journal of Business, pp.351-381.

Bharath, S.T., Dahiya, S., Saunders, A. and Srinivasan, A., 2011. Lending relationships and loan contract terms. The Review of Financial Studies, 24(4), pp.1141–1203.

Benmelech, E. and Moskowitz T. J., 2010. The political economy of financial regulation: evidence from us state usury laws in the 19th century. The Journal of Finance, 65 (3), 1029–1073.

Bemanke, B. and Gertler, M., 1989. Agency costs, net worth, and business fluctuations. American Economic Review, 79(1), pp.14–31.

Bond, S. and Meghir, C., 1994. Dynamic investment models and the firm's financial policy. The Review of Economic Studies, 61(2), pp.197–222.

Botsch, M. and Vanasco, V., 2019. Learning by lending. Journal of Financial Intermediation, 37, pp.1–14.

Cooper, R.W. and Haltiwanger, J.C., 2006. On the nature of capital adjustment costs. The Review of Economic Studies, 73(3), pp.611–633.

Crawford, G.S., Pavanini, N. and Schivardi, F., 2018. Asymmetric information and imperfect competition in lending markets. American Economic Review, 108(7), pp.1659–1701.

David, J.M., Schmid, L. and Zeke, D., 2020. Risk-adjusted capital allocation and misallocation.

De Loecker, J. and Warzynski, F., 2012. Markups and firm-level export status. American Economic Review, 102(6), pp.2437–71.

Degryse, H. and Ongena, S., 2005. Distance, lending relationships, and competition. The Journal of Finance, 60(1), pp.231–266.

Diamond, D.W., 1991. Monitoring and reputation: The choice between bank loans and directly placed debt. Journal of Political Economy, 99(4), pp.689–721.

Fama, E.F. and French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics, 33(1), pp.3–56.

Galor, O. and J. Zeira, 1993, Income distribution and macroeconomics. Review of Economic Studies 60, 35–52.

Gandhi, A., Navarro, S. and Rivers, D.A., 2020. On the identification of gross output production functions. Journal of Political Economy, 128(8), pp.2973–3016.

Gilchrist, S. and Zakrajšek, E., 2007. Investment and the cost of capital: New evidence from the corporate bond market.

Guiso, L., Pistaferri, L. and Schivardi, F., 2013. Credit within the Firm. Review of Economic Studies, 80(1), pp.211–247.

Guiso, L., Sapienza, P. and Zingales, L., 2004. Does local financial development matter?. The Quarterly Journal of Economics, 119(3), pp.929–969.

Hadlock, C.J. and Pierce, J.R., 2010. New evidence on measuring financial constraints: Moving beyond the KZ index. The Review of Financial Studies, 23(5), pp.1909–1940.

Haltiwanger, J., 2012. Job creation and firm dynamics in the United States. Innovation Policy and the Economy, 12(1), pp.17–38.

Heckman, J. J., 1974. Shadow Prices, Market Wages, and Labor Supply. Econometrica, 42, 679–693.

Hsieh, C.-T. and Klenow P. J., 2009. Misallocation and manufacturing TFP in China and India. The Quarterly Journal of Economics 124 (4), pp.1403–1448.

Hubbard, R.G., Kashyap, A. and Whited, T., 1995. Internal Finance and Firm Investment. Journal of Money, Credit, and Banking, 27, pp.683–701.

Jiménez, G., et al. 2012. Credit Supply and Monetary Policy: Identifying the Bank Balance-Sheet Channel with Loan Applications. American Economic Review 102(5), pp.2301–2326.

Kaplan, S.N. and Zingales, L., 1997. Do investment-cash flow sensitivities provide useful measures of financing constraints?. The Quarterly Journal of Economics, 112(1), pp.169–215.

Kiyotaki, N. and Moore, J., 1997. Credit cycles. Journal of Political Economy, 105(2), pp.211–248.

Lenzu, S. and Manaresi, F., 2019. Sources and implications of resource misallocation: New evidence from firm-level marginal products and user costs.

Love, I., 2003. Financial development and financing constraints: International evidence from the structural investment model. The Review of Financial Studies, 16(3), pp.765–791.

Marschak, J. and Andrews, W.H., 1944. Random simultaneous equations and the theory of production. Econometrica, pp.143–205.

Midrigan, V. and D. Y. Xu (2014). Finance and misallocation: Evidence from plant-level data. American Economic Review, 104 (2), pp.422–458.

Olley, G.S. and Pakes, A., 1996. The Dynamics of Productivity in the Telecommunications Equipment. Econometrica, 64(6), pp.1263-1297.

Petersen, M.A. and Rajan, R.G., 1995. The effect of credit market competition on lending relationships. The Quarterly Journal of Economics, 110(2), pp.407–443.

Petersen, M.A. and Rajan, R.G., 2002. Does distance still matter? The information revolution in small business lending. The Journal of Finance, 57(6), pp.2533–2570.

Restuccia, D. and Rogerson R., 2008. Policy distortions and aggregate productivity with heterogeneous establishments. Review of Economic Dynamics, 11 (4), pp.707–720.

Restuccia, D. and Rogerson R., 2013. Misallocation and productivity. Review of Economic Dynamics, 16 (1), pp.1–10.

Stiglitz, J.E. and Weiss, A., 1981. Credit rationing in markets with imperfect information. The American Economic Review, 71(3), pp.393–410.

Stiglitz, J.E. and Weiss, A., 1992. Asymmetric information in credit markets and its implications for macro-economics. Oxford Economic Papers, 44(4), pp.694–724.

Whited, T.M., 1992. Debt, liquidity constraints, and corporate investment: Evidence from panel data. The Journal of Finance, 47(4), pp.1425–1460.

Whited, T.M. and Wu, G., 2006. Financial constraints risk. The Review of Financial Studies, 19(2), pp.531–559.